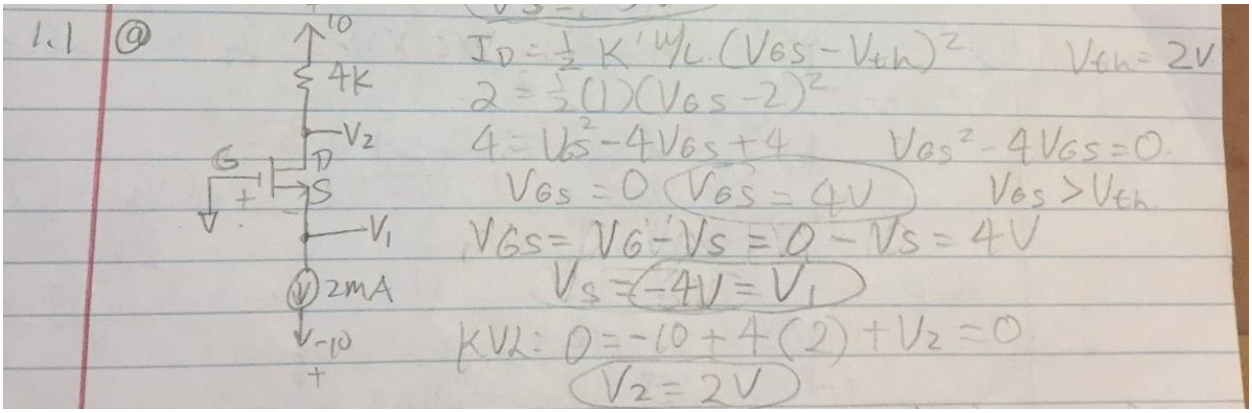
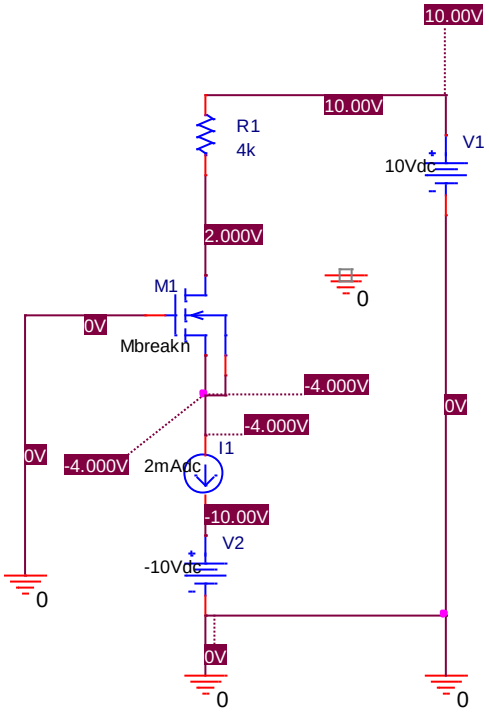
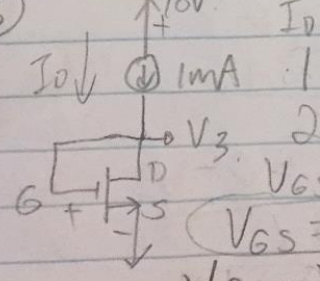
Rong Zheng

Homework 4





1.1 (b)



$I_D = \frac{1}{2} K' W/L (V_{GS} - V_{th})^2 = \frac{1}{2}(1)(V_{GS} - 2)^2$

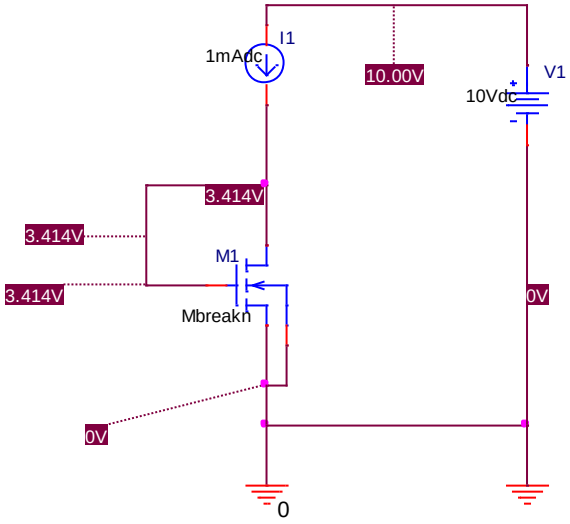
$1 = \frac{1}{2}(1)(V_{GS}^2 - 4V_{GS} + 4)$

$2 = V_{GS}^2 - 4V_{GS} + 4$

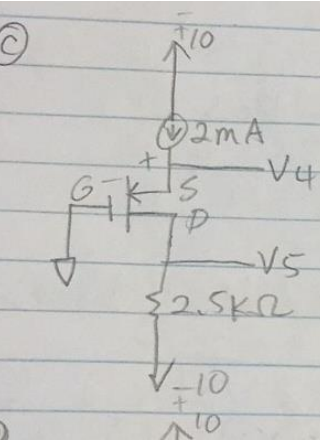
$V_{GS}^2 - 4V_{GS} + 2 = 0$

$V_{GS} = 3.4V$ $V_{GS} = 0.585V$ $V_{GS} > V_{th}$

$V_{GS} = V_G - V_S = 3.4 -$ $V_3 = 3.4V$



1.1 (c)



$V_7 = 10 - 2(4) = 2V = V_7$

$I_D = \frac{1}{2} k_p' W/L (V_{GS} - V_{TH})^2$

$2 = \frac{1}{2}(1)(V_{GS} + 2)^2$

$4 = V_{GS}^2 + 4V_G + 4$

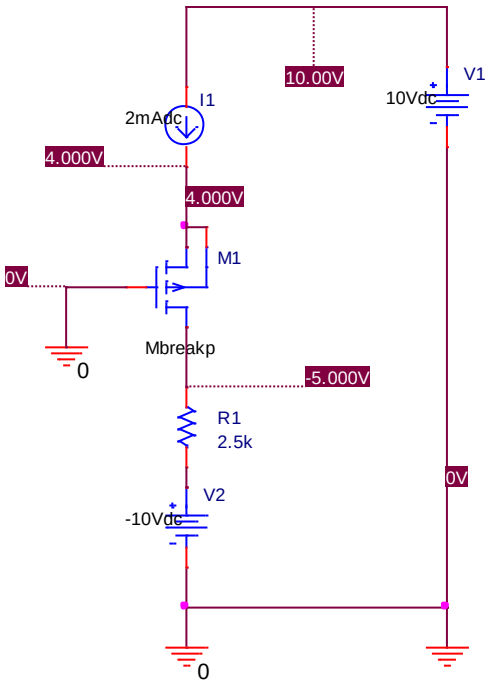
$V_{GS} = -4V$ $V_{GS} = 0$ $V_{GS} < V_{th}$

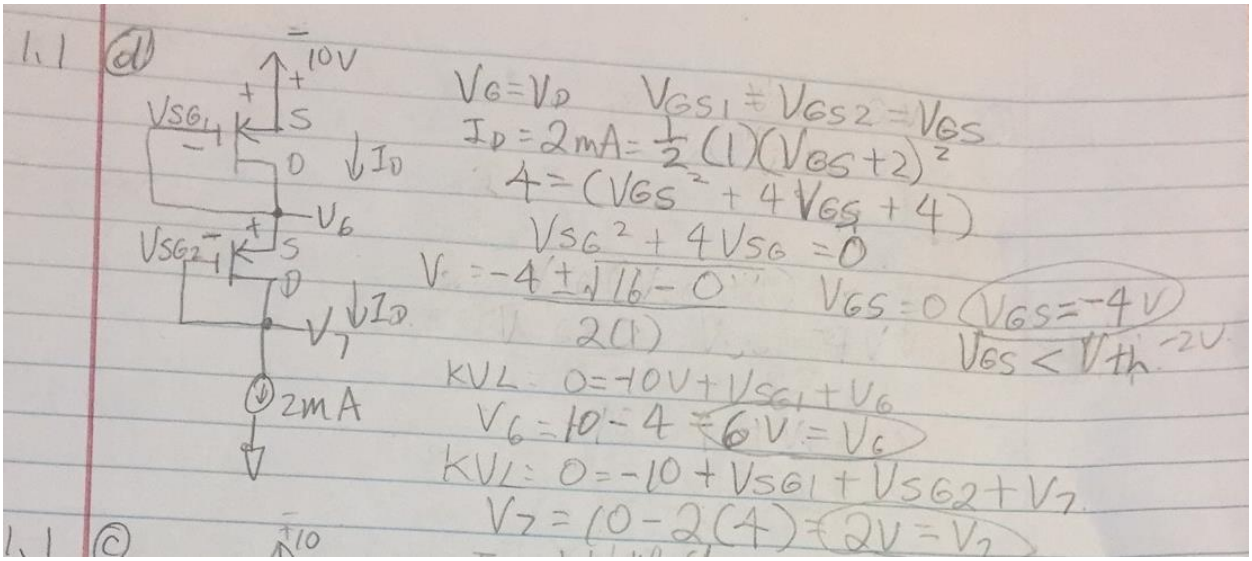
KVL: $0 = -V_{GS} - V_4$

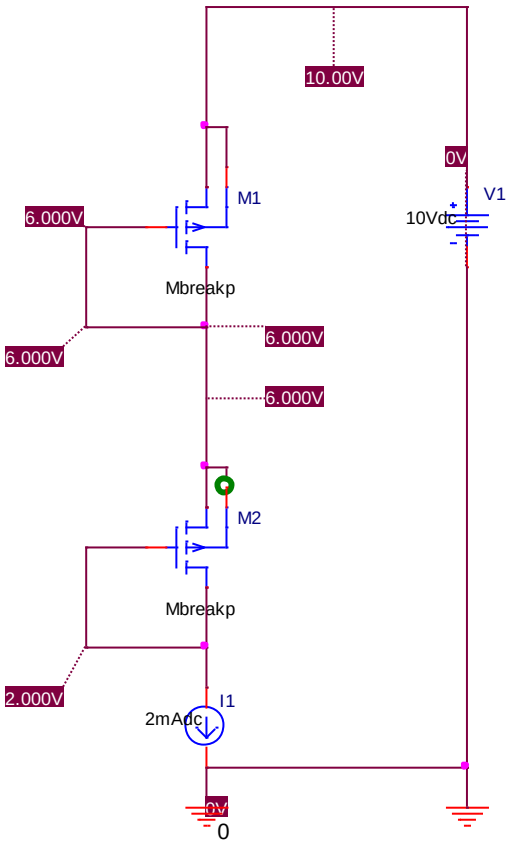
$V_4 = 4V$

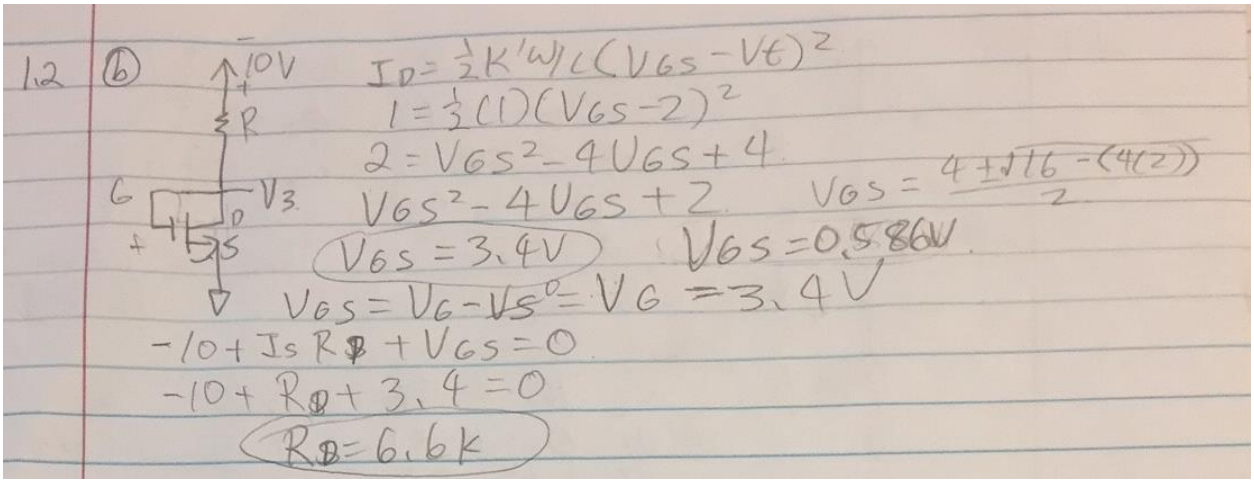
KVL: $0 = 10 - I_D(2.5) + V_5$

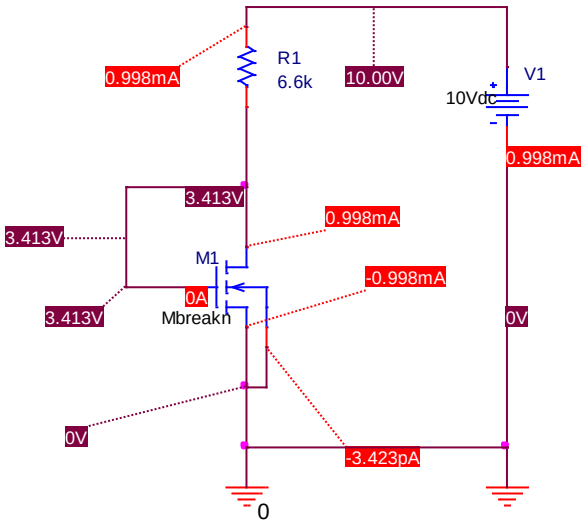
$V_5 = -5V$


10.00V
V1
10Vdc
I1
2mAdc
4.000V
4.000V
M1
0V
Mbreakp
0
-5.000V
R1
2.5k
0V
V2
-10Vdc
0

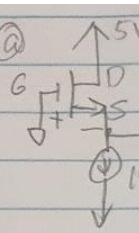

1.1 (d)
10V
VSG1
S
D
ID
V6
VSG2
S
D
V7
ID
2mA
VG=VD
VGS1 = VGS2 = VGS
ID = 2mA = ½ (1)(VGS+2)²
4 = (VGS² + 4VGS + 4)
VSG² + 4VSG = 0
VGS = 0 (VGS = -4 V)
VGS < Vth -2V
KVL: 0 = -10V + VSG1 + V6
V6 = 10 - 4 = 6V = V6
KVL: 0 = -10 + VSG1 + VSG2 + V7
V7 = (0 - 2(4) = 2V = V7
1.1 (c)


10.00V
0V
V1
10Vdc
M1
6.000V
Mbreakp
6.000V
6.000V
6.000V
M2
Mbreakp
2.000V
I1
2mAdc
0V
0


12 (b)
10V
R
G
D
S
V_3
$I_D = \frac{1}{2}K'W/L(V_{GS}-V_t)^2$
$1 = \frac{1}{2}(1)(V_{GS}-2)^2$
$2 = V_{GS}^2 - 4V_{GS} + 4$
$V_{GS}^2 - 4V_{GS} + 2$
$V_{GS} = \frac{4 \pm \sqrt{16-(4(2))}}{2}$
$V_{GS} = 3.4V$
$V_{GS} = 0.586V$
$V_{GS} = V_G - V_S = V_G = 3.4V$
$-10 + I_S R_D + V_{GS} = 0$
$-10 + R_D + 3.4 = 0$
$R_D = 6.6k$



2.1 @



$I_D = \frac{1}{2}(0.4)(V_{GS} - V_t)^2 = 0.2(V_{GS} - 1)^2$

$10mA = 0.2(V_{GS}^2 - 2V_{GS} + 1)$

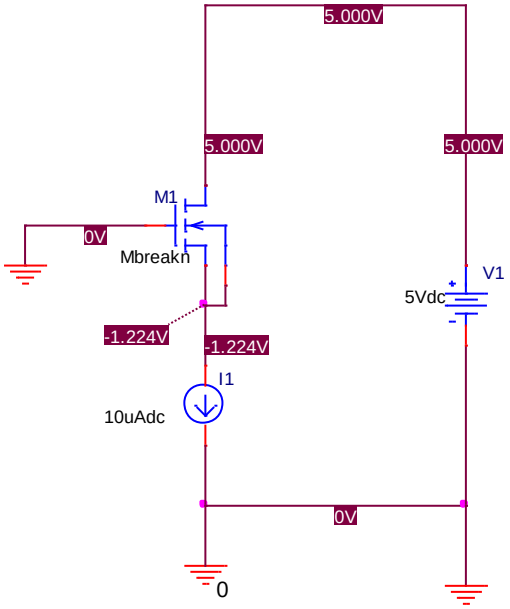
$V_{GS}^2 - 2V_{GS} + 0.95 = 0$

$V_{GS} = \frac{2 \pm \sqrt{4 - 4(0.95)}}{2}$

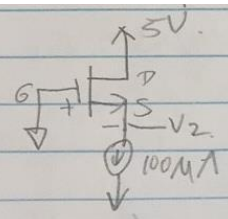
$V_{GS} = 1.22V$ $V_{GS} = 0.776V$ $V_{GS} > V_t$

KVL: $0 = V_{GS} + V_1 = 0$

$V_1 = -V_{GS} = -1.22V = V_1$



2.1 (b)



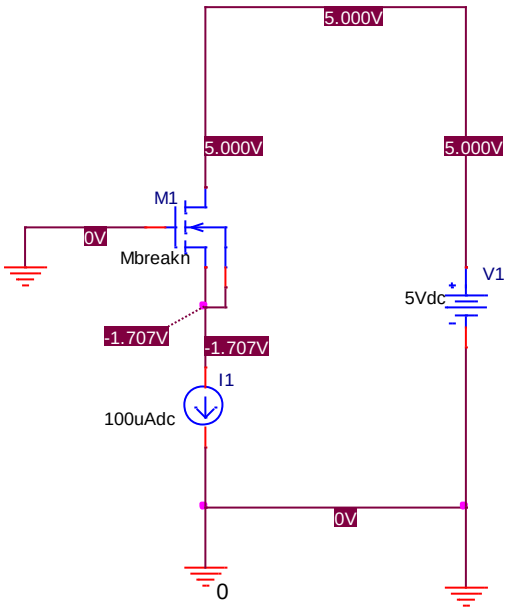
$100\mu A = \frac{1}{2}(0.4)(V_{GS}-V_t)^2 = 0.2(V_{GS}-1)^2$

$V_{GS}^2 - 2V_{GS} + 0.5 = 0$ $\quad V_{GS} > V_t$

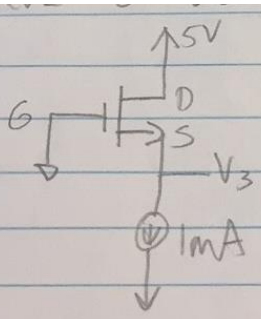
$V_{GS} = \frac{2 \pm \sqrt{4-4(0.5)}}{2}$ $\quad V_{GS} = 1.7$

$V_{GS} = 0.29$

KVL: $0 = V_{GS} + V_2 = 0$ $\quad V_2 = -V_{GS} = -1.7V = V_2$



2.1 (c)



$I = \frac{1}{2}(0.4)(V_{GS}-2V_{GS}+1)$

$5 = V_{GS} - 2V_{GS} + 1$

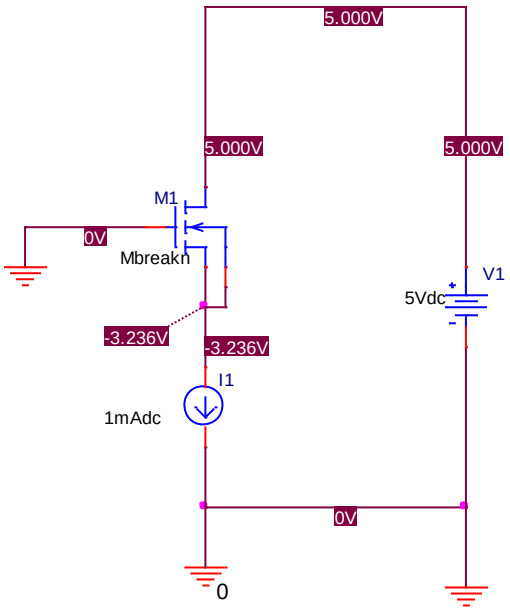
$V_{GS}^2 - 2V_{GS} - 4$ $\quad V_{GS} > V_t$

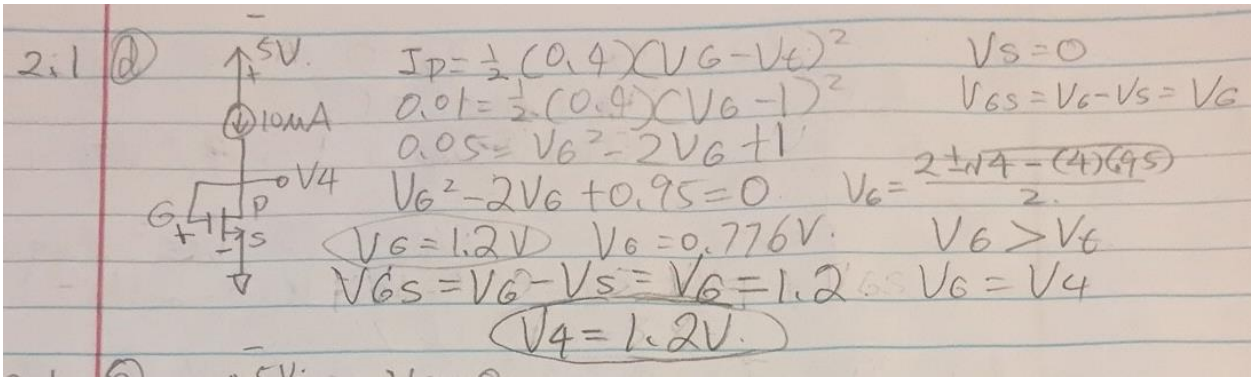
$V_{GS} = \frac{2 \pm \sqrt{4-4(-4)}}{2}$ $\quad V_{GS} = 3.23\ V$

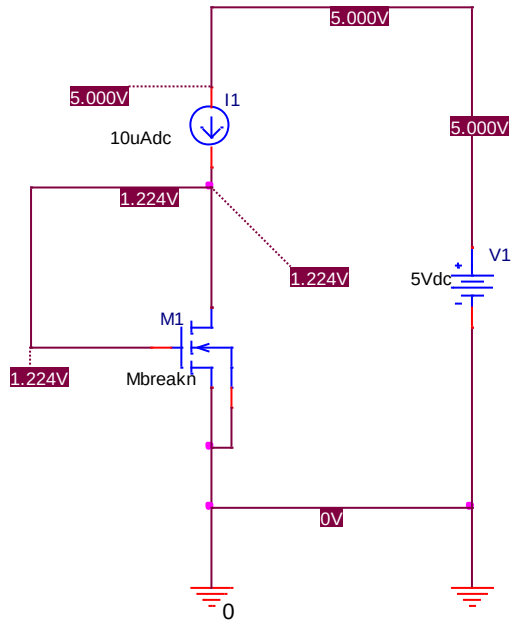
$V_{GS} = -1.35V$

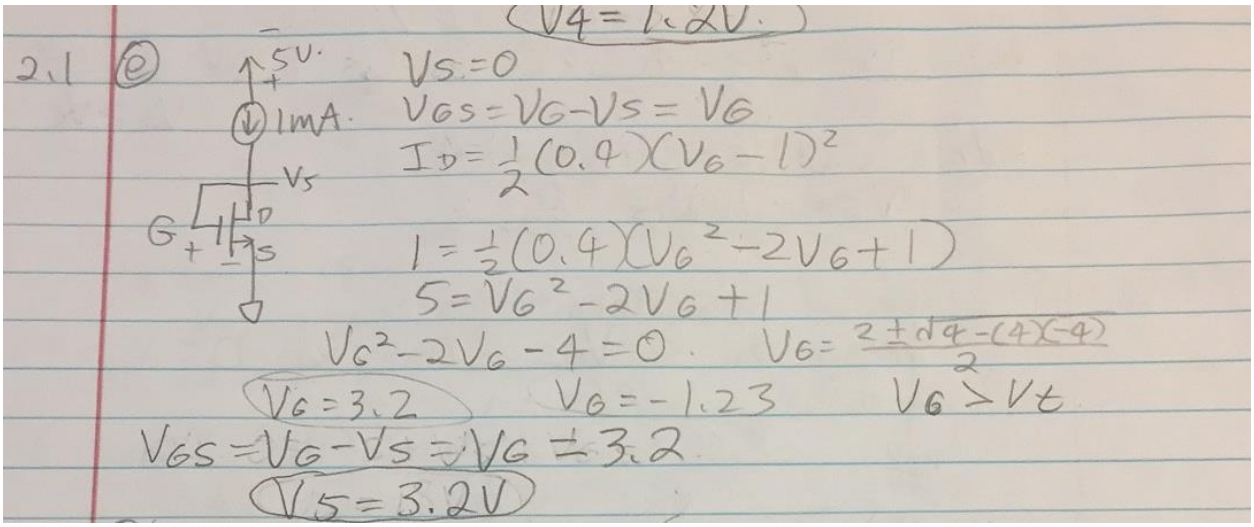
$V_{GS} = V_G - V_S = 0 - V_S = 3.23V$

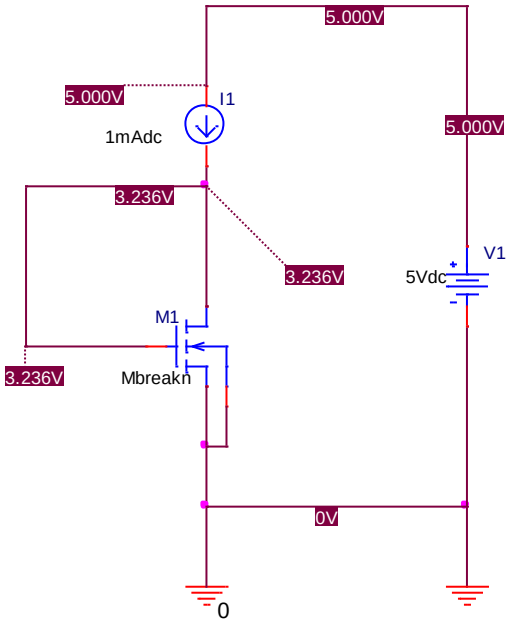
$V_S = -3.23V = V_3$


5.000V
5.000V
5.000V
M1
0V
Mbreakn
V1
5Vdc
-3.236V
-3.236V
I1
1mAdc
0V
0

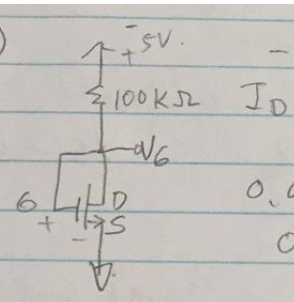

2.1
5V
10mA
V4
G
D
S
$I_D = \frac{1}{2}(0.4)(V_G - V_t)^2$
$0.01 = \frac{1}{2}(0.4)(V_G - 1)^2$
$0.05 = V_G^2 - 2V_G + 1$
$V_G^2 - 2V_G + 0.95 = 0$
$V_G = \frac{2 \pm \sqrt{4 - (4)(.95)}}{2}$
$V_G = 1.2V$
$V_G = 0.776V$
$V_{GS} = V_G - V_S = V_G = 1.2$
$V_4 = 1.2V$
$V_S = 0$
$V_{GS} = V_G - V_S = V_G$
$V_G > V_t$
$V_G = V_4$


5.000V
5.000V
I1
10uAdc
5.000V
1.224V
1.224V
V1
5Vdc
M1
1.224V
Mbreakn
0V
0


(V4 = 1.2V.)
2.1 (e)
5V.
1mA.
Vs
G
D
S
Vs = 0
VGS = VG - Vs = VG
ID = 1/2 (0.4)(VG - 1)^2
1 = 1/2 (0.4)(VG^2 - 2VG + 1)
5 = VG^2 - 2VG + 1
VG^2 - 2VG - 4 = 0
VG = 2 ± √4 - (4)(-4) / 2
VG = 3.2
VG = -1.23
VG ≥ Vt
VGS = VG - Vs = VG = 3.2
Vs = 3.2V


5.000V
5.000V
I1
1mAdc
5.000V
3.236V
3.236V
V1
5Vdc
M1
3.236V
Mbreakn
0V
0

2.1 (f)



$-5 + I_D R_D + V_{GS} = 0$

$I_D = \frac{5 - V_{GS}}{100} = \frac{1}{2}(0.4)(V_{GS}-1)^2$

$0.05 - 0.01V_{GS} = 0.2(V_{GS}^2 - 2V_{GS} + 1)$

$0.25 - 0.05V_{GS} = V_{GS}^2 - 2V_{GS} + 1$

$V_{GS}^2 - 1.95V_{GS} + 0.75 = 0$

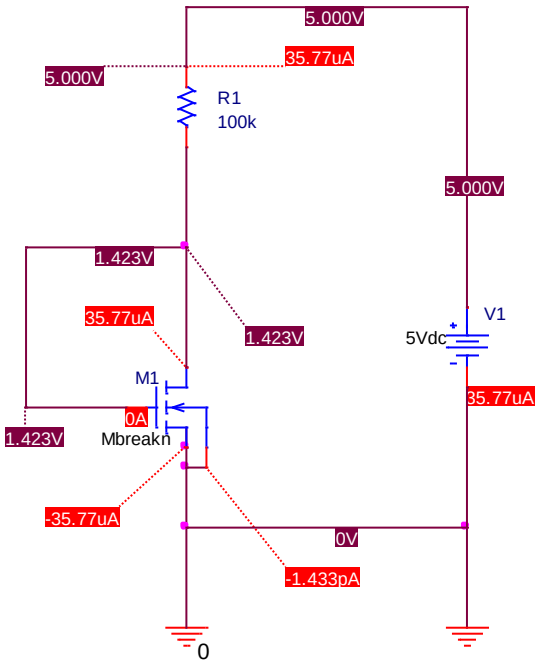
$V_{GS} = \frac{1.95 \pm \sqrt{3.8 - 4(0.75)}}{2}$

$V_{GS} = 1.4V$ $\quad V_{GS} \geq V_t$

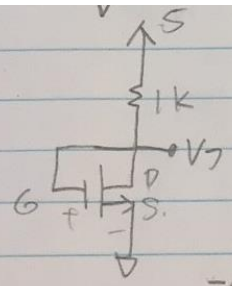
$V_{GS} = 0.528V$

$V_{GS} = V_G - V_S = V_G = 1.4V$

$V_G = 1.4V$



2.1 (a)



Short cut

$R_D = \frac{1}{K(V_7 - V_t)} = \frac{1}{0.4(V_7 - 1)} = 1k$

$0.4V_7 - 0.4 = 1$

$V_7 = 3.5V$

Long way

$-5 + I_D R_D + V_{GS} = 0$

$I_D = \frac{5 - V_{GS}}{1k} = \frac{1}{2}(0.4)(V_{GS} - 1)^2 = 0.2(V_{GS}^2 - 2V_{GS} + 1)$

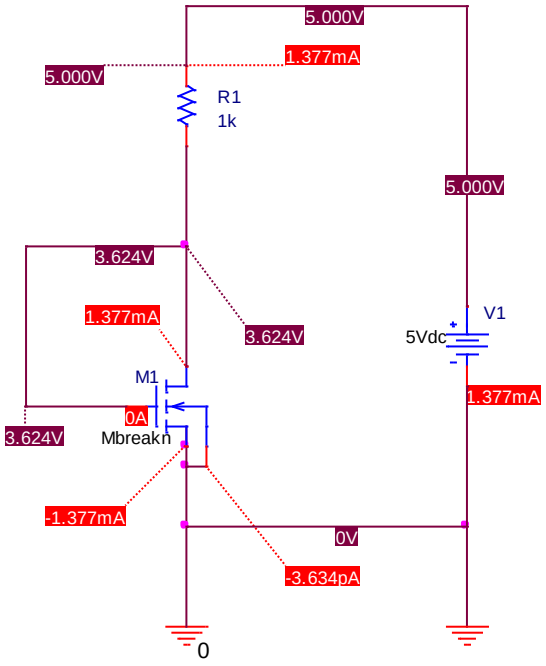
$25 - 5V_{GS} = V_{GS}^2 - 2V_{GS} + 1$

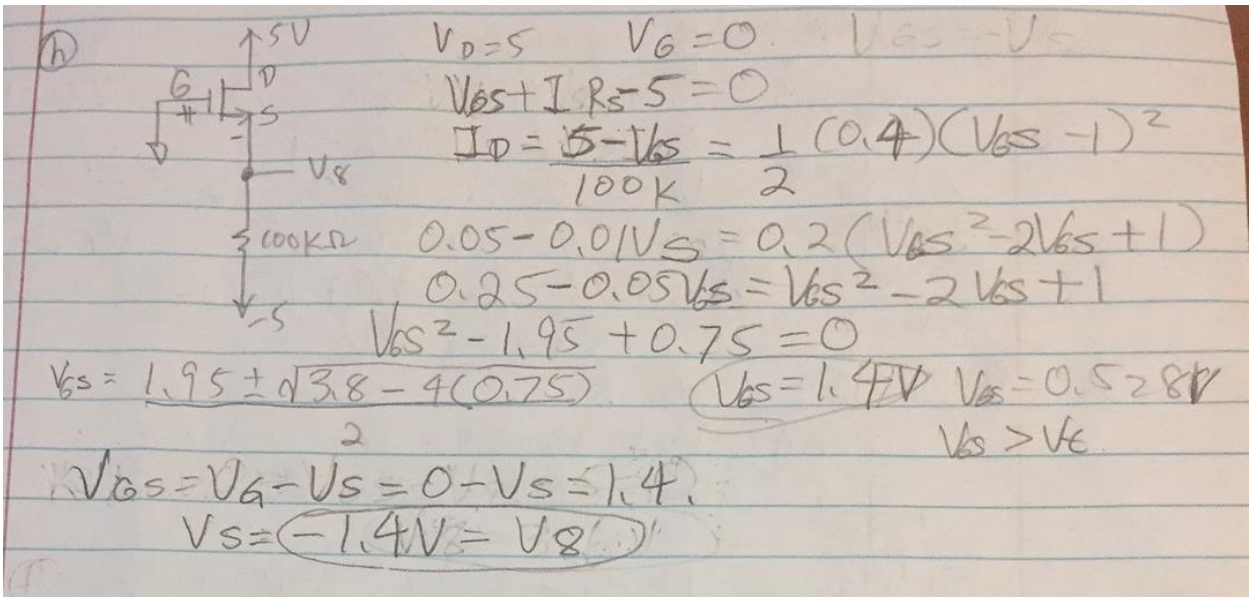
$V_{GS}^2 + 3V_{GS} - 24 = 0$

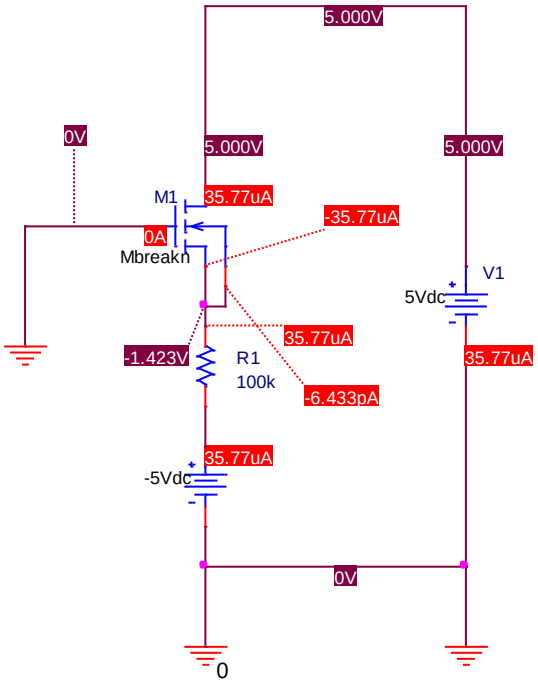
$V_G = V_7 \Rightarrow 3.6V$

$V_{GS} = \frac{-3 \pm \sqrt{9 - 4(-24)}}{2}$

$V_{GS} = 3.6V$ $V_{GS} = -6.6V$

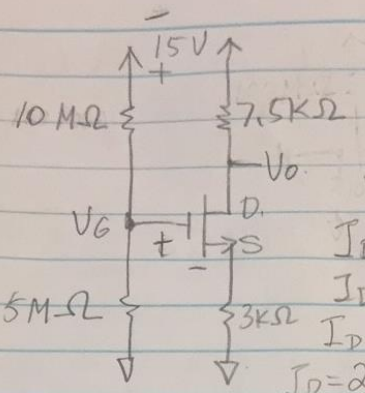



h
5V
D
G
S
V_8
100KΩ
-5
$V_D = 5$ $V_G = 0$
$V_{GS} + I_D R_S - 5 = 0$
$I_D = \frac{5 - V_{GS}}{100K} = \frac{1}{2}(0.4)(V_{GS} - 1)^2$
$0.05 - 0.01V_S = 0.2(V_{GS}^2 - 2V_{GS} + 1)$
$0.25 - 0.05V_{GS} = V_{GS}^2 - 2V_{GS} + 1$
$V_{GS}^2 - 1.95 + 0.75 = 0$
$V_{GS} = \frac{1.95 \pm \sqrt{3.8 - 4(0.75)}}{2}$
$V_{GS} = 1.4V$ $V_{GS} = 0.528V$
$V_{GS} > V_t$
$V_{GS} = V_G - V_S = 0 - V_S = 1.4$
$V_S = -1.4V = V_8$


5.000V
0V
5.000V
5.000V
M1
35.77uA
-35.77uA
0A
Mbreakn
V1
5Vdc
35.77uA
-1.423V
R1
100k
-6.433pA
35.77uA
35.77uA
-5Vdc
0V
0

3.1

(i)



$V_G = \frac{R_2}{R_1+R_2}(15V) = \frac{5}{10+5}(15) = 5V$

$V_S = I_D R_S = 3I_D$

$V_{GS} = 5 - 3I_D$

$I_D = \frac{1}{2}K'_n(V_{GS}-V_{th})^2$

$I_D = \frac{1}{2}(2)(V_{GS}-1)^2 = V_{GS}^2 - 2V_{GS} + 1$

$I_D = (5-3I_D)^2 - 2(5-3I_D) + 1$

$I_D = 25 - 30I_D + 9I_D^2 - 10 + 6I_D + 1$

$I_D = 16 - 24I_D + 9I_D^2$ $\quad 0 = 9I_D^2 - 25I_D + 16$

$I_D = \frac{25 \pm \sqrt{625 - 4(9)(16)}}{2(9)}$ $\quad I_{D1} = 1.78\,mA$ $\quad I_{D2} = 1\,mA$

If $I_{D1} = 1.78mA$

$V_{GS} = 5 - 3(1.78) = -0.34V$ < Vth not saturation

If $I_{D2} = 1mA$

$V_{GS} = 5 - 3(1) = 2V > V_{th}$ ✓ $\quad V_S = 3V$ $\quad V_D = 7.5V$

$KVL = -15 + I_D R_D + V_{DS} + I_D R_S$

$V_{DS} = 15 - I_D(7.5) - I_D(3) = 15 - 10.5(1) = 4.5V$

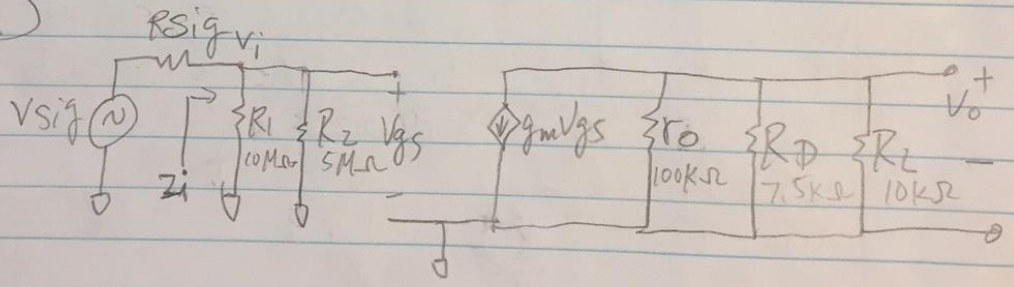
$V_{DS} = 4.5V > V_{GS} - V_t = 1V$ ✓

$I_D = I_{D2} = 1mA$

(ii) $g_m = \frac{2I_D}{V_{GS}-V_t} = \frac{2(1)}{2-1} = \frac{2}{1} = 2\frac{mA}{V} = g_m$

$r_o = \frac{V_A}{I_D} = \frac{100}{1mA} = 100K\Omega = r_o$

(iii)



(iv) $A_v = \frac{V_o}{V_{in}} = -g_m(r_o \| R_D \| R_L)$ $\quad -7.89$

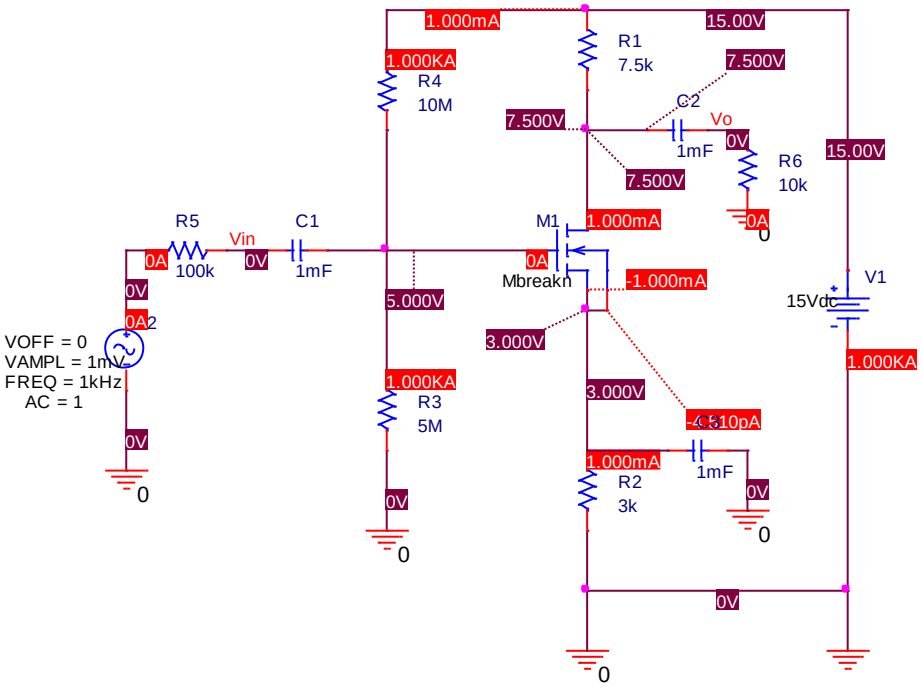
$A_v = -2(100 \| 7.5 \| 10) = -8.22 = A_v$

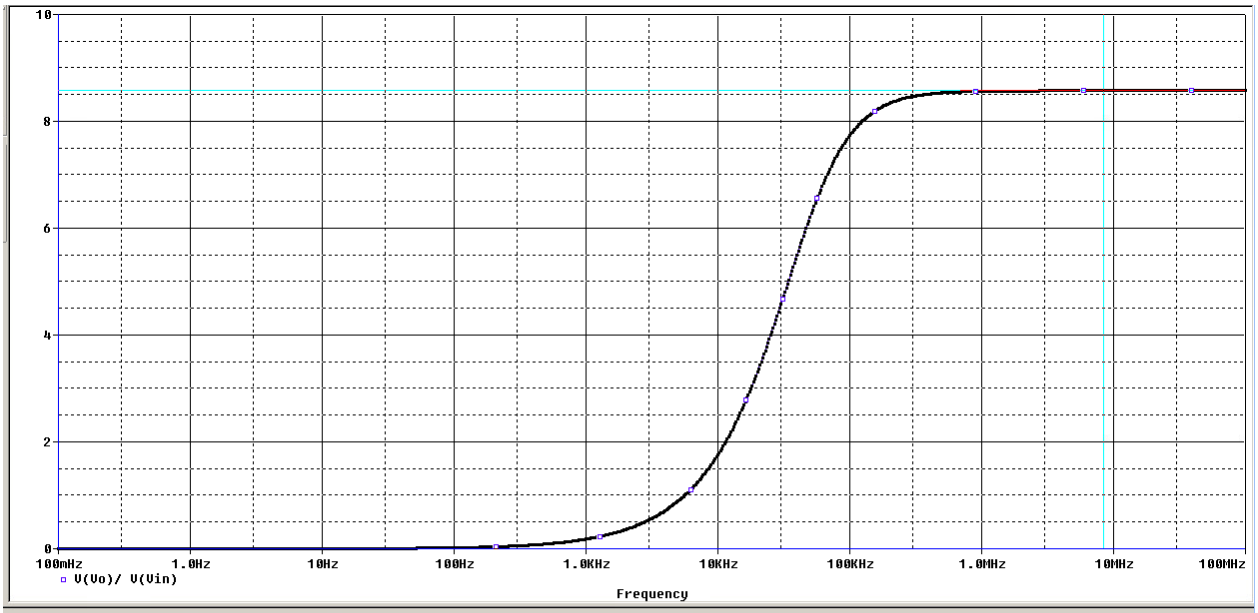
$V_i = V_{sig}\left(\frac{R_1 \| R_2}{R_{sig} + R_1 \| R_2}\right)$ $\quad \frac{V_i}{V_{sig}} = \left(\frac{R_1 \| R_2}{R_{sig} + R_1 \| R_2}\right)$

$A_{VS} = \frac{V_o}{V_{sig}} = \frac{V_i}{V_{sig}} \cdot \frac{V_o}{V_i} = \frac{R_1 \| R_2}{R_{sig} + R_1 \| R_2}$, $A_V = \left(\frac{10M \| 5M}{100_K + (10M \| 5M)} \right)$ *8.2

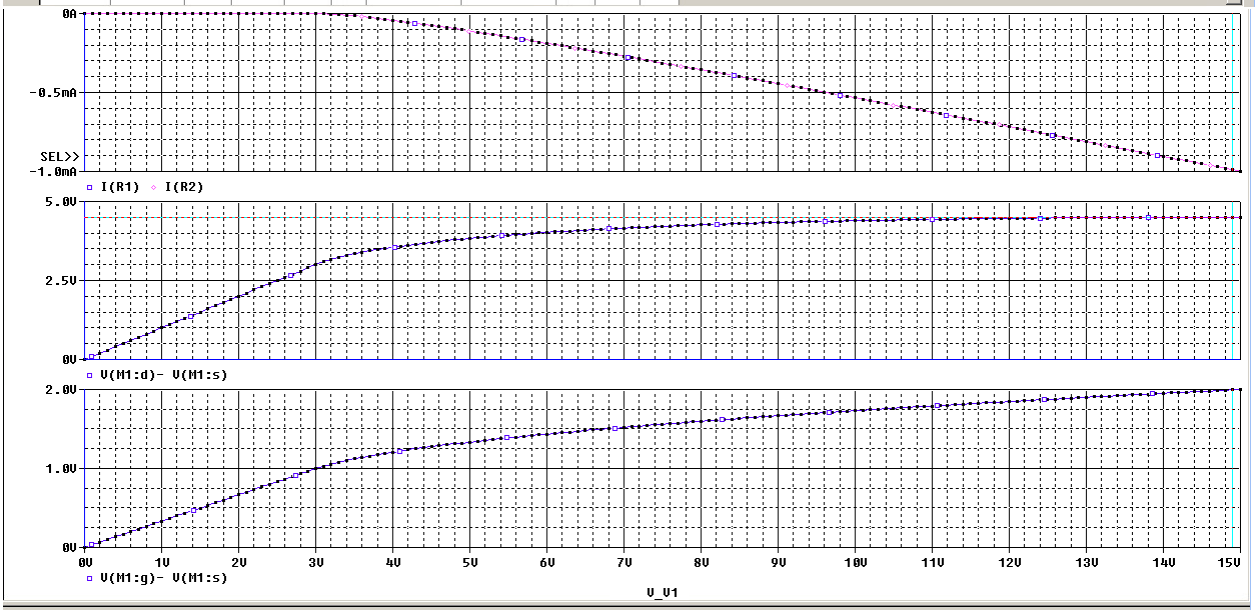
$A_{VS} = -7.98$

$R_{in} = R_G = R_1 \| R_2 = 10 \| 5 = 3.3\,M\Omega = R_{in}$



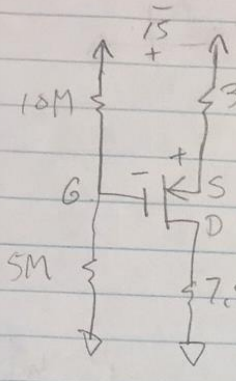


Trace Color	Trace Name	Y1	Y2	Y1 - Y2		Y1(Cursor1) - Y2(Cursor2)		8.5713		
	X Values	8.3993M	100.000m	8.3993M		Y1 - Y1(Cursor1)	Y2 - Y2(Cursor2)	Max Y	Min Y	Avg Y
CURSOR 1,2	V(Vo)/ V(Vin)	8.5713	5.2623u	8.5713		0.000	0.000	8.5713	5.2623u	4.2857



Trace Color	Trace Name	Y1	Y2	Y1 - Y2		Y1(Cursor1) - Y2(Cursor2)		4.5000		
	X Values	14.900	0.000	14.900		Y1 - Y1(Cursor1)	Y2 - Y2(Cursor2)	Max Y	Min Y	Avg Y
	V(M1:g)- V(M1:s)	1.9951	0.000	1.9951		-2.5049	0.000	1.9951	0.000	0.9976
CURSOR 1,2	V(M1:d)- V(M1:s)	4.5000	0.000	4.5000		0.000	0.000	4.5000	0.000	2.2500
	I(R1)	-990.236u	0.000	-990.236u		-4.5010	0.000	0.000	-990.236u	-495.118u
	I(R2)	-990.236u	0.000	-990.236u		-4.5010	0.000	0.000	-990.236u	-495.118u

3.2 (1)



$V_G = 5V$. $V_S = 15 - I_D R_S$

$V_{GS} = 5 - (15 - I_D R_S) = 3I_D - 10$

$I_D = \frac{1}{2}(2)(V_{GS}+1)^2$

$I_D = \frac{1}{2}(2)(V_{GS}^2 + 2V_{GS} + 1)$

$I_D = (3I_D - 10)^2 + 2(3I_D - 10) + 1$

$0 = 9I_D^2 - 55I_D + 81$

$I_D = \frac{55 \pm \sqrt{55^2 - 4(9)(81)}}{18}$ $I_{D1} = 3.6$ $I_{D2} = 2.5$

if $I_{D1} = 3.6mA$

$V_{GS} = 3I_D - 10 = 3(3.6) - 10 = 0.8V > V_{th}$ ✗

if $I_{D2} = 2.476mA$

$V_{GS} = 3I_D - 10 = 3(2.476) - 10 = -2.57 < V_{th}$ ✓

KVL: $-15 + I_D R_S + V_{SD} + I_D R_D = 0$

$V_{SD} = 15 - I_D(R_S + R_D) = 15 - (2.476)(3+7.5)$

$V_{SD} = -11$ $V_{SD} < V_{SG} + V_{th} = 2.57 - 1 = 1.57V$.

Triode Mode

$I_D = K'\frac{W}{L}\left[(V_{SG} + V_{th}) + \frac{V_{SD}}{2}\right]V_{SD}$ $V_{SD} = V_S - V_D$

$V_D = I_D R_D$

$V_{sd} = V_S - I_D R_D$

$V_{SD} = V_S - 7.5I_D$

$I_D = 2\left[(V_S - V_G - 1) + \left(\frac{V_S - 7.5I_D}{2}\right)\right](V_S - 7.5I_D)$

$I_D = 2\left[((15 - 3I_D) - 5 - 1) + \left(\frac{(15 - 3I_D) - 7.5I_D}{2}\right)\right](15 - 3I_D - 7.5I_D)$

$173.25 I_D^2 - 595 I_D + 495 = 0$

$I_D = \frac{595 \pm \sqrt{595^2 - 4(173.25)(495)}}{2(173.25)}$

$I_{D1} = 2.02\,mA \quad I_{D2} = 1.41\,mA$

If $I_{D1} = 2.02\,mA$

$V_{GS} = 3(2.02) - 10 = -3.94 < V_{th}$ ← 1

$V_{SD} = 15 - 2.02(3 + 7.5) = -6.21 < V_{SG} + V_{th} = 1.57V$

If $I_{D2} = 1.41\,mA$

$V_{GS} = 3(1.41) - 10 = -5.77 < V_{th}$

$V_{SD} = 15 - 1.41(3 + 7.5) = 0.195 < V_{SG} + V_{th} = 1.57V$

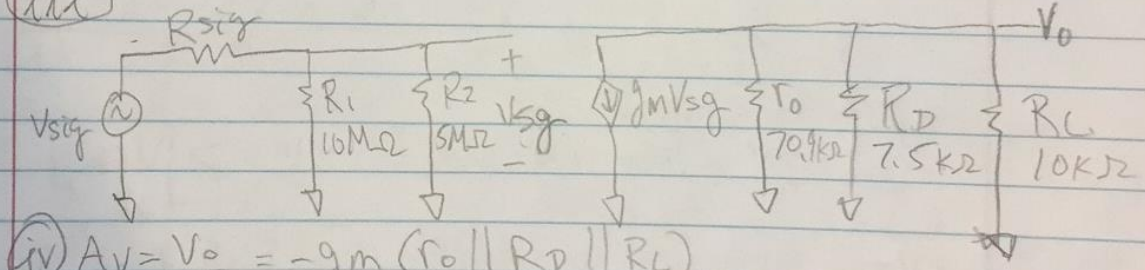
$I_D = 1.41\,mA \qquad V_{DS} = -0.195V$

(ii) $g_m = K_p' W/L (V_{GS} + V_t) = 2(-5.77 + 1)$

$g_m = +9.54\,mA/V$

$r_o = \frac{V_A}{I_D} = \frac{100}{1.41} = 70.9\,K\Omega = r_o$

(iii)

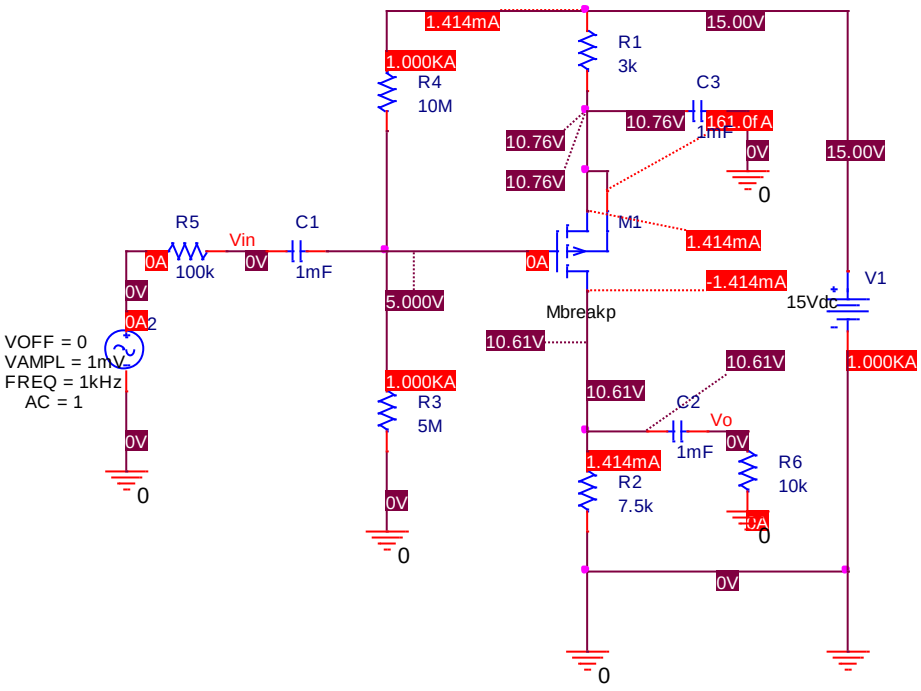


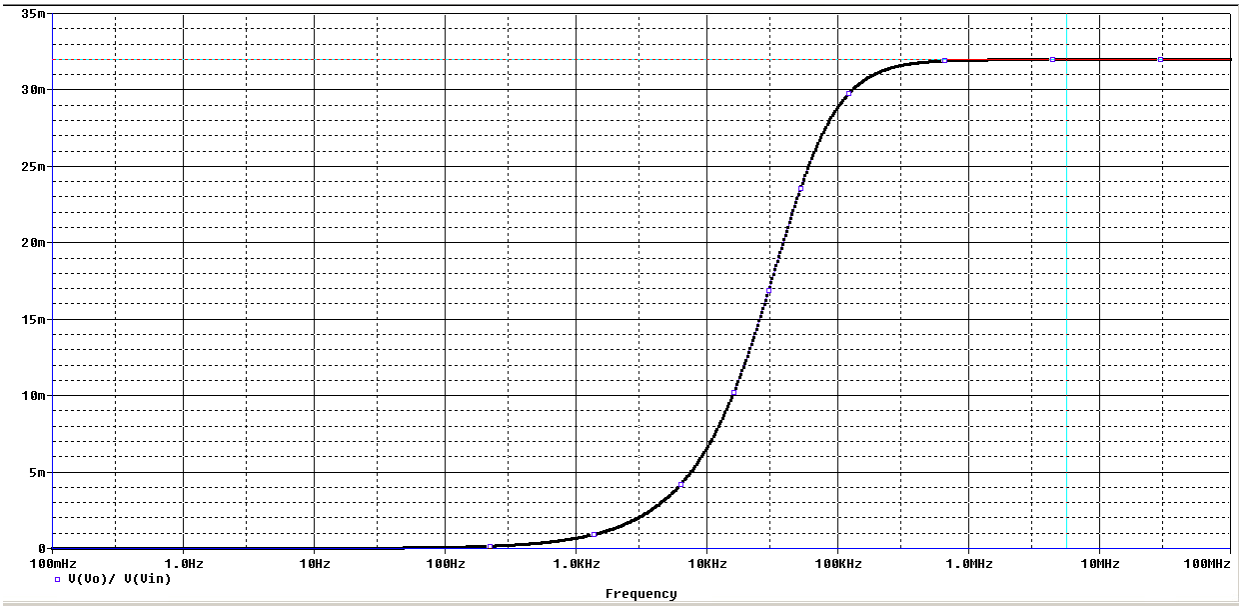
(iv) $A_V = \frac{V_o}{V_{in}} = -g_m (r_o \| R_D \| R_L)$

$A_V = -9.54(70.9 \| 7.5 \| 10)$

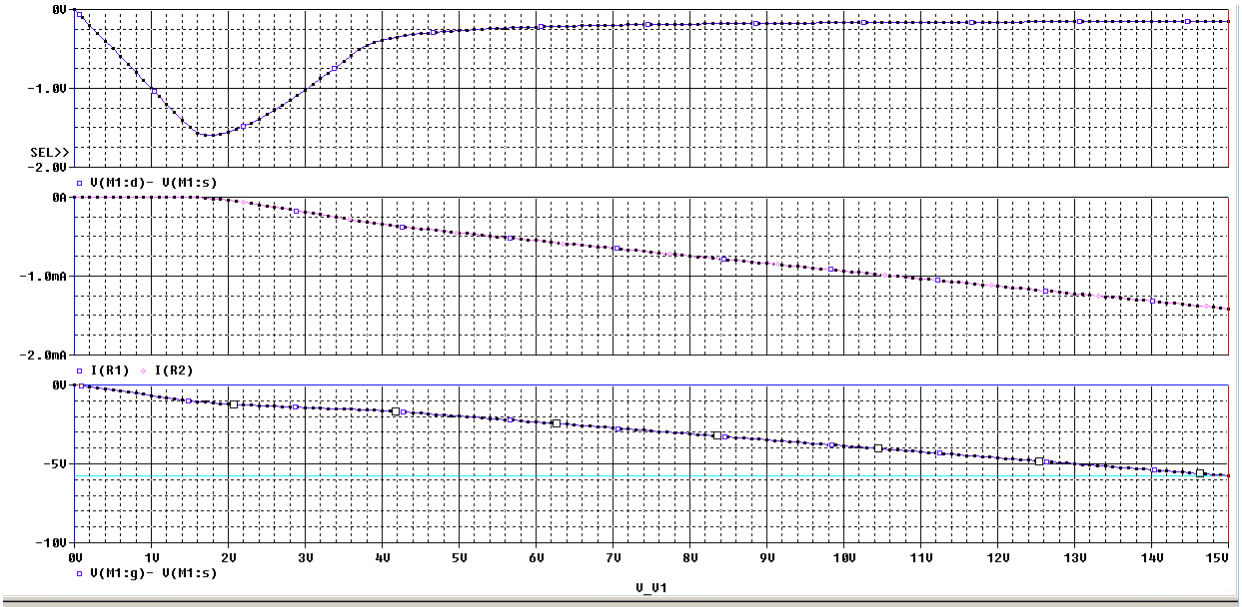
$A_V = -38.55$

$R_{in} = R_G = R_1 \| R_2 = 10 \| 5 = 3.3\,M\Omega$





Trace Color	Trace Name	Y1	Y2	Y1 - Y2		Y1(Cursor1) - Y2(Cursor2)		31.975m		
	X Values	5.5988M	100.000m	5.5988M		Y1 - Y1(Cursor1)	Y2 - Y2(Cursor2)	Max Y	Min Y	Avg Y
CURSOR 1,2	V(Vo)/ V(Vin)	31.975m	54.904n	31.975m		0.000	0.000	31.975m	54.904n	15.988m



Trace Color	Trace Name	Y1	Y2	Y1 - Y2		Y1(Cursor1) - Y2(Cursor2)		-5.7574		
	X Values	15.000	0.000	15.000		Y1 - Y1(Cursor1)	Y2 - Y2(Cursor2)	Max Y	Min Y	Avg Y
CURSOR 1,2	V(M1:g)- V(M1:s)	-5.7574	0.000	-5.7574		0.000	0.000	0.000	-5.7574	-2.8787
	I(R1)	-1.4046m	0.000	-1.4046m		5.7560	0.000	0.000	-1.4046m	-702.322u
	I(R2)	-1.4046m	0.000	-1.4046m		5.7560	0.000	0.000	-1.4046m	-702.322u
	V(M1:d)- V(M1:s)	-151.239m	0.000	-151.239m		5.6062	0.000	0.000	-151.239m	-75.620m