Examples and Intuitions II

The $\Theta^{(1)}$ matrices for AND, NOR, and OR are:

AND:

$\Theta^{(1)}$ **=[−30 20 20]**

NOR:

$\Theta^{(1)}$ **=[10 −20 −20]**

OR:

$\Theta^{(1)}$ **=[−10 20 20]**

We can combine these to get the XNOR logical operator (which gives 1 if $\mathbf{x_1}$ and $\mathbf{x_2}$ are both 0 or both 1).

$$\begin{bmatrix} \mathbf{x_0} \\ \mathbf{x_1} \\ \mathbf{x_2} \end{bmatrix} \rightarrow \begin{bmatrix} \mathbf{a_1^{(2)}} \\ \mathbf{a_2^{(2)}} \end{bmatrix} \rightarrow [\mathbf{a^{(3)}}] \rightarrow \mathrm{h}_\Theta(x)$$

For the transition between the first and second layer, we'll use a $\Theta^{(1)}$ matrix that combines the values for AND and NOR:

$$\Theta^{(1)} = \begin{bmatrix} -30 & 20 & 20 \\ 10 & -20 & -20 \end{bmatrix}$$

For the transition between the second and third layer, we'll use a $\Theta^{(2)}$ matrix that uses the value for OR:

$\Theta^{(1)}$ **=[−10 20 20]**

Let's write out the values for all our nodes:

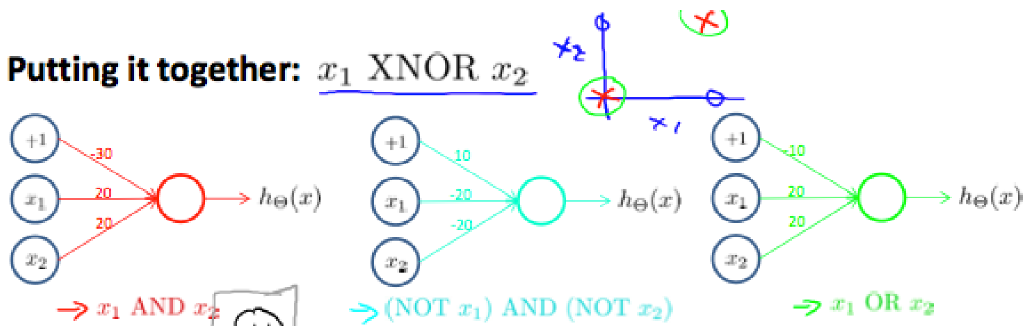
$\mathbf{a^{(2)}}$ **= g ($\mathbf{\Theta^{(1)}} \cdot$x)**

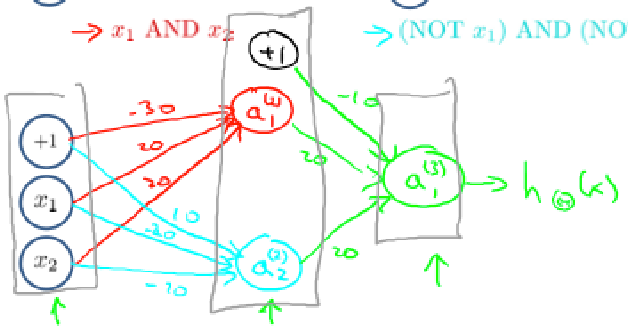
$\mathbf{a^{(3)}}$ **= g ($\mathbf{\Theta^{(2)}} \cdot \mathbf{a^{(2)}}$)**

$\mathbf{h_\Theta(x)}$= $\mathbf{a^{(3)}}$

And there we have the XNOR operator using a hidden layer with two nodes! The following summarizes the above algorithm:

Putting it together: $x_1 \text{ XNOR } x_2$





x_1	x_2	$a_1^{(2)}$	$a_2^{(2)}$	$h_\Theta(x)$
0	0	0	1	1
0	1	0	0	0
1	0	0	0	0
1	1	1	0	1