A Strategy-Proof Combinatorial Heterogeneous Channel Auction Framework in Noncooperative Wireless Networks

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Abstract—Auction is believed to be an effective way to solve or relieve the problem of radio spectrum shortage, by dynamically redistributing idle wireless channels of primary users to secondary users. However, to design a practical channel auction mechanism, we have to consider five challenges, including strategy-proofness, channel spatial reusability, channel heterogeneity, bid diversity, and social welfare maximization. Unfortunately, none of the existing works fully considered the five design challenges. In this paper, we present the first in-depth study on the problem of dynamic channel redistribution jointly considering the five design challenges, and present SMASHER, which is a family of Strategy-proof coMbinatorial Auction mechaniSms for HEterogeneous channel Redistribution. SMASHER contains two strategy-proof auction mechanisms, namely SMASHER-AP and SMASHER-GR. SMASHER-AP is a strategy-proof, approximately efficient combinatorial auction mechanism for indivisible channel redistribution. We further consider the case, in which channels can be shared by the users in a paradigm of time-division multiplexing and propose SMASHER-GR, which is a strategy-proof channel allocation and scheduling mechanism. We have extensively evaluated our designs. The evaluation results show that our designs achieve much better performance than existing works.

Index Terms—Wireless network, channel allocation, combinatorial auction

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1 INTRODUCTION

THE last two decades have witnessed a rapid development of wireless communication technology. Unfortunately, naturally limited radio spectrum is becoming a more and more serious bottleneck of the ongoing growth of wireless applications and services. Most of the countries have specific departments to regulate spectrum usage, e.g., Federal Communications Commission (FCC) [1] in the US and Radio Administration Bureau (RAB) in China [2]. They statically allocate spectrum to wireless application service providers on a long term basis for large geographical regions. Such static management leads to low spectrum utilization in the spatial and temporal dimensions. Large chunks of radio spectrum are left idle most of the time at a lot of places, while new wireless applications are starving for the radio spectrum. Therefore, an open and market-based framework is highly needed to dynamically redistribute the radio spectrum, and thus improve the utilization of the radio spectrum [3].

Auctions are the most well-known market-based mechanisms to redistribute resources [4], [5]. Since 1994, FCC has conducted a series of auctions for the licenses of radio spectrum. While FCC auctions target only at large wireless service providers, our focus is on small wireless applications,

such as community wireless networks or home wireless networks.

There exist many challenges in designing a practical channel auction mechanism [11], [12]. We list five major challenges:

- *Strategy-Proofness*. In strategy-proof auction mechanisms (please refer to Section 2.1 for the definition), simply submitting truthful channel demands (e.g., valuation of the channels) maximizes each participant's utility. Since the participants are normally rational and selfish, they always tend to strategically manipulate the auction, if doing so can increase their utilities. Such selfish behavior inevitably hurts the other participants' utilities. Therefore, it discourages truthfully behaving participants from joining the auction, if strategy-proofness is not guaranteed.
- *Spatial Reusability*. Spatial reusability differentiates the wireless channels from conventional goods. Two wireless users can use the same wireless channel simultaneously, if they are well-separated (i.e., out of the interference range of each other). Exploiting spatial reusability can highly improve spectrum utilization.
- *Channel Heterogeneity*. The nature of wireless channels makes the goods in the channel auction heterogeneous. The channel heterogeneity comes from both *spatial heterogeneity* and *frequency heterogeneity*. On one hand, the availability and quality of a channel vary at different locations. On the other hand, channels with different central frequency may have different propagation and penetration characteristics.

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TABLE 1
Comparison with Existing Channel Auction Mechanisms

Existing Works	Strategy-Proofness	Spatial Reusability	Channel Heterogeneity	Bid Diversity	Social Welfare
VERITAS [6]	✓	✓	✗	✓	No Guarantee
TRUST [7]	✓	✓	✗	✗	No Guarantee
SMALL [8]	✓	✓	✗	✓	No Guarantee
TAHES [9]	✓	✓	✓	✗	No Guarantee
CRWDP [10]	✓	✗	✓	✗	Approximately Efficient
SMASHER-AP	✓	✓	✓	✓	Approximately Efficient
SMASHER-GR	✓	✓	✓	✓	No Guarantee

- *Bid Diversity*. Wireless devices may be equipped with multiple radios, each of which can work on a different channel at the same time. Consequently, a wireless user may request multiple channels, according to her quality of service (QoS) requirement. Buyers have higher opportunities to obtain channels by submitting multiple channel bundles, which makes the channel redistribution more flexible. Therefore, it is necessary to allow users to express diverse demands for channels.
- *Social Welfare*. The basic and common objective of auctions is to maximize social welfare, which is the sum of the auction winners' valuations of the allocated goods (please refer to Section 2.1 for the definition).

A number of related works (e.g., [6], [7], [8], [9], [10]) exist in the literature. Unfortunately, none of these works fully consider the five design challenges (as shown in Table 1). Some of strategy-proof channel auction mechanisms (e.g., VERITAS [6], TRUST [7], SMALL [8]) consider channel spatial reusability, but only work when the trading channels are homogenous. Two recent works TAHES [9] and CRWDP [10] consider the heterogeneity of channels, but TAHES restricts each buyer to bid for a single channel while CRWDP ignores the spatial reusability of channels.

In this paper, we conduct an in-depth study on the problem of dynamic channel redistribution jointly considering the five design challenges, and present SMASHER, which is a family of <u>S</u>trategy-proof co<u>M</u>binatorial <u>A</u>uction mechani<u>S</u>ms for <u>HE</u>terogeneous channel <u>R</u>edistribution. SMASHER contains two distinct auction mechanisms, namely SMASHER-AP and SMASHER-GR. Specifically, SMASHER-AP is a novel combinatorial auction mechanism for indivisible heterogeneous channel redistribution, and achieves both strategy-proofness and approximately efficient social welfare. SMASHER-GR jointly considers channel allocation and scheduling when channels can be shared in a paradigm of time-division multiplexing. We use Table 1 to show the comparison of our designs with closely related works.

We make the following contributions in this paper:

- First, we present a general model of combinatorial auction for heterogeneous channel redistribution. The auction model is powerful enough to express channel spatial reusability and heterogeneity, as well as bid diversity.
- Second, we introduce the concept of *virtual channel* to capture the conflicts of channel usage among different auction participants. By using virtual channels,

we transform the problem of heterogeneous channel allocation to a classic combinatorial auction.
- Third, we propose SMASHER-AP, which is a combinatorial auction mechanism for heterogeneous channel redistribution, achieving both strategy-proofness and approximately efficient social welfare.
- Fourth, we further consider the case, in which channels can be shared in a paradigm of time-division multiplexing, and propose SMASHER-GR, which is a strategy-proof combinatorial auction mechanism for channel allocation and scheduling.
- Finally, we evaluate the performance of our designs. Our simulation results show that our designs achieve much better performance than closely related works, in terms of social welfare, buyer satisfaction ratio, and channel utilization.

The rest of this paper is organized as follows. In Section 2, we present the model of combinatorial auction for heterogeneous channel redistribution. In Section 3, we introduce the concept of virtual channel and convert the problem of heterogeneous channel allocation to a classic combinatorial auction. In Section 4, we present the design of SMASHER-AP. In Section 5, we propose SMASHER-GR. In Section 6, we report evaluation results. In Section 7, we review related works. In Section 8, we conclude the paper and discuss future works.

2 PRELIMINARIES AND PROBLEM FORMULATION

In this section, we present the auction model for the problem of heterogeneous channel allocation, and review some important solution concepts.

2.1 Auction Model

We consider a static scenario, in which there is a primary spectrum user, called "seller", who wants to lease out her temporarily unused wireless channels, and some secondary users (e.g., WiFi access points), called "buyers", who want to lease channels to provide services to their customers at certain quality of service. We consider that the channels for leasing are *heterogeneous*, and thus the buyers have their own preference over the channels due to spatial variance (e.g., background noise, temperature, and landform). Since wireless devices can be equipped with multiple radios, the buyers may request more than one channel according to their requirements of QoS. Considering the diversity of QoS demand and the heterogeneity of channels, we allow the buyers to submit multiple channel requests, among which

one of the requests can be granted.[1] We assume that buyer have uniform valuation over any of her channel requests, because the buyer's requirement of QoS can be satisfied if one of her requested bundles is allocated. Different from the allocation of traditional goods, wireless channels can be spatially reused, meaning that well-separated buyers can work on the same channel simultaneously, if they do not have interference between each other.

We model the process of heterogeneous channel redistribution as a sealed-bid combinatorial auction, in which buyers simultaneously submit their demands for channels to a trustworthy auctioneer, such that no buyer can know other participants' information. The auctioneer makes the decision on channel allocation and the charge to each winner. We denote the set of orthogonal and heterogeneous channels for leasing by $\mathbb{C} \triangleq \{c_1, c_2, \dots, c_m\}$, and the set of buyers by $\mathbb{N} \triangleq \{1, 2, \dots, n\}$. We list useful notations in our model of combinatorial channel auction as follows:

Channel Request R_i: Each buyer $i \in \mathbb{N}$ submits a vector of requested channel bundles

$$R_i \triangleq \left(S_i^1, S_i^2, \dots, S_i^{\phi_i} \right)$$

to the auctioneer. Any channel bundle $S_i^l \subseteq \mathbb{C}, 1 \leq l \leq \phi_i$ can satisfy her QoS. We assume that the request is strict, meaning that the buyer is only interested in winning a whole bundle S_i^l in her request vector. Although the buyer i can submit a request vector R_i with more than one channel bundle, only one channel bundle can be granted by the auctioneer. We call buyer i, who submits a request vector of ϕ_i channel bundles, and is interested in winning one of the bundles, as ϕ_i-minded buyer. If $\phi_i = 1$, then the buyer i is single-minded. Note that our auction model is a generalization of existing models with single-minded buyers (e.g., [9], [10]). The maximum number of submitted channel bundles among all buyers is denoted by $\Phi \triangleq \max_{i \in \mathbb{N}} \phi_i$. We denote the channel request vector $\vec{\mathbb{R}}$ of all the buyers as

$$\vec{\mathbb{R}} \triangleq (R_1, R_2, \dots, R_n).$$

Valuation v_i: Each buyer $i \in \mathbb{N}$ has a uniform valuation v_i over any requested channel bundles in R_i. Here, v_i is the private information of the buyer i. This is also known as *type* in mechanism design. The buyer valuation has two properties: *Free Disposal* and *Normalization*. Free disposal means that for any two subsets of channels S and T, if $S \subseteq T$, then $v_i(S) \leq v_i(T)$; while normalization means that $v_i(\varnothing) = 0$. We denote the valuation vector $\vec{\mathbb{V}}$ of all the buyers as

$$\vec{\mathbb{V}} \triangleq (v_1, v_2, \dots, v_n).$$

Bid b_i: Each buyer $i \in \mathbb{N}$ submits a bid b_i to the auctioneer, meaning that if she wins any channel bundle S_i^l, she would like to pay no more than b_i for it. Here, the bid b_i may not

necessarily be equal to her valuation v_i. Let vector $\vec{\mathbb{B}}$ represent the bids of all the buyers

$$\vec{\mathbb{B}} \triangleq (b_1, b_2, \dots, b_n).$$

Clearing price p_i: The auctioneer charges each winning buyer $i \in \mathbb{N}$ a clearing price p_i. The loser in the auction is free of any charge. We use vector

$$\vec{\mathbb{P}} \triangleq (p_1, p_2, \dots, p_n)$$

to represent the clearing prices of all the buyers.

Utility u_i: The utility of a buyer $i \in \mathbb{N}$ is defined as the difference between her valuation on the bundle of winning channels and her clearing price p_i

$$u_i \triangleq v_i - p_i. \tag{1}$$

We consider that the buyers are rational and selfish, thus their goals are to maximize their own utilities. In contrast to the buyers, the auctioneer's objective is to maximize *social welfare*. Here social welfare is defined as follows.

Definition 1 (Social Welfare). *The social welfare in a channel auction is the sum of winning buyers' valuations on their allocated bundles of channels.*

$$SW \triangleq \sum_{i \in \mathbb{W}} v_i, \tag{2}$$

where $\mathbb{W}$ is the set of winners.

In this paper, we assume that buyers do not collude with each other and do not cheat about their channel bundles,[2] while leaving these problems to our future works.

2.2 Solution Concepts

We briefly review the solution concepts used in this paper.

A strong solution concept from game theory is *dominant strategy*.

Definition 2 (Dominant Strategy [20], [21]). *Strategy s_i is player i's dominant strategy, if for any strategy $s_i' \neq s_i$ and any other player's strategy profile s_{-i}:*

$$u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}).$$

Intuitively, a dominant strategy of a player is a strategy that maximizes her utility, regardless of what strategy profile the other players choose.

The concept of dominant strategy is the basis of *incentive-compatibility*, which means that there is no incentive for any player to lie about her private information, and thus revealing truthful information is the dominant strategy for every

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1. We discuss this model in Section 4 and extend to the scenario, in which each buyer can be allocated multiple bundles to reach her QoS in Section 5.

2. When both valuations and channel bundles are private information, buyers will have more power to manipulate the auction market, i.e., they can further improve their utilities by cheating on channel bundles, and our auction model falls into the general combinatorial auctions with multi-parameter domain, which is still an open problem in algorithmic mechanism design [13]. Papers [14], [15] have characterized the truthfulness for mechanisms in multiple parameter domain, and some negative results are demonstrated [16], [17], [18], [19].

player. An accompanying concept is *individual-rationality*, which means that every player participating in the game expects to gain no less utility than staying outside. We now can introduce the definition of *Strategy-Proof Mechanism*.

Definition 3 (Strategy-Proof Mechanism [22], [23]). *A mechanism is strategy-proof when it satisfies both incentive-compatibility and individual-rationality.*

The objective of this work is to design strategy-proof combinatorial auction mechanisms for heterogeneous channel redistribution.

3 COMBINATORIAL CHANNEL AUCTION

Different from existing works on strategy-proof channel allocation, we introduce a novel concept of *virtual channel* to represent the conflicts of channel usage among the buyers. By introducing virtual channels, we transform the problem of heterogeneous channel allocation to a classic combinatorial auction, which is computationally intractable. Therefore, we propose strategy-proof and approximately efficient combinatorial auction mechanisms for heterogeneous channel redistribution in the following sections.

3.1 Virtual Channel

We introduce *virtual channel* to capture the interference among the buyers on different channels. Specifically, a virtual channel $vc_{i,j}^k$ denotes that the buyer i and the buyer j may cause interference between each other on channel c_k, and thus they cannot work on channel c_k simultaneously. Since virtual channel $vc_{i,j}^k$ represents the exclusive usage of channel c_k between the buyer i and j, its quantity is set to 1. When virtual channel $vc_{i,j}^k$ is added to the requested bundle(s) that contains channel c_k from the buyer i and j, at most one of the requests containing channel c_k from the two buyers can be granted. Consequently, the exclusive usage of channel c_k between the buyer i and j is guaranteed. The heterogeneous channel redistribution problem can be converted to the problem of exclusive virtual channels allocation. We present the definition of virtual channel as follows.

Definition 4 (Virtual Channel). *There is a virtual channel $vc_{i,j}^k$, if the buyer i and buyer j are within the interference range of each other on channel c_k.*

In most of existing works on channel auction, a single conflict graph is used to represent the interference among buyers [6], [7]. However, in case of heterogeneous channels, each channel may have a distinctive conflict graph. Let $G_k \triangleq (O_k, E_k)$ denote the conflict graph on channel c_k, where $O_k \subseteq \mathbb{N}$ is the set of buyers who can access channel c_k, and each edge $(i, j) \in E_k$ represents the interference between the buyer i and j on channel c_k. Let $\mathbb{G} \triangleq \{G_k | c_k \in \mathbb{C}\}$ denote the set of conflict graphs. We also denote the maximum degree of all the conflict graphs as δ. These conflict graphs can be built by the auctioneer through some measurement methods, e.g., measurement calibrated method [24]. We note that the conflict graphs used in this paper belong to binary interference model, such as the protocol model. The problem of channel redistribution under physical

interference model is totally different, and please refer to papers [25], [26] for more discussion.

Since the conflict graph is commonly assumed to be available in wireless networks, we construct the virtual channel from the conflict graph. The process of converting the edges in the conflict graphs to virtual channels with unit quantity is shown by Algorithm 1. We create a virtual channel $vc_{i,j}^k$ (Line 4), if there is an edge between the buyer i and j in conflict graph G_k, and append $vc_{i,j}^k$ to the requested bundle(s) containing channel c_k from the buyer i and j, while remaining the corresponding bid(s) unchanged (Lines 6-11). After adding virtual channels into the channel bundles, we remove the original channels from all updated channel bundles (Line 14). Let $\mathbb{VC}$ be the set of virtual channels (Line 5). Let $S_i''^l$ be the lth updated channel bundle of buyer i. Since the maximum degree of conflict graphs is δ and there are at most m trading channels, we have $|S_i''^l| \leq \delta \times m, \; \forall i \in \mathbb{N}, 1 \leq l \leq \phi_i$.

Algorithm 1. Virtual Channel Generation

Input: A set of conflict graph $\mathbb{G}$, a vector of channel requests $\vec{\mathbb{R}}$.

Output: A set of virtual channels $\mathbb{VC}$, a vector of updated requests $\vec{\mathbb{R}}'$.

1. $\mathbb{VC} \leftarrow \varnothing; \vec{\mathbb{R}}' \leftarrow \vec{\mathbb{R}}$;
2. **foreach** $G_k = (O_k, E_k) \in \mathbb{G}$ **do**
3.   **foreach** $(i, j) \in E_k$ **do**
4.   Create virtual channel $vc_{i,j}^k$;
5. $\mathbb{VC} \leftarrow \mathbb{VC} \bigcup \{vc_{i,j}^k\}$
6.   **foreach** $S_i''^l \in R_i'$
       s.t. $c_k \in S_i''^l \wedge \left(\exists S_j''^t \in R_j', c_k \in S_j''^t \right)$ **do**
7. $S_i''^l \leftarrow S_i''^l \bigcup \{vc_{i,j}^k\}$;
8.   **end**
9.   **foreach** $S_j''^l \in R_j'$
       s.t. $c_k \in S_j''^l \wedge \left(\exists S_i''^t \in R_i', c_k \in S_i''^t \right)$ **do**
10. $S_j''^l \leftarrow S_j''^l \bigcup \{vc_{i,j}^k\}$;
11.   **end**
12.   **end**
13. **end**
14. Remove the original channels $\mathbb{C}$ from updated channel bundles $\vec{\mathbb{R}}'$;
15. **return** $\mathbb{VC}$ and $\vec{\mathbb{R}}'$;

We use a simple example in Fig. 1 to explain the concept of virtual channel. In Fig. 1, there are two channels and four buyers. The two conflict graphs show the interference among buyers on two heterogeneous channels c_1 and c_2. The upper right table shows the buyers' channel demands. Both single-minded and multi-minded buyers exist in this example. Here, the buyer 2 is a single-minded buyer, and only bids a bundle of channels ($\{c_1, c_2\}$) for 15; the buyer 3 is a multi-minded buyer, and submits three requests, i.e., ($\{c_1\}, \{c_2\}, \{c_1, c_2\}$), and a uniform valuation 13. After running Algorithm 1, the updated request vectors with virtual channels are shown in the lower right table. Let's see buyer 2's updated request as an example. Since both buyer 1 and buyer 2 bid for channel c_1 and they interfere with each other on this channel, we add a virtual channel $vc_{1,2}^1$ with unit quantity to buyer 2's requested bundle.
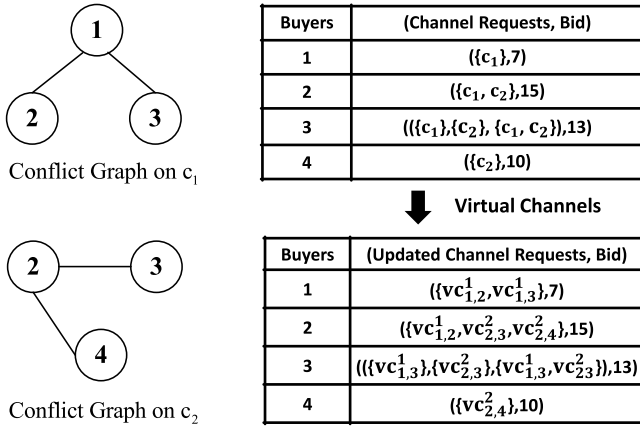
Buyers	(Channel Requests, Bid)
1	$(\{c_1\},7)$
2	$(\{c_1, c_2\},15)$
3	$(\{\{c_1\},\{c_2\}, \{c_1, c_2\}\},13)$
4	$(\{c_2\},10)$

Virtual Channels

Buyers	(Updated Channel Requests, Bid)
1	$(\{vc_{1,2}^1,vc_{1,3}^1\},7)$
2	$(\{vc_{1,2}^1,vc_{2,3}^2,vc_{2,4}^2\},15)$
3	$(\{\{vc_{1,3}^1\},\{vc_{2,3}^2\},\{vc_{1,3}^1,vc_{23}^2\}\},13)$
4	$(\{vc_{2,4}^2\},10)$

Fig. 1. An example showing the generation of virtual channels.

3.2 Problem Formulation

Given the virtual channel introduced in the last section, we are ready to transform the problem of heterogenous channel allocation to a classic combinatorial auction. The outcome of the auction is the set of winning buyers and their assigned channel bundles.

The goods in the combinatorial channel auction are the virtual channels. The quantity of each virtual channel $vc_{i,j}^k \in \mathbb{VC}$ is 1. Given the vector of requests with virtual channels $\vec{\mathbb{R}}'$ and the bid vector $\vec{\mathbb{B}}$, the auctioneer determines the winners and which channel bundles to grant. Let $x(i, S_i''^l) = 1$ denote that the channel set $S_i''^l$ is granted to the buyer i; otherwise, $x(i, S_i''^l) = 0$. The process of winner determination can be modeled as a binary program. The objective is to maximize the social welfare. We use b_i, instead of v_i, because the strategy-proof mechanisms shown in later sections will guarantee that bidding truthfully is the dominant strategy of each buyer $i \in \mathbb{N}$. *Objective:*

$$Maximize \quad \sum_{i \in \mathbb{N}} \sum_{l=1}^{\phi_i} x(i, S_i''^l) \times b_i$$

Subject to:

$$\sum_{i \in \mathbb{N}} \sum_{S_i''^l \in R_i', S_i''^l \ni vc_k} x(i, S_i''^l) \leq 1 \quad \forall vc_k \in \mathbb{VC} \quad (3)$$

$$\sum_{l=1}^{\phi_i} x(i, S_i''^l) \leq 1 \quad \forall i \in \mathbb{N} \quad (4)$$

$$x(i, S_i''^l) \in \{0,1\} \quad \forall i \in \mathbb{N}, 1 \leq l \leq \phi_i. \quad (5)$$

Here constraint (3) indicates the quantity limitation of virtual channel. As the original channels have been removed from the updated channel bundles, we do not have quantity constraints on the original channels. Constraint (4) indicates that each buyer can win at most one bundle of channels out of her submitted requests. Constraint (5) indicates the binary value of the auctioneer's decision of allocation.

If the optimal social welfare can be achieved by solving the above binary program, then the celebrated VCG mechanism (named after Vickrey [27], Clark [28], and Groves [29])

can be applied to calculate clearing prices that can ensure the strategy-proofness of the auction mechanism. Unfortunately, the above winner determination problem can be proven to NP-hard by reducing from the *exact cover* problem [30] in polynomial time. Considering the computational intractability of the winner determination problem, we present an alternative solution with greedy channel allocation to achieve approximately efficient social welfare in next section. Furthermore, we integrate the greedy allocation algorithm with a novel pricing mechanism to provide a strategy-proof and approximately efficient combinatorial auction mechanism for heterogeneous channel redistribution.

4 EXCLUSIVE CHANNEL REDISTRIBUTION

We consider the case of indivisible channels, which can only be allocated exclusively to non-interfering buyers, in this section. As shown in Section 3.2, finding the optimal auction decision is computationally intractable. Furthermore, existing works [17], [18] show that it is impossible to design a strategy-proof approximation combinatorial auction mechanism in the general case, even if the goods are not spatially reusable. We assume that buyers have uniform valuation on their multiple channel requests, and present SMASHER-AP, which is a strategy-proof and approximately efficient combinatorial auction mechanism for heterogeneous channel redistribution.

4.1 Design of SMASHER-AP

SMASHER-AP consists of the following three major components: virtual channel generation, winner determination, and clearing price calculation. We briefly describe the design rationale of SMASHER-AP. We first generate virtual channels to capture the interference of channel usage among buyers, and transform the problem of channel redistribution into the exclusive virtual channel allocation. After that, we propose a greedy channel allocation algorithm to determine winning buyers, which leads to a good approximation ratio. Finally, a clearing price calculation scheme based on critical virtual bid is designed to guarantee the economic properties of SMASHER-AP.

4.1.1 Virtual Channel Generation

The process of virtual channel generation is the same as that of Algorithm 1 shown in Section 3.1, except that we add one more virtual channel vc_i with unit quantity to each requested bundle of buyer $i \in \mathbb{N}$. Virtual channel vc_i is used to ensure that at most one of the requested bundles from the buyer i can be granted.

$$S_i''^l = S_i''^l \bigcup \{vc_i\}, i \in \mathbb{N}, 1 \leq l \leq \phi_i,$$

where $S_i''^l$ is updated bundle with virtual channels. The set of virtual channels is also updated

$$\mathbb{VC} = \mathbb{VC} \bigcup \{vc_i | i \in \mathbb{N}\}.$$

4.1.2 Winner Determination

Before presenting the approximation algorithm for winner determination, we introduce *virtual bid*. The uniform virtual

bid $\tilde{b}_i$ over any of requested bundles from the buyer i is defined as

$$\tilde{b}_i \triangleq \frac{b_i}{\max\limits_{1 \le l \le \phi_i}\left(\sqrt{|S_i''^l|}\right)}. \tag{6}$$

SMASHER-AP sorts all the buyers according to their virtual bids in non-increasing order:

$$\mathbb{L}_1 : \tilde{b}_1 \ge \tilde{b}_2 \ge \cdots \ge \tilde{b}_n.$$

In case of a tie, SMASHER-AP breaks the tie following a bid-independent rule, such as lexicographic order of buyers' IDs or channel number. Following the order in $\mathbb{L}_1$, SMASHER-AP greedily grants the smallest channel bundle, in which no virtual channel has already been allocated, to each buyer.[3]

Algorithm 2 shows the pseudo-code of above winner determination process. In practice, the number of buyers n is much larger than Φ, thus the time complexity of Algorithm 2 is $\mathcal{O}(n \log n)$.

Algorithm 2. Approximation Algorithm for Winner Determination

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Input: Vector of updated channel requests $\vec{\mathbb{R}}'$, vector of bids $\vec{\mathbb{B}}$.
Output: A pair of sets of winning buyers and allocated bundles of channels $(\mathbb{W}, \mathbb{S})$.
1. $(\mathbb{W}, \mathbb{S}) \leftarrow (\varnothing, \varnothing); \mathcal{V} \leftarrow \varnothing;$
2. **foreach** $i \in \mathbb{N}$ **do**
3. $\tilde{b}_i \leftarrow b_i / \max_{1 \le l \le \phi_i}\left(\sqrt{|S_i''^l|}\right);$
4. **end**
5. Sort $\tilde{b}_i$ in non-increasing order:
    $\mathbb{L}_1 : \tilde{b}_1 \ge \tilde{b}_2 \ge \cdots \ge \tilde{b}_n;$
6. **for** $i = 1$ to n **do**
7.   Sort $S_i''^l$ in non-decreasing order of bundle
       size: $\mathbb{L}_2 : |S_i'^1| \le |S_i'^2| \le \cdots \le |S_i'^{\phi_i}|;$
8.   **for** $l = 1$ to ϕ_i **do**
9.   **If** $S_i''^l \cap \mathcal{V} = \varnothing$ **then**
10. $\mathcal{V} \leftarrow \mathcal{V} \bigcup S_i''^l;$
11. $(\mathbb{W}, \mathbb{S}) \leftarrow \left(\mathbb{W} \bigcup \{i\}, \mathbb{S} \bigcup \{S_i''^l\}\right);$
12.   **break**
13.   **end**
14.   **end**
15. **end**
16. **return**$(\mathbb{W}, \mathbb{S});$

4.1.3 Clearing Price Calculation

The clearing price is calculated based on *critical virtual bid*.

Definition 5 (Critical Virtual Bid). *The critical virtual bid $cr(i) \in \mathbb{L}_1$ of buyer $i \in \mathbb{N}$ is the minimum virtual bid that the buyer i must exceed to be allocated one of her channel bundles,*

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3. Actually, we allocate the original channel bundle S_i^l to the winning buyer i, when she is granted the updated channel bundle $S_i''^l$ in Algorithm 2.

i.e., if the virtual bid of the buyer i is higher than $cr(i)$, she wins the auction; otherwise, she loses.

We note that according to the definition of critical virtual bid, no matter which channel bundles of buyer i is granted in the auction, the critical virtual bid $cr(i)$ is always the same.

The critical virtual bid of buyer $i \in \mathbb{N}$ can be calculated by the following procedure. Given other buyers' requests and bids $(\vec{\mathbb{R}}'_{-i}, \vec{\mathbb{B}}_{-i})$, we greedily select virtual bids by rerunning Algorithm 2 until none of buyer i's requests can be satisfied. The threshold virtual bid $cr(i)$ we select finally is regarded as the critical virtual bid of the buyer i. We now show the method of calculating the clearing price of the buyer i by distinguishing two cases:

1) If the buyer i loses in the auction or $cr(i)$ does not exist (denoted by $cr(i) = 0$), then her clearing price is 0.
2) If the buyer i is granted channel bundle $S_i''^l$ and there exists a critical virtual bid $cr(i)$, the clearing price p_i of buyer i is set to

$$p_i \triangleq cr(i) \times \max\limits_{1 \le l \le \phi_i}\left(\sqrt{|S_i''^l|}\right). \tag{7}$$

4.2 Analysis

We prove the strategy-proofness and analyze the approximation ratio of SMASHER-AP in this section.

4.2.1 Strategy-Proofness

Theorem 1. *SMASHER-AP is a strategy-proof combinatorial auction mechanism for heterogeneous indivisible channel redistribution.*

Proof. We first show that buyer $i \in \mathbb{N}$ cannot obtain higher utility by bidding untruthfully.
We discuss the problem in the following two cases:

- The buyer i wins bundle $\hat{S}_i''^l$ and gets utility $u_i \ge 0$ when bidding truthfully, i.e., $b_i = v_i$. Let $\hat{S}_i''^t \ne \hat{S}_i''^l$ be the bundle won by the buyer i, when she cheats the bid, i.e., $b_i' \ne v_i$. The utility of the buyer i remains the same:

$$\begin{aligned} u_i' &= v_i - p_i' \\ &= v_i - cr(i) \times \max\limits_{1 \le l \le \phi_i}\left(\sqrt{|S_i''^l|}\right) \\ &= u_i. \end{aligned}$$

  If the buyer i loses the auction when she cheats the bid, her utility is 0, which is not better than that gained when bidding truthfully.

- The buyer i loses in the auction when bidding truthfully. Then, her utility $u_i = 0$. If she still loses when bidding untruthfully, her utility cannot be changed. We consider the case, in which she cheats the bid $b_i' \ne v_i$ and wins a bundle $\hat{S}_i''^t \ne \varnothing$. We denote virtual bid $\tilde{b}_i$ and $\tilde{b}_i'$ for channel bundle $\hat{S}_i''^t$ when the buyer i bids truthfully and untruthfully, respectively. Then, we have $\tilde{b}_i' \ge cr(i) \ge \tilde{b}_i$,

because otherwise, she still cannot win any bundle. Her utility now becomes non-positive:

$$
\begin{aligned}
u_i' &= v_i - p_i' \\
&= v_i - cr(i) \times \max_{1 \le l \le \phi_i} \left(\sqrt{\left| S_i'^l \right|} \right) \\
&\le v_i - \tilde{b}_i \times \max_{1 \le l \le \phi_i} \left(\sqrt{\left| S_i'^l \right|} \right) \\
&= v_i - \frac{v_i}{\max_{1 \le l \le \phi_i} \left(\sqrt{\left| S_i'^l \right|} \right)} \times \max_{1 \le l \le \phi_i} \left(\sqrt{\left| S_i'^l \right|} \right) \\
&= v_i - v_i \\
&= 0.
\end{aligned}
$$

From the above analysis of two cases, we can see that the buyer i cannot increase her utility by bidding any other value than v_i, and thus bidding truthfully is a dominant strategy for each buyer. Therefore, SMASHER-AP satisfies incentive compatibility.

We now prove that SMASHER-AP also satisfies individual rationality. On one hand, buyer i's utility is zero if she loses in the auction. On the other hand, winning buyer i gets utility:

$$
\begin{aligned}
u_i &= v_i - p_i \\
&= v_i - cr(i) \times \max_{1 \le l \le \phi_i} \left(\sqrt{\left| S_i^l \right|} \right) \\
&= \left(\frac{v_i}{\max_{1 \le l \le \phi_i} \left(\sqrt{\left| S_i^l \right|} \right)} - cr(i) \right) \times \max_{1 \le l \le \phi_i} \left(\sqrt{\left| S_i^l \right|} \right) \\
&= \left(\tilde{b}_i - cr(i) \right) \times \max_{1 \le l \le \phi_i} \left(\sqrt{\left| S_i^l \right|} \right),
\end{aligned}
$$

where $\tilde{b}_i$ is the virtual bid of buyer i, Since the buyer i is a winner, we have $\tilde{b}_i \ge cr(i)$, and thus $u_i \ge 0$. Buyer utility is always non-negative, which is not worse than staying outside the auction (i.e., the utility is 0). Therefore, SMASHER-AP satisfies individual rationality.

Since SMASHER-AP satisfies both incentive compatibility and individual rationality, according to Definition 3, SMASHER-AP is a strategy-proof mechanism. Our claim holds. Since our mechanism belongs to single-parameter mechanism, we can also obtain the property of strategy-proofness by using Myerson's well known characterization [31]. □

4.2.2 Approximation Ratio

We now present the approximation ratio of SMASHER-AP.

Theorem 2. *The approximation ratio of SMASHER-AP is $\mathcal{O}(\delta m)$, where δ is the maximum degree of conflict graphs and m is the number of channels.*

Proof. Let $(\mathbb{W}_{OPT}, \mathbb{S}_{OPT})$ be the optimal channel allocation, and $(\mathbb{W}_{APP}, \mathbb{S}_{APP})$ be the allocation achieved by SMASHER-AP. The social welfare of the optimal solution and SMASHER-AP is $\sum_{i \in \mathbb{W}_{OPT}} v_i$ and $\sum_{i \in \mathbb{W}_{APP}} v_i$, respectively.

For each buyer $i \in \mathbb{W}_{APP}$, we define

$$
\mathbb{W}_{OPT}^i \triangleq \left\{ j \in \mathbb{W}_{OPT} \,\middle|\, \frac{b_j}{\max_{1 \le l \le \phi_j} \left(\sqrt{\left| S_j'^l \right|} \right)} \right.
$$

$$
\left. \le \frac{b_i}{\max_{1 \le l \le \phi_i} \left(\sqrt{\left| S_i'^l \right|} \right)}, \left(\mathbb{S}_{OPT}^j \cap \mathbb{S}_{APP}^i \cap \mathbb{VC} \ne \varnothing \right) \right\}
$$

to represent the buyers in $\mathbb{W}_{OPT}$, whose bundles in $\mathbb{S}_{OPT}$ cannot be granted in SMASHER-AP because of the existence of i.

Since every $j \in \mathbb{W}_{OPT}^i$ appears after i in the ordered list $\mathbb{L}_1$, we have

$$
v_j \le \frac{v_i \times \max_{1 \le l \le \phi_j} \left(\sqrt{\left| S_j'^l \right|} \right)}{\max_{1 \le l \le \phi_i} \left(\sqrt{\left| S_i'^l \right|} \right)}.
$$

Summing over all $j \in \mathbb{W}_{OPT}^i$, we can get

$$
\sum_{j \in \mathbb{W}_{OPT}^i} v_j \le \frac{v_i}{\max_{1 \le l \le \phi_i} \left(\sqrt{\left| S_i'^l \right|} \right)} \sum_{j \in \mathbb{W}_{OPT}^i} \max_{1 \le l \le \phi_j} \left(\sqrt{\left| S_j'^l \right|} \right). \quad (8)
$$

Using the Cauchy-Schwarz inequality, we can bound

$$
\sum_{j \in \mathbb{W}_{OPT}^i} \max_{1 \le l \le \phi_j} \left(\sqrt{\left| S_j'^l \right|} \right) \le \sqrt{\left| \mathbb{W}_{OPT}^i \right|} \sqrt{\sum_{j \in \mathbb{W}_{OPT}^i} \max_{1 \le l \le \phi_j} \left(\left| S_j'^l \right| \right)}. \quad (9)
$$

By integrating inequations (8) and (9), we get

$$
\sum_{j \in \mathbb{W}_{OPT}^i} v_j \le \frac{v_i \sqrt{\left| \mathbb{W}_{OPT}^i \right|} \sqrt{\sum_{j \in \mathbb{W}_{OPT}^i} \max_{1 \le l \le \phi_j} \left(\left| S_j'^l \right| \right)}}{\max_{1 \le l \le \phi_i} \left(\sqrt{\left| S_i'^l \right|} \right)}. \quad (10)
$$

Since $(\mathbb{W}_{OPT}, \mathbb{S}_{OPT})$ is the optimal channel allocation, the channel bundles allocated to any pair of buyers $i, j \in \mathbb{W}_{OPT}$ cannot overlap on any virtual channel: $\mathbb{S}_{OPT}^i \cap \mathbb{S}_{OPT}^j \cap \mathbb{VC} = \varnothing$. Every bundle allocated to $j \in \mathbb{W}_{OPT}^i$ in the optimal allocation intersects with $\mathbb{S}_{APP}^i$ at least one virtual channel. Consequently, there are at most $\max_{1 \le l \le \phi_i} \left(\left| S_i'^l \right| \right)$ buyers in $\mathbb{W}_{OPT}^i$

$$
\left| \mathbb{W}_{OPT}^i \right| \le \max_{1 \le l \le \phi_i} \left(\left| S_i'^l \right| \right) \Rightarrow \sqrt{\left| \mathbb{W}_{OPT}^i \right|} \le \max_{1 \le l \le \phi_i} \left(\sqrt{\left| S_i'^l \right|} \right). \quad (11)
$$

Since $\max_{1 \le l \le \phi_j} \left(\left| S_j'^l \right| \right) \le \delta m + 1$, we also have

$$
\sum_{j \in \mathbb{W}_{OPT}^i} \max_{1 \le l \le \phi_j} \left(\left| S_j'^l \right| \right) \le \delta m (\delta m + 1). \quad (12)
$$

By integrating inequations (10), (11) and (12), we get

$$\sum_{j \in \mathbb{W}_{OPT}^i} v_j \leq \sqrt{\delta m(\delta m + 1)} v_i. \qquad (13)$$

Since $\mathbb{W}_{OPT} = \bigcup_{i \in \mathbb{W}_{APP}} \mathbb{W}_{OPT}^i$, we finally get

$$\sum_{i \in \mathbb{W}_{OPT}} v_i \leq \sum_{i \in \mathbb{W}_{APP}} \sum_{j \in \mathbb{W}_{OPT}^i} v_j \leq \sqrt{\delta m(\delta m + 1)} \sum_{i \in \mathbb{W}_{APP}} v_i. \qquad (14)$$

Therefore, the approximation ratio of SMASHER-AP is $\mathcal{O}(\delta m)$. □

5 CHANNEL REDISTRIBUTION WITH TIME SCHEDULING

In this section, we consider the scenario, in which the clocks of buyers are synchronized [32] and the radios on buyers' devices can switch among different channels within very short time [33]. Therefore, a channel can be shared by wireless devices in a paradigm of time-division multiplexing, which is similar to the time-frequency model in [10]. We extend SMASHER-AP to the channel redistribution with time flexibility, and design SMASHER-GR, which is a strategy-proof combinatorial auction mechanism for heterogeneous channel redistribution, jointly considering spatial and temporal channel reusability.

SMASHER-GR divides the time into a series of slots with a fixed length of duration τ. A time slot of a channel can be scheduled to multiple buyers using time-division multiplexing, by which each buyer uses a certain fraction of the slot. In other words, the channel is considered as a kind of divisible goods. During a time slot, buyer $i \in \mathbb{N}$ has a requested data throughput Q_i, which can be derived from her subscribers' QoS. We use vector $\vec{\mathbb{Q}}$ to denote the data throughput of all the buyers

$$\vec{\mathbb{Q}} \triangleq (Q_1, Q_2, \ldots, Q_n).$$

Due to the heterogeneity of channels, different channel bundles may provide different data rates to different buyers. The channel requests from the buyer i can now be expressed as

$$R_i \triangleq \left(\left(S_i^1, d_i^1 \right), \left(S_i^2, d_i^2 \right), \ldots, \left(S_i^{\phi_i}, d_i^{\phi_i} \right) \right),$$

where d_i^l denotes the data rate achieved by the buyer i if she operates on the channel bundle S_i^l. In this model, a multi-minded buyer can work on multiple channel bundles sequentially to reach her data throughput requirement Q_i in each time slot. Here, we assume that buyers do not cheat on data throughput and data rate, and we will relax these assumptions in our future works.[4]

---

4. Similarly, if we relax this assumption, the auction model will fall into the general combinatorial auction with multiple parameter domain, and the deterministic strategy-proof combinatorial auction mechanisms are still unknown for multiple parameters scenarios.

Let binary variable $x_i = 1$ denote that buyer i can obtain throughput Q_i by working on required channel bundles in time duration τ; otherwise, $x_i = 0$. We use h_i^l and t_i^l to denote the starting time and lasting time of channel bundle S_i^l from buyer i, respectively. Buyer i can get throughput $d_i^l \times t_i^l$ if she operates on bundle S_i^l for t_i^l time. We jointly consider allocation and scheduling algorithm to determine which channel bundles to grant, and when and how long winning buyers can access allocated channel bundles. Let R_i' be the updated channel requests with virtual channels for buyer i. We denote the updated request vector $\vec{\mathbb{R}}'$ of all the buyers as

$$\vec{\mathbb{R}}' = (R_1', R_2', \ldots, R_n').$$

We formalize the process of channel allocation and scheduling as the following *mixed-integer nonlinear program (MINLP)*.

Objective:

$$Maximize \qquad \sum_{i \in \mathbb{N}} (x_i \times b_i)$$

Subject to:

$$\left(h_i^l - h_{i'}^{l'} \right) \times \left(h_i^l + x_i \times x_{i'} \times t_i^l - h_{i'}^{l'} \right) > 0,$$
$$\forall S_i^{\prime l}, S_{i'}^{\prime l'} \in \vec{\mathbb{R}}', \ S_i^{\prime l} \cap S_{i'}^{\prime l'} \cap \mathbb{VC} \neq \varnothing \qquad (15)$$

$$\left(h_{i'}^{l'} - h_i^l \right) \times \left(h_{i'}^{l'} + x_i \times x_{i'} \times t_{i'}^{l'} - h_i^l \right) > 0,$$
$$\forall S_i^{\prime l}, S_{i'}^{\prime l'} \in \vec{\mathbb{R}}', \ S_i^{\prime l} \cap S_{i'}^{\prime l'} \cap \mathbb{VC} \neq \varnothing \qquad (16)$$

$$\sum_{l=1}^{\phi_i} \left(d_i^l \times t_i^l \right) = x_i \times Q_i \qquad \forall i \in \mathbb{N} \qquad (17)$$

$$h_i^l \geq 0 \qquad \forall i \in \mathbb{N}, 1 \leq l \leq \phi_i \qquad (18)$$

$$h_i^l + x_i \times t_i^l \leq \tau \qquad \forall i \in \mathbb{N}, 1 \leq l \leq \phi_i \qquad (19)$$

$$x_i \in \{0, 1\} \qquad \forall i \in \mathbb{N}. \qquad (20)$$

Same as before, the objective is to maximize the social welfare. Constraints (15) and (16) indicate that any two winning channel bundles containing the same virtual channel should be carefully scheduled to avoid interference in the time dimension. Specifically, we use intervals $[h_i^l, h_i^l + t_i^l]$ and $[h_{i'}^{l'}, h_{i'}^{l'} + t_{i'}^{l'}]$ to denote the working time duration of two allocated channel bundles S_i^l and $S_{i'}^{l'}$, respectively. Constraints (15) and (16) guarantee that these two intervals are non-overlapping. Constraint (17) indicates that the sum of throughput obtained from multiple channel bundles should be equal to the requested throughput of each winning buyer. Constraints (18) and (19) guarantee that the starting time should be larger than 0, and the ending time should be less than τ. Constraint (20) shows the binary value of x_i.

The above allocation and scheduling problem is NP-hard [34], and thus is computational intractable. So we follow the design rationale of SMASHER-AP, and design SMASHER-GR, including greedy channel allocation, scheduling and pricing calculation, to adapt to channel redistribution with time flexibility.

5.1 Design of SMASHER-GR

The design rationale of SMASHER-GR is briefly described as follows. SMASHER-GR first generates virtual channels according to the conflict graphs to represent the spatial interference of heterogeneous channels. Then it greedily selects the available and non-overlapping time intervals for winning buyers, considering the channel interference in the time dimension. Finally, SMASHER-GR applies a pricing mechanism to guarantee economic properties.

SMASHER-GR consists of three parts: virtual channel generation, winner determination and clearing price calculation.

5.1.1 Virtual Channel Generation

SMASHER-GR generates the virtual channels by running Algorithm 1, described in Section 3.1. Different from SMASHER-AP, SMASHER-GR does not generate virtual channel vc_i for each buyer $i \in \mathbb{N}$, because buyer i may be allocated multiple channel bundles to achieve her throughput during a time slot.

5.1.2 Winner Determination

The winner determination algorithm consists of two parts, i.e., channel allocation and channel bundle scheduling. Before presenting the algorithm, we redefine the virtual bid, considering the different data rates of channel bundles. The virtual bid of the buyer i is

$$\tilde{b}_i \triangleq \frac{b_i}{\max\limits_{1 \leq l \leq \phi_i} \left(\sqrt{|S_i'^l| \times Q_i / d_i^l} \right)}. \tag{21}$$

Intuitively, a channel bundle with larger size ($|S_i'^l|$) and longer occupation time (Q_i/d_i^l), has lower priority to be granted, because it may lead to more spatial and temporal conflicts with other requests.

SMASHER-GR sorts all the buyers by their virtual bids in non-increasing order:

$$\mathbb{L}_3 : \tilde{b}_1 \geq \tilde{b}_2 \geq \cdots \geq \tilde{b}_n,$$

and breaks the tie by using a bid-independent rule.

We use $\mathcal{F}$ to denote the $<virtual\ channel, time>$ dimensional space, which records the available working time intervals of virtual channels. Following the order in list $\mathbb{L}_3$, SMASHER-GR first checks whether the buyer i can fulfill her claimed throughput Q_i by being allocated the remaining time of virtual channels in $\mathcal{F}$. If the buyer i is a winner, SMASHER-GR then greedily selects the available and non-overlapping time intervals for channel bundle $S_i'^l$, and "packs" these intervals into space $\mathcal{F}$. SMASHER-GR iteratively allocates time intervals to each bundle $S_i'^l$ of buyer i until achieving her requested data throughput Q_i.

Algorithm 3 shows the pseudo-code of greedy algorithm for winner determination, including channel allocation and scheduling. The function $Satisfiable(Q_i, R_i', \mathcal{F})$ checks whether the buyer i can satisfy her data throughput Q_i by being allocated the non-overlapping time

intervals in current space $\mathcal{F}$. Function $Scheduling(S_i'^l, \mathcal{F})$ returns triple tuple $(Schedulable, \mathbb{H}(i, l), \mathbb{T}(i, l))$, in which the binary variable $Schedulable$ indicates whether there exist non-overlapping time intervals in $\mathcal{F}$ for bundle $S_i'^l$. If the variable $Schedulable$ is $true$, $Scheduling$ also returns the starting time $\mathbb{H}(i, l)$ and lasting time $\mathbb{T}(i, l)$ for bundle $S_i'^l$.[5] Function $Scheduling$ finally "packs" the time interval $[\mathbb{H}(i, l), \mathbb{H}(i, l) + \mathbb{T}(i, l)]$ into space $\mathcal{F}$. The process of scheduling can be done linearly, then the complexity of Algorithm 3 is $\mathcal{O}(n \log n)$.

Algorithm 3. Greedy Algorithm for Winner Determination and Scheduling.

Input: Vector of updated channel requests $\vec{\mathbb{R}}'$,
vector of data throughput $\vec{\mathbb{Q}}$, vector of bids $\vec{\mathbb{B}}$,
$<virtual\ channel, time>$ space $\mathcal{F}$.

Output: Sets of winning buyers $\mathbb{W}$, allocated bundle of channels $\mathbb{S}$, scheduling matrix $(\mathbb{H}, \mathbb{T})$.

1. $\mathbb{W} \leftarrow \varnothing; \mathbb{S} \leftarrow \varnothing; (\mathbb{H}, \mathbb{T}) \leftarrow (0^{n,\Phi}, 0^{n,\Phi})$;
2. **foreach** $i \in \mathbb{N}$ **do**
3. $\tilde{b}_i \leftarrow b_i / \max_{1 \leq l \leq \phi_i} \left(\sqrt{|S_i'^l| \times Q_i / d_i^l} \right)$;
4. **end**
5. Sort $\tilde{b}_i$ in non-increasing order:
   $\mathbb{L}_3 : \tilde{b}_1 \geq \tilde{b}_2 \geq \cdots \geq \tilde{b}_n$;
6. **for** $i = 1$ to n **do**
7.   **If** $Satisfiable(Q_i, R_i', \mathcal{F})$ is $true$ **then**
8. $\mathbb{W} \leftarrow \mathbb{W} \cup \{i\}$;
9. $l \leftarrow 1; Q_i' \leftarrow Q_i$;
10.   **while** $Q_i' > 0$ **do**
11. $(Schedulable, \mathbb{H}(i, l), \mathbb{T}(i, l)) \leftarrow$
        $Scheduling(S_i'^l, \mathcal{F})$;
12.   **if** $Schedulable$ is $true$ **then**
13. $\mathbb{S} \leftarrow \mathbb{S} \cup \{S_i'^l\}$;
14. $Q_i' \leftarrow (Q_i' - d_i^l \times \mathbb{T}(i, l))$;
15.   **end**
16. $l \leftarrow l + 1$;
17.   **end**
18.   **end**
19. **end**
20. **return** $\mathbb{W}, \mathbb{S}, (\mathbb{H}, \mathbb{T})$;

5.1.3 Clearing Price Calculation

The pricing mechanism is also based on critical virtual bid.

Definition 6 (Critical Virtual Bid). *The critical virtual bid $cr(i) \in \mathbb{L}_3$ of winning buyer $i \in \mathbb{W}$ is the minimum virtual bid that buyer i must exceed in order to fulfil her requested data throughput, i.e., if the virtual bid of the buyer i is higher than $cr(i)$, her requested data throughput would be satisfied; otherwise, she would lose the auction.*

The critical virtual bid $cr(i)$ of buyer i can be obtained by the following steps. Given the other buyers' channel

---

5. Similar to Algorithm 2, the updated channel bundle $S_i'^l$ is schedulable means that the corresponding original channel bundle S_i^l is also schedulable in Algorithm 3.

demands $\left(\vec{\mathbb{R}}'_{-i}, \vec{\mathbb{Q}}_{-i}, \vec{\mathbb{B}}_{-i}\right)$, we greedily select the virtual bid from L_3 by running Algorithm 3 until buyer i's requested data throughput cannot be fulfilled. The last virtual bid we selected is considered as the critical virtual bid of buyer i. Now we can calculate the clearing price of buyer i by distinguishing the following two cases:

- If the buyer i loses in the auction or there exist no critical virtual bids for her, then her clearing price is 0.

- If the buyer i is a winner and her critical bid is $cr(i)$, we can calculate her clearing price p_i:

$$p_i \triangleq cr(i) \times \max_{1 \leq l \leq \phi_i} \left(\sqrt{|S_i''^l| \times Q_i/d_i^l} \right). \tag{22}$$

5.2 Analysis

By combining the channel allocation, scheduling and pricing mechanisms together, SMASHER-GR achieves the following property.

Theorem 3. *SMASHER-GR is a joint allocation and scheduling strategy-proof combinatorial auction mechanism for heterogeneous channel redistribution with time flexibility.*

Proof. We first prove that buyer $i \in \mathbb{N}$ cannot increase her utility by bidding untruthfully, i.e., reporting true valuation is a dominant strategy for the buyer i.

We distinguish two cases:

- Buyer i achieves her throughput Q_i and gets her utility $u_i \geq 0$ when bidding truthfully, i.e., $b_i = v_i$. She gains channel bundle set $\mathbb{S}_i \subseteq \mathbb{S}$ and scheduling matrix $(\mathbb{H}_i, \mathbb{T}_i) \subseteq (\mathbb{H}, \mathbb{T})$. Buyer i wins another channel bundle set $\mathbb{S}'_i \neq \mathbb{S}_i, \mathbb{S}'_i \subseteq \mathbb{S}$ and scheduling matrix $(\mathbb{H}'_i, \mathbb{T}'_i) \subseteq (\mathbb{H}, \mathbb{T})$ when she reports another bid $b'_i \neq v_i$. Her utility is not changed:

$$
\begin{aligned}
u'_i &= v_i - p'_i \\
&= v_i - cr(i) \times \max_{1 \leq l \leq \phi_i} \left(\sqrt{|S_i^l| \times Q_i/d_i^l} \right) \\
&= u_i.
\end{aligned}
$$

If buyer i loses in the auction when she cheats the bid, her utility is 0, which is no more than when buyer i bids truthfully.

- We consider the other case, in which the buyer i cannot fulfill her throughput Q_i when bidding truthfully. Then her utility $u_i = 0$. The only way to improve her utility is to cheat the bid $b'_i \neq v_i$ and become a winner. We denote the buyer i's winning channel bundle set $\mathbb{S}'_i \subseteq \mathbb{S}$ and the corresponding scheduling matrix $(\mathbb{H}'_i, \mathbb{T}'_i) \subseteq (\mathbb{H}, \mathbb{T})$ when she cheats on bid. Let $\tilde{b}_i$ and $\tilde{b}'_i$ denote the virtual bid of buyer i when she bids truthfully and untruthfully, respectively. Then, we have $\tilde{b}'_i \geq cr(i) \geq \tilde{b}_i$, because otherwise, she still cannot fulfil her throughput. Her utility now becomes non-positive

$$
\begin{aligned}
u'_i &= v_i - p'_i = v_i - cr(i) \times \max_{1 \leq l \leq \phi_i} \left(\sqrt{|S_i''^l| \times Q_i/d_i^l} \right) \\
&\leq v_i - \tilde{b}_i \times \max_{1 \leq l \leq \phi_i} \left(\sqrt{|S_i''^l| \times Q_i/d_i^l} \right) \\
&= v_i - \frac{v_i \times \max\limits_{1 \leq l \leq \phi_i} \left(\sqrt{|S_i''^l| \times Q_i/d_i^l} \right)}{\max\limits_{1 \leq l \leq \phi_i} \left(\sqrt{|S_i''^l| \times Q_i/d_i^l} \right)} = v_i - v_i = 0.
\end{aligned}
$$

Therefore, bidding the true valuation is a dominant strategy for each buyer. We can conclude that SMASHER-GR satisfies incentive compatibility.

We now prove that SMASHER-GR also satisfies individual rationality. On one hand, the buyers losing in the auction get zero utility. On the other hand, the winning buyer i's utility

$$
\begin{aligned}
u_i &= v_i - p_i = v_i - cr(i) \times \max_{1 \leq l \leq \phi_i} \left(\sqrt{|S_i^l| \times Q_i/d_i^l} \right) \\
&= \left(\frac{v_i}{\max\limits_{1 \leq l \leq \phi_i} \left(\sqrt{|S_i^l| \times Q_i/d_i^l} \right)} - cr(i) \right) \\
&\quad \times \max_{1 \leq l \leq \phi_i} \left(\sqrt{|S_i^l| \times Q_i/d_i^l} \right) \\
&= \left(\tilde{b}_i - cr(i) \right) \times \max_{1 \leq l \leq \phi_i} \left(\sqrt{|S_i^l| \times Q_i/d_i^l} \right),
\end{aligned}
$$

where $\tilde{b}_i$ is the virtual bid of buyer i. From the definition of critical virtual bid, we have $\tilde{b}_i \geq cr(i)$ for winning buyer i. Then, winning buyer i gets non-negative utility. Buyer utility is always non-negative, which is not worse than staying outside the auction (i.e., utility is equal to zero). Therefore, we can conclude that SMASHER-GR satisfies individual rationality.

Since SMASHER-GR satisfies both incentive compatibility and individual rationality, SMASHER-GR is a strategy-proof mechanism, and then our claim holds. Similar to SMASHER-AP, the property of strategy-proofness can also be analyzed by applying Myerson's well known characterization [31]. □

We note that the performance of SMASHER-GR can be arbitrarily bad in some special cases. But the evaluation results show that SMASHER-GR performs quite well in most of cases.

6 EVALUATION RESULTS

In this section, we show our evaluation results.

6.1 Methodology

We implement SMASHER-AP and compare its performance with TAHES [9] and CRWDP [10]. We also show the performance of SMASHER-GR. Buyers are randomly distributed in a terrain area of 2000×2000 meters. The number of buyers varies from 20 to 400 with increment of 20. The number of leasing channels can be one of the three values: 6, 12
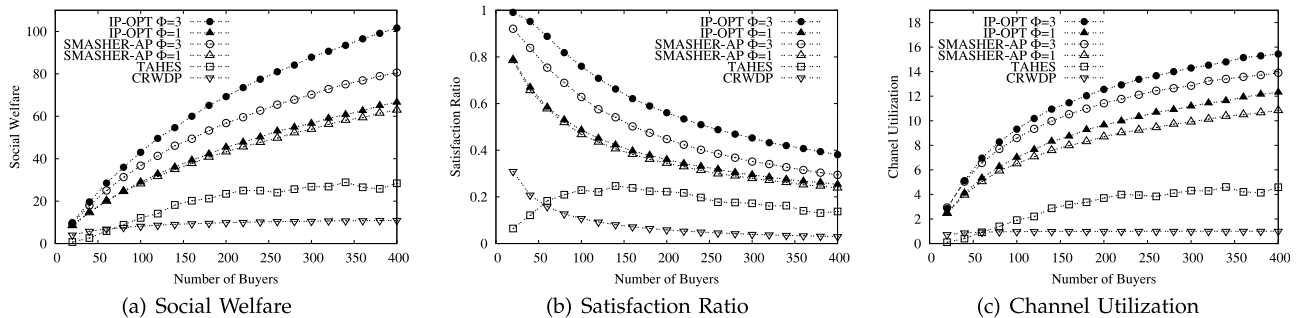
Fig. 2. Performance of SMASHER-AP, TAHES, CRWDP and IP-OPT, when there are 12 channels.

and 24. The heterogeneous channels have different interference ranges, spanning from 250 meters to 450 meters. We allow buyers to be equipped with different number of radios in our auctions, but limit the maximum size of requested channel bundle to 3. We assume that the buyers' valuations are randomly distributed over $(0, 1]$. We consider the case of single-minded buyers (i.e., $\Phi = 1$), and the case of multi-minded buyers who can submit up to three channel bundles (i.e., $\Phi = 3$). In SMASHER-GR, we normalize the length of time slot to 1. We assume the throughput of buyers and data rate of channel bundle are uniformly distributed in the interval $(0, 1]$. All the results of performance are averaged over 200 runs.[6]

Metrics. We evaluate three metrics:

- *Social Welfare:* Social welfare is the sum of winning buyers' valuations on their allocated bundles of channels.
- *Satisfactory Ratio:* Satisfactory ratio is the percentage of buyers who obtain one of their demanded channel bundles in SMASHER-AP or achieve their throughput in SMASHER-GR.
- *Channel Utilization:* Channel utilization is the average number of radios allocated to each channel.

6.2 Performance of SMASHER-AP

We compare the performance of SMASHER-AP with two strategy-proof heterogeneous channel auction mechanisms, TAHES and CRWDP. We evaluate the outcome of SMASHER-AP when the buyers are single-minded ($\Phi = 1$) and multi-minded ($\Phi = 3$). We also show the optimal results with tolerance 10^{-4}, denoted by IP-OPT, computed by solving the binary integer program in Section 3.2, as references of upper bound.

Fig. 2 shows the evaluation results when there are 12 channels and different number of buyers. We can see that SMASHER-AP always outperforms the other two auction mechanisms, and its performance approaches the optimum, especially when $\Phi = 1$. When the number of nodes is smaller than 60, TAHES cannot form sufficient buyer groups with a large number of bids, and thus does not perform well in this case. When the number of buyer is larger than 60, CRWDP's performance is not good because CRWDP does not consider channel spatial reusability

(i.e., the channel utilization of CRWDP is equal to 1 in all cases). Fig. 2 also shows that when the buyer number increases, the social welfare and channel utilization increase, but the satisfaction ratio decreases. On one hand, the larger number of buyers leads to more intense competition on limited channels, thus decreasing the satisfaction ratio. On the other hand, SMASHER-AP can allocate channels more efficiently among more buyers, hence the social welfare and channel utilization increase.

Fig. 3 shows the evaluation results when there are 200 buyers and the number of channels is 6, 12, 24. Again, SMASHER-AP always achieves better performance than TAHES and CRWDP, whenever $\Phi = 1$ or $\Phi = 3$. Fig. 3 also shows that when the number of channels increases, the social welfare and satisfaction ratio increase and the channel utilization decreases. The reason is that larger supply of leasing channels leads to more trades in the auction, thus the social welfare and satisfaction ratio increase when there exists a fixed number of buyers. The channel utilization decreases because buyers' radios can be allocated to more channels when the number of channels increases.

From Figs. 2 and 3, we can see that SMASHER-AP sacrifices limited system performance to achieve economic robustness. Although IP-OPT achieves near optimal social welfare, we cannot apply it to channel redistribution problem, because IP-OPT has not any guarantee on economic properties. We observe that SMASHER-AP with multi-minded buyers (i.e., $\Phi = 3$) always performs better than SMASHER-AP with single-minded buyers (i.e., $\Phi = 1$), on all the three metrics. This is because multi-minded buyers have higher chance to obtain channel bundles than single-minded buyers. This leads to more trades in the auction. Therefore, allowing buyers to submit multiple spectrum requests indeed improves the auction performance.

6.3 Performance of SMASHER-GR

Solving the mixed integer nonlinear program, shown in Section 5, is computational intractable even in small scale network scenarios. CRWDP only allocates time-frequency blocks, without considering the channel scheduling problem while TAHES does not take time multiplexing of channels into account, regarding channels as indivisible goods. Therefore, we only show the performance results of SMASHER-GR in Fig. 4.

Similarly, when the buyer number increases, the social welfare and channel utilization increase and the satisfaction ratio decreases. We can also observe that when the number of leasing channels increases from 12 to 24, the social

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6. All parameters can be different from the ones used here. However, the evaluation results of using different parameters are identical. Therefore, we only show the results for these parameters in this paper.
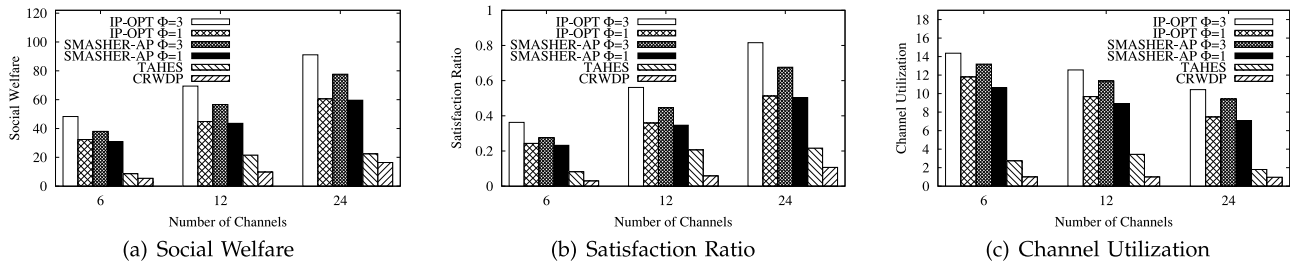
Fig. 3. Performance of SMASHER-AP, TAHES, CRWDP and IP-OPT, when there are 200 buyers.

welfare and satisfaction ratio increase while the channel utilization decreases. The reasons of the relation between system performance and the number of buyers or the number of channels is similar to those analyzed in SMASHER-AP. Fig. 4 also shows that SMASHER-GR with multi-minded buyers (i.e., $\Phi = 3$) always outperforms SMASHER-GR with single-minded buyers (i.e., $\Phi = 1$), on all the three metrics. This result verifies that diverse spectrum requests lead to higher auction performance. We can draw a conclusion that bid diversity is an effective strategy to improve the performance of channel redistribution system.

Compared with SMASHER-AP, SMASHER-GR performs better on all the three metrics. This is because channels can be shared among buyers in a paradigm of time-division multiplexing in SMASHER-GR. SMASHER-GR is an effective auction mechanism to channel redistribution, considering both spatial and temporal channel reusability.

7 RELATED WORKS

In this section, we briefly review related works on channel auction and auction mechanism design.

7.1 Channel Allocation with Selfish Participants

A number of works model the problem of channel allocation by game theory. Felegyhazi et al. [35] studied Nash Equilibria in a static multi-radio multi-channel allocation game. Later, Wu et al. [36] designed an incentive scheme for the multi-radio multi-channel allocation game, making the system converge to a much stronger equilibrium state. Gao and Wang studied the multi-radio channel allocation problem in multi-hop wireless networks, and proposed the min-max coalition-proof Nash Equilibrium channel allocation scheme in the cooperative game [37]. Yang et al. considered the channel allocation in multi-radio multi-channel wireless networks with multiple collision domains [38]. Chen and Huang proposed distributed spectrum sharing schemes to

coverage Nash equilibrium in spectrum access game [39], [40]. In cognitive radio networks, Kasbekar and Sarkar analyzed spectrum pricing game and computed Nash Equilibrium in different scenarios [41], [42], [43]. Byun et al., computed a market equilibrium, which is defined in context of extended Fisher model, for spectrum sharing in cognitive radio networks [44]. The paper [45] proposed a Quality of Experience driven channel allocation scheme for secondary users in cognitive radio networks. Resource allocation among selfish participants has been studied in different network scenarios, such as wireless mesh networks [46], OFDMA femtocell networks [47], and LTE networks [48].

The most closely related works are VERITAS [6], TRUST [7], and SMALL [8], all of which are auction-based strategy-proof channel allocation mechanisms. VERITAS and SMALL are single-sided auctions both supporting multiple channel requests. In contrast, TRUST elegantly extends double auction to consider both channel sellers and buyers' incentives. Recently, TAHES [9] was proposed to solve the problem of heterogeneous channel allocation. Besides, there are some other related works on channel auction, such as online channel auctions [49], [50], [51], collusion-resistant channel auction [52], revenue generation for spectrum auction [53], and approximate algorithms for different models of interference and different formats of valuations [25], [26].

7.2 Mechanism Design

A large number of works on combinatorial auctions have been proposed during the last decades. Dobzinski [17], Buchfuhrer et al. [54], and Papadimitriou et al. [16] proved that getting optimal social welfare and ensuring strategy-proofness cannot be achieved simultaneously in general combinatorial auctions. Lehmann et al. [18] even asserted that there is no payment scheme to make greedy allocation algorithm strategy-proof in general combinatorial auctions with multiminded buyers. Considering the intractability of combinatorial
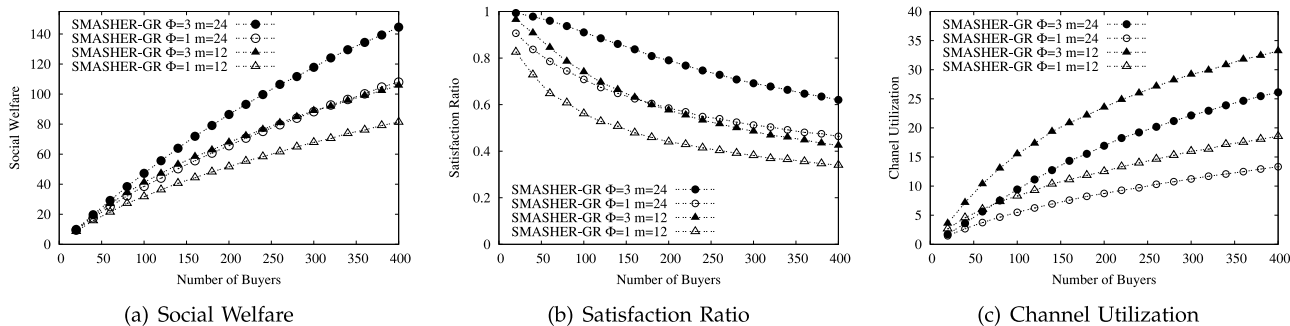

Fig. 4. Performance of SMASHER-GR in different network scenarios.

auction, a number of strategy-proof auction mechanisms with well bounded approximation ratios were proposed [55], [56],[57],[58]. In [10], the author modeled the time-frequency allocation problem as a combinatorial auction with single-minded buyers, and proposed a greedy allocation algorithm to achieve approximately efficient social welfare in a single collision domain. However, none of the above combinatorial auction considers the spectrum spatial reusability.

Auction mechanisms have been proposed to address different kinds of resource allocation problems, such as resource management in cloud computing [59], [60], sensing tasks distribution in mobile crowdsensing [61], [62], and cooperation dynamics on collaborative social networks [63].

Scheduling theory has received a growing interest since its origins, and there are various types of scheduling problems [64]. Recently, some works have studied the scheduling problems in the mechanism design context. Nisan and Ronen [65] were the first to consider makespan-minimization on unrelated machines. Later, Archer and Tardos [66] considered the related-machine problem and gave a 3-approximation truthful-in-expectation mechanism. However, the objective of these works is to minimize the makespan, while our objective is to maximize social welfare and the scheduling model we considered is more complicated.

8 CONCLUSION AND FUTURE WORKS

In this paper, we have made an in-depth study on channel redistribution problem by jointly considering the five design challenges. We have presented two closely related strategy-proof combinatorial auction mechanisms for dynamic heterogeneous channel redistribution, namely SMASHER-AP and SMASHER-GR. SMASHER-AP is a combinatorial auction mechanism for indivisible channel redistribution, achieving strategy-proofness and approximately efficient social welfare. SMASHER-GR is a strategy-proof combinatorial auction for joint channel allocation and scheduling for the scenarios, in which channels can be shared in a time-multiplexing way. We have also evaluated the performance of our designs. The simulation results have shown that our designs achieve good performance, in terms of social welfare, buyer satisfaction ratio, and channel utilization.

As for future works, we are interested in designing combinatorial channel auction mechanisms that can prevent collusion among multiple buyers. Designing a strategy-proof combinatorial channel auction mechanism to prevent buyers from cheating on channel bundle is also an interesting problem. Finally, there are additional economic properties that could be considered, such as fairness and false-name bidding resistent, in combinatorial auction mechanism design.

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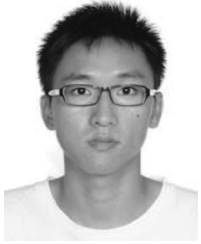
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