Online Pricing with Reserve Price Constraint for Personal Data Markets

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Abstract—The society's insatiable appetites for personal data are driving the emergence of data markets, allowing data consumers to launch customized queries over the datasets collected by a data broker from data owners. In this paper, we study how the data broker can maximize its cumulative revenue by posting reasonable prices for sequential queries. We thus propose a contextual dynamic pricing mechanism with the reserve price constraint, which features the properties of ellipsoid for efficient online optimization and can support linear and non-linear market value models with uncertainty. In particular, under low uncertainty, the proposed pricing mechanism attains a worst-case cumulative regret logarithmic in the number of queries. We further extend our approach to support other similar application scenarios, including hospitality service and online advertising, and extensively evaluate all three use cases over MovieLens 20M dataset, Airbnb listings in U.S. major cities, and Avazu mobile ad click dataset, respectively. The analysis and evaluation results reveal that: (1) our pricing mechanism incurs low practical regret, while the latency and memory overhead incurred is low enough for online applications; and (2) the existence of reserve price can mitigate the cold-start problem in a posted price mechanism, thereby reducing the cumulative regret.

Index Terms—personal data market, revenue maximization, contextual dynamic pricing, reserve price, ellipsoid

1 Introduction

NOWADAYS, tremendous volumes of diverse data are collected to seamlessly monitor human behaviors, such as product ratings, electrical usages, social media data, web cookies, health records, and driving trajectories. However, for the sake of security, privacy, or business competition, most of data owners are reluctant to share their data, resulting in a large number of data islands. Because of data isolation, potential data consumers (e.g., commercial companies, financial institutions, medical practitioners, and researchers) cannot benefit from private data. To facilitate personal data circulation, more and more data brokers have emerged to build bridges between the data owners and the data consumers. Typical data brokers in industry include Factual [2], DataSift [3], Datacoup [4], CitizenMe [5], and CoverUS [6]. On the one hand, a data broker needs to adequately compensate the data owners for the breach of their privacy caused by using their data to answer any data consumer's query, thereby incentivizing active data sharing. On the other hand, the data broker should properly charge the online data consumers for their sequential queries over the collected datasets, because both underpricing and overpricing may result in loss of revenue for the data broker. The data circulation ecosystem is conventionally called "data market" in the literature [7].

In this paper, we study how to trade personal data for revenue maximization from the data broker's standpoint in online data markets. We summarize three major design challenges as follows. The first and the thorniest challenge is that the objective function for optimization is quite complicated. The principal goal of a data broker in data markets is to maximize its cumulative revenue, which is defined as the difference between the prices of queries charged from the data consumers and the privacy compensations allocated to the data owners. Let's examine one round of data trading. Given a query, the privacy leakages together with the total privacy compensation, regarded as the reserve price of the query, are virtually fixed. Thus, for revenue maximization, an ideal way for the data broker is to post a price, taking the larger value of the query's reserve price and market value. However, the reality is that the data broker does not know the exact market value and can only estimate it from the context of the current query and the historical transaction records. Of course, a loose estimation will lead to different levels of regret: (1) if the reserve price is higher than the market value, implying that the posted price must be higher than the market value, the query definitely cannot be sold, no matter whether the data broker knows the market value or not. Thus, the regret is zero; and (2) if the reserve price is no more than the market value, a slight underestimation of the market value incurs a low regret, whereas a slight overestimation causes the query not to be sold, generating a high regret. Therefore, the initial goal of revenue maximization can be equivalently converted to minimizing the cumulative regret, particularly, the difference between the data broker's cumulative revenues with and without the knowledge of the market values. Considering even the single-round regret function is piecewise and highly asymmetric, it is nontrivial to perform optimization for multiple rounds.

Another challenge lies in how to model the market values of the customized queries from the data consumers. For regret minimization in pricing online queries, the pivotal step for the data broker is to gain a good knowledge of their market values. However, markets for personal data significantly differ from conventional markets in that each data consumer as a buyer rather than the data broker as a seller can determine the product, namely, a query. In general, each query involves a concrete data analysis method and a tolerable level of noise added to the true

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- *An early version of this work with the same title appeared as a 4-page poster paper in IEEE ICDE 2020 [1]. This journal version has added the principles, details, and analysis of our design, the evaluation results, the related work, as well as substantial illustrations and revisions.*

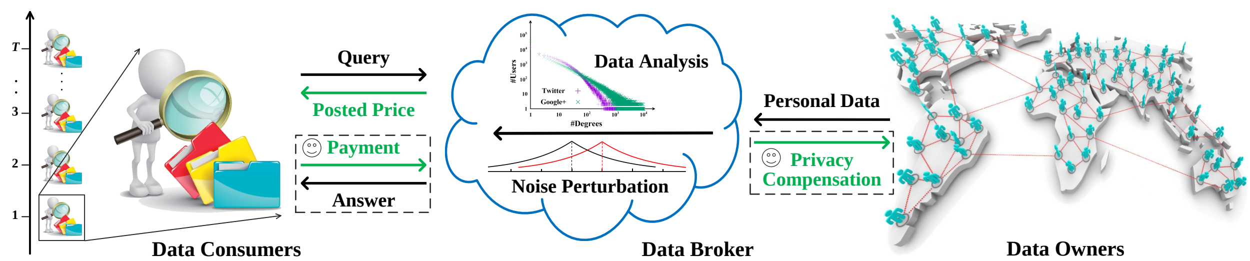


Fig. 1. A general system model of online personal data markets. The smile indicates that the posted price is accepted and a deal is made.

answer, which are both customized by a data consumer [8], [9]. Hence, the queries from different data consumers are highly differentiated and are uncontrollable by the data broker. This striking property further implies that most of the dynamic pricing mechanisms, which target identical products or a manageable number of distinct products, cannot apply here. In addition, existing work on data market design either considered a single query [10] or investigated the determinacy relation among multiple queries [9], [11]–[17], but ignored whether the data consumers accept or reject the marked prices. Thus, these work omitted modeling the market values of queries and is parallel to this work.

The ultimate challenge comes from the novel online pricing with reserve price setting. For the estimation of a query's market value, the data broker can exploit only the current and historical queries. Thus, the pricing of sequential queries can be viewed as an online learning process. Besides the usual tension between exploitation and exploration, our pricing problem has three atypical aspects: (1) the feedback after trading one query is very limited. The data broker can observe only whether the posted price for the query is higher than its market value or not, but cannot obtain the exact market value, which makes standard online learning algorithms [18] inapplicable; (2) the reserve price essentially imposes a lower bound on the posted price beyond the market value estimation, while the ordering between the reserve price and the market value is unknown. In addition, the impact of such a lower bound on the whole learning process has not been studied as of yet; and (3) the online mode requires our design of the posted price mechanism to be quite efficient. In other words, the data broker needs to choose each posted price and further update its knowledge about the market value model with low latency.

We outline the key contributions in this work as follows.

• To the best of our knowledge, we are the first to study trading personal data for revenue maximization from the perspective of a data broker in online data markets. In addition, we formulate it into a contextual dynamic pricing problem with the reserve price constraint.

• The proposed pricing mechanism features the properties of ellipsoid to exploit and explore the market values of sequential queries effectively and efficiently. It supports both linear and non-linear market value models and tolerates some uncertainty. The worst-case cumulative regret under low uncertainty is $O(\max(n^2 \log(T/n), n^3 \log(T/n)/T))$, where n is the dimension of feature vector and T is the total number of rounds. The time and space complexities are both $O(n^2)$. Further, our market framework can also support trading other similar products, which share customization, existence of reserve price, and timeliness with online queries.

• We evaluate three use cases over three real-world datasets. The major results are: (1) for the pricing of noisy linear query under the linear model, when $n = 100$ and the number of rounds t is 10^5, the regret ratio of our pricing mechanism with reserve price (resp., with reserve price and uncertainty) is 7.77% (resp., 9.87%), reducing 57.19% (resp., 45.64%) of the regret ratio than a risk-averse baseline, where the reserve price is posted in each round; (2) for the pricing of accommodation rental under the log-linear model, when $n = 55$, $t = 74,111$, and the ratio between the natural logarithms of the reserve price and market value is set to 0.6, the regret ratio of our pricing mechanism is 3.83%, reducing 77.46% of the regret ratio compared with the baseline; (3) for the pricing of impression under the logistic model, when $n = 1024$ and $t = 10^5$, the regret ratios of our pure pricing mechanism are 8.04% and 0.89% in the sparse and dense cases, respectively; and (4) the latency of three applications per round is each in the magnitude of millisecond (ms for short), while the memory overhead is each less than 160 MB.

• We instructively demonstrate that the reserve price can mitigate the cold-start problem in a posted price mechanism, thereby reducing the cumulative regret. Specifically, (1) for the pricing of noisy linear query, when $n = 20$ and $t = 10^4$, our pricing mechanism with reserve price (resp., with reserve price and uncertainty) reduces 13.16% (resp., 10.92%) of the cumulative regret than without reserve price; and (2) for the pricing of accommodation rental, as the reserve price approaches the market value, its impact on mitigating cold start is more evident.

2 Technical Overview

In this section, we introduce system model, problem formulation, and design principles.

2.1 System Model

As shown in Fig. 1, we consider a general system model for online personal data markets. There are three kinds of entities: data owners, a data broker, and data consumers.

The data broker first collects massive personal data from the data owners. Then, the data consumers come to the data market in an online fashion. In round $t \in [T]$, a data consumer arrives and makes a customized query Q_t over the collected dataset. Specifically, the query Q_t comprises a concrete data analysis method and a tolerable level of noise added to the true answer [8], [9]. Here, the noise perturbation not only can allow the data consumer to control the accuracy of a returned answer but also can preserve the privacy of the data owners.

Depending on Q_t and the underlying dataset, the data broker quantifies the privacy leakage of each data owner and needs to compensate it if a deal occurs. The data broker then offers a price p_t to the data consumer. If p_t is no more than the market value v_t of Q_t, this posted price will be accepted. The data broker charges the data consumer p_t, returns the noisy answer, and compensates the data owners as planned. Otherwise, this deal is aborted, and the data consumer goes away. To guarantee non-negative utility for the data broker no matter whether a deal occurs in round t or not, the posted price p_t should be no less than the total privacy compensation q_t. q_t functions as the *reserve price* and can be pre-computed when given Q_t.

We next give the online trading of noisy linear queries for example. A static market framework for trading the same products with marked prices was studied in [9].

Example 1. *A data broker, called Bob, maintains a vector* $(2, 1, 4, 3)$*, where each value is contributed by a data owner (e.g., denoting a student's rating for some course). Each data owner also signs a digital contract with Bob with respect to different levels of privacy leakage and corresponding compensations. In round 1, a data consumer, called Alice, launches a query* Q_1*, including "How many data owners have values higher than 3?" and "The variance of tolerable noise is no more than 0.1.". The level of noise guarantees an error of 1 with* 90% *confidence for the counting answer by Chebyshev's inequality. Given* Q_1*, Bob quantifies the privacy leakage of each data owner (e.g., using differential privacy-based method in [9]) and computes its privacy compensation under the contract. For example, the privacy compensations of 4 data owners are* $(0.3, 0.25, 0.2, 0.25)$*. Bob obtains the total privacy compensation* $q_1 = 1$ *and posts a price* p_1 *to Alice. Here,* p_1 *must be higher than the reserve price* q_1 *(e.g.,* $p_1 = 1.2$*). If Alice accepts (resp., rejects)* p_1*, Bob will know that the posted price is no more than (resp., higher than) the market value of* Q_1*, namely,* $p_1 \leq v_1$ *(resp.,* $p_1 > v_1$*). In round t, another data consumer launches another query* Q_t*, comprising a different type of statistic analysis (e.g., "What is the mean?") and a different tolerable variance of noise (e.g., 0.01). The holistic trading process is the same as that of round 1.*

2.2 Problem Formulation

We now formulate the regret minimization problem for pricing sequential queries in online personal data markets.

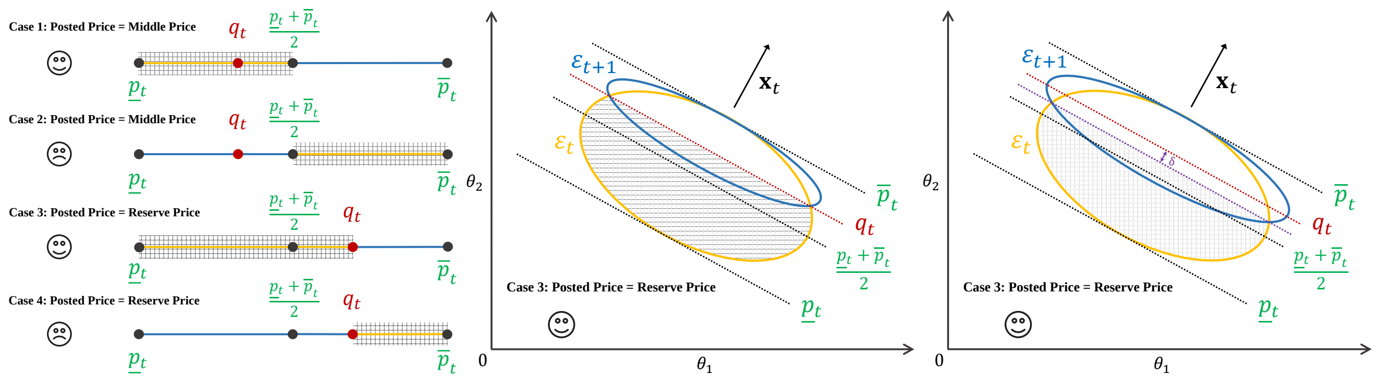
We first model the market values of customized and highly differentiated queries. We use an elementary assumption from *contextual pricing* in computational economics [19]–[21] and *hedonic pricing* in marketing [22], [23], which states that the market value of a product is a deterministic function of its features. Here, the product is a query, and the function can be linear or non-linear. To make the pricing model more robust, we allow for some uncertainty in the market value of each query. In particular, for a query Q_t, we let $\mathbf{x}_t \in \mathbb{R}^n$ denote its n-dimensional feature vector, let $f : \mathbb{R}^n \mapsto \mathbb{R}$ denote the mapping from the feature vector $\mathbf{x}_t$ to the deterministic part in its market value, and let $\delta_t \in \mathbb{R}$ denote the random variable in its market value, which is independent of $\mathbf{x}_t$. In a nutshell, $v_t = f(\mathbf{x}_t) + \delta_t$.

We next identify the features of a query for measuring its market value. One naïve way is to directly encode the contents of the whole query, including the data analysis method and the noise level. However, the query alone, especially the abstract data analysis method, is hard to embody its economic value. Let's examine the same type of simple queries in Example 1 for easy illustration: it is nontrivial to directly compare the economic values of the counting and mean statistics, let alone incorporating different levels of accuracy. Thus, we turn to leveraging the underlying valuations of the data owners about the query, namely, the privacy compensations, as the feature vector. We explain the rationality and feasibility of this feature representation: (1) the market value of a query depending on the privacy compensations inherits the core principle of *cost-plus pricing* [24], [25] and has been widely used in personal data pricing under the static market framework [9], [16], [17]. In particular, cost-plus pricing states that the market value of a product is determined by adding a specific amount of markup to its cost. Here, the cost is the total privacy compensation, the determinacy is reflected in the feature representation, and the markup is realized by setting the reserve price constraint; (2) the privacy compensations are observable by the data broker and can help it to discriminate the economic values of distinct queries. For example, the privacy compensations are higher, which implies that the privacy leakages of the data owners are larger, the knowledge discovered by the data consumer is richer, and thus the market value of the query to the data consumer should be higher; and (3) considering the scale of individual data owners can be large in practice, the dimension of the feature vector call be high as well. We can apply some celebrated dimension reduction techniques (e.g., Principal Components Analysis (PCA) [26]). We can also apply aggregation/clustering to the privacy compensations and regard the aggregate results as the feature vector, where the dimension n controls the granularity of aggregation. One extreme case is $n = 1$, where the only feature is the total privacy compensation; the other extreme case is n equal to the number of data owners, where every feature corresponds to a data owner's individual privacy compensation. Intuitively, we can interpret the aggregation technique as the introduction of n "master" data owners. Each master data owner represents and manages a group of "child" data owners for unified privacy compensation. We still examine Example 1 and set $n = 2$. We assume that one master data owner manages the first two data owners, while the other master data owner manages the last two data owners. Then, the feature vector of Q_1 is $\mathbf{x}_1 = (0.55, 0.45)$.

We finally define the cumulative regret of the data broker due to its limited knowledge of market values. We consider a game between the data broker and an adversary. During this game, the adversary chooses the sequence of queries $Q_1, Q_2, \ldots, Q_T$, selects the mapping f, but cannot control the uncertainty δ_t in each round t, namely, the adversary can determine the part $f(\mathbf{x}_t)$ in the market value v_t. In contrast, the data broker only can passively receive each query Q_t and then post a price p_t. If the posted price is no more than the market value (i.e., $p_t \leq v_t$), a deal occurs, and the data broker earns a revenue of p_t. Otherwise, the deal is aborted, and the data broker gains no revenue. We define the regret r_t in round t as the difference between the adversary's revenue and the data broker's revenue for trading the query Q_t. The detailed formula of r_t is

$$r_t = \begin{cases} 0 & \text{if } q_t > v_t, \\ \max\limits_{p_t^*} p_t^* \Pr\limits_{\delta_t}(p_t^* \leq v_t) - p_t \mathbf{1}\{p_t \leq v_t\} & \text{otherwise.} \end{cases}$$

In the first branch (as $q_t > v_t$), if the reserve price and thus the posted price are higher than the market value, there is no regret. This is because under such a circumstance, no matter whether the adversary knows the market value in advance or the data broker does not, there is definitely no deal and zero revenue. Let's consider Q_1 in Example 1: if the reserve price $q_1 = 1$ is higher than the market value $v_1 = 0.8$, then the posted price $p_1 > q_1 = 1$ must be higher than $v_1 = 0.8$, implying that Alice certainly rejects p_1. In the second branch (as $q_t \leq v_t$), p_t^* is the adversary's optimal



(a) One-Dimensional Scenario (b) Multi-Dimensional Scenario (c) Multi-Dimensional Scenario with Uncertainty

Fig. 2. Illustrations of (effective) exploratory posted prices under the linear market value model.

posted price to maximize its expected revenue in round t, where the expectation is taken over δ_t. When δ_t is omitted, the adversary will just post the market value if the reserve price is no more than the market value (i.e., $q_t \le p_t^* = v_t$), and r_t will change to:

$$r_t = \begin{cases} 0 & \text{if } q_t > v_t, \\ v_t - p_t \mathbf{1}\{p_t \le v_t\} & \text{otherwise.} \end{cases} \quad (1)$$

At last, considering the sequential queries can be chosen adversarially (e.g., by other competitive data brokers or malicious data consumers), our design goal is to minimize the total worst-case regret accumulated over T rounds.

2.3 Design Principles

We overview our pricing framework and illustrate its key principles. We first consider the deterministic linear market value model, where f is a linear function, parameterized by a weight vector $\theta^* \in \mathbb{R}^n$. In other words, the market value of the query Q_t is $v_t = \mathbf{x}_t{}^T\theta^*$. We then consider extensions to the uncertain setting and non-linear models.

We start with a special case of the linear model, where each feature vector $\mathbf{x}_t$ is one-dimensional (i.e., $n = 1$). For example, the single feature can be the total privacy compensation or the reserve price q_t, and the weight θ^* denotes some fixed but unknown revenue-to-cost ratio. We note that to minimize the regret in pricing the query Q_t, the data broker needs to have a good estimation of its market value v_t, which can be equivalently converted to gaining a good knowledge of the observable feature $\mathbf{x}_t$'s market value, namely θ^*. We let $\mathcal{K}_t$ denote the data broker's knowledge set of θ^* in round t. In addition, the initial knowledge set $\mathcal{K}_1$ can be an interval $[\ell, u]$ for some $\ell, u \in \mathbb{R}$. Moreover, after round t, if the posted price p_t is rejected (resp., accepted), the data broker will update its knowledge set $\mathcal{K}_t$ to $\mathcal{K}_{t+1} = \mathcal{K}_t \bigcap \{\theta \in \mathbb{R} | p_t \ge \mathbf{x}_t{}^T\theta\}$ (resp., $\mathcal{K}_{t+1} = \mathcal{K}_t \bigcap \{\theta \in \mathbb{R} | p_t \le \mathbf{x}_t{}^T\theta\}$). Now, the key problem for the data broker is how to set the posted price p_t. In fact, the knowledge set $\mathcal{K}_t$ can impose a lower bound $\underline{p}_t = \min_{\theta \in \mathcal{K}_t} \mathbf{x}_t{}^T\theta$ and an upper bound $\bar{p}_t = \max_{\theta \in \mathcal{K}_t} \mathbf{x}_t{}^T\theta$ on estimating the market value v_t and thus on the posted price p_t, while the reserve price q_t imposes the other lower bound on the posted price p_t. If the posted price p_t is $\max(q_t, \underline{p}_t)$, the data broker can sell the query Q_t with the highest probability. However, in the worst case, where $q_t \le \underline{p}_t$, this deal will not refine the knowledge set (i.e., $\mathcal{K}_{t+1} = \mathcal{K}_t$) and thus cannot benefit the following rounds.

We call such a price $\max(q_t, \underline{p}_t)$ a *conservative price*. On the other hand, as shown in Fig. 2a, inspired by bisection, we define the larger value of the reserve price and the middle price (i.e., $\max(q_t, \frac{\underline{p}_t + \bar{p}_t}{2})$) as an *exploratory price*. In the worst case, the feedback from posting this price can narrow down the knowledge set $\mathcal{K}_t$ by most and thus can benefit the following rounds most. Of course, compared with the conservative price, the exploratory price would suffer a higher risk of no sale or losing the current revenue. We note that both the conservative price and the exploratory price have adequately exploited the experience from the previous rounds (i.e., the latest knowledge set $\mathcal{K}_t$), and the difference is that these two types of posted prices give distinct biases to the immediate rewards (exploitation) and the future rewards (exploration). Accompanied with the key problem of setting posted prices, another problem is when the data broker should choose which price. Our strategy is to measure the size of the knowledge set $\mathcal{K}_t$ (e.g., the width of interval in the one-dimensional case). If it exceeds some threshold, the data broker chooses the exploratory price to further improve its knowledge set; otherwise, its knowledge set is near optimal, and the data broker chooses the conservative price. In our real design, we use $\bar{p}_t - \underline{p}_t$ to capture the size of $\mathcal{K}_t$ and let $\epsilon > 0$ denote the threshold.

We next take Example 1 as a running instance of our one-dimensional design. We set the revenue-to-cost ratio $\theta^* = 1.4$, set Bob's initial knowledge set $\mathcal{K}_1 = [1, 2]$, and set $\epsilon = 0.07$. In round 1, given the feature of Q_1 (i.e., $\mathbf{x}_1 = q_1 = 1$), Bob computes the lower bound and the upper bound on estimating the market value, namely, $\underline{p}_t = 1 \times 1 = 1$ and $\bar{p}_t = 1 \times 2 = 2$. Thus, the conservative price is $\max(1, 1) = 1$, and the exploratory price is $\max(1, \frac{1+2}{2}) = 1.5$. Considering $\bar{p}_t - \underline{p}_t = 1 > \epsilon$, Bob posts the exploratory price $p_t = 1.5$, which is higher than the market value $v_1 = 1 \times 1.4 = 1.4$ and is rejected by Alice. Bob has a regret of $r_1 = 1.4$, but narrows its knowledge set $\mathcal{K}_1$ to $\mathcal{K}_2 = [1, 1.5)$, significantly benefiting the following $T - 1$ rounds. Assume that Bob posted the conservative price 1, which is lower than v_1 and would be accepted by Alice. Bob would have a lower regret of $1.4 - 1 = 0.4$, but cannot refine its knowledge set to benefit the following rounds.

We further consider the general linear model with multiple features (i.e., $n \ge 2$). The holistic process is the same. The difference lies in the concrete form of the knowledge set $\mathcal{K}_t$. In the one-dimensional case, $\mathcal{K}_t$ is an interval, while the minimum and maximum possible market values (i.e., $\underline{p}_t$ and

$\bar{p}_t$) can be efficiently computed from $\mathcal{K}_t$. However, when extended to the multi-dimensional case, we assume that the initial knowledge set is $\mathcal{K}_1 = \{\theta \in \mathbb{R}^n | \ell_i \leq \theta_i \leq u_i, \ell_i, u_i \in \mathbb{R}\}$. After each round, the knowledge set is updated by adding a linear inequality. Thus, the knowledge set $\mathcal{K}_t$ can be viewed as a set of linear inequalities, the cardinality of which is non-decreasing with the number of rounds t. To post a price in round t, it suffices to solve two linear programs under $\mathcal{K}_t$, which is quite time-consuming and can be computationally infeasible in online mode. Therefore, we turn to borrowing some key principles from the celebrated ellipsoid method for solving online linear programs, which was first proposed by Khachiyan in 1979 [27]. The key idea is to replace the raw knowledge set $\mathcal{K}_t$, viewed as a polytope in geometry, with the ellipsoid $\mathcal{E}_t$ of the minimum volume that contains $\mathcal{K}_t$. $\mathcal{E}_t$ is called the Löwner-John ellipsoid of the convex body $\mathcal{K}_t$. By leveraging the property that every ellipsoid is an image of the unit ball under a bijective affine transformation [28], the data broker can efficiently determine the posted price and further update its knowledge set in each round, requiring only a few matrix-vector and vector-vector multiplications. Fig. 2b gives an illustration of the exploratory posted price in the two-dimensional case.

We finally consider the uncertain setting and non-linear models. First, for tractability, we make a common assumption on the randomness δ_t in the market value v_t, where the distribution of δ_t belongs to subGaussian. We thus bound the absolute value of any δ_t in all T rounds by δ with probability near 1. We regard δ as a "buffer" in posting the price and updating the knowledge set, which can circumvent the randomness δ_t in each round. Second, we mainly investigate four classic non-linear models in market value estimation, whose pattern is first applying an inner feature mapping to the feature vector, then performing dot product with the weight vector, and finally applying an outer non-decreasing and continuous function. By still focusing on the discovery of the weight vector rather than the inner and outer non-linear functions, we can extend our pricing mechanism to support this class of non-linear market value models.

Algorithm 1: An Online Pricing Mechanism for Personal Data Markets

Input: $\mathbf{A}_1 = R^2\mathbf{I}_{n\times n}$, $\mathbf{c}_1 = \mathbf{0}_{n\times 1}$, an uncertainty parameter $\delta = \sqrt{2\log C}\sigma \log T$, a threshold ϵ.
Output: The posted price p_t in each round $t \in [T]$.

for $t = 1, 2, \ldots, T$ **do**
- $\mathcal{E}_t = \{\theta \in \mathbb{R}^n | (\theta - \mathbf{c}_t)^T \mathbf{A}_t^{-1} (\theta - \mathbf{c}_t) \leq 1\}$;
- Receives a query Q_t with the feature vector $\mathbf{x}_t \in \mathbb{R}^n$;
- Determines the reserve price q_t of Q_t;
- $\mathbf{b}_t = \frac{\mathbf{A}_t\mathbf{x}_t}{\sqrt{{\mathbf{x}_t}^T\mathbf{A}_t\mathbf{x}_t}}$;
- $\underline{p}_t = \min_{\theta\in\mathcal{E}_t} {\mathbf{x}_t}^T\theta = {\mathbf{x}_t}^T(\mathbf{c}_t - \mathbf{b}_t)$;
- $\bar{p}_t = \max_{\theta\in\mathcal{E}_t} {\mathbf{x}_t}^T\theta = {\mathbf{x}_t}^T(\mathbf{c}_t + \mathbf{b}_t)$;
- **if** $q_t \geq \bar{p}_t + \delta$ **then**
  - $\mathbf{A}_{t+1} = \mathbf{A}_t$; $\mathbf{c}_{t+1} = \mathbf{c}_t$;
  - **continue**;
- **else**
  - **if** $\bar{p}_t - \underline{p}_t = 2\sqrt{{\mathbf{x}_t}^T\mathbf{A}_t\mathbf{x}_t} > \epsilon$ **then**
    - Posts a price $p_t = \max\left\{q_t, \frac{\underline{p}_t + \bar{p}_t}{2} = {\mathbf{x}_t}^T\mathbf{c}_t\right\}$;
    - **if** p_t is rejected **then**
      - $\alpha_t = \frac{\frac{\underline{p}_t+\bar{p}_t}{2} - (p_t + \delta)}{\sqrt{{\mathbf{x}_t}^T\mathbf{A}_t\mathbf{x}_t}} = \frac{{\mathbf{x}_t}^T\mathbf{c}_t - p_t - \delta}{\sqrt{{\mathbf{x}_t}^T\mathbf{A}_t\mathbf{x}_t}}$;
      - **if** $-\frac{1}{n} \leq \alpha_t \leq 1$ **then**
        - $\mathbf{A}_{t+1} = \frac{n^2(1-{\alpha_t}^2)}{n^2-1}\left(\mathbf{A}_t - \frac{2(1+n\alpha_t)}{(n+1)(1+\alpha_t)}\mathbf{b}_t{\mathbf{b}_t}^T\right)$;
        - $\mathbf{c}_{t+1} = \mathbf{c}_t - \frac{1+n\alpha_t}{n+1}\mathbf{b}_t$;
      - **else**
        - $\mathbf{A}_{t+1} = \mathbf{A}_t$; $\mathbf{c}_{t+1} = \mathbf{c}_t$;
    - **else**
      - $\alpha_t = \frac{\frac{\underline{p}_t+\bar{p}_t}{2} - (p_t - \delta)}{\sqrt{{\mathbf{x}_t}^T\mathbf{A}_t\mathbf{x}_t}} = \frac{{\mathbf{x}_t}^T\mathbf{c}_t - p_t + \delta}{\sqrt{{\mathbf{x}_t}^T\mathbf{A}_t\mathbf{x}_t}}$;
      - **if** $-\frac{1}{n} \leq -\alpha_t \leq 1$ **then**
        - $\mathbf{A}_{t+1} = \frac{n^2(1-{\alpha_t}^2)}{n^2-1}\left(\mathbf{A}_t - \frac{2(1-n\alpha_t)}{(n+1)(1-\alpha_t)}\mathbf{b}_t{\mathbf{b}_t}^T\right)$;
        - $\mathbf{c}_{t+1} = \mathbf{c}_t + \frac{1-n\alpha_t}{n+1}\mathbf{b}_t$;
      - **else**
        - $\mathbf{A}_{t+1} = \mathbf{A}_t$; $\mathbf{c}_{t+1} = \mathbf{c}_t$;
  - **else**
    - Posts a price $p_t = \max\{q_t, \underline{p}_t - \delta\}$;
    - $\mathbf{A}_{t+1} = \mathbf{A}_t$; $\mathbf{c}_{t+1} = \mathbf{c}_t$;

3 Fundamental Design Under Linear Market Value Model

In this section, we propose an ellipsoid-based pricing mechanism under the deterministic linear model and then extend it to tolerate uncertainty. We also analyze the time and space complexities as well as the worst-case cumulative regret.

3.1 Ellipsoid-Based Pricing Mechanism

As an appetizer, we first briefly review the definition of an ellipsoid and some of its key properties.

Definition 1. *$\mathcal{E} \subseteq \mathbb{R}^n$ is an ellipsoid, if there exists a vector $\mathbf{c} \in \mathbb{R}^n$ and a positive definite matrix $\mathbf{A} \in \mathbb{R}^{n\times n}$ such that:*

$$\mathcal{E} = \left\{\theta \in \mathbb{R}^n \middle| (\theta - \mathbf{c})^T \mathbf{A}^{-1} (\theta - \mathbf{c}) \leq 1\right\}. \tag{2}$$

Intuitively, $\mathbf{c}$ represents the center of the ellipsoid $\mathcal{E}$, while $\mathbf{A}$ portrays its shape. In particular, there are some useful connections between the geometric properties of $\mathcal{E}$ and the algebraic properties of $\mathbf{A}$. We let $\gamma_i(\mathbf{A}) > 0$ denote the i-th largest eigenvalue of $\mathbf{A}$. Then, the i-th widest axis (resp., its width) of the ellipsoid $\mathcal{E}$ corresponds to the i-th eigenvector (resp., $2\sqrt{\gamma_i(\mathbf{A})}$). In addition, the volume of the ellipsoid $\mathcal{E}$, denoted as $V(\mathcal{E})$, depends only on the eigenvalues of $\mathbf{A}$ and the dimension n. Specifically, $V(\mathcal{E}) = V_n\sqrt{\prod_{i\in[n]}\gamma_i(\mathbf{A})}$, where V_n is the volume of the unit ball in $\mathbb{R}^n$ and is a constant that hinges only on n.

We now present the ellipsoid-based posted price mechanism with the reserve price constraint for online personal data markets in Algorithm 1 (omitting the uncertainty parameter δ here, also called "the version with reserve price" in our evaluation part). We recall that the initial knowledge set of the data broker about the weight vector θ^* is $\mathcal{K}_1 = \{\theta \in \mathbb{R}^n | \ell_i \leq \theta_i \leq u_i, \ell_i, u_i \in \mathbb{R}\}$. We choose a ball centered at the origin with radius $R = \sqrt{\sum_{i\in[n]}\max({\ell_i}^2, {u_i}^2)}$ to enclose $\mathcal{K}_1$. This ball can serve as the initial ellipsoid $\mathcal{E}_1$, where $\mathbf{A}_1 = R^2\mathbf{I}_{n\times n}$ and $\mathbf{c}_1 = \mathbf{0}_{n\times 1}$ (Input). In what follows, we focus on a concrete round t.

The data broker receives a query Q_t with the feature vector $\mathbf{x}_t$ from a data consumer. Without loss of generality, we assume that $\forall t \in [T], \|\mathbf{x}_t\| \leq S$ for some $S \geq 1$. Then, the data broker virtually computes the total privacy compensation allocated to the data owners as the reserve price q_t, which imposes a strict lower bound on the posted price p_t. Based on the knowledge set $\mathcal{E}_t$, the data broker can elicit that the market value of the query Q_t falls into a certain interval, namely, $v_t = {\mathbf{x}_t}^T\theta^* \in [\underline{p}_t, \bar{p}_t]$ (Lines 5–7). If the reserve price is no less than the maximum possible market value, implying that the posted price should be no less than the market value, namely, $p_t \geq q_t \geq \bar{p}_t \geq v_t$, the query Q_t cannot be sold (Lines 8–10); otherwise, the data broker judges whether the difference between the maximum and minimum possible market values (i.e., $\bar{p}_t - \underline{p}_t$) exceeds a threshold ϵ. If yes, the data broker posts the exploratory price (Lines 12–13); otherwise, it posts the conservative price (Lines 26–27). In fact, the posted price places a cut on the ellipsoid $\mathcal{E}_t$ and splits it into two parts, where the cutting hyperplane is $\{\theta \in \mathbb{R}^n | p_t = {\mathbf{x}_t}^T\theta\}$. In addition, the data broker can compute a parameter α_t to locate the position of the cut (Line 15 or 21). Formally, α_t is interpreted as the signed distance from the center $\mathbf{c}_t$ to the cutting hyperplane, measured in the space $\mathbb{R}^n$ endowed with the ellipsoidal norm $\|\cdot\|_{{\mathbf{A}_t}^{-1}}$. For example, if the posted price is the middle price (i.e., $p_t = \frac{\underline{p}_t+\bar{p}_t}{2} = {\mathbf{x}_t}^T\mathbf{c}_t$), the center $\mathbf{c}_t$ is on the cutting hyperplane, and $\alpha_t = 0$. Moreover, according to the feedback from the data consumer, the data broker can decide to retain which side of the ellipsoid $\mathcal{E}_t$ and update to its Löwner-John ellipsoid $\mathcal{E}_{t+1}$ by computing the new shape $\mathbf{A}_{t+1}$ and center $\mathbf{c}_{t+1}$ (Lines 14–25). In particular, Grötschel et al. [28] have offered the formulas of $\mathbf{A}_{t+1}$ and $\mathbf{c}_{t+1}$, when the remaining part of $\mathcal{E}_t$ is contained in the halfspace like $\{\theta \in \mathbb{R}^n | p_t \geq {\mathbf{x}_t}^T\theta\}$. This corresponds to the rejection branch (Lines 14–19). By the symmetry of ellipsoid, we can obtain the formulas in the acceptance branch (Lines 20–25). Furthermore, if the remaining part after a cut is exactly half of the ellipsoid $\mathcal{E}_t$, we call the cut a *central cut*; if the remaining part is less than half, we call it a *deep cut*; and if the remaining part is more than half, we call it a *shallow cut*. Last, it is worth noting that the data broker is prohibited from refining the ellipsoid with the conservative price (Line 28). The reason is that $\bar{p}_t - \underline{p}_t$ essentially probes the ellipsoid's width along the direction given by the feature vector $\mathbf{x}_t$ (Please see Fig. 2b for an intuition.), which is very small ($\leq \epsilon$) when posting the conservative price. Suppose the data broker is allowed to cut along this direction. By adversarially setting the reserve prices, the width of ellipsoid along this direction can shrink successively, while the widths along the other directions can expand exponentially, which can result in $O(T)$ worst-case cumulative regret. Details about the adversarial example and its regret analysis are reserved in our technical report [29].

We finally discuss a special case by executing the above pricing mechanism without the reserve price constraint (omitting both δ and q_t in Algorithm 1, also called "the pure version" in our evaluation part). First, the exploratory posted price takes the middle price $\frac{\underline{p}_t+\bar{p}_t}{2}$ and poses a central cut over the ellipsoid $\mathcal{E}_t$. Second, the conservative posted price takes the minimum possible market value $\underline{p}_t$, which is definitely no more than the real market value v_t and must be accepted by the data consumer. In addition, the conservative posted price does not refine $\mathcal{E}_t$ and incurs a shallow cut. In a nutshell, there is no deep cut in this special case.

3.2 Incorporating Uncertainty

We extend our online pricing mechanism under the deterministic linear model to the uncertain setting. We make an assumption on the random variable δ_t in the market value model. We assume that the distribution of δ_t is σ-subGaussian, i.e., there exists a constant $C \in \mathbb{R}$ such that:

$$\forall z > 0, \Pr\left(|\delta_t| > z\right) \leq C \exp\left(-\frac{z^2}{2\sigma^2}\right). \tag{3}$$

This is a common assumption widely used in modeling uncertainty [30], [31]. In particular, many celebrated probability distributions, including normal distribution, uniform distribution, Rademacher distribution, and bounded random variables are subGaussian. For example, normal distribution is σ-subGaussian for its standard deviation σ and for $C = 2$ [30]. By assigning a value $\delta = \sqrt{2\log C}\sigma \log T$ to the variable z in Equation (3), we obtain:

$$\Pr\left(|\delta_t| > \delta\right) \leq T^{-\log T}. \tag{4}$$

We further apply Boole's inequality to the above inequality for all $t \in [T]$ and derive:

$$\begin{aligned} &\exists t \in [T], \Pr\left(|\delta_t| > \delta\right) \leq T^{1-\log T} \\ \Rightarrow \quad &\forall t \in [T], \Pr\left(|\delta_t| \leq \delta\right) \geq 1 - T^{1-\log T} \geq 1 - 1/T, \end{aligned} \tag{5}$$

where the last inequality holds for $T \geq 8$.

From Equation (5), we can draw that in each round t, the randomness δ_t in the market value v_t is bounded by δ in absolute value with probability at least $1 - 1/T$. Therefore, when posting the price and updating the knowledge set, we let the data broker introduce a "buffer" of size δ to circumvent the randomness δ_t. Specifically, if the data broker posts the price p_t and observes a rejection, it can no longer infer that $p_t \geq {\mathbf{x}_t}^T\theta^*$. Instead, it should infer that $p_t \geq v_t = {\mathbf{x}_t}^T\theta^* - \delta_t \geq {\mathbf{x}_t}^T\theta^* - \delta$. In a similar way, if the data broker observes an acceptance, it will infer that $p_t \leq v_t = {\mathbf{x}_t}^T\theta^* + \delta_t \leq {\mathbf{x}_t}^T\theta^* + \delta$ rather than $p_t \leq {\mathbf{x}_t}^T\theta^*$. Intuitively, in the case of rejection (resp., acceptance), the data broker imagines that it had posted $p_t + \delta$ (resp., $p_t - \delta$). We call $p_t + \delta$ (resp., $p_t - \delta$) the *effective posted price* in the case of rejection (resp., acceptance).

We now present the robust pricing mechanism in Algorithm 1 (called "the version with reserve price and uncertainty" in our evaluation part). For conciseness, we illustrate the differences after introducing uncertainty. First, in Lines 8–10, the condition for a certain no deal changes into $q_t \geq \bar{p}_t + \delta$. Only under this condition, the posted price must be no less than the market value, since $p_t \geq q_t \geq \bar{p}_t + \delta \geq v_t = {\mathbf{x}_t}^T\theta^* + \delta_t$. Second, in Lines 15 and 21, we use the effective exploratory prices to compute the positions of the cutting hyperplanes. In particular, due to the uncertainty in the market value, if the data broker posts the same price, the feedback from the data consumer can result in a smaller refinement of the knowledge set. We provide Fig. 2c for a visual comparison with Fig. 2b. Third, in Line 27, the conservative posted price, involving $\underline{p}_t$, decreases by δ to keep its high acceptance ratio.

We finally investigate Algorithm 1 without the reserve price constraint, denoted as Algorithm 1* (also called "the version with uncertainty" in our evaluation part). First, the exploratory posted price is the middle price $\frac{\underline{p}_t+\bar{p}_t}{2}$. The effective exploratory price used in refining the ellipsoid is $\frac{\underline{p}_t+\bar{p}_t}{2} + \delta$ (resp., $\frac{\underline{p}_t+\bar{p}_t}{2} - \delta$) in the case of rejection (resp., acceptance), and the corresponding position parameter α_t is

$-\delta/\sqrt{\mathbf{x}_t{}^T\mathbf{A}_t\mathbf{x}_t}$ (resp., $\delta/\sqrt{\mathbf{x}_t{}^T\mathbf{A}_t\mathbf{x}_t}$). As $\delta > 0$, the effective exploratory prices will refine the ellipsoid less than half. Second, the conservative posted price is $\underline{p}_t - \delta$ and can be either rejected or accepted. Here, the rejection case happens when the market value is outside the interval $[\underline{p}_t - \delta, \bar{p}_t + \delta]$ and has probability no more than $1/T$ by Equation (5). In addition, the conservative price keeps the ellipsoid unchanged. Jointly considering two types of posted prices, we can find that Algorithm 1* only has shallow cuts.

3.3 Performance Analysis

We analyze the time and space complexities, and the worst-case cumulative regret of Algorithm 1.

3.3.1 Time and Space Complexities

Considering the data broker needs to run the posted price mechanism online, Algorithm 1 should be quite efficient. We analyze single-round time and space complexities. First, the computation overhead of the data broker in round t mainly comes from two parts: (1) determining the posted price p_t, which roughly consumes 2 matrix-vector and 3 vector-vector multiplications; and (2) updating the shape and the center of the ellipsoid, which roughly consumes 1 vector-vector multiplication in the worst case. Thus, the time complexity is $O(n^2)$. Second, the memory overhead of the data broker is mainly caused by maintaining the knowledge set $\mathcal{E}_t$, or alternatively, the shape and the center of the ellipsoid, which requires 1 $n \times n$ matrix and 1 $n \times 1$ vector, respectively. Hence, the space complexity is $O(n^2)$.

3.3.2 Worst-Case Cumulative Regret

We analyze the worst-case cumulative regret of Algorithm 1, which is $O(\max(n^2 \log(T/n), n^3 \log(T/n)/T))$ under the low uncertain setting $\delta = O(n/T)$, namely, Theorem 1. We first prove that the existence of reserve price cannot increase the regret of a posted price mechanism in single round (Lemma 1). Thus, we can use Algorithm 1 without the reserve price constraint, namely, Algorithm 1*, as a springboard. In particular, to get an upper bound on the cumulative regret of Algorithm 1, we need to derive an upper bound on the number of rounds where the exploratory prices are posted, denoted as T_e. We derive this upper bound in a roundabout way: we first obtain the upper bound in Algorithm 1* (Lemma 5) and further prove that it still holds in Algorithm 1 by reduction and analyzing the impact of reserve price (Lemma 6). We elicit Lemma 5 in a squeezing manner, particularly, through constructing an upper bound and a lower bound on the final volume of the ellipsoid. For the upper bound, we adopt a core technique in proving the convergence of the traditional ellipsoid method: the ratio between the volumes of an ellipsoid and the Löwner-John ellipsoid after a cut has an upper bound (Lemma 2) [28]. Regarding the lower bound, we resort to the formula for computing an ellipsoid's volume by multiplying all the eigenvalues of its shape matrix. Thus, we can find a lower bound on the volume, by constructing a lower bound on the smallest eigenvalue (Lemmas 3 and 4). We present the detailed lemmas and theorem as follows, while reserving the proofs of Lemmas 3, 4, and 5 in our technical report [29].

Lemma 1. *The existence of reserve price cannot increase the regret of a posted price mechanism in single round.*

Proof. For round t, we still let v_t denote the market value and let q_t denote the reserve price. We introduce p'_t as the pure posted price and still let p_t denote the posted price with the reserve price constraint, where $p_t = \max(q_t, p'_t)$. We can express the regret of the posted price mechanism without reserve price in round t as:

$$r'_t = v_t - p'_t \mathbf{1}\left\{p'_t \le v_t\right\}. \tag{6}$$

After introducing the reserve price constraint, the regret changes to r_t given in Equation (1). We now prove $r_t \le r'_t$ in two complementary cases: $q_t > v_t$ and $q_t \le v_t$.

Case 1 ($q_t > v_t$): We can derive that $r_t = 0 \le r'_t$.

Case 2 ($q_t \le v_t$): We can derive that:

$$\begin{aligned} r_t &= v_t - p_t \mathbf{1}\left\{p_t \le v_t\right\} \\ &= v_t - \max\left(q_t, p'_t\right) \mathbf{1}\left\{\max\left(q_t, p'_t\right) \le v_t\right\} \\ &= v_t - \max\left(q_t, p'_t\right) \mathbf{1}\left\{p'_t \le v_t\right\} &\quad (7)\\ &\le v_t - p'_t \mathbf{1}\left\{p'_t \le v_t\right\} = r'_t, &\quad (8) \end{aligned}$$

where Equation (7) follows from that under the antecedent $q_t \le v_t$, the conditional statement $\{\max(q_t, p'_t) \le v_t \Leftrightarrow q_t \le v_t$ and $p'_t \le v_t\}$ can be simplified to $p'_t \le v_t$. Additionally, the inequality in Equation (8) follows from the maximum function and takes equal sign when $q_t \le p'_t$.

Jointly considering two cases, we complete the proof. □

Lemma 2. *Let $\mathcal{E}_{t+1}$ denote the Löwner-John ellipsoid obtained after a cut over the ellipsoid $\mathcal{E}_t$ with the position parameter α_t. If $\alpha_t \in [-1/n, 0]$, then $\frac{V(\mathcal{E}_{t+1})}{V(\mathcal{E}_t)} \le \exp\left(-\frac{(1+n\alpha_t)^2}{5n}\right)$.*

Lemma 3. *In Algorithm 1* ($\epsilon \ge 4n\delta$), there exists $\tau \in \mathbb{R}$ such that $\gamma_n(\mathbf{A}_t) \le \tau\epsilon^2, \mathbf{x}_t{}^T\mathbf{A}_t\mathbf{x}_t > \epsilon^2/4 \Rightarrow \gamma_n(\mathbf{A}_{t+1}) \ge \gamma_n(\mathbf{A}_t)$. In addition, $\tau = \frac{1}{400n^2S^4}$ is a feasible solution.*

Lemma 4. *For any round t in Algorithm 1* ($\epsilon \ge 4n\delta$) where the exploratory price is posted, $\gamma_n(\mathbf{A}_{t+1}) \ge \frac{n^2(1-\alpha_t)^2}{(n+1)^2}\gamma_n(\mathbf{A}_t)$.*

We interpret the intuitions behind Lemmas 3 and 4. Lemma 3 says that if the smallest eigenvalue is below some threshold (i.e., $\tau\epsilon^2$), it can no longer decrease. Lemma 4 says that in each round, the smallest eigenvalue cannot decrease sharply, to its $\frac{n^2(1-\alpha_t)^2}{(n+1)^2}$ at most. Therefore, the smallest eigenvalue is bounded below by $\tau\epsilon^2\frac{n^2(1-\alpha_t)^2}{(n+1)^2}$. In terms of geometry, these two lemmas follow from that the difference $\bar{p}_t - \underline{p}_t$ monitors the width of the ellipsoid along the direction given by the feature vector $\mathbf{x}_t$, and if it is below the threshold ϵ, the data broker will post the conservative price rather than the exploratory price to avoid shortening the width along this direction. Hence, the smallest eigenvalue, having a correspondence with the width of the ellipsoid's narrowest axis, cannot become too small.

By combining all above three lemmas, we can derive an upper bound on T_e in Algorithm 1*.

Lemma 5. *Algorithm 1* ($\epsilon \ge 4n\delta$) chooses the exploratory prices in at most $20n^2\log(20RS^2(n+1)/\epsilon)$ rounds.*

We restate Lemma 5 for Algorithm 1, by analyzing the impact of the reserve price constraint on T_e.

Lemma 6. *Algorithm 1 ($\epsilon \ge 4n\delta$) chooses the exploratory prices in at most $20n^2\log(20RS^2(n+1)/\epsilon)$ rounds.*

Proof. For conciseness, we here focus only on the rejection branch of Algorithm 1. The analysis of the acceptance branch can be derived by the symmetry of ellipsoid. We recall that if the reserve price q_t is introduced in round t, the exploratory posted price is $p_t = \max(q_t, \frac{\underline{p}_t+\bar{p}_t}{2})$, the effective exploratory price is $p_t + \delta$ in the rejection case, and its position parameter can be computed via $\alpha_t = (\frac{\underline{p}_t+\bar{p}_t}{2} - (p_t + \delta))/\sqrt{\mathbf{x}_t{}^T \mathbf{A}_t \mathbf{x}_t}$ (Algorithm 1, Line 15). We now prove Lemma 6 in two complementary cases:

Case 1 ($\frac{\underline{p}_t+\bar{p}_t}{2} \geq q_t$): The posted price is the middle price (i.e., $p_t = \frac{\underline{p}_t+\bar{p}_t}{2}$). Algorithm 1 degenerates to Algorithm 1*, and Lemma 6 holds from Lemma 5.

Case 2 ($q_t > \frac{\underline{p}_t+\bar{p}_t}{2}$): The posted price is the reserve price (i.e., $p_t = q_t$). Given the reserve price is rejected, we can draw that the reserve price is higher than the market value (i.e., $p_t = q_t > v_t$), which further implies $r_t = 0$ from Equation (1). Suppose the data broker does not use the reserve price to refine the ellipsoid in this round. The analysis of Algorithm 1 can be reduced to analyzing Algorithm 1* with the total number of rounds $T - 1$ plus one dummy round inserted in the t-th round. Considering Lemma 5 does not rely on the total number of rounds, $T_e \leq 20n^2 \log(20RS^2(n+1)/\epsilon)$ still holds in Algorithm 1. However, in Algorithm 1 (Lines 14–19), the data broker needs to cut the ellipsoid using the effective exploratory price (i.e., $q_t + \delta$ here). We thus need to analyze the impact of such a cut on T_e. Following the guidelines in proving Lemma 5, to prove Lemma 6, it suffices to prove that this cut cannot increase the upper bound on the final volume of the ellipsoid, and meanwhile, cannot decrease the lower bound. First, the effective exploratory price imposes a cut over the ellipsoid and thus cannot increase the final volume together with the upper bound on the final volume. Second, the lower bound on the smallest eigenvalue of the final ellipsoid's shape matrix (i.e., $\tau\epsilon^2 \frac{n^2(1-\alpha_t)^2}{(n+1)^2}$) takes its minimum at $\alpha_t = 0$. This corresponds to the lower bound on the ellipsoid's final volume used in proving Lemma 5. Additionally, a negative α_t can increase the lower bound. Thus, the effective exploratory price $q_t + \delta$ here, holding a negative $\alpha_t = (\frac{\underline{p}_t+\bar{p}_t}{2} - (q_t + \delta))/\sqrt{\mathbf{x}_t{}^T \mathbf{A}_t \mathbf{x}_t} < -\delta/\sqrt{\mathbf{x}_t{}^T \mathbf{A}_t \mathbf{x}_t} < 0$, cannot decrease the lower bound on the final volume.

By summarizing two cases, we complete the proof. □

We finally obtain Theorem 1 as follows.

Theorem 1. *If $\delta = O(n/T)$, then the worst-case cumulative regret of Algorithm 1 is $O(\max(n^2 \log(T/n), n^3 \log(T/n)/T))$.*

Proof. First, as we illustrated below Equation (5): in each round t, the absolute value of the random variable δ_t has probability at most $1/T$ outside δ. Thus, the cumulative regret incurred by removing the weight vector θ^* from the knowledge set is at most $\max_{\mathbf{x}_t,\theta^*} \mathbf{x}_t{}^T \theta^* T/T = RS$.

Second, we analyze the cumulative regret due to the posted prices. In round t, the regret incurred by posting the exploratory (resp., conservative) price can be bounded above by $\bar{p}_t + \delta$ (resp., $(\bar{p}_t + \delta) - (\underline{p}_t - \delta)$), which can be further bounded above by $RS + \delta$ (resp., $\epsilon + 2\delta$). Thus, the cumulative regret is no more than $T_e(RS + \delta) + (T - T_e)(\epsilon + 2\delta)$. When $\delta = O(n/T)$, T_e takes its upper bound $20n^2 \log(20RS^2\ (n+1)/\epsilon)$ from Lemma 6, and ϵ is set to $\max(n^2/T, 4n\delta) = O(n^2/T)$, the worst-case cumulative regret incurred by the posted prices is $O(\max(n^2 \log(T/n), n^3 \log(T/n)/T))$.

By adding two parts, the worst-case cumulative regret of Algorithm 1 is $O(\max(n^2 \log(T/n), n^3 \log(T/n)/T))$. □

4 Extensions

In this section, we extend the proposed pricing mechanism under the fundamental linear model to support some common non-linear models. We also discuss how to support several other similar application scenarios.

4.1 Supporting Non-Linear Market Value Models

We mainly investigate four kinds of non-linear models commonly used in measuring market values. The first two are the log-log and log-linear models in hedonic pricing from real estate and property studies [22], [23], which can be formalized as $\log v_t = \sum_{i\in[n]} \log(x_{t,i})\theta_i^*$ and $\log v_t = \mathbf{x}_t{}^T \theta^*$, respectively. Here, $x_{t,i}$ and θ_i^* denote the i-th elements of the feature vector $\mathbf{x}_t$ and the weight vector θ^*, respectively. The other two models are the logistic model [32], [33] and the kernelized model [34] in online advertising, which can be formalized as $v_t = 1/(1 + \exp(\mathbf{x}_t{}^T \theta^*))$ and $v_t = \sum_{k=1}^{t-1} K(\mathbf{x}_t, \mathbf{x}_k)\theta_k^*$, respectively. Here, $K(\cdot, \cdot)$ is a Mercer kernel operator.

We can further observe that the above four non-linear models can be unified to a general class of non-linear models $v_t = g(\phi(\mathbf{x}_t)^T \theta^*)$. Here, $g : \mathbb{R} \mapsto \mathbb{R}$ is a non-decreasing and continuous function. For example, in the two hedonic pricing models, g is the natural exponential function; in the logistic model, g is the logistic sigmoid function; and in the kernelized model, g is the identity function. Additionally, $\phi : \mathbb{R}^n \mapsto \mathbb{R}^m$ represents a feature mapping of the original feature vector $\mathbf{x}_t$ and intends to capture non-linear correlations/dependencies among the different features of $\mathbf{x}_t$ and the different feature vectors within t rounds. For example, in the log-log model, ϕ denotes applying the natural logarithm function to each element of $\mathbf{x}_t$; in the kernelized model, $m = t - 1$, and ϕ stands for the kernel function K; and in the other two models, ϕ denotes the identity map. Furthermore, we note that both g and ϕ are public knowledge, and only the weight vector θ^* is unknown. Therefore, by regarding the domain of θ^* as the knowledge set to be refined, our proposed pricing mechanism under the linear model can still apply to the above class of non-linear models. Specifically, $\phi(\mathbf{x}_t)$ now functions as the new feature vector, and the threshold ϵ is used to control $\bar{p}_t - \underline{p}_t$, which denotes the difference between the maximum and minimum possible values of $\phi(\mathbf{x}_t)^T \theta$, where θ belongs to the data broker's knowledge set. In addition, the data broker will post the price $g(p_t)$ rather than the original p_t. Due to the limitation of space, the worst-case regret analysis of the adapted Algorithm 1 under the above class of non-linear models is put into our technical report [29].

4.2 Supporting Other Application Scenarios

We first summarize the characteristics of the pricing problem in online personal data markets. We then point out some other similar application scenarios in practice and further illustrate how to support them with our proposed pricing mechanism under different market value models.

In personal data markets, the data broker is the seller, and each data consumer is a buyer. The sequential queries, as the products to be sold, have three atypical characteristics: (1) *Customization*: The queries, requested by different data consumers, are highly differentiated; (2) *Existence of reserve price*: The total privacy compensation, allocated to the underlying data owners, serves as the reserve price of a query; and (3) *Timeliness*: If no deal occurs in a round, the query will vanish, generating regret for the data broker.

Several other products in practice share one or more characteristics listed above, which implies that our proposed pricing mechanism for personal data markets can be extended to support these scenarios. One example is the hospitality service on booking platforms (e.g., Airbnb, Wimdu, and Workaway). A tourist can raise some requirements on his/her desirable accommodation, such as location, the numbers of bedrooms and bathrooms, amenities, reviews, historical occupancy rate, and so on. Meanwhile, the host of the house can set a minimum/reserve price for the accommodation. If the house is not rented out at a certain date, it may cause regret for both the host and the booking platform. We note that the host, the booking platform, and the tourist play similar roles to the data owner, the data broker, and the data consumer in data markets, respectively. In addition, the market value of the accommodation can be well interpreted by the linear or log-linear model [23]. Another example is the online advertising on web publishers. We consider a novel scenario, where the impressions are traded through posting prices rather than the ad auctions already adopted by Internet giants (e.g., Google, Microsoft, Facebook, and Alibaba). In particular, an advertiser can customize its/his/her need of an impression (e.g., position and target audience). If the impression is not sold within a given time frame, it will generate regret for the web publisher. We note that the web visitors who generate impressions, the web publisher, and the advertiser play similar roles to the data owners, the data broker, and the data consumer in data markets, respectively. In addition, the market value of an impression is normally measured by its click-through rate (CTR), which can be effectively captured by the logistic [32], [33] or kernelized model [34].

In conclusion, our proposed pricing mechanism is not just limited to online personal data markets and can also support other similar application scenarios.

5 Evaluation Results

In this section, we present the evaluation results of our pricing mechanism from practical regret and overhead.

We use three real-world datasets, including MovieLens 20M dataset [35], Airbnb listings in U.S. major cities [36], and Avazu mobile ad click dataset [37], to evaluate our pricing mechanism over noisy linear queries, accommodation rentals, and impressions under the linear, log-linear, and logistic market value models, respectively. First, the MovieLens dataset contains $20,000,263$ ratings of $27,278$ movies made by $138,493$ users. Second, the Airbnb dataset provides $74,111$ booking records in 6 U.S. cities (e.g., New York and Los Angeles). Each record contains a user id, the logarithmic lodging price, house type, location, amenities, host response rate, cancellation policy, and so on. Third, the Avazu dataset comprises 10 days of click-through data, in total $404,289,670$ ad displaying samples. Each sample covers information of an ad and the corresponding mobile user (e.g., the ad id, click or non-click reaction, position, device id, device ip, and internet access type).

5.1 Pricing of Noisy Linear Query

We first introduce our setup details for trading noisy linear queries, the workflow of which has been briefly introduced in Example 1. On the one hand, we regard the MovieLens users, who contributed the ratings, as the data owners in data markets. We adopt the differential privacy-based privacy leakage quantification mechanism and the tanh-based privacy compensation functions from [9] for each data owner. On the other hand, we simulate the noisy linear queries from online data consumers. To validate the adaptability of our pricing mechanism, the parameters of each linear query are randomly drawn either from a multivariate normal distribution with zero mean vector and identity covariance matrix or from a uniform distribution within the interval $[-1,1]$. Meanwhile, the variance of Laplace noise added to the true answer is randomly selected from $\{10^k | k \in \mathbb{Z}, |k| \leq 4\}$. For each noisy linear query Q_t, we compute the privacy compensations of all data owners and then generate an n-dimensional feature vector with the aggregation technique: we first sort the privacy compensations, then evenly divide them into n partitions, and finally sum the privacy compensations falling into a certain partition, thereby obtaining a feature. For the sake of normalization, we scale each feature vector such that its L_2 norm is 1 (i.e., $\forall t \in [T], \|\mathbf{x}_t\| = 1$ and $S = 1$). Additionally, we set the reserve price of a query to be the total privacy compensation (i.e., $q_t = \sum_{i\in[n]} x_{t,i}$ here). In nature, the L_2 norm of the weight vector for deriving q_t is $\sqrt{n}$. Moreover, we draw the weight vector θ^* for modeling the market values of queries in a similar way to sample the query's parameters. The difference is that we further scale θ^* such that its L_2 norm is $\sqrt{2n}$ (i.e., $\|\theta^*\| = \sqrt{2n}$). This guarantees that the market value of each query $v_t = {\mathbf{x}_t}^T\theta^*$ is no less than its reserve price q_t with a high probability. Furthermore, we set the data broker's initial knowledge set of θ^* to $\mathcal{E}_1 = \{\theta \in \mathbb{R}^n | \|\theta\| \leq 2\sqrt{n}\}$, geometrically, the ball centered at the origin with radius $R = 2\sqrt{n}$.

TABLE 1
Statistics over pricing of noisy linear query per pound under the version with reserve price.

n	T	Market Value	Reserve Price	Posted Price	Regret
1	10^2	1.414	1	1.409 (0.045)	0.035 (0.202)
20	10^4	*3.874 (1.278)	3.388 (0.776)	3.685 (1.631)	0.166 (0.824)
40	10^4	5.246 (1.616)	4.739 (1.188)	5.254 (1.614)	0.743 (1.933)
60	10^5	7.098 (1.910)	5.733 (1.491)	7.089 (1.912)	0.220 (1.257)
80	10^5	7.266 (2.046)	6.531 (1.761)	7.243 (2.091)	0.387 (1.690)
100	10^5	8.824 (2.235)	7.221 (1.985)	8.820 (2.242)	0.686 (2.461)

*The entry is stored in the format: mean (standard deviation).

In Fig. 3, we plot the cumulative regrets of four versions of our pricing mechanism under the linear model, including the pure version (omitting the reserve price q_t and the uncertainty parameter δ in Algorithm 1), the version with uncertainty (Algorithm 1*), the version with reserve price (omitting δ in Algorithm 1), and the version with reserve price and uncertainty (Algorithm 1). Here, the dimension of feature vector n first takes 1 and then increases from 20 to 100 with a step of 20. In addition, δ is fixed at 0.01, which is in the pre-analyzed order of $O(n/T)$ for $n = 1$, but is much larger than $O(n/T)$ for $n \neq 1$. Moreover, in each round t, the randomness δ_t in the market value v_t is drawn from the normal distribution with mean 0 and standard deviation $\sigma = \delta/(\sqrt{2\log 2}\log T)$. Furthermore, the threshold ϵ is set to n^2/T. As a complement to Fig. 3, Table 1 lists some precise statistic information about the version with reserve price, where the market value column can work as a baseline for relatively measuring the levels of uncertainty (particularly, in the magnitude of 0.1% of the market value) and regret.

We first observe Fig. 3 holistically. We can see that under a specific version, the cumulative regret after a certain number of rounds increases with the dimension n. The reason is that as n grows, the data broker needs to post exploratory prices in more rounds to obtain a good knowledge of

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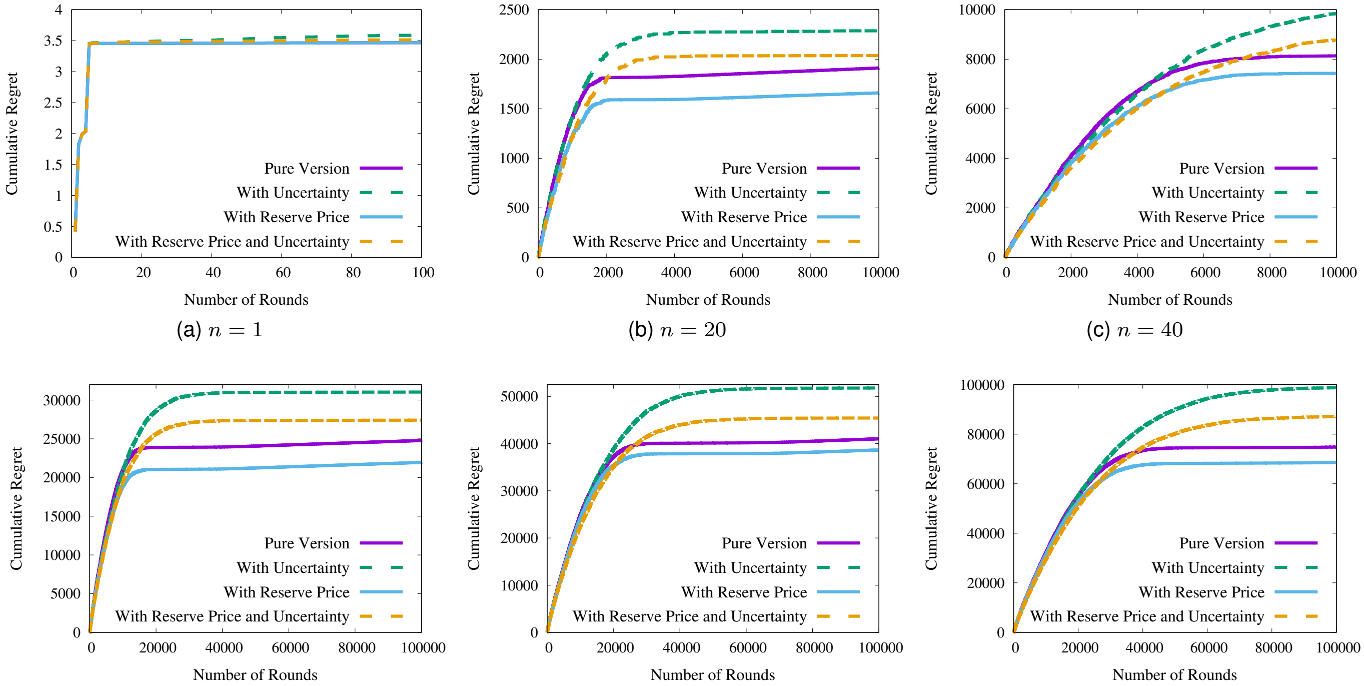

(a) $n = 1$ (b) $n = 20$ (c) $n = 40$

(d) $n = 60$ (e) $n = 80$ (f) $n = 100$

Fig. 3. Cumulative regrets with varying dimensions of feature vector in pricing of noisy linear query.

the weight vector θ^*, thus accumulating more regret. This conforms to our theoretic regret analysis.

We then observe the one-dimensional case in Fig. 3a and the multi-dimensional cases from Fig. 3b to Fig. 3f more carefully. We start with the one-dimensional case. From Fig. 3a, we can see that the introduction of the reserve price constraint has no effect on the pure version of our pricing mechanism. When $n = 1$, the reserve price and the market value of each query are constants 1 and $\sqrt{2}$, respectively. In addition, the data broker's initial knowledge of the market value is the interval $[0, 2]$. Thus, in the first round, no matter the data broker considers or ignores the reserve price 1, it posts the exploratory price 1, which is less than the market value $\sqrt{2}$ and is accepted by the data consumer. After this round, the interval is refined to $[1, 2]$, which indicates that the reserve price 1 can no longer affect the posted prices. From Fig. 3a, we can also see that the introduction of low uncertainty will slightly increase the cumulative regrets in the pure version and the version with reserve price.

We next focus on the multi-dimensional cases. Once again, we examine how the reserve price constraint can affect our posted price mechanism. We can find that the incorporation of reserve price can dramatically reduce the cumulative regret. In particular, when $n = 20$ and the number of rounds t is 10^4, the version with reserve price (resp., the version with reserve price and uncertainty) reduces 13.16% (resp., 10.92%) of the cumulative regret than the pure version (resp., the version with uncertainty). We further examine the impact of uncertainty. We can see that the existence of uncertainty accumulates more regret, especially when t is large. This is because in the case of a large t, the data broker already has a good knowledge of the weight vector θ^* and posts the conservative price with a high probability. In addition, we recall that to circumvent uncertainty, the conservative price, involving the minimum possible market value $\underline{p}_t$, decreases by δ to keep its acceptance ratio, which can generate a higher regret.

We finally provide an intuition of the regret level of our pricing mechanism. We introduce a metric, called *regret ratio*, defined as the ratio between the cumulative regret and the cumulative market value, namely, $\sum_{k=1}^{t} r_k / \sum_{k=1}^{t} v_k$ at the end of t rounds. For example, in Table 1, we can divide the mean values in the regret column by those in the market value column and obtain the regret ratios of the version with reserve price for different n's at the end of T rounds. Coupled with Fig. 3f, which depicts the cumulative regrets of four versions for $n = 100$ at the end of different rounds, Fig. 4a further plots the regret ratios.

One key observation from Fig. 4a is that when the number of rounds t is small, the regret ratio of the version with reserve price (resp., the version with reserve price and uncertainty) is much lower than that of the pure version (resp., the version with uncertainty). This reflects a critical functionality of reserve price: it can mitigate the cold-start problem in a posted price mechanism. More specifically, in the beginning, the data broker holds a broad knowledge set of the weight vector θ^*, and thus the estimation of a query's market value is coarse, which implies a high regret ratio. However, with the help of reserve price, the data broker can improve the market value estimation, through imposing an additional lower bound and refining the knowledge set more quickly. The mitigation of cold start can be a factor underlying our aforementioned observation that the reserve price constraint reduces the cumulative regret.

The second key observation from Fig. 4a is that as t grows, the difference between the regret ratios of the versions with and without reserve price shrinks. In addition, when t is very large, the regret ratios of all four versions are very low. In particular, at the end of $T = 10^5$ rounds, the regret ratios of the pure version, the version with uncer-

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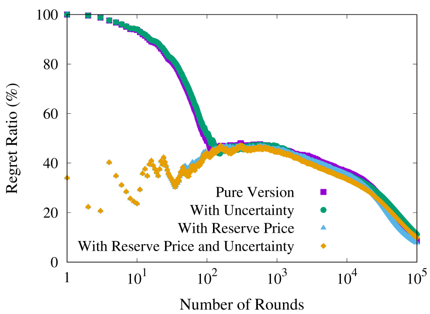
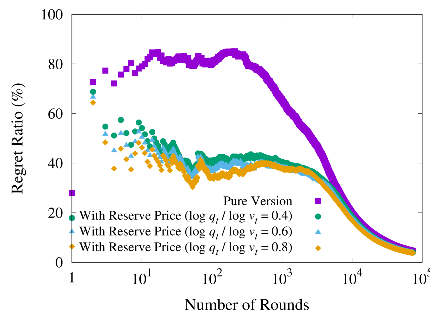
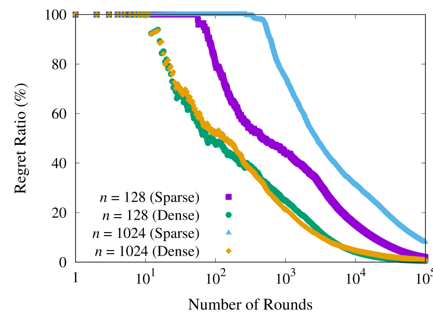


(a) Noisy Linear Query ($n = 100$)



(b) Accommodation Rental



(c) Impression

Fig. 4. Regret ratios in pricings of noisy linear query, accommodation rental, and impression.

tainty, the version with reserve price, and the version with reserve price and uncertainty are 8.48%, 11.19%, 7.77%, and 9.87%, respectively. The reason is that after enough rounds, the data broker will have a good estimation of any query's market value, and the effect of reserve price on the posted price diminishes. An extreme example happens in the one-dimensional case presented above, where after the first round, the reserve price has already been excluded from the estimated interval. At last, we provide a risk-averse baseline, which consistently posts the reserve price in each round, for the versions involving the reserve price constraint. The regret ratio of such a baseline is 18.16%. Compared with this baseline, our pricing mechanism can further reduce 57.19% (resp., 45.64%) of the regret ratio in the version with reserve price (resp., the version with reserve price and uncertainty).

These results demonstrate that our pricing mechanism under the fundamental linear model can indeed reduce the practical regret of the data broker in online data markets.

5.2 Pricing of Accommodation Rental

We first describe how to preprocess the Airbnb dataset and then present the setup details for pricing accommodation rentals under the log-linear model. First, to obtain the feature vector of each booking record, we process the categorical features with the pandas library in Python, which can handle the missing values and return an integer array of codes for all categories. In addition, we add some interaction features to enhance model capacity. The final dimension of each feature vector n is 55. Second, to obtain the weight vector θ^* in modeling the market values of accommodations, we regard the logarithmic lodging prices as target variables in supervised learning and then apply linear regression to learn the coefficients of different features, which play the role of θ^* here. Specifically, the mean squared error (MSE) over the test set, which occupies 20% of the Airbnb dataset, is 0.226. Third, to investigate how different settings of reserve price can affect the posted price mechanism, we vary the ratio between the natural logarithms of reserve price and market value (i.e., $\log q_t / \log v_t$). Fourth, when computing the regret ratios, we use the real rather than the logarithmic posted prices and market values. Fig. 4b depicts the regret ratios of the pure version of our pricing mechanism under the log-linear model, as well as the version with reserve price where $\log q_t / \log v_t$ ranges from 0.4, to 0.6, and to 0.8.

From Fig. 4b, we can see that when the reserve price is set to be closer to the market value, the regret ratio decreases, especially when the number of rounds t is small. In other words, as the reserve price approaches the market value, its impact on mitigating the cold-start problem in a posted price mechanism is more evident. We can also see from Fig. 4b that at the end of $T = 74,111$ rounds, the regret ratios are very low. In particular, the regret ratios of the pure version and the version with reserve price where $\log q_t / \log v_t = 0.4, 0.6$, and 0.8, are 4.57%, 4.01%, 3.83%, and 3.79%, respectively. We still consider the risk-averse baseline, where the reserve price is posted in each round, for comparison. The regret ratios of this baseline are 23.40%, 17.00%, and 9.33% in the version with reserve price where $\log q_t / \log v_t$ = 0.4, 0.6, and 0.8, respectively. Compared with this baseline, our pricing mechanism can further reduce 82.88%, 77.46%, and 59.39% of the regret ratios when $\log q_t / \log v_t$ = 0.4, 0.6, and 0.8, respectively.

The above fine-grained evaluation results provide a deeper understanding of the reserve price's role in reducing the practical regret of a posted price mechanism. In addition, our proposed pricing mechanism significantly outperforms the baseline which merely exploits the reserve price.

5.3 Pricing of Impression in Advertising

We first introduce data preprocessing and setup for pricing impressions under the logistic model. First, to handle the categorical data fields in ad displaying samples, we use one-hot encoding with the hashing trick, where the dimension of the feature vector n serves as the modulus after hashing. Second, we regard the click/non-click states as target variables, further apply Follow The Proximally Regularized Leader (FTRL-Proximal)-based logistic regression (which has been deployed at Google's advertising platform [33]), thereby obtaining the weight vector θ^* for capturing CTRs. In particular, FTRL-Proximal is an online learning algorithm with per-coordinate learning rates and L_1, L_2 regularizations, and it can preserve excellent performance and sparsity. When testing over the samples in the last two days, the logistic loss is 0.420 (resp., 0.406) for $n = 128$ (resp., $n = 1024$). Additionally, the learnt weight vector θ^* is quite sparse. Specifically, the number of nonzero elements in θ^* is 21 (resp., 23) for $n = 128$ (resp., $n = 1024$). In what follows, we investigate two different cases to validate the feasibility of our pricing mechanism over both sparse and dense feature vectors. In the sparse case, all the features are kept no matter whether their corresponding weights are zero or not. In the dense case, the features are omitted if their corresponding weights are zero.

In Fig. 4c, we plot the regret ratios of the pure version of our pricing mechanism in both sparse and dense cases for

$n = 128$ and $n = 1024$. We can observe from Fig. 4c that the regret ratio in the sparse case decreases more slowly than that in the dense case, especially when the number of rounds t is smaller than 10^3. This outcome stems from that the starting rounds are mainly dedicated to eliminating those zero elements in the weight vector, which implies a larger regret ratio in the beginning. This reason can also account for the phenomenon that in the sparse case, the regret ratio for $n = 1024$ decreases more slowly than that for $n = 128$. Even so, after 10^5 rounds, the regret ratios are 2.02% and 0.41% (resp., 8.04% and 0.89%) for $n = 128$ (resp., $n = 1024$) in the sparse and dense cases, respectively.

These evaluation results reveal that our pricing mechanism performs well over both sparse and dense feature vectors. By further combining with the pricing of accommodation rental, we can conclude that our pricing mechanism has a good extensibility to non-linear market value models.

5.4 Details on Implementation and Overhead

We implemented our pricing mechanism in Python 2.7.15. The running environment is a Broadwell-E workstation with 64-bit Ubuntu 16.04.5 OS. In particular, the processor is Intel(R) Core(TM) i7-6900K with 8 cores, the base frequency is 3.20 GHz, the memory size is 64 GB, and the cache size is 20 MB. Our source code is online available from [38].

We report the computation and memory overhead of three use cases: (1) for the pricing of noisy linear query under the version with reserve price, when $n = 100$, the latency of the data broker in determining the posted price and updating its knowledge set is 0.115 ms per query. In addition, the memory overhead is 151 MB; (2) for the pricing of accommodation rental under the version with reserve price where $\log q_t / \log v_t = 0.6$, the latency is 0.019 ms per booking request, and the memory overhead is 105 MB; and (3) for the pricing of impression, when $n = 1024$, the latency is 3.509 ms (resp., 0.024 ms) per ad displaying sample in the sparse (resp., dense) case. Additionally, the memory overhead is 106 MB (resp., 75 MB) in the sparse (resp., dense) case.

In a nutshell, our pricing mechanism has a light load under both linear and non-linear models. It can be employed to dynamically price the products with customization, existence of reserve price, and timeliness properties.

6 Related Work

In this section, we briefly review related work.

6.1 Data Market Design

An explosive demand for sharing data contributes to growing interest in data market design. We here focus only on the design of pricing mechanisms. We direct interested readers to the comprehensive surveys [39]–[41] and the vision papers [7], [42] for more perspectives. For example, Fernandez et al. [42] provided a vision for the design and implementation of data markets mainly from data sharing, discovery, and integration.

First regards general (insensitive) data trading. The researchers from the database community (e.g., Koutris et al. [11]–[14], Lin and Kifer [15]) mainly focused on arbitrage freeness in pricing queries over the relational databases. The existence of arbitrage means that the data consumer can buy a query with a lower price than the marked price through combining a bundle of other cheaper queries. Thus, the data broker needs to rule out arbitrage opportunities to preserve its revenue. Stahl et al. surveyed several empirical pricing strategies in practical data markets [43]. Their later work [44]–[46] introduced data quality as a criterion of pricing and allowed the data consumers to suggest their own prices. Chawla et al. [47] considered the static revenue maximization problem with the prior knowledge of the data consumers' queries and valuations, while leaving the online setting as an open problem. They mainly adopted two static pricing strategies, called uniform bundle pricing and item pricing. Agarwal et al. [48] proposed a combinatorial auction mechanism to trade data for machine learning tasks.

Specific to personal data trading, the researchers routinely adopted the cost-plus pricing strategy, where the data broker first compensates each data owner for its privacy leakage and then scales up the total privacy compensation to determine the price of query for the data consumer. Different researchers investigated distinct types of queries from the data consumers. Ghosh and Roth [10] considered single counting query. The follow-up work by Li et al. [9] further extended to multiple noisy linear queries. We considered the queries of noisy aggregate statistics over private correlated data [16], [17]. Hynes et al. [49] investigated model training requests. Chen et al. [50] studied how to price a trained model with different levels of noise perturbation, by an analogy to the queries over personal data. They also considered how to statically optimize the data broker's revenue under the assumption that the error demands and corresponding valuations of the data consumers are known.

Our work advances previous data trading work in that: (1) we model the unknown valuations and demands of the data consumers, namely, the market values of customized and highly differentiated queries, which were assumed as priors in previous work; (2) we consider a posted price setting and incorporate the response of either an acceptance or a rejection from each data consumer in sequence, whereas the previous work normally used a marked price setting and ignored the responses; and (3) we optimize the data broker's cumulative revenue in an online and dynamic manner, whereas previous work optimized in a static way.

6.2 Contextual Dynamic Pricing

The dynamic pricing problem has been extensively studied in diverse contexts. The pioneering work by Kleinberg and Leighton [51] considered markets for identical products and designed several optimal posted pricing strategies. However, the products in practical markets (e.g., online commerce and advertising) tend to differ from each other. This further motivated the emergence of contextual pricing, where the seller intends to sell a sequence of highly differentiated products, posts a price for each product, and then observes whether the buyer accepts or not. More specifically, each product is represented by a feature vector for differentiation, while its market value is typically assumed to linear in the feature vector. The researchers thus turned to online learning the weight vector from feedbacks and further converted this task to a multi-dimensional binary search problem. Amin et al. [34] first proposed a stochastic gradient descent (SGD)-based solution, which can attain $O(T^{2/3})$ strategic regret by ignoring logarithmic terms. However, their solution requires an independent and identically distributed (i.i.d.) assumption on the feature vectors. Cohen et al. [19] abandoned this strict requirement. They approximated the polytope-shaped knowledge set with ellipsoid and provided $O(n^2 \log T)$ worst-case cumulative regret, which is essentially the pure version of our pricing mechanism. Lobel et al. [20] further reduced regret to $O(n \log T)$ by projecting and cylindrifying the polytope. Leme et al. [21] borrowed a key concept from geometric probability, called the intrinsic volumes of a convex body, and achieved a regret guarantee of $O(n^4 \log\log(nT))$. The key principle behind this line of work is to identify the centroid of the knowledge set or its projection/transformation, such that each exploratory posted price can roughly impose a central cut in terms of different measures (e.g., volume, surface area, and width). In addition, although the most recent two work optimized the regret, they are too computationally complex to be deployed in practical online markets.

It is still worth noting that the contextual dynamic pricing mechanisms significantly differ from the classical cutting-plane or localization algorithms in the field of convex optimization (e.g., the original ellipsoid method [27] and the analytic center cutting-plane method [52]). In particular, the purpose of a cutting-plane method is to find a point in a convex set for optimizing a preset objective function. In contrast, the goal of a contextual dynamic pricing mechanism is to minimize the cumulative regret during the process of locating a preset point (i.e., the weight vector here). Furthermore, under contextual dynamic pricing, the direction of each cut is fixed by the feature vector of a product requested by a buyer, while the seller can choose only the position of the cut through posting a certain price. This setting distinguishes contextual dynamic pricing from a majority of ellipsoid-based designs [53]–[55], which allow the seller to control the direction of each cut. In fact, the contextual dynamic pricing problem can also be modeled into contextual multi-armed bandit (MAB), where the arms/actions to be exploited and explored are the domain of the weight vector. However, given the domain of the weight vector is continuous, we need to apply the discretization technique, which makes the number of bandits extremely large. In addition to inefficiency, since the payoff/regret function is piecewise and highly asymmetric, this sort of solutions can be oracle-based (e.g., [56]–[61]) and inevitably incurs polynomial rather than logarithmic cumulative regret in the total number of rounds T [20].

Our work advances contextual dynamic pricing in that: (1) we, for the first time, incorporate the reserve price constraint; (2) due to the existence of reserve price, we support an arbitrary position of the cut over the ellipsoid-shaped knowledge set, whereas previous designs normally adopted central cuts; and (3) we analyze and verify the impact of reserve price on a posted price mechanism, particularly, mitigating the cold-start problem and thus reducing the cumulative regret.

7 Conclusion

In this paper, we have proposed the first contextual dynamic pricing mechanism with the reserve price constraint, for the data broker to maximize its cumulative revenue in online personal data markets. Our posted price mechanism features the properties of ellipsoid to perform online optimization effectively and efficiently and can support both linear and non-linear market value models, while allowing some uncertainty. We further have illustrated how to support two other similar application scenarios and extensively evaluated all three use cases over three practical datasets. Empirical results have demonstrated the feasibility and extensibility of our pricing mechanism as well as the functionality of the reserve price constraint.

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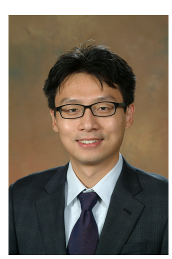

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