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Master-Thesis

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Altruistic committee election rules

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1 Introduction

2 Definitions

Assume we have n voters and m candidates for a committee election case with the committee size of g . Let V = { V_1, V_2, ..., V_n } be a set of all voters. Let C = { C_1, C_2, ..., C_m } be a set of all candidates. This way we can describe a committee election case as $E(C, V, g)$ where (m = | C |, n= | V |).

We define $F_{V_i} \subseteq V$ (i = 1, 2, ..., n) as a subset of V contains all the friends of V_i. We say W is a friend of V_i if $W \subseteq F_{V_i}$
Besides, we define $\succeq$ = ($\succeq_{V_1}$, $\succeq_{V_2}$, ..., $\succeq_{V_n}$) as the set of preferences that each voter has over all the candidates in C.

We use S to denote the score a candidate receives. We assume $S_{V_i}(C_j)$ is the borda score that candidate C_j receives from voter V_i. Under the circumstance that we have m candidates all together, the borda score can be calculated as follows: if voter V_i ranks candidate C_j at the position k in $\succeq_{V_j}$, then:

$$S_{V_i}(C_j) = m - k + 1 \tag{1}$$

2.1 score based rules

score based selfish rule

We propose a score based selfish rule where the committee election is only dependent on each voter's personal opinion. For a committee election case $E(C, V, g)$ where (m = | C |, n= | V |).

In this setting, each voter will first decide his preference $\succeq_{V_i}$ ($i = 1, 2, \ldots, n$) and this is reflected by the borda score (**??**). Without considering the opinion of friends, we calculate the final score of a candidate as the sum of the score it received from all voters:

$$S^{SF}(C_j) = \sum_{i=1}^{n} S_{V_i}(C_j) = S_{V_1}(C_j) + S_{V_2}(C_j) + \cdots + S_{V_n}(C_j) \tag{2}$$

The top g candidates with the highest scores will be elected into the final committee.

score based altruistic rule

We propose a score based altruistic rule where the committee election is only dependent on each voter's friends' opinion and ignore the voter's personal opinion. For a committee election case $E(C, V, g)$ where (m = | C |, n= | V |). We define $F_{V_i} \subseteq V$ (i = 1, 2, ..., n) as a subset of V contains all the friends of V_i. We say W is a friend of V_i if $W \subseteq F_{V_i}$.

Similarly, each voter will first decide his preference $\succeq_{V_i}$ ($i = 1, 2, \ldots, n$) and this is reflected by the borda score (**??**).

Then we will consider the opinion of friends and calculate the altruistic score based on the original scoring of each voter. For this, we need to first introduce a relation variant

$f^{rel}_{v_i}(v_j)$, which indicates the relationship between two voters $V_i \subseteq V$ and $V_j \subseteq V$.

$$f^{rel}_{v_i}(v_j) = \begin{cases} 1, & V_j \subseteq F_{V_i} \\ 0, & otherwise \end{cases} \tag{3}$$

The altruistic score is the average of the score a candidate C_k receives from the voter V_i's friends F_{V_i}:

$$\begin{aligned} S^{AL}_{V_i}(C_k) &= \frac{1}{\mid F_{V_i} \mid} \sum_{j=1}^{n} f^{rel}_{v_i}(v_j) S_{V_j}(C_k) \\ &= \frac{1}{\mid F_{V_i} \mid} [f^{rel}_{v_i}(v_1) S_{V_1}(C_k) + \cdots + f^{rel}_{v_i}(v_n) S_{V_n}(C_k)] \end{aligned} \tag{4}$$

The final score of a candidate C_k is calculated as the sum of the score it received from all voters

$$S^{AL}(C_k) = \sum_{i=1}^{n} S^{AL}_{V_i}(C_k) = S^{AL}_{V_1}(C_k) + S^{AL}_{V_2}(C_k) + \cdots + S^{AL}_{V_n}(C_k) \tag{5}$$

The top g candidates with the highest scores will be elected into the final committee.

score based equal treatment rule

In the equal treatment rule, we combine the selfish rule and the altruistic rules and calculate the score of each candidate aggregating half of the altruistic score and half of the selfish score:

$$S^{EQ}_{V_i}(C_k) = \frac{1}{2} S^{AL}_{V_i}(C_k) + \frac{1}{2} S_{V_i}(C_k) \tag{6}$$

We can reformulate the Equation (**??**) using Equation (**??**) and Equation (**??**):

$$\begin{aligned} S^{EQ}_{V_i}(C_k) &= \frac{1}{2 \mid F_{V_i} \mid} \sum_{j=1}^{n} f^{rel}_{v_i}(v_j) S_{V_j}(C_k) + \frac{1}{2} S_{V_i}(C_k) \\ &= \frac{1}{2}(\frac{1}{\mid F_{V_i} \mid}(f^{rel}_{v_i}(v_1) S_{V_1}(C_k) + \cdots + f^{rel}_{v_i}(v_n) S_{V_n}(C_k)) + S_{V_i}(C_k)) \end{aligned} \tag{7}$$

Similarly, the final score of a candidate C_k is calculated as the sum of the score it received from all voters:

$$S^{EQ}(C_k) = \sum_{i=1}^{n} S^{EQ}_{V_i}(C_k) = S^{EQ}_{V_1}(C_k) + S^{EQ}_{V_2}(C_k) + \cdots + S^{EQ}_{V_n}(C_k) \tag{8}$$

We elect, same as all cases above, the top g candidates with the highest scores into the final committee.

score based most popular first rule

In practise, there are often opinion leaders(this is a cited concept) who can influence the majority of the friends circle. We introduce therefore the following rule that consider only the opinion of the most popular friend within each friends circle. For this, we need a specific relation variant $f_{v_i}^{mst}(v_j)$, which indicates the relationship between two voters $V_i \subseteq V$ and $V_j \subseteq V$.

$$f_{v_i}^{mst}(v_j) = \begin{cases} 1, & \forall V_p \subseteq F_{V_i}, \mid F_{V_j} \mid \geqslant \mid F_{V_p} \mid \\ 0, & otherwise \end{cases} \tag{9}$$

The most popular score $S_{V_i}^{MST}(C_k)$ is the score of the most popular voter in V_i's friends F_{V_i}:

$$\begin{aligned} S_{V_i}^{MST}(C_k) &= \frac{1}{\sum_{j=1}^{n} f_{v_i}^{mst}(v_j)} \sum_{j=1}^{n} f_{v_i}^{mst}(v_j) S_{V_j}(C_k) \\ &= \frac{1}{\sum_{j=1}^{n} f_{v_i}^{mst}(v_j)} (f_{v_i}^{mst}(v_1) S_{V_1}(C_k) + \cdots + f_{v_i}^{mst}(v_n) S_{V_n}(C_k)) \end{aligned} \tag{10}$$

The final score of a candidate C_k is calculated as the sum of the score it received from all voters. The top g candidates with the highest scores will be elected into the final committee.

score based more popular first rule

For this, we need a specific relation variant $f_{v_i}^{mst}(v_j)$, which indicates the relationship between two voters $V_i \subseteq V$ and $V_j \subseteq V$.

$$f_{v_i}^{mre}(v_j) = \begin{cases} 1, & \mid F_{V_i} \mid \leqslant \mid F_{V_j} \mid \\ 0, & otherwise \end{cases} \tag{11}$$

The more popular score $S_{V_i}^{MRE}(C_k)$ is the average score of all more popular voters in V_i's friends F_{V_i}:

$$\begin{aligned} S_{V_i}^{MRE}(C_k) &= \frac{1}{\sum_{j=1}^{n} f_{v_i}^{mre}(v_j)} \sum_{j=1}^{n} f_{v_i}^{mst}(v_j) S_{V_j}(C_k) \\ &= \frac{1}{\sum_{j=1}^{n} f_{v_i}^{mre}(v_j)} (f_{v_i}^{mst}(v_1) S_{V_1}(C_k) + \cdots + f_{v_i}^{mre}(v_n) S_{V_n}(C_k)) \end{aligned} \tag{12}$$

The final score of a candidate C_k is calculated as the sum of the score it received from all voters. The top g candidates with the highest scores will be elected into the final committee.

2.2 approval based rules

approval based selfish rule

Correspondent to the score based selfish rule we propose a approval based selfish rule where the committee election is only dependent on each voter's personal opinion. For a committee election case $E(C, V, g)$ where (m = | C |, n= | V |). We define $A_{V_i} \subseteq C$ (i = 1, 2, . . . , n) as a subset of C contains all the approved candidates by V_i. We say Y is a approved candidate by V_i if $Y \subseteq A_{V_i}$.

We use T to denote the times that a candidate is approved by a voter. We assume $T_{V_i}(C_k)$ is the time that candidate C_k is approved by voter V_i. As defined earlier, $C_k \subseteq A_{V_i}$ if voter V_i approves C_k, then:

$$T_{v_i}(C_k) = \begin{cases} 1, & C_k \subseteq A_{V_i} \\ 0, & otherwise \end{cases} \tag{13}$$

Without considering the opinion of friends, we calculate the final times of approval a candidate C_k receives as the sum of the approval it received from all voters:

$$T^{SF}(C_k) = \sum_{i=1}^{n} T_{V_i}(C_k) = T_{V_1}(C_k) + T_{V_2}(C_k) + \cdots + T_{V_n}(C_k) \tag{14}$$

The top g candidates with the highest times of approval will be elected into the final committee.

approval based altruistic rule

In the altruistic setting, we will first calculate the times of approval for each candidate by each voter as with Equation (**??**). The friend circle of each voter V_i is F_{V_i}, a voter V_i will calculate his altruistic approval times T^{AL} as how many times a candidate C_k is approved by all friends of his $V_j \subseteq F_{V_i}$. For this, we use the relation variant $f_{v_i}^{rel}(v_j)$ as of (**??**) again:

$$\begin{aligned} T_{V_i}^{AL}(C_k) &= \frac{1}{\mid F_{V_i} \mid} \sum_{j=1}^{n} f_{v_i}^{rel}(v_j) T_{V_j}(C_k) \\ &= \frac{1}{\mid F_{V_i} \mid} (f_{v_i}^{rel}(v_1) T_{V_1}(C_k) + \cdots + f_{v_i}^{rel}(v_n) T_{V_n}(C_k)) \end{aligned} \tag{15}$$

For each candidate, the final times of approval $T^{AL}(C_k)$ can be calculated as:

$$T^{AL}(C_k) = \sum_{i=1}^{n} T_{V_i}^{AL}(C_k) = T_{V_1}^{AL}(C_k) + T_{V_2}^{AL}(C_k) + \cdots + T_{V_n}^{AL}(C_k) \tag{16}$$

The top g candidates with the highest times of approval will be elected into the final committee.

approval based equal treatment rule

In the altruistic setting, we will combine the selfish rule and the altruistic rule and calculate the new times of approval as follows:

$$\begin{aligned} T_{V_i}^{EQ}(C_k) &= \frac{1}{2}T_{V_i}^{EQ}(C_k) + \frac{1}{2}T_{V_i}(C_k) \\ &= \frac{1}{2}(\frac{1}{\mid F_{V_i} \mid}\sum_{j=1}^{n} f_{v_i}^{rel}(v_j)T_{V_j}(C_k) + T_{V_i}(C_k)) \\ &= \frac{1}{2}(\frac{1}{\mid F_{V_i} \mid}(f_{v_i}^{rel}(v_1)T_{V_1}(C_k) + \cdots + f_{v_i}^{rel}(v_n)T_{V_n}(C_k)) + T_{V_i}(C_k)) \end{aligned} \tag{17}$$

For each candidate, the final times of approval $T^{AL}(C_k)$ can be calculated as:

$$T^{EQ}(C_k) = \sum_{i=1}^{n} T_{V_i}^{EQ}(C_k) = T_{V_1}^{EQ}(C_k) + T_{V_2}^{EQ}(C_k) + \cdots + T_{V_n}^{EQ}(C_k) \tag{18}$$

The top g candidates with the highest times of approval will be elected into the final committee.

2.3 non-weighted approval based rules

In this case, the voters will only change his approval for a certain candidate C_k, if certain percentage of his friends are holding the opposite opinion towards this candidate, we denote this percentage as ρ. We define these friends of V_i as $F_{V_i}^{opp}$: $W \in F_{V_i}^{opp}$ if $W \in F_{V_i}$ and $T_W(C_k) \neq T_{V_i}(C_k)$, then we will let V_i adopt the majority opinion of all his friends and therefore:

$$T_{v_i}(C_k) = \begin{cases} T_{v_i}(C_k) \oplus 1, & \frac{|F_{V_i}^{opp}|}{|F_{V_i}|} \geqslant \rho \\ T_{v_i}(C_k), & otherwise \end{cases} \tag{19}$$

We calculate the final times of approval a candidate C_k receives as the sum of the approval it received from all voters. The top g candidates with the highest times of approval will be elected.

2.4 more complex rules

limit personal dissatisfaction rule

sometimes a voter can make to a certain extend compromises for his friends. For this case, we introduce a voting rule with limited personal dissatisfaction. We first describe the degree of personal dissatisfaction with a Personal Dissatisfaction Score (PDS). For approval based setting, the PDS is valued with the number of candidates in the final voting decision, that a voter does not approval originally. For a committee election case $E(C, V, g)$ where (m = $\mid C \mid$, n= $\mid V \mid$), we denote the approval ballot of voter V_i as

$B_{V_i}^{org}$ and the final decision approval ballot $B_{V_i}^{fnl}$. Then the PDS score of voter V_i which is denoted as $S_{V_i}^{PDS}$ can be defined as:

$$S_{V_i}^{PDS} = \mid (C \setminus B_{V_i}^{org}) \cap B_{V_i}^{fnl} \mid \tag{20}$$

On the other hand, we want to measure how happy are the friends of a voter regarding the final voting decision of a voter. We describe the degree of friends' satisfaction with a Friends' Satisfaction Score (FSS). $FSS \subseteq [0, 1]$. The FSS score of V_i is dependent to F_{V_i}, and how each of the friends is satisfied with the final decision of this voter. We use the relation variant $f_{v_i}^{rel}(v_j)$ to keep the friends and filter the non-friends. The final decision approval ballot of V_i covers certain number of candidates which is in the original approval ballot of voter V_j as: $B_{V_i}^{org}$ is $B_{V_i}^{cov}(V_j)$:

$$B_{V_i}^{cov}(V_j) = \mid B_{V_j}^{org} \cap B_{V_i}^{fnl} \mid \tag{21}$$

With this notion, we can then describe how satisfied is a voter V_j with the final decision of voter V_i:

$$S_{V_i}^{FSS}(V_j) = \frac{B_{V_i}^{cov}(V_j)}{g} \tag{22}$$

We weight each friend differently according to how ïnfluentialä voter is. Depending on how many friends each voter has, the weight of the voter V_j in the friends circle of V_i is:

$$W_{F_{V_i}}(V_j) = \frac{\mid F_{V_j} \mid}{\sum_{r=1}^{n} f_{v_i}^{rel}(v_r) \mid F_{V_r} \mid} \tag{23}$$

The final FSS of a voter V_i for his decision $B_{V_i}^{fnl}$ can be calculated as follows:

$$S_{V_i}^{FSS} = \sum_{j=1}^{n} f_{v_i}^{rel}(v_j) S_{V_i}^{FSS}(V_j) W_{F_{V_i}}(V_j) \tag{24}$$

In general, if a voter make some compromises for his friends, this will rise the FSS and also the PDS at the same time. We aim to limit the PDS to a certain value and within this PDS, we try to find the highest FSS possible.