Decision Boundary

In order to get our discrete 0 or 1 classification, we can translate the output of the hypothesis function as follows:

$$\mathbf{h}_{\boldsymbol{\theta}}(\boldsymbol{x}) \geq 0.5 \rightarrow y=1$$
$$\mathbf{h}_{\boldsymbol{\theta}}(\boldsymbol{x}) < 0.5 \rightarrow y=0$$

The way our logistic function g behaves is that when its input is greater than or equal to zero, its output is greater than or equal to 0.5:

$$g(z) \geq 0.5$$
$$when\ z \geq 0$$

Remember.

$$z=0,\ \ \mathbf{e}^{\mathbf{0}}=1 \Rightarrow g(z)=1/2$$

$$z \rightarrow \infty,\ \ \mathbf{e}^{-\boldsymbol{\infty}} \rightarrow 0 \Rightarrow g(z)=1$$

$$z \rightarrow -\infty,\ \ \mathbf{e}^{\boldsymbol{\infty}} \rightarrow \infty \Rightarrow g(z)=0$$

So if our input to g is $\boldsymbol{\theta}^{\boldsymbol{T}}\boldsymbol{X}$, then that means:

$$\mathbf{h}_{\boldsymbol{\theta}}(\boldsymbol{x}) = \boldsymbol{g}(\boldsymbol{\theta}^{\boldsymbol{T}}\boldsymbol{x}) \geq 0.5$$
$$When\ \ \boldsymbol{\theta}^{\boldsymbol{T}}\boldsymbol{x} \geq 0$$

From these statements we can now say:

$$\boldsymbol{\theta}^{\boldsymbol{T}}\boldsymbol{x} \geq 0 \Rightarrow y=1$$
$$\boldsymbol{\theta}^{\boldsymbol{T}}\boldsymbol{x} < 0 \Rightarrow y=0$$

The **decision boundary** is the line that separates the area where y = 0 and where y = 1. It is created by our hypothesis function.

Example:

$$\boldsymbol{\theta} = \begin{bmatrix} \mathbf{5} \\ \mathbf{-1} \\ \mathbf{0} \end{bmatrix}$$
$$\boldsymbol{y}=\mathbf{1}\ \boldsymbol{If}\ \mathbf{5}+(\mathbf{-1})\ \mathbf{x}_{\mathbf{1}}+\mathbf{0}\ \mathbf{x}_{\mathbf{2}} \geq \mathbf{0}$$

$$5 - x_1 \geq 0$$
$$- x_1 \geq -5$$
$$x_1 \leq 5$$

In this case, our decision boundary is a straight vertical line placed on the graph where $x_1 = 5$, and everything to the left of that denotes y = 1, while everything to the right denotes y = 0.

Again, the input to the sigmoid function g(z) (e.g. $\theta^T X$) doesn't need to be linear, and could be a function that describes a circle (e.g. $z = \theta_0 + \theta_1 x_1^2 + \theta_2 x_1^2$) or any shape to fit our data.