芯片制程統計基礎篇
_照妖镜海底捞针SPC

Cusum & EWMA Charts for SPC

王不老說半导

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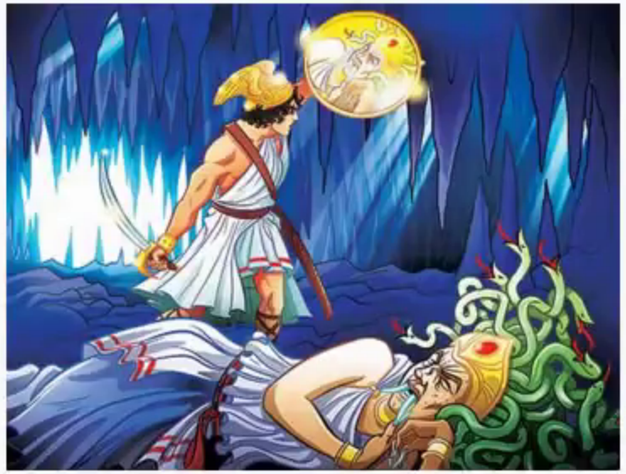
古代照妖镜

希臘神話

- 珀尔修斯(Perseus)腳穿爱马仕的高級金色有翼凉鞋，頭載黑德斯的隐形头盔，以雅典娜的照妖镜盾的反射(避免直视)，一劍砍掉了滿頭蛇髮超級悲衰魔女美杜莎(Medusa)的頭。

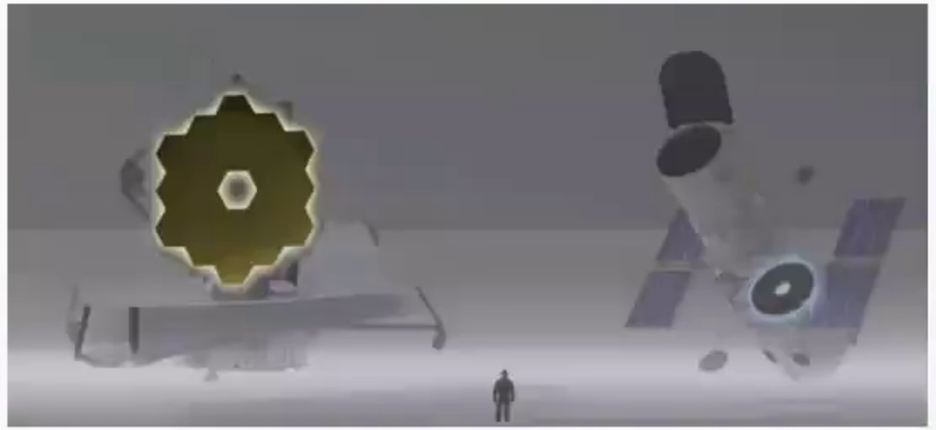


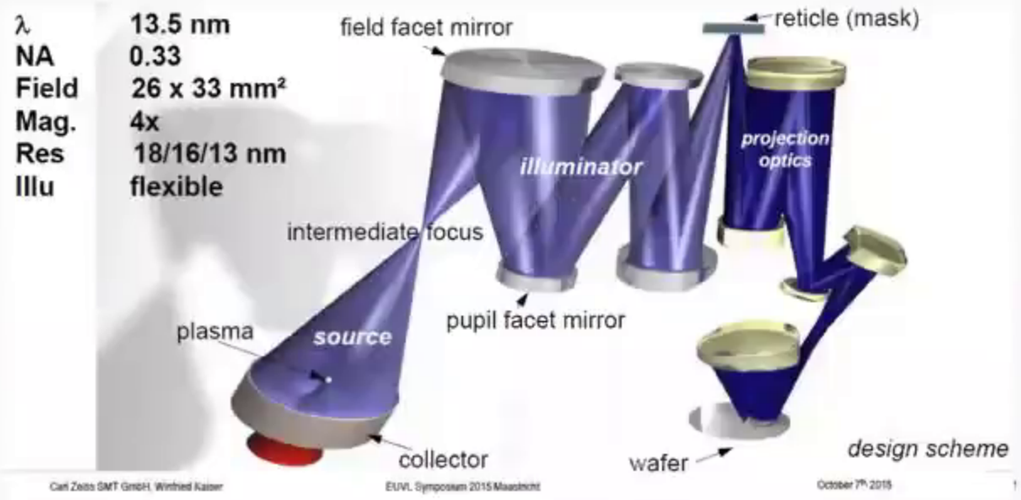
古代中国

- 大禹治水有三宝：定海针(孙悟空的金箍棒)、降魔铁索，照妖镜
- 《抱朴子内篇·登涉》：万物之老者，其精悉能假托人形，以眩惑人目而尝试人，唯不能于(照妖)镜中易(改變)其真形耳

現代照妖镜

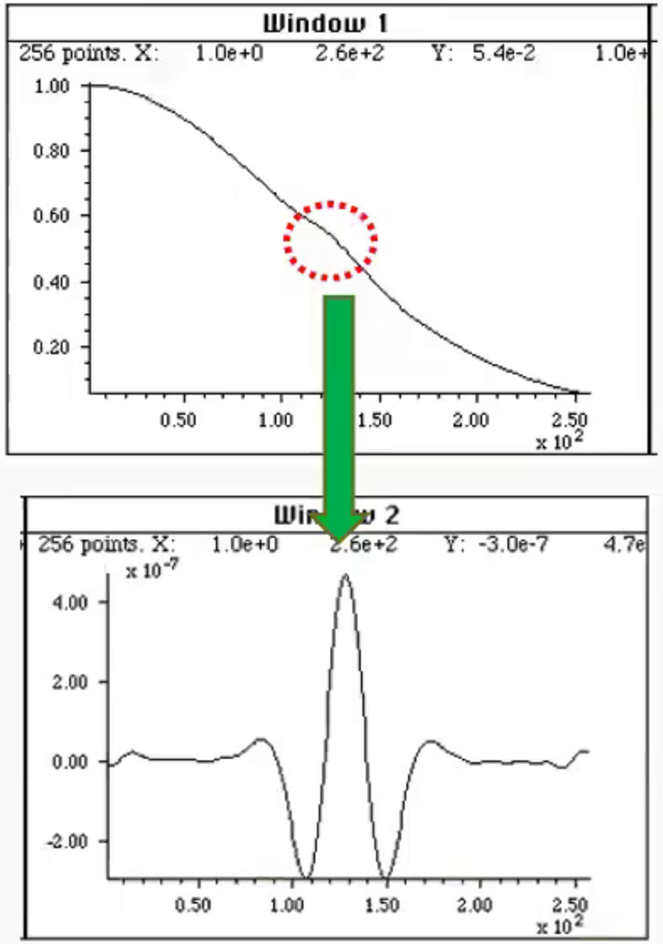
- 哈伯天文鏡可觀察遙遠的宇宙起源，其主鏡值徑約2.4米(見右上圖)
- 精密半导体工藝使用最昂贵的EUV光刻机，裡面使用的鏡子(見右下圖)皆耗資近千萬美金，它們被用來光刻世界上最小的電晶體(數十納米)
- 還有啥特別的照妖镜呢?
  - 特別是一種海底捞针型照妖镜!





物數照妖镜

- **海底捞针**
  - 當真正信號非常微弱，想找出他來，猶如海底捞针(見右圖紅色圓虛線內鼓起的小小信號)，那如何將此信號放大，使得其清晰呢?
- **解決方案**
  - 將信號微分，微分再微分即可(右下就是四次微分的結果)!
  - 此技巧早已被廣泛使用中，如各式 **Derivative Spectroscopy**

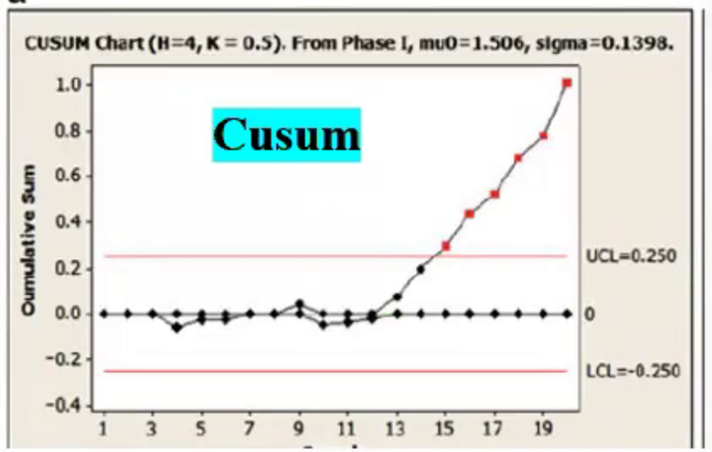


https://terpconnect.umd.edu/~toh/spectrum/Differentiation.html

SPC照妖镜

- 海底捞针
  - 當SPC變化信號不大(< ±1σ)，想找出他來猶如海底捞针，那如何將信號放大，使得其清晰呢?
- 解決方案
  - Cusum (見右上圖)
  - EWMA (見右下圖)

a



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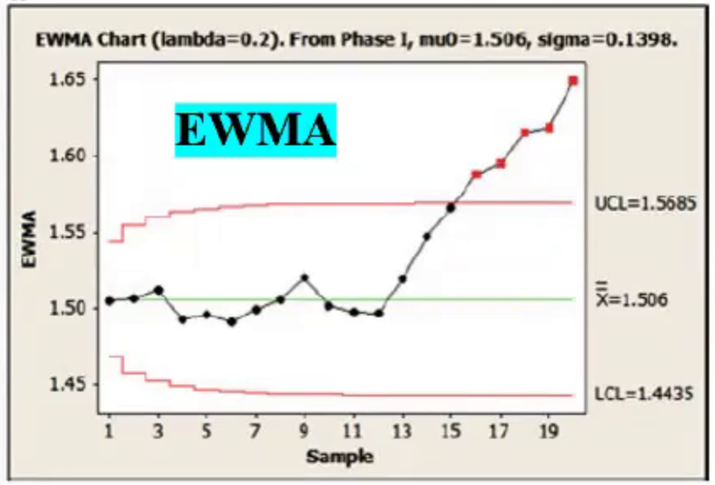

Cusum: 累积和照妖镜

- 累积和(Cumulative Sum, CUSUM):
  - 它充分利用了数据变化之顺序与大小，适合用于侦测制程的微量平均值变化(small shifts in the mean)

$$C_i = \sum_{j=1}^{i} \left(\bar{x}_j - \mu_0 \right)$$

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  - 但右上圖中，小小的平均值變化(μ = 10 → μ = 11)，就根本看不出來了 (大近視眼!)
- 累积和照妖镜出手→ 問題立馬現形(右下圖)!
  - Cusum可以侦测制程平均值(μ)的微量(±1σ)变化，所以可說是SPC界的照妖镜!

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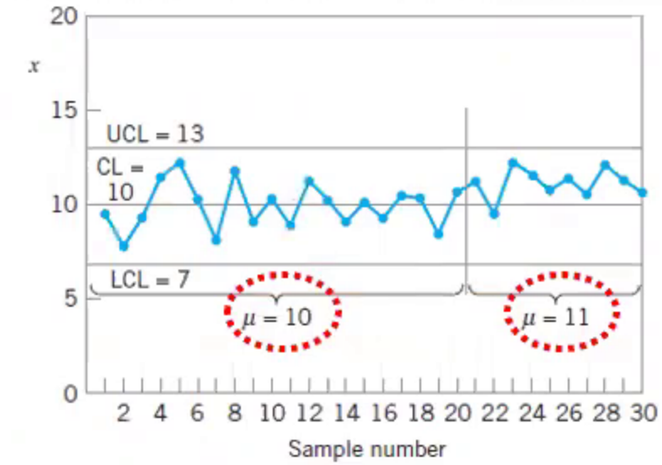

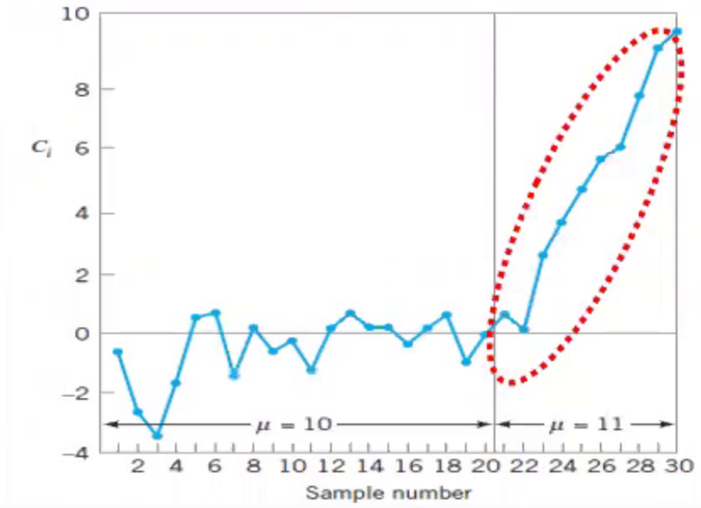

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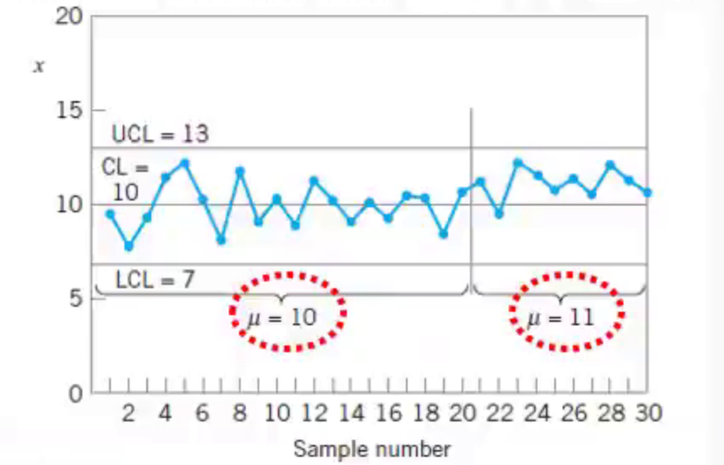

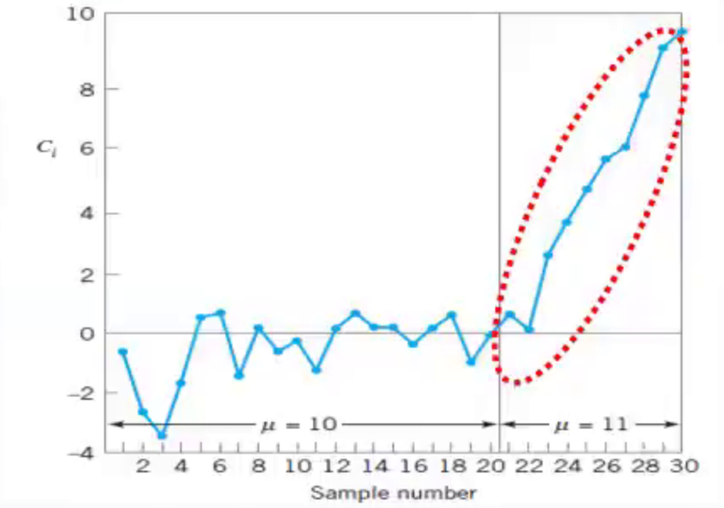

Cusum: 累积和照妖镜

已知: 製程數據如右原始平均值 μ = 10

試問:製程在第幾個樣本開始出現1σ平均值偏移的問題?

解答:

- 筆算如右上圖
  - **23**出現問題

$$C_i = \sum_{j=1}^{i} (x_j - 10)$$
$$= (x_i - 10) + \sum_{j=1}^{i-1} (x_j - 10)$$
$$= (x_i - 10) + C_{i-1}$$

- 太麻煩了，我們偷懶用**Minitab**代勞
  - *Stat/Control charts/time average charts/Cusum (enter data column, subgroup size=1，target mean =10)*
  - 右下圖輕鬆看出平均值在第**23**樣本，開始出現1σ偏移的問題

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Sample, i	(a) x_i	(b) $x_i - 10$	(c) $C_i = (x_i - 10) + C_{i-1}$
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3	9.29	−0.71	−3.27
4	11.66	1.66	−1.61
5	12.16	2.16	0.55
6	10.18	0.18	0.73
7	8.04	−1.96	−1.23
8	11.46	1.46	0.23
9	9.20	−0.80	−0.57
10	10.34	0.34	−0.23
11	9.03	−0.97	−1.20
12	11.47	1.47	0.27
13	10.51	0.51	0.78
14	9.40	−0.60	0.18
15	10.08	0.08	0.26
16	9.37	−0.63	−0.37
17	10.62	0.62	0.25
18	10.31	0.31	0.56
19	8.52	−1.48	−0.92
20	10.84	0.84	−0.08
21	10.90	0.90	0.82
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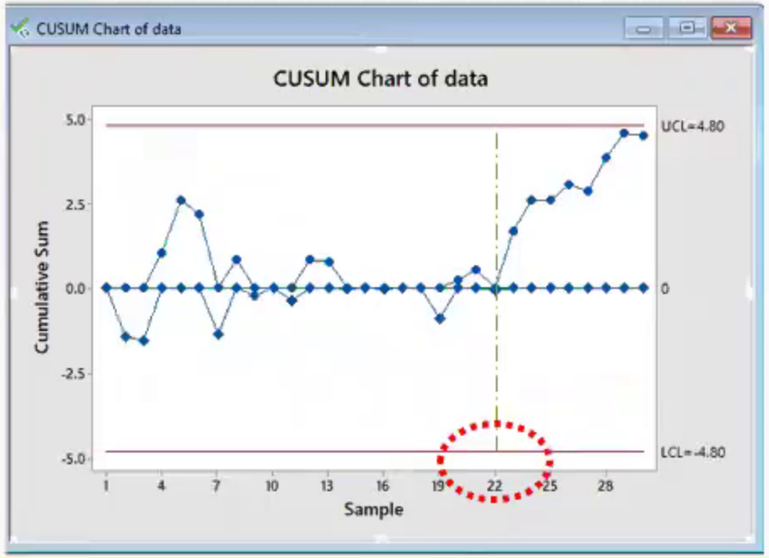

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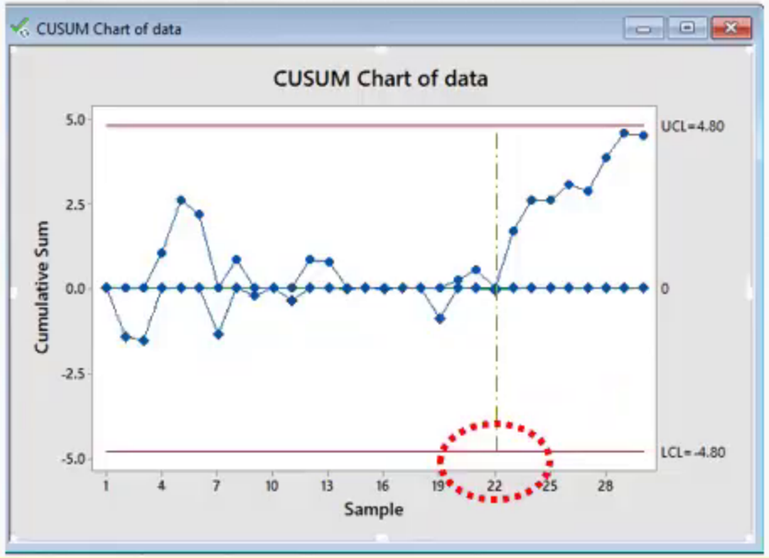

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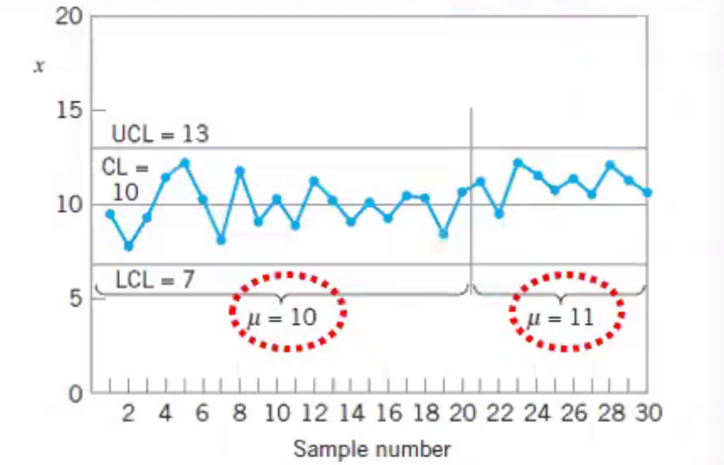

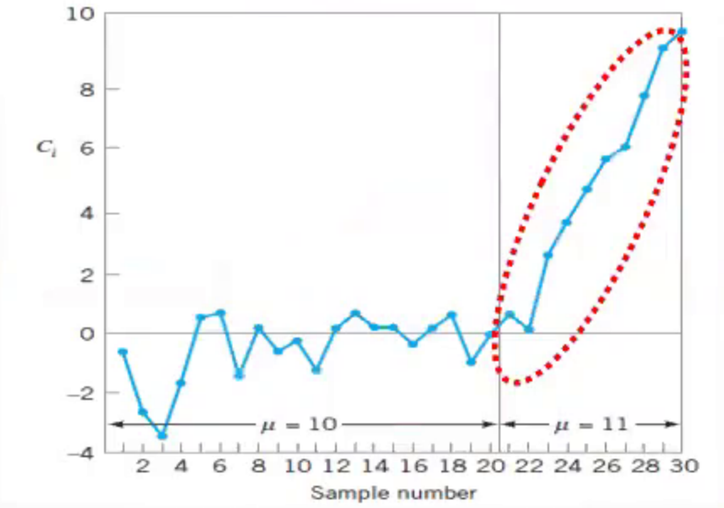

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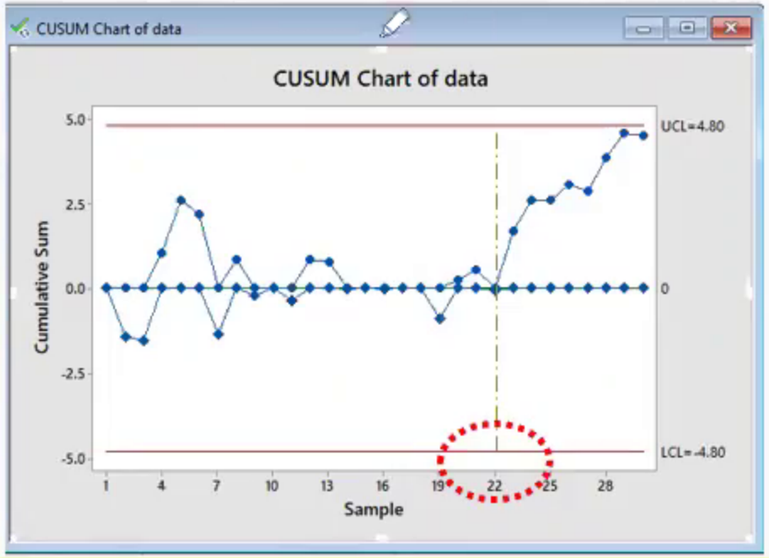

Minitab - 22. Cusum and EWMA.MPJ

File Edit Data Calc Stat Graph Editor Tools Window Help Assistant

Session

Worksheet 1 ***

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16
	data															
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Current Worksheet: Worksheet 1

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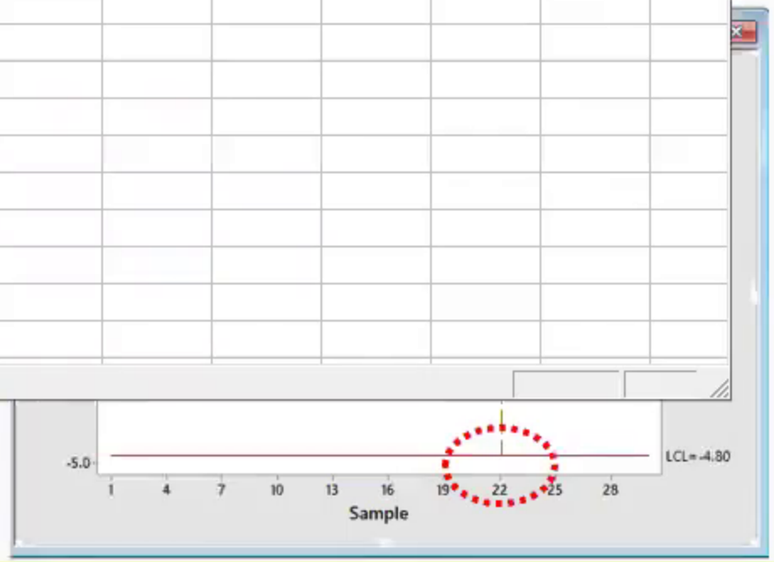

開始出現1σ偏移的問題

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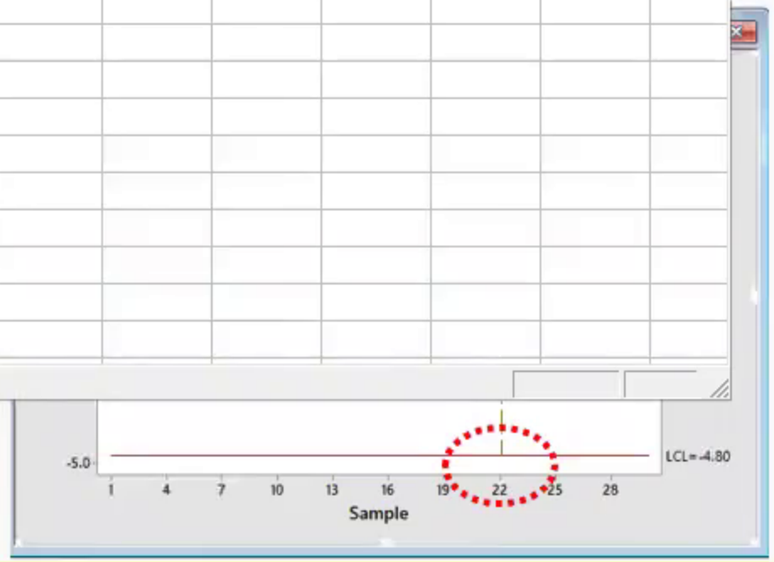

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Stat menu: Basic Statistics, Regression, ANOVA, DOE, Control Charts, Quality Tools, Reliability/Survival, Multivariate, Time Series, Tables, Nonparametrics, Equivalence Tests, Power and Sample Size

Control Charts menu: Box-Cox Transformation..., Variables Charts for Subgroups, Variables Charts for Individuals, Attributes Charts, Time-Weighted Charts, Multivariate Charts, Rare Event Charts

Time-Weighted Charts menu: Moving Average..., EWMA..., CUSUM...

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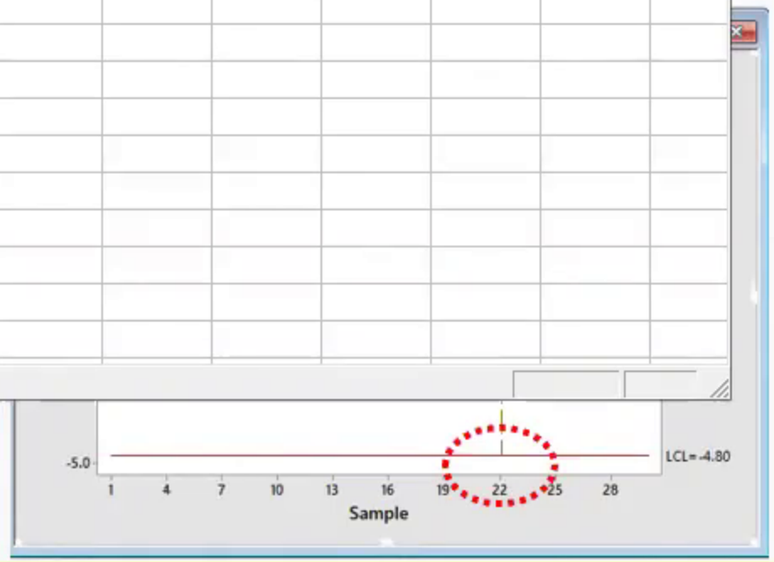

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4	11.66															
5	12.16															
6	10.18															
7	8.04															
8	11.46															
9	9.20															
10	10.34															
11	9.03															
12	11.47															
13	10.51															
14	9.40															
15	10.08															

Current Worksheet: Worksheet 1

CUSUM Chart

All observations for a chart are in one column:

data

Subgroup sizes: 1 (enter a number or ID column)

Target: 10

Scale... Labels...

Multiple Graphs... Data Options... CUSUM Options...

Select

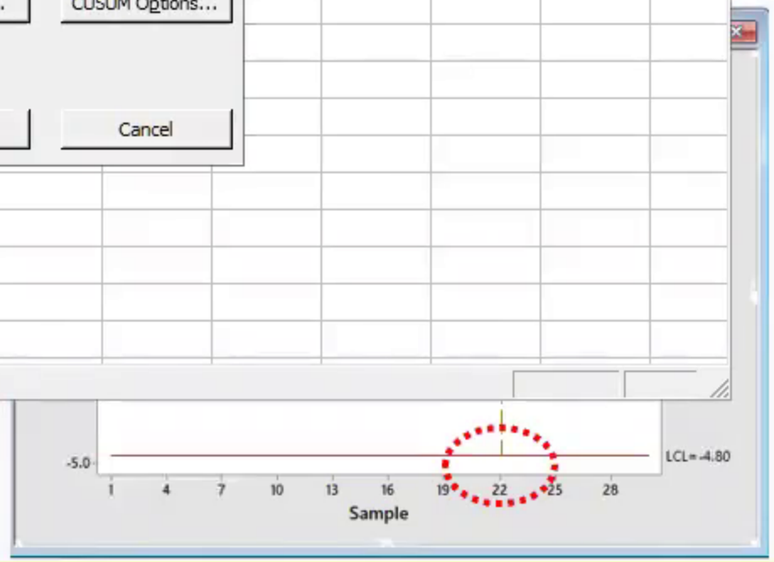
Help OK Cancel

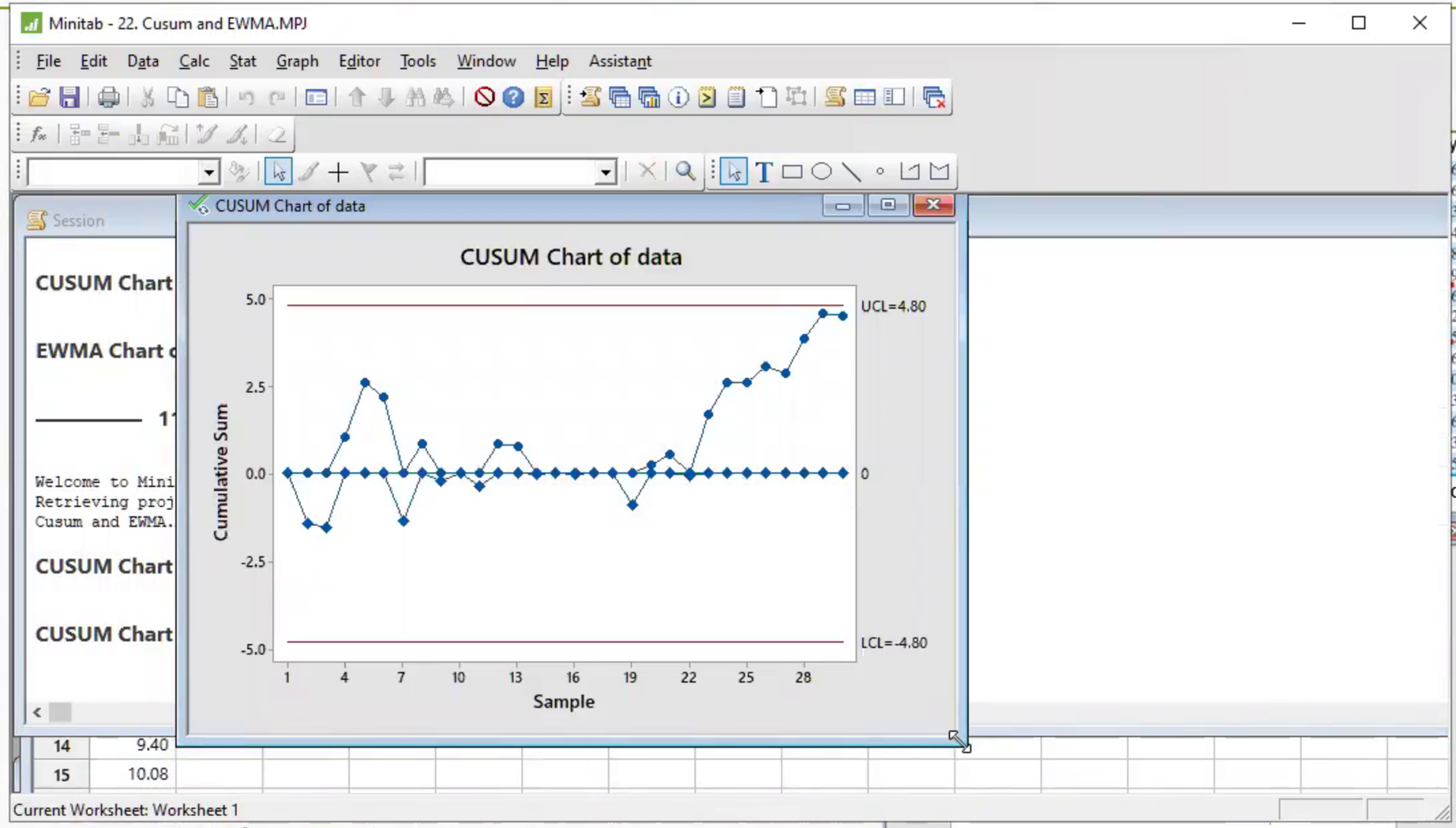
開始出現1σ偏移的問題

y Control, D.C. Montgomery

63	−0.37
62	0.25
31	0.56
48	−0.92
84	−0.08
90	0.82
67	0.15
29	2.44
50	3.94
60	4.54
08	5.62
38	6.00
62	7.62
31	8.93
52	9.45

omery





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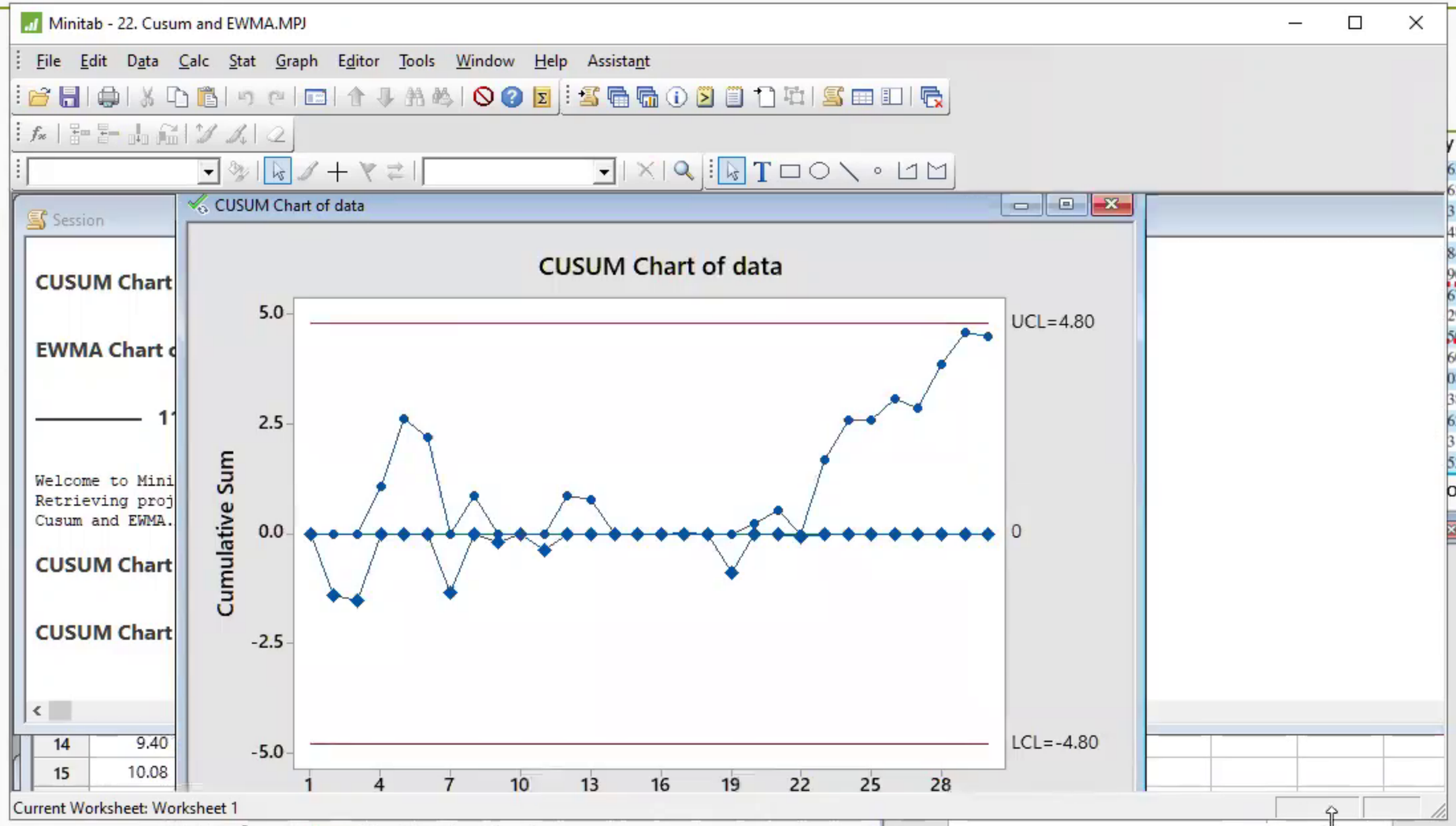

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omery

14	9.40
15	10.08

開始出現1σ偏移的問題

Minitab - 22. Cusum and EWMA.MPJ

File Edit Data Calc Stat Graph Editor Tools Window Help Assistant

Session

CUSUM Chart

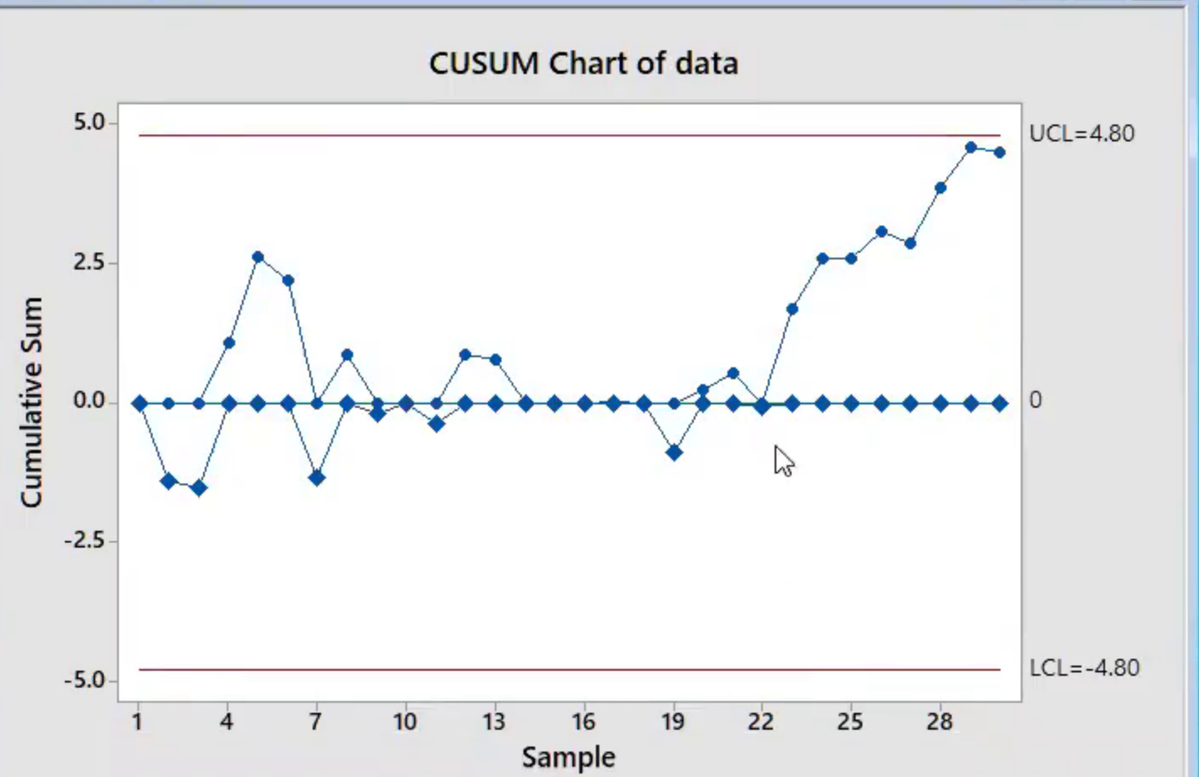
EWMA Chart

Welcome to Mini
Retrieving proj
Cusum and EWMA.

CUSUM Chart

CUSUM Chart

CUSUM Chart of data



14	9.40
15	10.08
16	9.37
17	10.62
18	10.31
19	8.52
20	10.84

Current Worksheet: Worksheet 1

y Control, D.C. Montgomery

.63	−0.37
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gomery

Cusum: 累积和照妖镜

已知: 製程數據如右原始平均值 μ = 10

試問:製程在第幾個樣本開始出現1σ平均值偏移的問題?

解答:

- 筆算如右上圖
  - **23**出現問題

$$C_i = \sum_{j=1}^{i} (x_j - 10)$$
$$= (x_i - 10) + \sum_{j=1}^{i-1} (x_j - 10)$$
$$= (x_i - 10) + C_{i-1}$$

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  - *Stat/Control charts/time average charts/Cusum (enter data column, subgroup size=1，target mean =10)*
  - 右下圖輕鬆看出平均值在第**23**樣本，開始出現1σ偏移的問題

Introduction to Statistical Quality Control，D.C. Montgomery

Data for the CUSUM Example

Sample, i	(a) x_i	(b) $x_i - 10$	(c) $C_i = (x_i - 10) + C_{i-1}$
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5	12.16	2.16	0.55
6	10.18	0.18	0.73
7	8.04	−1.96	−1.23
8	11.46	1.46	0.23
9	9.20	−0.80	−0.57
10	10.34	0.34	−0.23
11	9.03	−0.97	−1.20
12	11.47	1.47	0.27
13	10.51	0.51	0.78
14	9.40	−0.60	0.18
15	10.08	0.08	0.26
16	9.37	−0.63	−0.37
17	10.62	0.62	0.25
18	10.31	0.31	0.56
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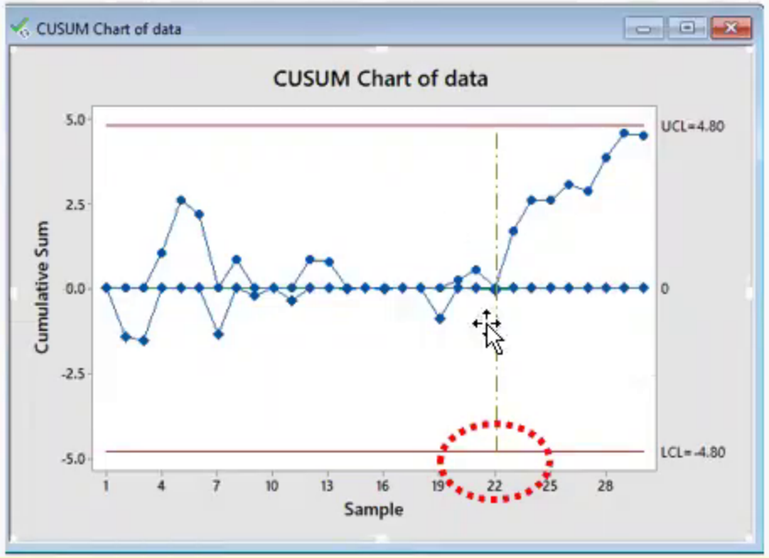
Introduction to Statistical Quality Control，D.C. Montgomery



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6	10.18	0.18	0.73
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8	11.46	1.46	0.23
9	9.20	−0.80	−0.57
10	10.34	0.34	−0.23
11	9.03	−0.97	−1.20
12	11.47	1.47	0.27
13	10.51	0.51	0.78
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15	10.08	0.08	0.26
16	9.37	−0.63	−0.37
17	10.62	0.62	0.25
18	10.31	0.31	0.56
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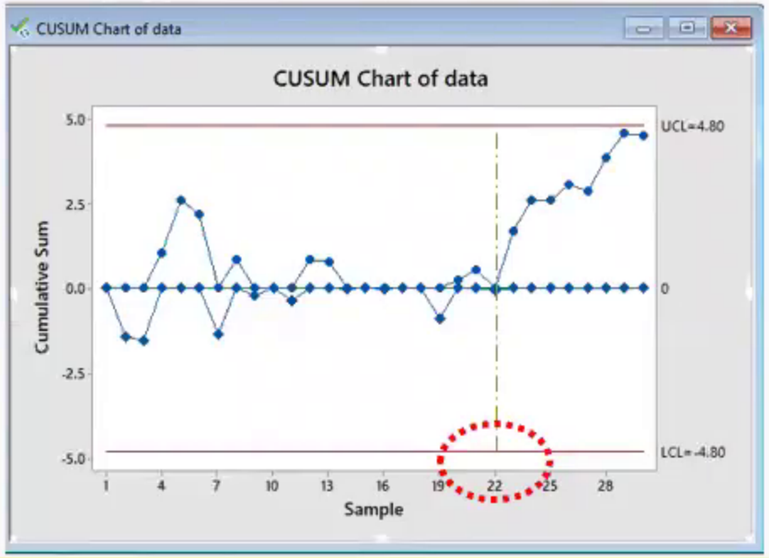
Introduction to Statistical Quality Control，D.C. Montgomery



Cusum: 累积和照妖镜

- 每年约有十四萬平方公里的热带雨林被摧毁
- 右圖即是以累积和(Cusum)處理優化衛星訊號，可以及時(甚至六個月以前)找出最嚴重區域

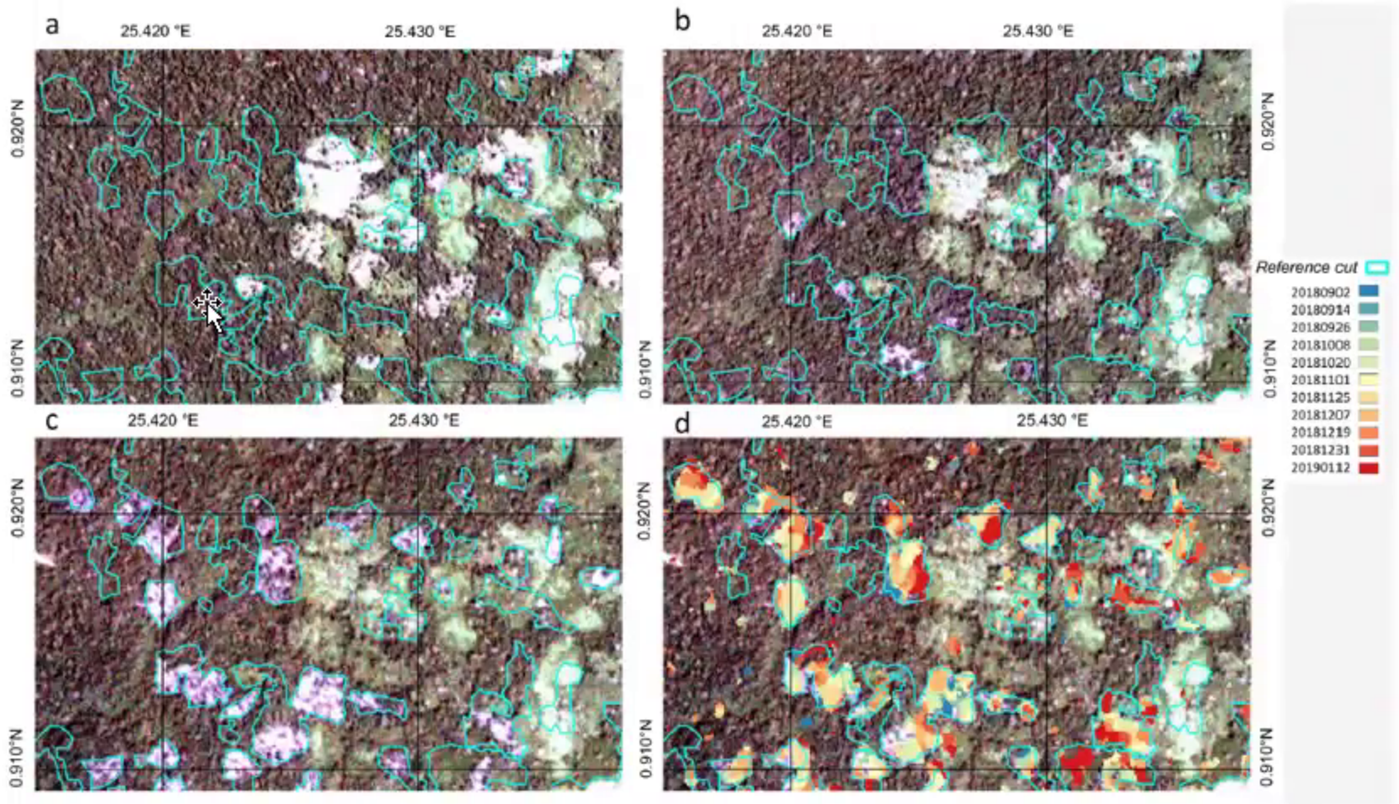


Fig. 9. (a) 10/09/2018 PlanetScope image, (b) 08/11/2018 PlanetScope image, (c) 03/01/2019 PlanetScope image, (d) CuSum results based on VV with $T_c = 0.75$. Blue polygons correspond to PlanetScope cut map. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

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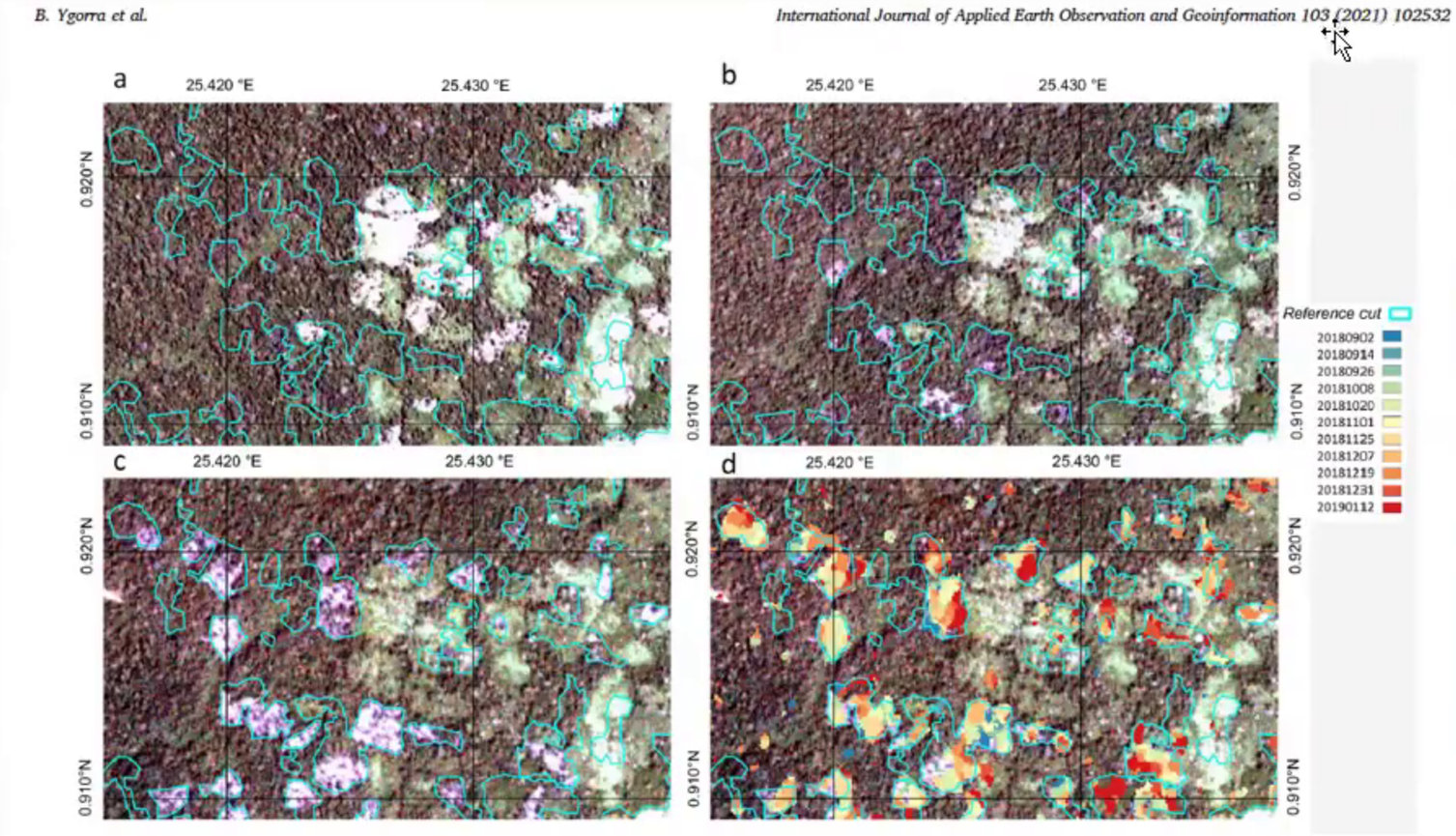

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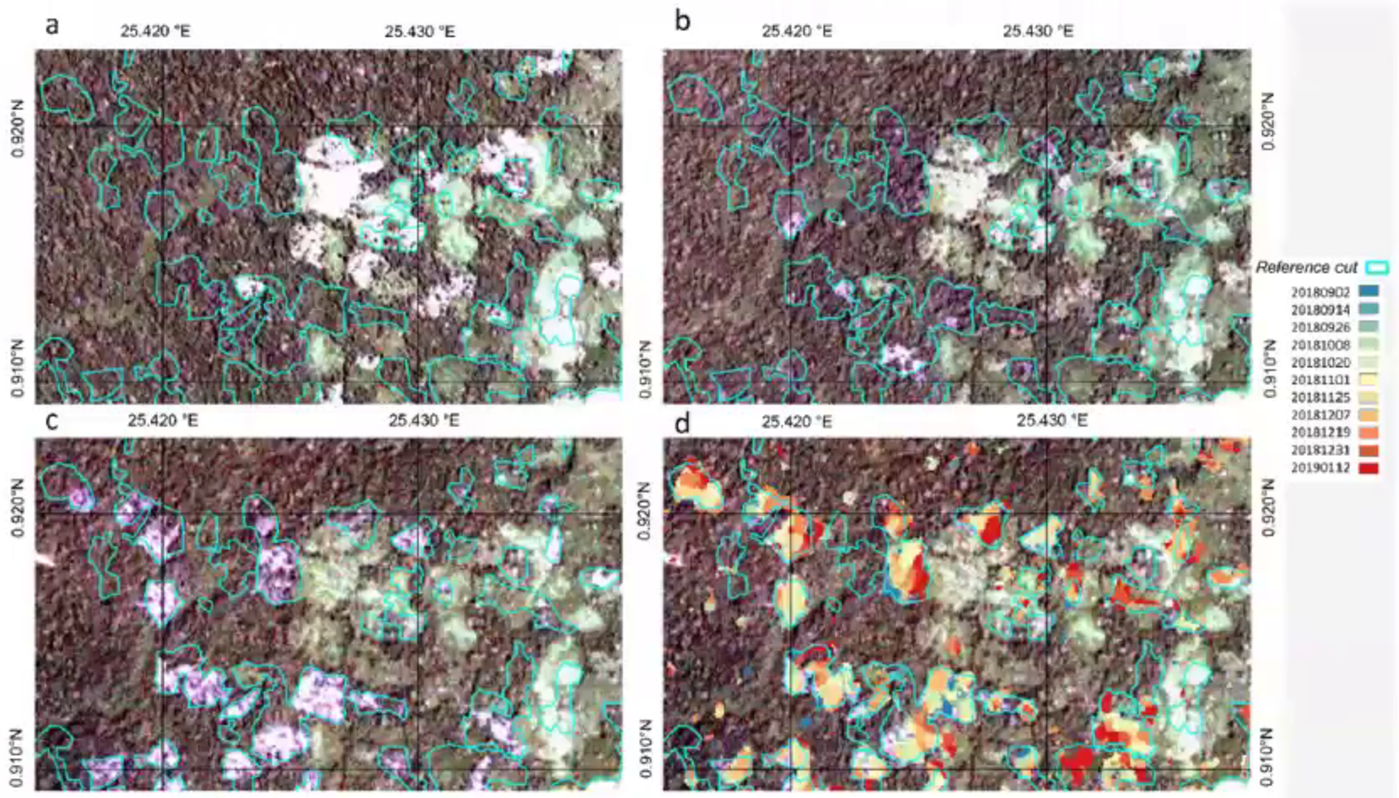

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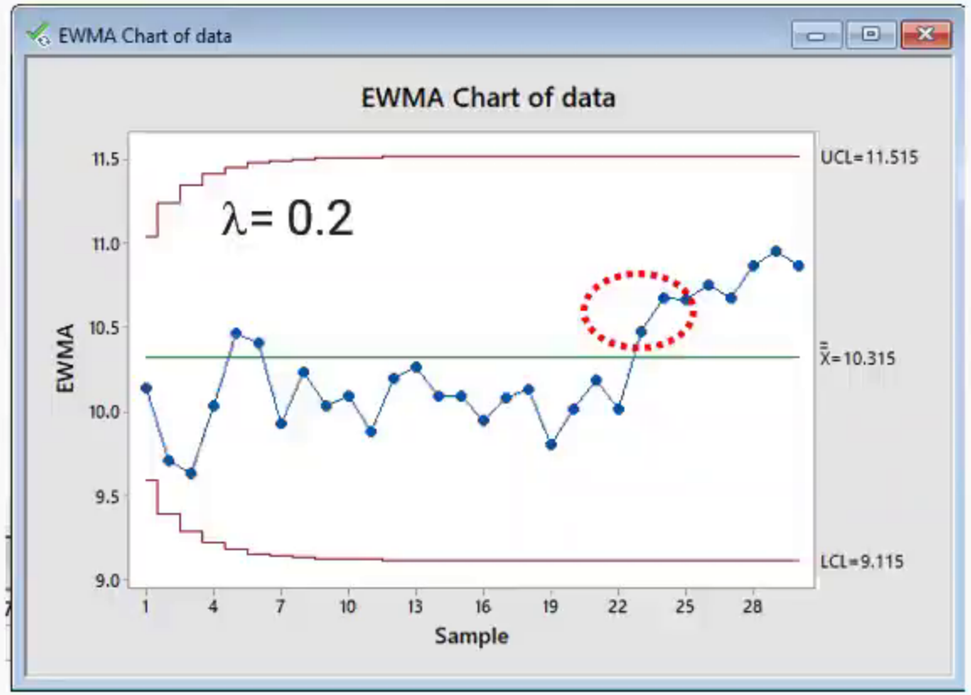
EWMA:指数加权移动照妖镜

EWMA (Exponentially Weighted Moving Average): 指数加权移动平均值的控制图

- 每个EWMA 点，都结合了来自之前所有子组或观测值的信息，可以检测过程中任意大小的偏移。
- 其實與Cusum差不多，只是公式不太一樣(λ =加权 = weight of EWMA)

$$z_i = \lambda \sum_{j=0}^{i-1} (1-\lambda)^j x_{i-j} + (1-\lambda)^i z_0$$

- 筆算太麻煩，我們偷懶用Minitab代勞
  - Stat/Control charts/time average charts/EWMA (enter data column, subgroup size=1，weight of EWMA=0.2)
- 平均值也在第23樣本出現1σ偏移的問題



Minitab - 22. Cusum and EWMA.MPJ

File Edit Data Calc Stat Graph Editor Tools Window Help Assistant

Session

Worksheet 1 ***

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16
	data															
1	9.45															
2	7.99															
3	9.29															
4	11.66															
5	12.16															
6	10.18															
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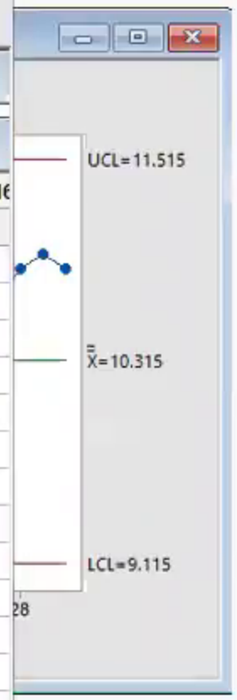
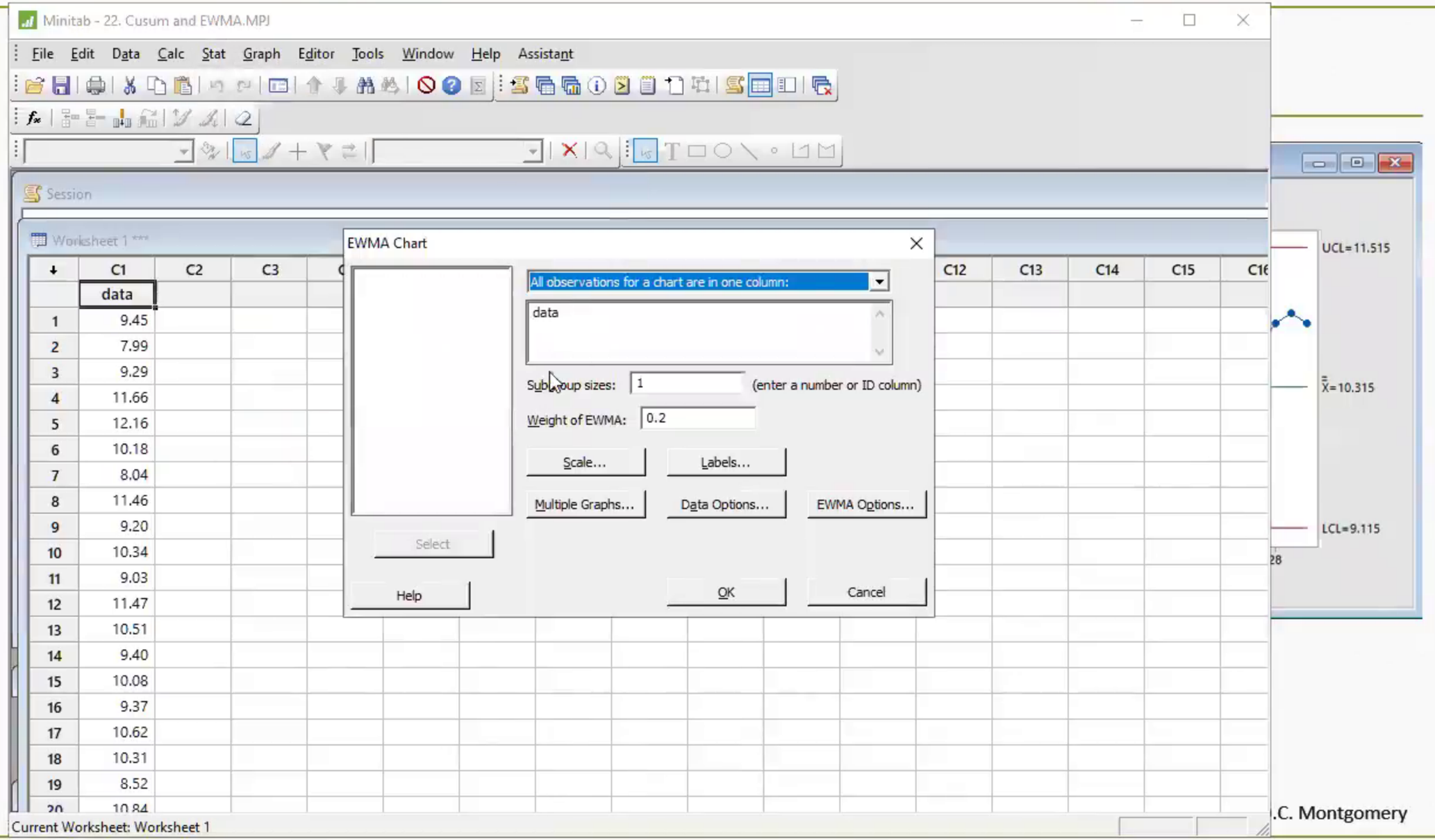
18	10.31															
19	8.52															
20	10.84															

Current Worksheet: Worksheet 1




Minitab - 22. Cusum and EWMA.MPJ
File Edit Data Calc Stat Graph Editor Tools Window Help Assistant
Session
Worksheet 1 ***
C1 data
1 9.45
2 7.99
3 9.29
4 11.66
5 12.16
6 10.18
7 8.04
8 11.46
9 9.20
10 10.34
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15 10.08
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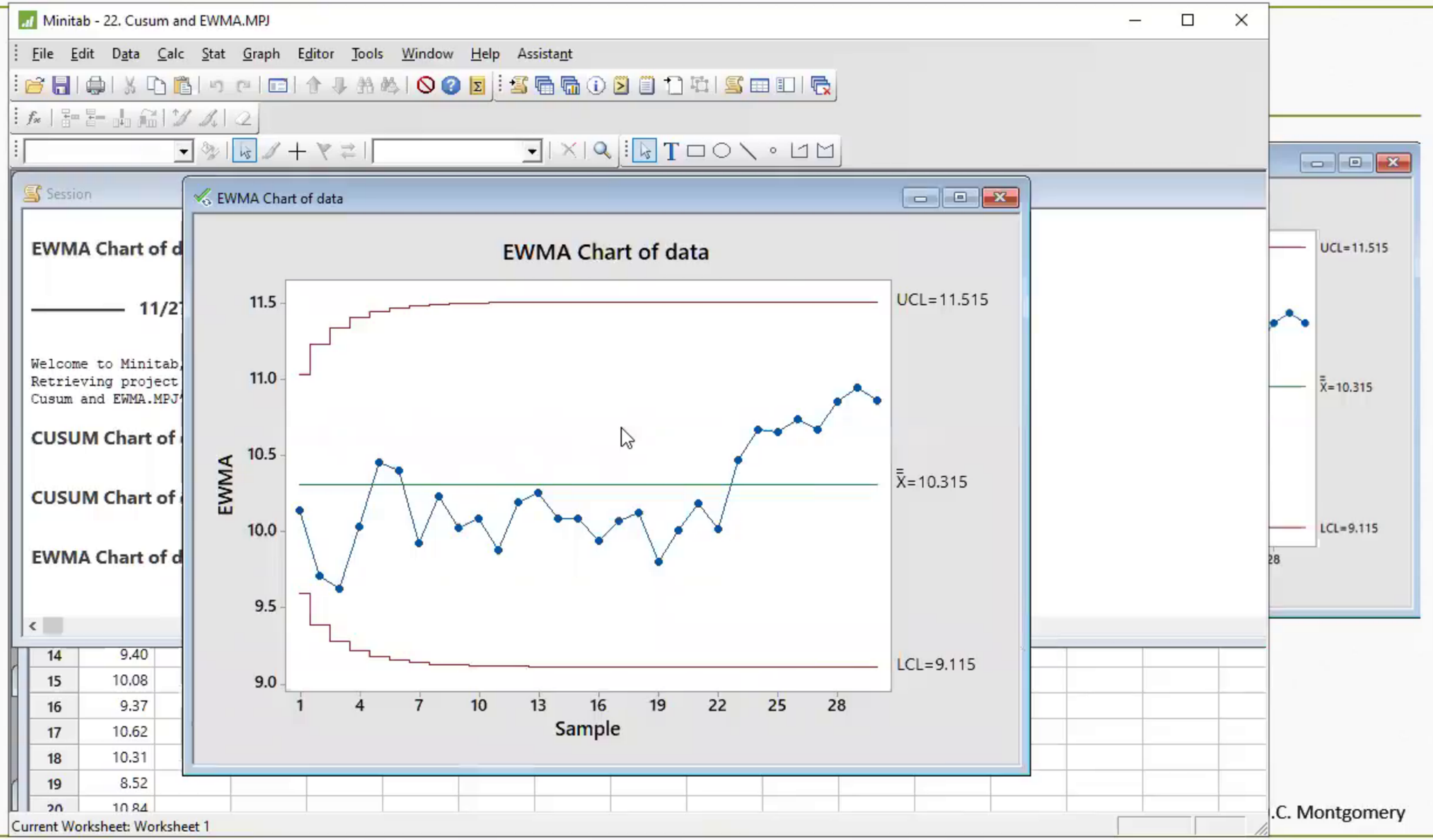
18 10.31
19 8.52
EWMA Chart
All observations for a chart are in one column:
data
Subgroup sizes: 1 (enter a number or ID column)
Weight of EWMA: 0.2
Scale... Labels...
Multiple Graphs... Data Options... EWMA Options...
Select
Help OK Cancel
UCL=11.515
X=10.315
LCL=9.115
Current Worksheet: Worksheet 1


Minitab - 22. Cusum and EWMA.MPJ
File Edit Data Calc Stat Graph Editor Tools Window Help Assistant
Session
EWMA Chart of d
11/27
Welcome to Minitab,
Retrieving project
Cusum and EWMA.MPJ
CUSUM Chart of
CUSUM Chart of
EWMA Chart of d
EWMA Chart of data
EWMA Chart of data
EWMA
Sample
UCL=11.515
X̿=10.315
LCL=9.115
11.5
11.0
10.5
10.0
9.5
9.0
1
4
7
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19
22
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28
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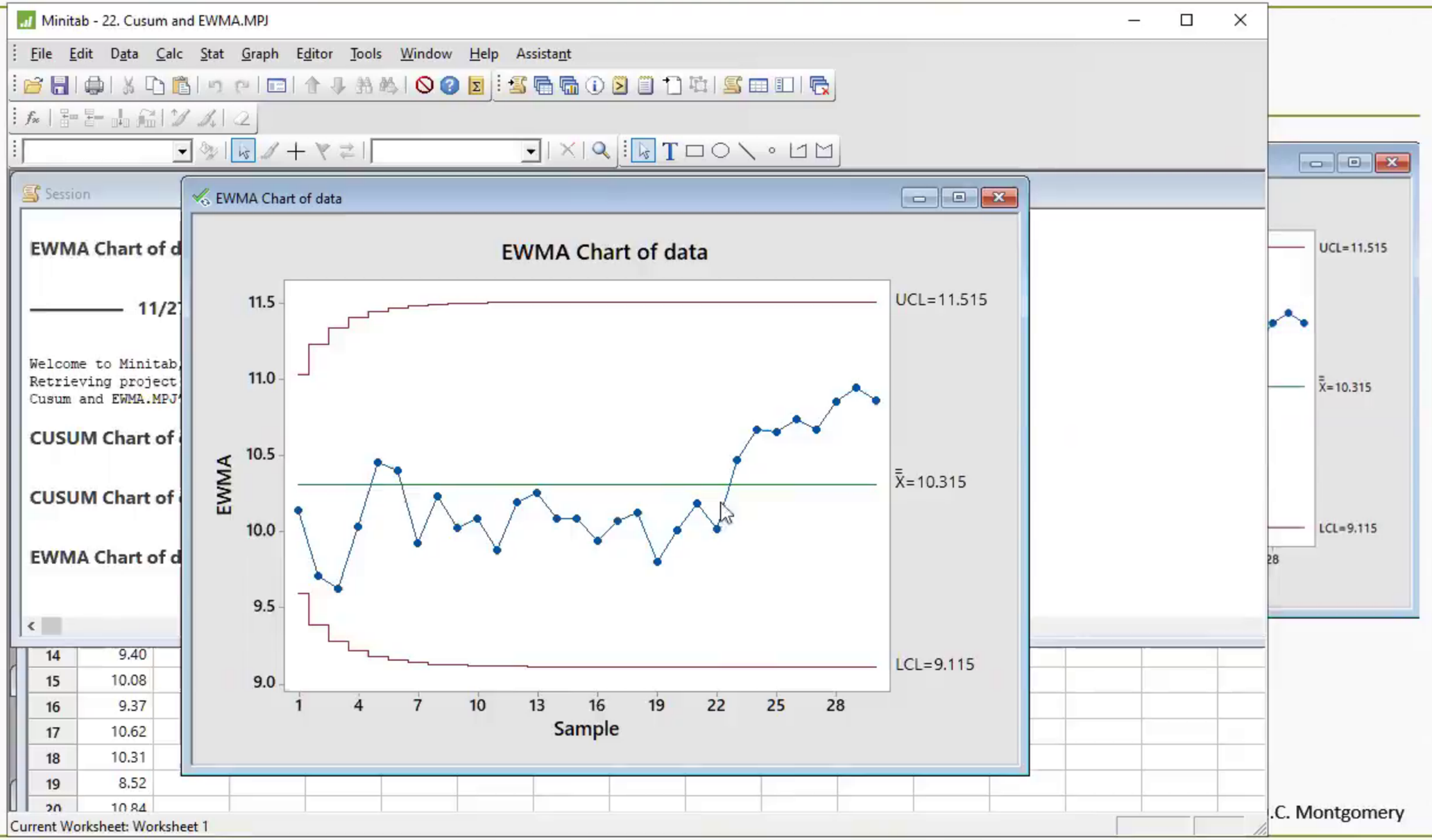
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CUSUM Chart of
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EWMA Chart of d
EWMA Chart of data
EWMA Chart of data
UCL=11.515
X̿=10.315
LCL=9.115
EWMA
Sample
11.5
11.0
10.5
10.0
9.5
9.0
1
4
7
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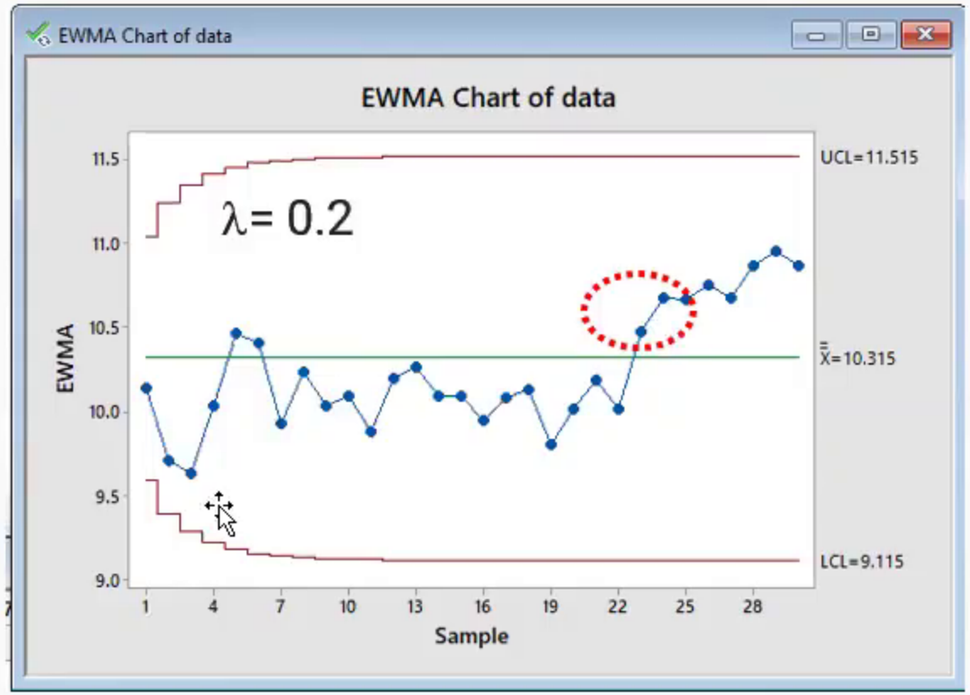

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21	10.90	0.90	0.82
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