Measurement of the Mass Ratio of Hydrogen and Deuterium Nuclei via Spectroscopy

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This study measured the spectra of hydrogen and deuterium using a reflective grating spectrometer; from the spectroscopy data of isotope electron de-excitation photon emission wavelength shift, nuclei mass ratio between hydrogen and deuterium is determined. We found the nuclei mass ratio to be $M_D/M_H = 1.996408 \pm 0.036956$; the published value of 1.99900750139 [1] is within 68% confidence interval of the measurement result. The major source of error is the amplification of background light noise and thermal excitation of electrons in the photo-multiplier tube when source intensity is low which reduces fit fidelity in data analysis and introduces systematic error.

PACS numbers:

I. INTRODUCTION

Electron transition from a higher to a lower energy level is accompanied by a single photon emission. The photon emitted possesses the same energy as the difference between the different energy levels. The Rydberg equation describes the wavelength of emitted photon:

$$\frac{1}{\lambda} = R_y \frac{\mu}{m_e}(\frac{1}{{n^2}_f} - \frac{1}{{n^2}_i}) \tag{1}$$

Here R_y is the Rydberg constant, with accepted value of $R_y = 1.0973731568160 \cdot 10^7\ m^{-1}$ [2]. μ is the reduced mass of the nucleus-electron system, where:

$$\mu = \frac{M \cdot m_e}{M + m_e} \tag{2}$$

Here M is mass of nucleus and m_e is mass of the electron has accepted value of $m_e = 9.1093837015 \cdot 10^{-31} kg$ [3], and n_f & n_i are final and initial energy levels.

A proton has mass of $m_p = 1.67262192369 \cdot 10^{-27}$ [4] and a neutron has mass of $m_n = 1.67492749804 \cdot 10^{-27}$ [5]. A hydrogen nucleus is a proton, thus $M_H = m_p$. Deuterium is an isotope of hydrogen with an additional neutron; obviously, M_D & μ_D are correspondingly greater than M_H & μ_H; and due to strong nuclear force that binds the nucleons together, the mass of deuterium nucleus M_D is less than sum of m_p & m_n.

Using spectroscopy, we can measure the relative wavelength shift of deuterium and hydrogen for certain final and initial energy levels n_f & n_i:

$$\Delta_\lambda = \left| \frac{m_e}{R_y} \cdot \frac{1}{\frac{1}{{n^2}_f} - \frac{1}{{n^2}_i}} \cdot (\frac{1}{\mu_D} - \frac{1}{\mu_H}) \right| \tag{3}$$

Focusing on Balmer series where $n_f = 2$ and define $n_i \equiv n$, we get the relationship between wavelength shift and initial energy level:

$$\Delta_\lambda(n) = \frac{m_e}{R_y} \cdot [\frac{\mu_D - \mu_H}{\mu_D \cdot \mu_H}] \cdot \frac{4n^2}{n^2 - 4} \tag{4}$$

Here the square-bracketed term can be quantitatively obtained through analyzing spectroscopy data; say, the value came to be b. For acquired b and m_p & m_e being known values, the deuterium hydrogen nuclei mass ratio can be solved for:

$$\frac{M_D}{M_H} = a = \frac{1}{1 - b \cdot m_p} \quad where \quad b = \frac{\mu_D - \mu_H}{\mu_D \cdot \mu_H} \tag{5}$$

II. METHOD

The set-up for measuring the spectra of hydrogen, deuterium is shown in [FIG.1]. The lamp was filled with gaseous atoms for which the spectra were to be measured. Electron energy level transitions, excitation and de-excitation, are induced when the lamp was turned on.

The de-excitation process release photons; a spectrum of wavelengths of light are emitted from different initial and final electron energy levels. The multi-chromatic light would then enter the monochrometer through a slit with width of 30 μm, reflect off of a mirror and hit a reflective diffraction grating.

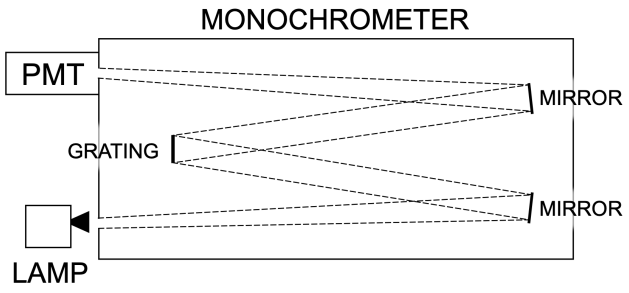

FIG. 1: Diagram of the monochrometer. The grating diffracts different wavelengths of light emitted by the lamp in different directions so that wavelengths are measured one at a time by the PMT. By rotating the grating, the monochrometer can scan through many different wavelengths.[6]

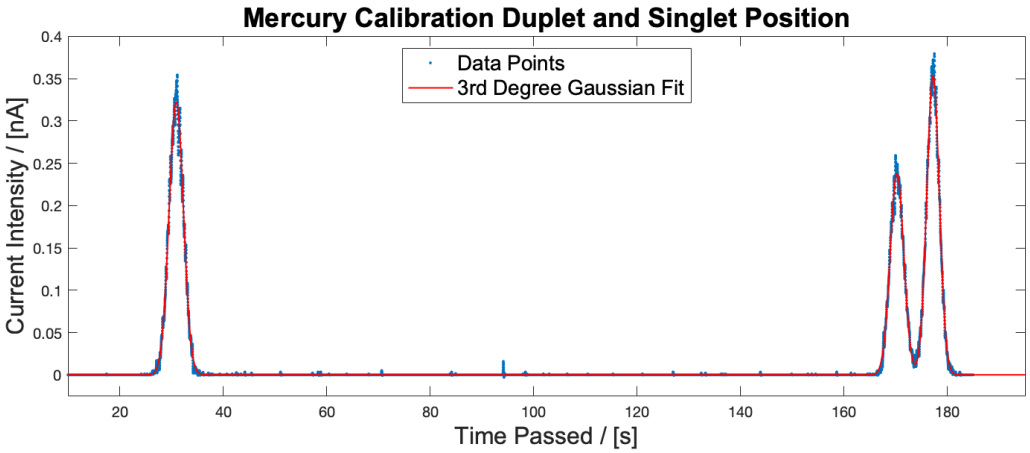

FIG. 2: Mercury lamp calibration fit. The three Gaussian peaks from left to right correspond spatially to the 6251.4 Å singlet and the 6263.0 Å / 6263.6 Å doublet. Hence spatial separation: 11.6 Å from left to middle peak and 12.2 Å from left to right peak. Fitted third degree Gaussian curves with $R^2 \simeq 0.9950$ suggests corresponding temporally separation: 139.562 s and 146.482 s. Hence calibration factor: (0.083202 ± 0.000899) Å$/s$.

The grating is made up of many small groves; it diffracts light to different angles with respect to wavelengths. Hence, the original multi-chromatic light would be spatially separated into beams of single wavelength (monochromatic) light. By rotating the grating, we can determine which wavelength to be reflected by the second mirror through the exit slit with width of 30 μm into the photo-multiplier tube.

The photo-multiplier tube can detect very faint light signals by taking advantage of the photoelectric effect, with gain on the order of 10^7 to 10^9. The amplified signal is in the form of a current and would then be measured by a pico-ammeter. The pico-ammeter outputs digital signal to a computer (via LabView program) that reads and records the current amplitude against time.

To calibrate x-scale from time into length, a mercury lamp with well defined peaks was used. By sweeping at rate listed as 5 Å/min in a range from 6248 Å to 6265 Å, we recorded the 6251.4 Å singlet and the 6263.0 Å / 6263.6 Å doublet. As shown by [FIG.2], the three peaks are fitted as three Gaussian functions. From such fit, we could determine x-scale length measurements with respect to time measurements.

After calibration, hydrogen-deuterium lamp was used. The lamp would emit light with wavelengths corresponding to the de-excitation of electrons of the two isotope. Since the reduce masses of hydrogen and deuterium are very similar, their corresponding emission wavelengths are spatially close, forming duplets.

Focusing on Balmer series for hydrogen and deuterium, the first six Balmer lines (initial energy level 3-8, final energy level 2) occur roughly as listed in [TABLE I]. Each duplet range was swept through five times and fitted

n	λ_H / Å	λ_D / Å
3	6564.70	6562.91
4	4862.74	4862.42
5	4341.73	4340.55
6	4102.94	4101.82
7	3971.24	3970.16
8	3890.19	3889.13

TABLE I: Expected first six Balmer lines for hydrogen and deuterium; for each initial energy level n, the two corresponding peaks occurs closely, forming a duplet.

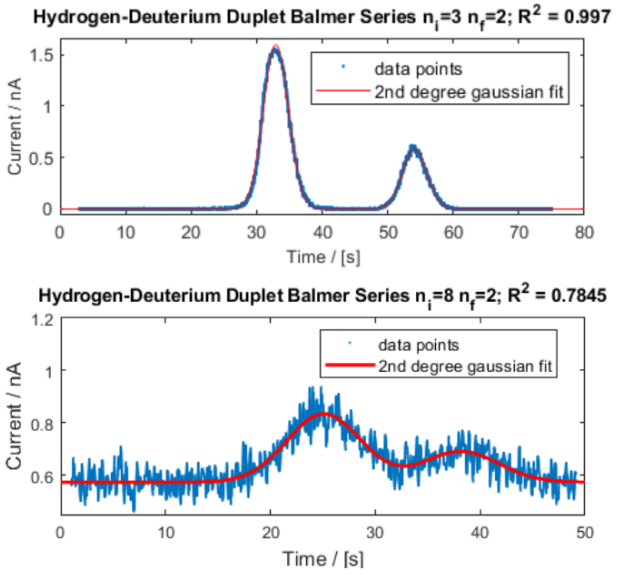

FIG. 3: n=3 & 8 Balmer series fitted plots. n=8 produces significantly lower signal intensity than n=3 hence poorer fit and greater systematic error.

with second degree Gaussian curves (with constant offset for low intensity to compensate for amplified background and thermal noise) that suggest temporal separation between duplet peaks. [FIG.3] shows examples of fitted plots for n=3 & 8. At n=8, the background and thermal noise was more significant and the fit was poorer compared to n=3; this demonstrated a main source of systematic error.

The temporal separations between duplet peaks were then turned into spatial separations via the acquired calibration factor: (0.083202 ± 0.000899) Å$/s$. The measurement result summarises into [TABLE II]:

n	Δ_λ / Å	σ_{Δ_λ} / Å
3	1.760719291	0.01430734583
4	1.317419396	0.02264560352
5	1.191784478	0.01206571461
6	1.149296025	0.00590284336
7	1.075246305	0.006927945342
8	1.084675857	0.01652209662

TABLE II: Measurement of first six Balmer hydrogen-deuterium duplet peak spatial separation and their standard deviations. Such separation represent the isotope wavelengths shift in Balmer series light emission.

III. ANALYSIS

Plotting the data points of wavelength shift against initial energy level and fitting according to the equation (4), the data points followed the expected pattern. Here the plot [FIG.4] follows:

$$\Delta_\lambda(n) = \alpha \cdot \frac{4n^2}{n^2-4} \tag{6}$$

$$where \quad \alpha = \frac{m_e}{R_y} \cdot [\frac{\mu_D - \mu_H}{\mu_D \cdot \mu_H}]$$

The fitted plot has R^2 value of 0.9924 indicating the fit had high fidelity despite having portions not included by the data points' error-bars at higher n's which were intrinsically not so accurate due to high noise-signal ratio. The fit suggests:

$$\alpha = (0.2485 \pm 0.0046) \text{ Å} \tag{7}$$

With m_e & R_y being known constants, the bracketed expression can be determined:

$$\frac{\mu_D - \mu_H}{\mu_D \cdot \mu_H} = (5.978644 \pm 0.110671) \cdot 10^{26} \ kg^{-1} \tag{8}$$

Plugging this result into equation (5), with m_p value also known, the nuclei mass ratio can be determined:

$$\frac{M_D}{M_H} = 1.996408437 \pm 0.036955649 \tag{9}$$

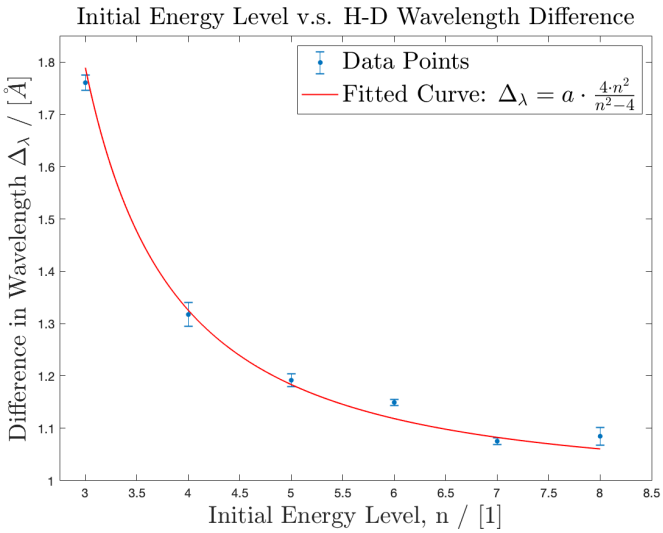

FIG. 4: Wavelength shift plotted against initial energy level fitted with expected pattern. $\alpha = (0.2485 \pm 0.0046)$ Å. The curve it fitted with $R^2 = 0.9924$.

IV. RESULT

By studying the spectra of deuterium and hydrogen, we have found the mass ratio of their nuclei to be $M_D/M_H = 1.996408437 \pm 0.036955649$. Accepted value of such ratio can be found as "deuteron-proton mass ratio" = 1.99900750139(11) from NIST database [1].

The accepted value is close to the experimentally acquired value and is within the 68% confidence interval. The accepted value and measured value show discrepancy of 0.002599 which is 0.13%.

V. DISCUSSION

The spectrometer produces accurate enough data at very fine increment of 0.01 s and 1 pA such that systematic error is negligible. Wavelengths shift data for each duplet is measured five times and a standard deviation is acquired as the statistical error-bar [TABLE II]. For all six data sets, five had statistical error less than 1% and one less than 2%. This is small enough to be ignored as the next step of analysis with curve fitting [FIG. 4] showed consistency with theoretical prediction. The standard deviation of the fitting model was then used as the error-bar and was propagated through steps described by equation (6) through (9).

Final result of $M_D/M_H = 1.996408437 \pm 0.036955649$, although accurate, has a non-trivial error-bar of 1.8%. This is mainly due the the deviation of the fitted curve from data points at high initial energy levels where noise strength was comparable to signal strength due to sys-

tematic limitations such as thermal excitation of electrons in the photo-multiplier tube.

Possible improvement to experimental precision includes: 1) Increasing hydrogen-deuterium lamp brightness by allowing higher voltage being applied. This will fix the high noise to signal ratio at high initial energy levels and thus decrease systematic error. 2) If time permits, instead of taking 5 initial measurements, taking 10 will decrease standard deviation and improve statistical precision by a factor of $\sqrt{2}$. 3) The calibration step used mercury peaks which only specified to single decimal place; more precise data with higher numbers of decimal places will allow better calibration and hence decrease systematic error.

[1] "deuteron-proton mass ratio". The NIST Reference on Constants, Units, and Uncertainty. NIST. 30 Jan 2020. Retrieved 2020-02-15.

[2] "Rydberg constant". The NIST Reference on Constants, Units, and Uncertainty. NIST. 30 Jan 2020. Retrieved 2020-02-15.

[3] "electron mass". The NIST Reference on Constants, Units, and Uncertainty. NIST. 30 Jan 2020. Retrieved 2020-02-15.

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[5] "neutron mass". The NIST Reference on Constants, Units, and Uncertainty. NIST. 30 Jan 2020. Retrieved 2020-02-15.

[6] PHYS 128AL/BL. (n.d.). Retrieved from http://web.physics.ucsb.edu/ phys128/