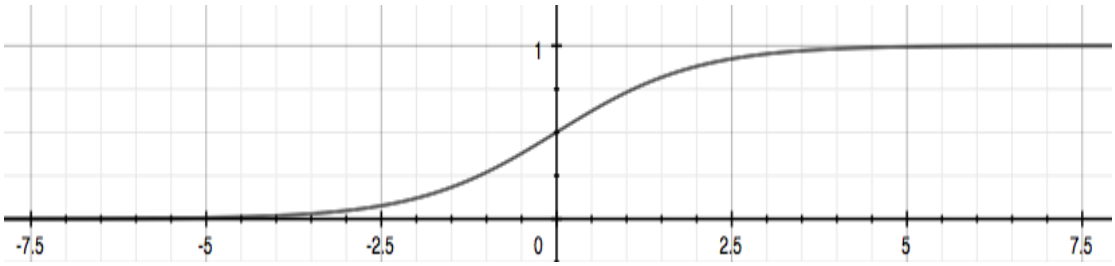
Hypothesis Representation

We could approach the classification problem ignoring the fact that y is discrete-valued, and use our old linear regression algorithm to try to predict y given x. However, it is easy to construct examples where this method performs very poorly. Intuitively, it also doesn't make sense for $\mathbf{h_\theta(x)}$ to take values larger than 1 or smaller than 0 when we know that $y \in \{0, 1\}$. To fix this, let's change the form for our hypotheses $\mathbf{h_\theta(x)}$ to satisfy $\mathbf{0 \leq h_\theta(x) \leq 1}$. This is accomplished by plugging $\mathbf{\theta^T x}$ into the Logistic Function. Our new form uses the "Sigmoid Function," also called the "Logistic Function":

$$\mathbf{h_\theta(x)} = g(\mathbf{\theta^T x})$$
$$z = \mathbf{\theta^T x}$$
$$g(z) = \frac{1}{1+e^{-z}}$$

The following image shows us what the sigmoid function looks like:



The function g(z), shown here, maps any real number to the (0, 1) interval, making it useful for transforming an arbitrary-valued function into a function better suited for classification.

$\mathbf{h_\theta(x)}$ will give us the **probability** that our output is 1. For example, $\mathbf{h_\theta(x)} = 0.7$ gives us a probability of 70% that our output is 1. Our probability that our prediction is 0 is just the complement of our probability that it is 1 (e.g. if probability that it is 1 is 70%, then the probability that it is 0 is 30%).

$$\mathbf{h_\theta(x)} = P(y=1|x;\theta) = 1 - P(y=0|x;\theta)$$
$$P(y=0|x;\theta) + P(y=1|x;\theta) = 1$$