

# Decision Boundary

In order to get our discrete 0 or 1 classification, we can translate the output of the hypothesis function as follows:

$$\begin{aligned} \mathbf{h}_{\theta}(\mathbf{x}) \geq 0.5 &\rightarrow y=1 \\ \mathbf{h}_{\theta}(\mathbf{x}) < 0.5 &\rightarrow y=0 \end{aligned}$$

The way our logistic function  $g$  behaves is that when its input is greater than or equal to zero, its output is greater than or equal to 0.5:

$$\begin{aligned} g(z) &\geq 0.5 \\ \text{when } z &\geq 0 \end{aligned}$$

Remember.

$$\begin{aligned} z=0, \quad \mathbf{e}^0=1 &\Rightarrow g(z)=1/2 \\ z \rightarrow \infty, \quad \mathbf{e}^{-\infty} \rightarrow 0 &\Rightarrow g(z)=1 \\ z \rightarrow -\infty, \quad \mathbf{e}^{\infty} \rightarrow \infty &\Rightarrow g(z)=0 \end{aligned}$$

So if our input to  $g$  is  $\theta^T \mathbf{X}$ , then that means:

$$\begin{aligned} \mathbf{h}_{\theta}(\mathbf{x}) = g(\theta^T \mathbf{x}) &\geq 0.5 \\ \text{When } \theta^T \mathbf{x} &\geq 0 \end{aligned}$$

From these statements we can now say:

$$\begin{aligned} \theta^T \mathbf{x} \geq 0 &\Rightarrow y=1 \\ \theta^T \mathbf{x} < 0 &\Rightarrow y=0 \end{aligned}$$

The **decision boundary** is the line that separates the area where  $y = 0$  and where  $y = 1$ . It is created by our hypothesis function.

**Example:**

$$\begin{aligned} \theta &= \begin{bmatrix} 5 \\ -1 \\ 0 \end{bmatrix} \\ y &= 1 \text{ If } 5 + (-1)x_1 + 0x_2 \geq 0 \end{aligned}$$

$$5 - \mathbf{x}_1 \geq 0$$

$$- \mathbf{x}_1 \geq -5$$

$$\mathbf{x}_1 \leq 5$$

In this case, our decision boundary is a straight vertical line placed on the graph where  $\mathbf{x}_1 = 5$ , and everything to the left of that denotes  $y = 1$ , while everything to the right denotes  $y = 0$ .

Again, the input to the sigmoid function  $g(z)$  (e.g.  $\boldsymbol{\theta}^T \mathbf{X}$ ) doesn't need to be linear, and could be a function that describes a circle (e.g.  $z = \boldsymbol{\theta}_0 + \boldsymbol{\theta}_1 \mathbf{x}_1^2 + \boldsymbol{\theta}_2 \mathbf{x}_1^2$ ) or any shape to fit our data.