Measurement of Electron Charge with Millikan Oil Drop Experiment

Zhengpu Zhao[1] and Will Nesbit[1]

[1] *Physics Department, University of California, Santa Barbara, CA 93106*

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In this study, the charge of electron was measured through the Millikan Oil Drop Experiment. The experiment setup includes a "Millikan Oil Drop Apparatus", an AC adaptor, an atomizer, and 120mL of non-volatile mineral oil. Specifically, the "Millikan Oil Drop Apparatus" includes a platform, a droplet viewing chamber, and a optical microscope. By analyzing the "rise velocity" and "fall velocity" of droplets across multiple trials of ionization and fitting the data to find the charge carried by each droplet in each trial, we were able to determine the quantization of electron charge and one elementary charge being $(1.5816 \pm .3581) \times 10^{-19}$C. Which constitutes an experimental error of 1.29 % comparing to the accepted value for the elementary charge.

INTRODUCTION

Electrons are generally considered to be elementary particles as they have no known substructure. Hence the magnitude of charge carried by one electron is defined to be the elementary charge. Historically, the first measurement was performed by Robert A. Millikan with the "Millikan Oil Drop Experiment" in 1909. [1]. The latest precise measurement agreed upon was published by The NIST Reference on Constants to be $1.602176634 \times 10^{-19}$C retrieved on 30 Jan 2020. [2]. This study measures the magnitude of the elementary charge through the "Millikan Oil Drop Experiment".

METHOD

The experimental setup had three main subdivisions: power-supply and voltage regulator that controls the charged plates, the experiment platform (FIG.1) and the optical microscope camera setup.

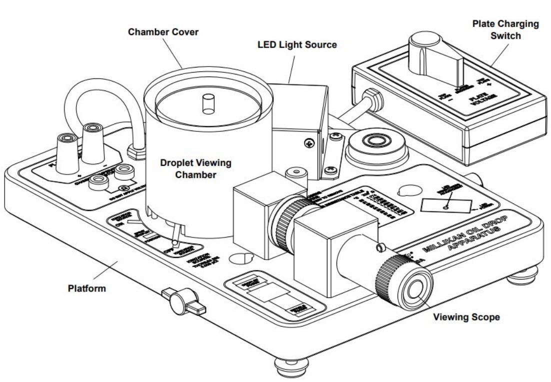

FIG. 1: Schematic of experiment platform

The experiment platform constitutes a thermistor section, an ionization source section connected to the power-supply setup, a LED light source, and a droplet viewing chamber that was connected with the camera setup which includes a viewing scope, a microscope, and a USB camera.

None volatile mineral oil droplets with density $\rho = 886kg/m^3$ were sprayed into the viewing chamber with an atomizer. Through a hole leading into the space between the charged plates, oil droplets entered the range of camera viewfinder by diffusion. By switching on and off the $V = 500V$ voltage supply connected to two charging plates separated by $d = 7.65 \times 10^{-3}m$, some droplets could undergone manipulable fall and uplifting motion.

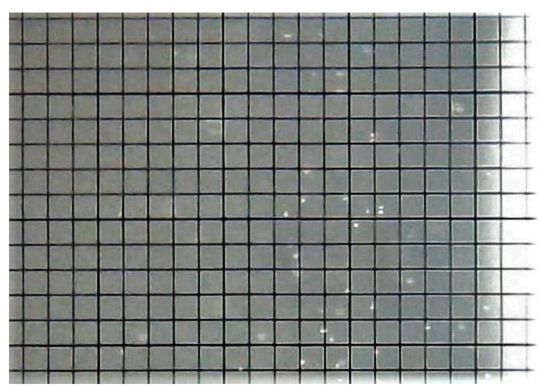

FIG. 2: Example of Video Acquired

To ensure the precision of video data extraction, the recording environment was optimized for visibility and contrast by strategically mirroring the LED light source. The camera was guided to focus via a focusing pin that was placed in the plane of droplet entries. Video recorded [Fig.2] have droplets well lit and focused against contrasting background and foreground along with focused grid lines to indicate position of droplets.

Focusing on one droplet at a time, the motion of the droplet is manipulated and tracked; first, the droplet goes through 0.15mm (3 major grids) of fall under only gravity and drag, then the plates are charged and the droplet goes through 0.15mm of ascending under uplifting electric force, gravity, and drag. Such fall and ascend are carried out three to five times as a *trial* and then the droplet is ionized by the ionization source and enters another *trial*. By repeating the trial and ionization process three to five times, same droplet with multiple charges is recorded.

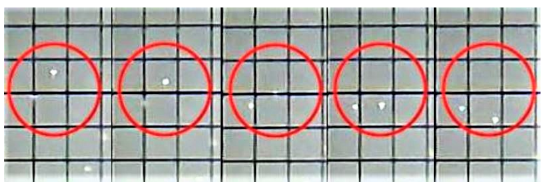

FIG. 3: Five Consecutive Frames of Droplet Crossing Grid-Line; Shows Sufficiency in Frame Rate and Video Quality

The video recordings were set to 30 frames per second allowing frame by frame viewing for optimal data acquisition while not being bottle-necked by computer RAM usage. Frame analysis [Fig.3] showed adequacy in precision of data extraction.

ANALYSIS

Balance of forces during fall and ascend respectively are shown in (FIG.4) where charge q can be determined as:

$$q = \frac{mg(v_f + v_r)}{Ev_f}$$

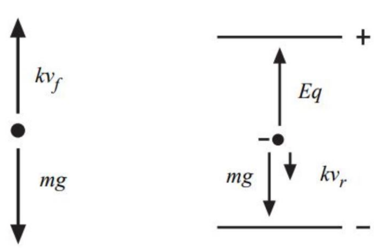

FIG. 4: Balance of Forces during Fall and Ascend

Since terminal velocity for both fall and ascend are slow, it's safe to assume near spherical droplet. With such assumption, mass of droplet can be replaced with:

$$mg = \frac{4}{3}\pi a^3 \rho g$$

where a is the radius of the droplet. Thus, q can be solved as:

$$q = \frac{4\pi a^3 \rho g(v_f + v_r)}{3(Ev_f)}$$

Due to low droplet velocity, Stoke's law:

$$a = \sqrt{\frac{9\eta v_f}{2\rho g}}$$

was modified with effective viscosity η_{eff}, where b is a constant given to be $8.20 \times 10^{-3} Pa \cdot m$ and η being temperature dependent viscosity in air measured to be $(1.2820 \pm 0.0040) \times 10^{-5} N \cdot s \cdot m^{-2}$:

$$\eta_{eff} = \eta\left(\frac{1}{1 + \frac{b}{pa}}\right)$$

By solving Stoke's law's quadratic equation for a:

$$a = \sqrt{\left(\frac{b}{2p}\right)^2 + \frac{9\eta v_f}{2\rho g}} - \frac{b}{2p}$$

Substituting a into previous equation for q:

$$q = \frac{4\pi}{3}\left[\sqrt{\left(\frac{b}{2p}\right)^2 + \frac{9\eta v_f}{2\rho g}} - \frac{b}{2p}\right]^3 \frac{\rho g d(v_f + v_r)}{Vv_f}$$

In this final expression[3], the only two remaining parameters to be determined were v_f and v_r. By analyzing the videos, multiple trials of v_f and v_r and their respective standard deviations δ_{v_f} and δ_{v_r} were extracted across three distinct droplets over multiple attempts of ionization.

By plugging in v_f, v_r, δ_{v_f}, δ_{v_r} and propagating error, charge carried q for each trial and respective standard deviation δ_q were computed. For each droplet, a set of differential charges were acquired by taking the difference between the charge q_n of each trial and q_{base}, the lowest charge measured. The same procedure was operated upon all three droplet to construct the final list of differential

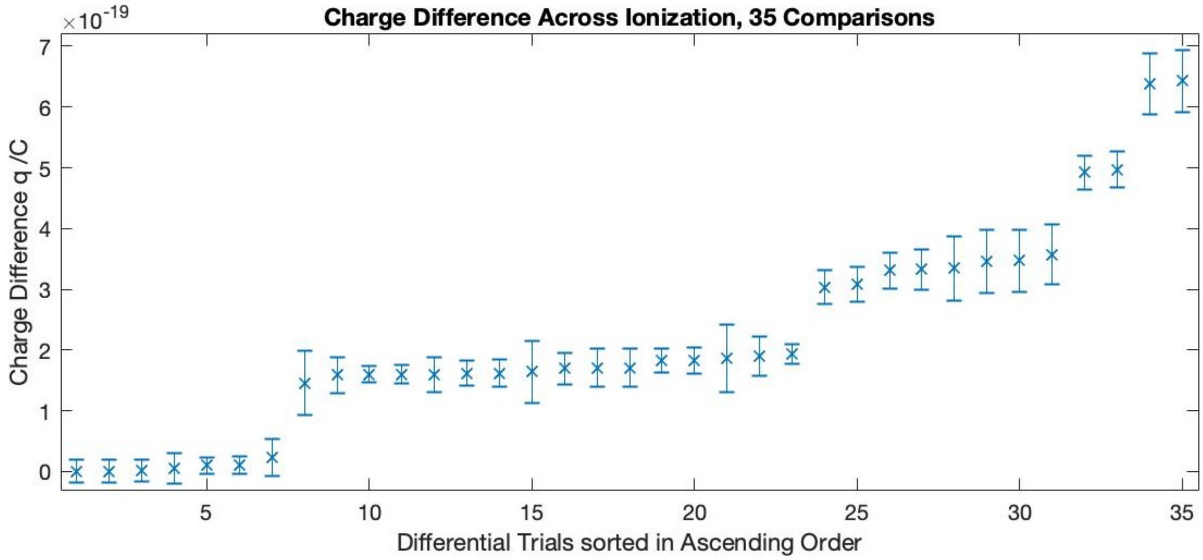

FIG. 5: Summary Plot of Differential Charge

charges. (FIG.5) shows a summary of all 35 differential charges and their respective error bars plotted in ascending order of magnitude.

RESULTS

(FIG.5) clearly showed five levels of charge differential and that for any level of charge clusters, the extension of error bars does not overlap with any from other levels of clusters. Such phenomenon indicates statistically significantly distinct levels of charges over ionization and hence suggested quantization of charge.

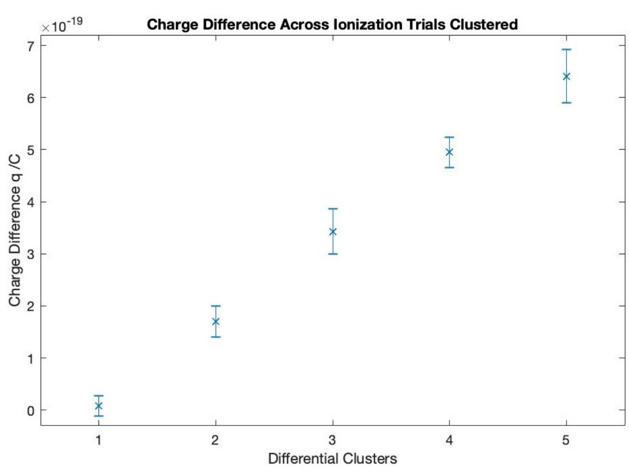

FIG. 6: Plot of Clustered Charge Differentials

Cluster	Δq / C	$\delta_{\Delta q}$/ C
1	7.88E-21	1.96E-20
2	1.70E-19	2.99E-20
3	3.43E-19	4.38E-20
4	4.95E-19	2.89E-20
5	6.41E-19	5.08E-20

Clustering for each level, an array of charge differential is acquired as (Fig.6) illustrates. The five distinct levels of charge differentials are shown in the table above.

Each cluster's charge value is close to an integer multiple of the second cluster's. Compute for difference across adjacent clusters gave the result:

$$\Delta q = (1.5816 \pm 0.3581) \times 10^{-19} C$$

It's interpreted that the charge difference across levels of charge clusters are the elementary charge. Since the ionization source used are negative electrons based on incoming alpha particles abolishing non-excess electrons, it can be inferred that the magnitude or negative of one electron charge according to the result is:

$$|e| = -e = (1.5816 \pm 0.3581) \times 10^{-19} C$$

DISCUSSION

This study serves as a tribute to the great physics discovery made by Robert Millikan more than a century ago. The Experimental setup is reminiscent of what it was long ago in the past, hard to master but effective.

The result acquired is close to the actual, scientifically recognized value published by The NIST Reference on Constants:

$$1.602176634 \times 10^{-19} C$$

. In fact, the actual value is within one standard error (uncertainty) or 68% confidence interval of experimental result:

$$(1.5816 \pm 0.3581) \times 10^{-19} C$$

The main shortcoming of the result is the size of the confidence interval; it makes up 23% of the actual value. This is caused by the lack of repetition. Only three droplets are analyzed and out of them the max number of ionization if four which provides limited amount of data to shrink the error margin. To combat this downfall, more and longer successful trials has to take place which requires better ionization source for higher chance of ionizing and better computer RAM for ability to take longer videos.

ACKNOWLEDGEMENT

Due to differences in time management and his unfortunate sickness, I and my Lab partner, Will Nesbit cooperated only on the first lab day. I still appreciate Will's effort in trying to work out a plausible arrangement with me despite ultimately unable to do so. I would also give credit to my T.A. Erin Yandel for Guidance and clarification. I also thank Professor Xiao Luo for the clear instruction on error analysis which helped with this "data analysis project".

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[1] Bishop, I., Xian, S., & Feller, S. (2019). Robert A. Millikan and the Oil Drop Experiment. The Physics Teacher, 57(7), 442–445. doi: 10.1119/1.5126819

[2] "2018 CODATA Value: elementary charge". The NIST Reference on Constants, Units, and Uncertainty. NIST. 30 Jan 2020. Retrieved 2020-01-30.

[3] Millikan Oil Drop Apparatus • AP-8210A. (n.d.). Retrieved from https://www.pasco.com/products/lab-apparatus/fundamental-constants/ap-8210