Some Notes on Position Based Fluids

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General

- the vector $\mathbf{p}$ is a concatenation of particle positions, i.e.

$$\mathbf{p} = (\mathbf{p}_1, \mathbf{p}_2, ..., \mathbf{p}_N) = (x_1, y_1, z_1, x_2, y_2, z_2, ..., x_N, y_N, z_N)$$

- similarly,

$$\begin{aligned} \Delta\mathbf{p} &= (\Delta\mathbf{p}_1, \Delta\mathbf{p}_2, ..., \Delta\mathbf{p}_N) \\ &= (\Delta x_1, \Delta y_1, \Delta z_1, \Delta x_2, \Delta y_2, \Delta z_2, ..., \Delta x_N, \Delta y_N, \Delta z_N) \end{aligned}$$

- Equation 4 expands to:

$$\begin{aligned} \begin{pmatrix} \Delta\mathbf{p}_1 \\ \Delta\mathbf{p}_2 \\ \vdots \\ \Delta\mathbf{p}_N \end{pmatrix} &\approx (\nabla C_1(\mathbf{p}); \nabla C_2(\mathbf{p}); \ldots; \nabla C_N(\mathbf{p})) \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_N \end{pmatrix} \\ &\approx \begin{pmatrix} \nabla_{\mathbf{p}_1} C_1(\mathbf{p}) & \nabla_{\mathbf{p}_1} C_2(\mathbf{p}) & \ldots & \nabla_{\mathbf{p}_1} C_N(\mathbf{p}) \\ \nabla_{\mathbf{p}_2} C_1(\mathbf{p}) & \nabla_{\mathbf{p}_2} C_2(\mathbf{p}) & \ldots & \nabla_{\mathbf{p}_2} C_N(\mathbf{p}) \\ \vdots & \vdots & & \vdots \\ \nabla_{\mathbf{p}_N} C_1(\mathbf{p}) & \nabla_{\mathbf{p}_N} C_2(\mathbf{p}) & \ldots & \nabla_{\mathbf{p}_N} C_N(\mathbf{p}) \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_N \end{pmatrix} \end{aligned}$$

  where $\nabla_{\mathbf{p}_k} C_i(\mathbf{p}) = \partial C_i(\mathbf{p})/\partial \mathbf{p}_k = (\partial C_i(\mathbf{p})/\partial x_k, \partial C_i(\mathbf{p})/\partial y_k, \partial C_i(\mathbf{p})/\partial z_k)$

- Equation 6 expands to

$$\begin{pmatrix} C_1(\mathbf{p}) \\ C_2(\mathbf{p}) \\ \vdots \\ C_N(\mathbf{p}) \end{pmatrix} + \begin{pmatrix} \nabla_{\mathbf{p}_1} C_1(\mathbf{p}) & \nabla_{\mathbf{p}_2} C_1(\mathbf{p}) & \ldots & \nabla_{\mathbf{p}_N} C_1(\mathbf{p}) \\ \nabla_{\mathbf{p}_1} C_2(\mathbf{p}) & \nabla_{\mathbf{p}_2} C_2(\mathbf{p}) & \ldots & \nabla_{\mathbf{p}_N} C_2(\mathbf{p}) \\ \vdots & \vdots & & \vdots \\ \nabla_{\mathbf{p}_1} C_N(\mathbf{p}) & \nabla_{\mathbf{p}_2} C_N(\mathbf{p}) & \ldots & \nabla_{\mathbf{p}_N} C_N(\mathbf{p}) \end{pmatrix} \cdot$$

$$\begin{pmatrix} \nabla_{\mathbf{p}_1} C_1(\mathbf{p}) & \nabla_{\mathbf{p}_1} C_2(\mathbf{p}) & \ldots & \nabla_{\mathbf{p}_1} C_N(\mathbf{p}) \\ \nabla_{\mathbf{p}_2} C_1(\mathbf{p}) & \nabla_{\mathbf{p}_2} C_2(\mathbf{p}) & \ldots & \nabla_{\mathbf{p}_2} C_N(\mathbf{p}) \\ \vdots & \vdots & & \vdots \\ \nabla_{\mathbf{p}_N} C_1(\mathbf{p}) & \nabla_{\mathbf{p}_N} C_2(\mathbf{p}) & \ldots & \nabla_{\mathbf{p}_N} C_N(\mathbf{p}) \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_N \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

Kernels

- *poly6* Kernel (2D)

$$W_{poly6\_2D}(r,h) = \frac{4}{\pi h^8} \begin{cases} (h^2 - r^2)^3 & \text{if } r < h \\ 0 & \text{otherwise} \end{cases} \tag{1}$$