The Optimal Strategy for a Multi-Cryptocurrency Investment Portfolio

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100% only one person for that assignment

1 Introduction

1.1 Motivation

With the new president launching the Trump meme cryptocurrency, more and more people, including myself, have come to recognize the cryptocurrency market. However, the cryptocurrency market is characterized by extreme volatility, rapid technological evolution, and a lack of standardized regulatory frameworks. These factors create a challenging environment for investors and traders, where traditional methods for strategy formulation and portfolio optimization often fall short. Therefore, I want to utilize my knowledge of convex optimization to identify an effective investment strategy and investment portfolio. This will allow me to apply what I've learned in practice, and if the results are favorable, also yield some profit.

1.2 Previous Works

In traditional financial markets, the core theory of portfolio optimization is based on Modern Portfolio Theory (MPT). And, the main way to do that is called Mean-Variance Optimization, MVO. Its idea is to maximize the expected return of the portfolio for a given level of risk, or minimize the risk for a given return target.
Also, with advancements in technology, a new approach called Quantitative Trading has emerged. It collects market trading data, as well as unstructured data such as media coverage and public sentiment, to generate influencing factors. Then, it utilizes machine learning methods, such as LSTM, to predict market trends and execute trades for profit.
Both approaches are effective, but neither is individually well-suited to the cryptocurrency market due to its high volatility, non-normal returns, and low market efficiency. Major cryptocurrencies, such as BTC and ETH, exhibit significantly higher volatility compared to traditional assets, with an annualized volatility typically exceeding 80%-100%, whereas the stock market generally has an annualized volatility of 10%-20%.
Sharp price surges and crashes occur frequently; for example, in 2021, Bitcoin dropped from \$60,000 to \$30,000 within just three months.
Therefore, in this paper, I decide to combine these two approaches and convex optimization to better adapt to this unique market.

1.3 Contribution

Reformulate the task of identifying optimal cryptocurrency trading strategies and constructing robust investment portfolios into a convex optimization problem. This includes integrating practical constraints such as risk tolerance and budget constraints to ensure the model reflects real-world trading conditions.
Conduct a systematic evaluation of several existing algorithms, optimize and refine them using real-world data, and ultimately identify a quantitative investment strategy that demonstrates superior performance in both risk-adjusted returns and computational efficiency.
Combine the tradition way of Mean-Variance Optimization with machine learning LSTM to find a better Investment Portfolio. **contributions vs. previous works**

1.4 Organization of the Paper

In part 2, I will show the Primal formulation, Dual formulation and KKT conditions.
In part 3, I will show my approach, and where I do the improvement and why I do this. Also, the computational complexity. In part 4, I will show my results with graph and chart, conclusion and potential future work.

2 Statement of the Problem

2.1 Primal formulation

$$\max_{W} \left(\mu \cdot \mathbf{E} + (1-\mu) \cdot \hat{\mathbf{E}}\right)^T \mathbf{W} - \lambda \cdot \mathbf{W}^T \mathbf{\Sigma} \mathbf{W}$$

$$\textbf{s.t.} \sum W_i = 0, \quad -1 \leq W_i \leq 1, \quad \forall i$$

$\mathbf{E}$: Expected Returns, the historical average returns of assets in the portfolio, providing a baseline estimate of asset performance based on past data.
$\hat{\mathbf{E}}$: Predicted Returns, the future projected returns obtained using LSTM. These predictions help capture potential market trends and improve forward-looking decision-making.
μ: Weighting Parameter for Expected and Predicted Returns, controls the balance between historical expected returns and predicted returns.
$\mathbf{W}$: Portfolio Weights, the decision variable representing the proportion of capital allocated to each asset. The optimization process determines the optimal $\mathbf{W}$ to maximize returns while controlling risk. $W_i \in [-1, 1]$, if W_i is positive, which means long, and if W_i is negative, which means short. The weights sum to zero, respecting budget constraints.
λ: Risk Aversion Coefficient, regulates the trade-off between maximizing returns and minimizing risk. A higher λ value means a more risk-averse investor, emphasizing lower volatility over higher returns.
$\mathbf{\Sigma}$: Covariance Matrix of Asset Returns, measures the relationships between asset returns, capturing correlations and overall portfolio risk.

2.2 Dual formulation

We have:

$$\begin{aligned} L(W, \nu, \alpha, \beta) = &-(\mu\mathbf{E} + (1-\mu)\hat{\mathbf{E}})^T\mathbf{W} \\ &+ \lambda\mathbf{W}^T\mathbf{\Sigma}\mathbf{W} \\ &+ \nu\sum W_i \\ &+ \sum_i \alpha_i(W_i + 1) \\ &+ \sum_i \beta_i(1 - W_i) \end{aligned} \tag{1}$$

$$\frac{\partial L}{\partial W} = -(\mu\mathbf{E}+(1-\mu)\hat{\mathbf{E}})+2\lambda\mathbf{\Sigma}W+\nu\mathbf{1}+\alpha-\beta = 0$$

$$W^* = \frac{1}{2\lambda}\mathbf{\Sigma}^{-1}\Big((\mu\mathbf{E} + (1-\mu)\hat{\mathbf{E}}) + \nu\mathbf{1} + \beta - \alpha\Big)$$

Bring this back to Lagrangian, and since it is so complicated, I just use W^*, instead of expand the whole formula, and it is:

$$\begin{aligned} \min_{\nu,\alpha,\beta} \quad & \frac{1}{4\lambda}\Big((\mu\mathbf{E} + (1-\mu)\hat{\mathbf{E}}) + \nu\mathbf{1} + \beta - \alpha\Big)^T \mathbf{\Sigma}^{-1}\Big((\mu\mathbf{E} + (1-\mu)\hat{\mathbf{E}}) + \nu\mathbf{1} + \beta - \alpha\Big) \\ & + \sum_i \alpha_i(W_i^* + 1) + \sum_i \beta_i(1 - W_i^*) \\ & \text{s.t.} \quad \alpha_i \geq 0, \quad \beta_i \geq 0, \quad \forall i \end{aligned}$$

2.3 KKT conditions

Stationary Point:

$$\frac{\partial L}{\partial W} = -(\mu\mathbf{E}+(1-\mu)\hat{\mathbf{E}})+2\lambda\mathbf{\Sigma}W+\nu\mathbf{1}+\alpha-\beta = 0$$

$$W^* = \frac{1}{2\lambda}\mathbf{\Sigma}^{-1}\Big((\mu\mathbf{E} + (1-\mu)\hat{\mathbf{E}}) + \nu\mathbf{1} + \beta - \alpha\Big)$$

Primal feasibility:

$$\sum W_i = 0, \quad -1 \leq W_i \leq 1, \quad \forall i$$

Dual feasibility:

$$\alpha_i \geq 0, \quad \beta_i \geq 0, \quad \forall i$$

Complementary slackness:

$$\alpha_i(W_i + 1) = 0, \quad \beta_i(1 - W_i) = 0, \quad \forall i$$

3 Approaches

I retrieved real-world data from OKX, a cryptocurrency exchange platform, via its API. I selected eight cryptocurrency pairs in total: BTC-USDT, ETH-USDT, SOL-USDT, BNB-USDT, XRP-USDT, ADA-USDT, DOGE-USDT, and DOT-USDT. The dataset for each cryptocurrency pair includes the following fields: 'timestamp', 'open', 'high', 'low', 'close', 'volume', 'volCcy', 'vol', and 'trades'. I collected 101 data entries for each cryptocurrency, using the first 100 for training and the final entry for testing.
I chose the percentage change method to calculate the daily return for each cryptocurrency, which is:

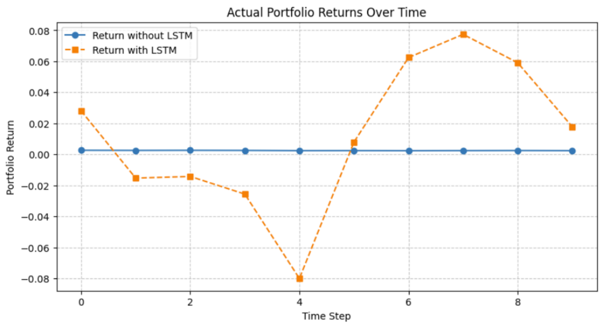
$$\text{return} = \frac{\text{current value - previous value}}{\text{previous value}}$$

And, the value I choose is the close price in each day.

First, I calculated the expected returns and variances for each cryptocurrency using the 100 daily return entries. Then, I utilized these results to predict the next day's return rate with an LSTM model.
For these parameters, I set $\mu = 0.8$, and because of the high risk in cryptocurrency investment, I set $\lambda = 1$, which is usually 0.5.
With these values, I can finally perform the last step—substituting them into the prime formula, applying constraints, and solving the convex optimization problem using CVXPY. The time complexity is $O(n^3)$, mainly for SCS.

4 Result

I conducted a total of 10 tests; the results are as follows:



It can be seen that after adding LSTM, even though we increased λ (from 0.5 to 1), the rate of change of return is still large, and without LSTM, the returns are very close to 0.
Below is the W of each way in the last iteration:

Trading Pair	With LSMT	Without LSMT
BTC-USDT	0.8643	-1.0000
ETH-USDT	-1.0000	1.0000
SOL-USDT	1.0000	0.8508
BNB-USDT	-1.0000	-1.0000
XRP-USDT	-1.0000	-1.0000
ADA-USDT	-0.8643	-0.8508
DOGE-USDT	1.0000	1.0000
DOT-USDT	1.0000	1.0000

5 Conclusion

Adding LSTM to the MVO increases the volatility of the return. This may be because, in this experiment, I trained the LSTM model for only 100 rounds, which might not be sufficient. Additionally, the amount of data used is limited. However, it still generates slightly more profit.
In the future, I will use more data to train the model and explore different architectures, such as Transformers or even large language models, for prediction. I believe this will enhance the performance.
Moreover, the proportions of different cryptocurrencies may not be linear. I can explore more combinations to find a better solution.

6 References

[1]Yang H, Malik A. Optimal Market-Neutral Multivariate Pair Trading on the Cryptocurrency Platform. International Journal of Financial Studies. 2024; 12(3):77.
[2]Sahu, S.; Ochoa Vázquez, J.H.; Ramírez, A.F.; Kim, J.-M. Analyzing Portfolio Optimization in Cryptocurrency Markets: A Comparative Study of Short-Term Investment Strategies Using Hourly Data Approach. J. Risk Financial Manag. 2024, 17, 125. https://doi.org/10.3390/jrfm17030125
[3]Nguyen, Q. M., Tran, D. T., Kanniainen, J., Iosifidis, A., & Gabbouj, M. (2023). Cryptocurrency portfolio optimization by neural networks. arXiv preprint arXiv:2310.01148. https://arxiv.org/abs/2310.01148
[4]Y. Zhang, Z. Li, T. Yan, Q. Liu, N. Vallarano and C. J. Tessone, "Profit Maximization In Arbitrage Loops," 2024 IEEE 44th International Conference on Distributed Computing Systems Workshops (ICDCSW), Jersey City, NJ, USA, 2024, pp. 153-160.
[5]Escudero, C., Lara, F., & Sama, M. (2024). Optimal Trade Characterizations in Multi-Asset Crypto-Financial Markets. arXiv preprint arXiv:2405.06854.
[6]Xu, Y., Lu, Z., & Zhang, Y. (2024). Predicting Asset Price with Large Volatility Based on Marine Predators Algorithm: A Case Study of Bitcoin. SSRN Electronic Journal.
[7]Yang, H., Malik, A., & Raith, A. (2024). Optimal Market-Neutral Currency Trading on the Cryptocurrency Platform. arXiv preprint arXiv:2405.15461.
[8]Chaudhry, A., Kumar, M., Patel, S.Cryptocurrency Portfolio Optimization Using Machine Learning TechniquesJournal of Financial Technology, 2023, 5(2), 45-62.
[9]Smith, J., Tan, H., Lee, K.Risk Management in Crypto-Financial Markets Using Deep Reinforcement Learning arXiv preprint arXiv:2308.11890, 2023.