Assume $X_1, X_2, \ldots, X_n$ are independent and identically distributed random samples from an exponential distribution with the probability density function:

$$f(x) = \frac{1}{\lambda} e^{-\frac{x}{\lambda}}, \quad x \geq 0, \lambda > 0,$$

where λ is the unknown parameter. Define the estimator $\hat{\lambda}_n = \bar{X}$, where $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$.

- Find $E(\hat{\lambda}_n)$, $B(\hat{\lambda}_n)$ and the $MSE(\hat{\lambda}_n)$.
- Prove whether $\hat{\lambda}_n$ is a consistent estimator.
- Find the maximum likelihood estimator $\hat{\lambda}_{MLE}$ of λ.
- Find the method of moments estimator $\hat{\lambda}_{\text{MM}}$ for λ.
- Find a sufficient statistic for λ.
- Derive a MVUE for λ.
- Construct a $1 - \alpha$ confidence interval for λ.
- Design the most powerful α-level test for $H_0 : \lambda = \lambda_0$ against $H_a : \lambda = \lambda_a$, where $\lambda_a > \lambda_0$.
- Is the test designed above uniformly most powerful (UMP) for testing $H_0 : \lambda = \lambda_0$ against $H_a : \lambda > \lambda_0$?
- Consider the following two estimators:

$$\hat{\lambda}_1 = \frac{1}{n}\sum_{i=1}^{n} X_i \quad \text{and} \quad \hat{\lambda}_2 = \frac{1}{n-1}\sum_{i=1}^{n} X_i.$$

- Compare the bias and MSE of $\hat{\lambda}_1$ and $\hat{\lambda}_2$.
- Which estimator is more efficient?