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# Airfoil Parameterization Techniques: A Review

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**Abstract** Optimization of airfoil shape using evolutionary algorithms is becoming a trend in design of blades for turbomachines and aircraft. Evolutionary algorithms work with parameterization of airfoil shape, i.e. representation of airfoil with the help of some parameters which control its shape. Thus, one of the challenges in this field is to describe the airfoil with suitable parameters and explicit or implicit mathematical functions. This paper discusses various parameterization techniques being used to parameterize the airfoil.

**Keywords:** airfoil optimization, parameterization of airfoil, blade optimization, cascade optimization, profile optimization

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## 1. Introduction

Many parameterization techniques are in use nowadays. Some techniques prove to be advantageous over others with respect to convergence rate, range of airfoil that could be represented, etc. The further sections deal with various parameterization techniques explained briefly.

Any parameterization technique must satisfy the following three objectives.

- It should minimize the number of degrees of freedom, i.e. number of parameters should be as less as possible.
- It should be able to represent a wide range of existing airfoils.
- Parameters should be simple to formulate and impose.

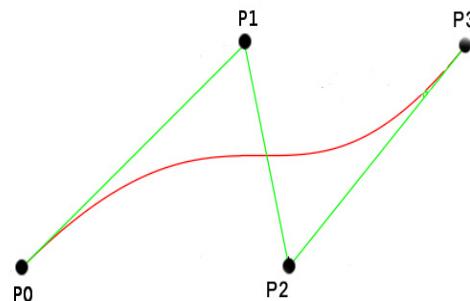


Figure 1. Cubic Bezier curve

The blending functions of Bezier curves are Bernstein polynomials. It is denoted by  $B_i^n$ , where

$$B_i^n = \binom{n}{i} (1-t)^{n-i} t^i \quad (1)$$

where  $i=0, 1, 2, 3$ .

and  $\binom{n}{i}$  is a binomial coefficient such that

$$\binom{n}{i} = \frac{n!}{i!(n-i)!} \quad (2)$$

For Bezier curve having  $n = 3$

$$\begin{aligned} B_0^3 &= (1-t)^3 \\ B_1^3 &= 3t(1-t)^2 \\ B_2^3 &= 3t^2(1-t) \\ B_3^3 &= t^3 \end{aligned} \quad (3)$$

Then, the Bezier curve associated with the given control points can be expressed as [3],

## 2. Bezier Parameterization

To represent an airfoil with the help of Bezier curve is one of the popular parameterization techniques. Airfoil consists of two curves namely camber line and thickness distribution. To obtain the upper and lower surfaces of the aerofoil, we add and subtract the thickness distribution to and from the camber line, respectively. A Bezier curve is controlled with the help of its control points in a plane. It passes through initial and final control points but it is not necessary for Bezier curve to pass through the each intermediate control point which is defining the shape of the aerofoil [1].

A  $n$ - degree Bezier curve is defined by  $(n+1)$  control points. A Three-degree Bezier curve will have four control points and will have shape as shown in Figure 1 depending on position of control points.

$$P(t) = \sum_{i=0}^n P_i B_i^n(t) \quad (4)$$

### 3. PARSEC Method

PARSEC is very common and highly effective method of airfoil parameterization. It uses eleven basic parameters to completely define the aerofoil shape as shown in Figure 2. The various parameters are leading edge radius ( $r_{LE}$ ), upper crest location ( $X_{UP}, Z_{UP}$ ), lower crest location ( $X_{LO}, Z_{LO}$ ), upper and lower curvature( $Z_{xxUP}, Z_{xxLO}$ ), trailing edge coordinate( $Z_{TE}$ ) and direction( $\alpha_{TE}$ ), trailing edge wedge angle( $\beta_{TE}$ ) and thickness( $\Delta Z_{TE}$ ).

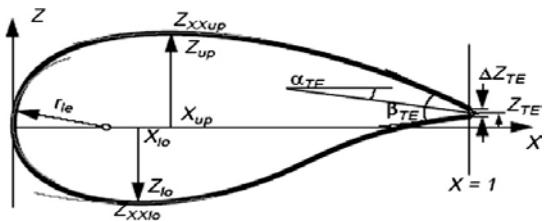


Figure 2. PARSEC method for airfoil parameterization

In this method, a linear combination of shape functions describes the aerofoil shape.

$$Z_k = \sum_{n=1}^6 a_{n,k} X_k^{\frac{n-1}{2}} \quad (5)$$

With the help of defined geometric parameters, one can find coefficients  $a_n$ . The subscript k will take values 1 and 2 for upper and lower surface respectively. In this way, using these Parameters, one can control the maximum curvature on upper and lower surfaces and their location, which greatly affect the occurrence of shock wave and its strength. However, PARSEC does not provide sufficient control over the trailing edge shape where important flow phenomenon can take place because it fits a smooth curve between the maximum thickness point and the trailing edge [2].

### 4. Sobieczky Method

As seen above, PARSEC method faces lack of control on trailing edge modeling, to overcome this, Sobieczky [2] proposed a way to give trailing edge a concave shape. Lower and upper surfaces observe the curvature increasing towards trailing edge. Such modified airfoil is known as divergent trailing edge [2].

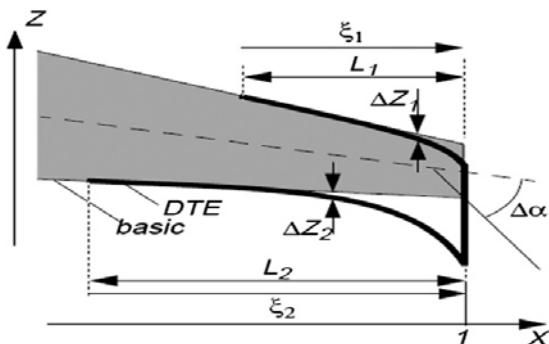


Figure 3. Sobieczky method for DTE

As illustrated in Figure 3, in Sobieczky method, some portion of length of airfoil near the trailing edge undergoes increment in trailing edge thickness  $\Delta Z$ . The parameters  $\Delta\alpha$ ,  $L_1$ ,  $L_2$  govern the thickness  $\Delta Z$  added to airfoil surface. Camber added to the upper and lower surfaces is controlled by the parameter  $\Delta\alpha$ .

Thickness added  $\Delta Z$  is given by,

$$\Delta Z_k = \frac{L_k \tan \Delta\alpha}{\mu n} \left[ 1 - \mu \cdot \xi_k^n - (1 - \xi_k^n)^\mu \right] \quad (6)$$

where k is the subscript which takes values 1 and 2 to indicate upper and lower Surfaces, respectively.  $\xi_k$  is the x coordinate variable indicating chord length. Parameters  $n$  and  $\mu$  can have various values.  $L_k$  is the length of airfoil which is modified on corresponding side. In this way, airfoil can be presented by the combination of both the methods. PARSEC method is used for much of the airfoil and Sobieczky method near the trailing edge.

Increment in curvature near the trailing edge can produce a flow near the trailing edge, which has a desirable pressure gradient on the airfoil surface [2]. This pressure distribution increases the lift, due to decrease in upper surface camber. This method was investigated by authors in [5].

### 5. Modified Sobieczky Method

As seen above, the Sobieczky method modifies airfoil near trailing edge, but it cannot ensure a physically acceptable or feasible trailing edge. This method may result in overlapping of upper and lower surfaces as shown in Figure 4 by continuous line.

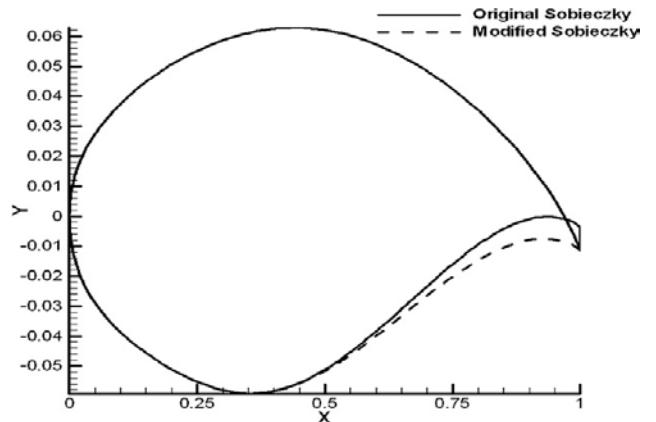


Figure 4. Original and modified Sobieczky method for DTE

To overcome this drawback, modified Sobieczky method is introduced. In this method, upper surface is created first and then constraint on lower surface is imposed to finish up at trailing edge of the upper surface. Airfoil with dotted line in Figure 4 shows the modified airfoil shape which is physically acceptable [2].

### 6. New Parameterization Method

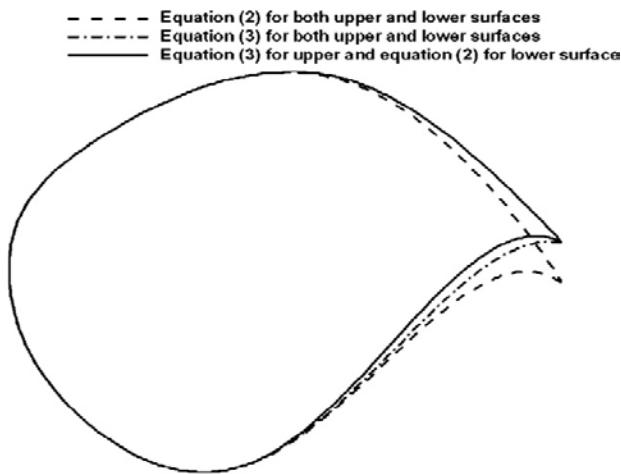
This method is proposed by the authors in Ref. [2]. The problem associated with the modified Sobieczky method is that it tends to pull the trailing edge downward to increase the curvature near the trailing edge. It may induce

the undesirable pressure gradient on the upper surface in viscous flow. It can be overcome by flattening the upper surface of the airfoil which creates weaker shock wave on the airfoil. Thus, for the shape of upper surface, new function for  $\Delta Z$  is introduced given by following equation.

$$\Delta Z_k = \frac{\tan \Delta \alpha}{\mu \cdot n} \left[ 1 - \eta \xi_k^n - (1 - \xi_k^n)^\mu \right] \quad (7)$$

Where,  $\eta$  is set to 0.8 and  $n$  is set to 6.

As can be seen in Eq.(7), parameter  $L_k$  of Eq.(6) is absent in this equation. It is due to fact that in this method not only a specified length near the trailing edge is modified but full upper surface is exposed to changes in  $Z$  coordinate. Now in this method  $\Delta Z$  is computed from Eq.(7) for both upper and lower surfaces. This method is proved to be advantageous for modeling of upper surface but not for the lower surface. This method suffers the inability to make sufficient changes in the curvature of the lower surface compared to the modified Sobecky method. It may reduce the lift. Hence to model lower surface, modified Sobecky method is used and upper surface is modeled with the help of new parameterization method. Figure 5 illustrates how different methods described above affect the shape of airfoil [2].



**Figure 5.** Airfoils created using different parameterization methods

## 7. Bezier-Parsec Parameterization

This method is a combination of Bezier and PARSEC parameterization techniques, which avails advantages of both the techniques. The Bezier-PARSEC parameterization uses PARSEC variables as parameters, which are used to define four separate Bezier curves. These four curves define the leading edge and trailing edge of the camber line and thickness distribution. The Bezier-PARSEC parameterization uses second order continuity to join the leading and trailing edges. Bezier- PARSEC parameterization is denoted by BP ijk where i and j represent the order of leading and trailing edge of thickness curve and k and l represent the order of leading and trailing edge of camber curve [1].

Oyama et al. [6] showed that Bezier- PARSEC parameterization increased the robustness and convergence speed for aerodynamic optimization using genetic algorithms [2].

### BP 3333 parameterization

As name suggests, all the four curves of the Bezier- PARSEC Parameterization, i.e. leading and trailing edges of thickness curve & leading and trailing edges of camber curve have polynomials of degree three.

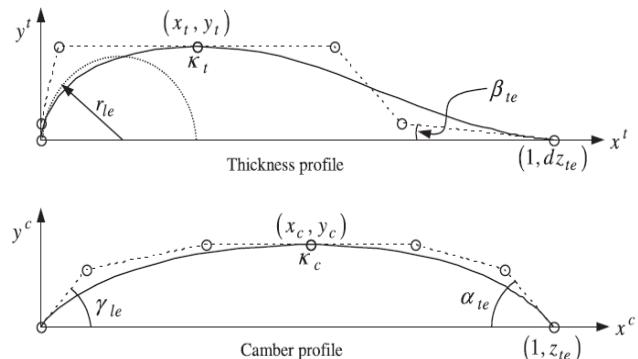
Parametrically a third degree Bezier curve is given by

$$x(u) = x_0(1-u)^3 + 3x_1u(1-u)^2 + 3x_2u^2(1-u) + x_3u^3$$

and

$$y(u) = y_0(1-u)^3 + 3y_1u(1-u)^2 + 3y_2u^2(1-u) + y_3u^3 \quad (8)$$

Where,  $u$  varies from 0 to 1. The BP 3333 parameterization depends on the 12 aerodynamic parameters shown in Figure 6 – there are no free Bezier points in BP 3333.



**Figure 6.** BP 3333 airfoil geometry and Bezier control points defined by twelve basic aerodynamic parameters

### Leading edge thickness curve

The control points are given by

$$\begin{aligned} x_0 &= 0 & y_0 &= 0 \\ x_1 &= 0 & y_1 &= 3k_t(x_t - r_t)^2/2 + y_t \\ x_2 &= r_t & y_2 &= y_t \\ x_3 &= x_t & y_3 &= y_t \end{aligned}$$

### Trailing edge thickness curve

The control points are given by

$$\begin{aligned} x_0 &= x_t & y_0 &= y_t \\ x_1 &= 2x_t - r_t & y_1 &= y_t \\ x_2 &= 1 + [dz_{te} - (3k_t(x_t - r_t)^2/2 + y_t)]\cot(\beta_{te}) & y_2 &= 3k_t(x_t - r_t)^2/2 + y_t \\ x_3 &= 1 & y_3 &= dz_{te} \end{aligned}$$

### Leading edge camber curve

The control points are given by

$$\begin{aligned} x_0 &= 0 & y_0 &= 0 \\ x_1 &= r_c \cot(\gamma_{le}) & y_1 &= r_c \\ x_2 &= x_c - \sqrt{2(r_c - y_c)/3k_c} & y_2 &= y_c \\ x_3 &= x_c & y_3 &= y_c \end{aligned}$$

### Trailing edge camber curve

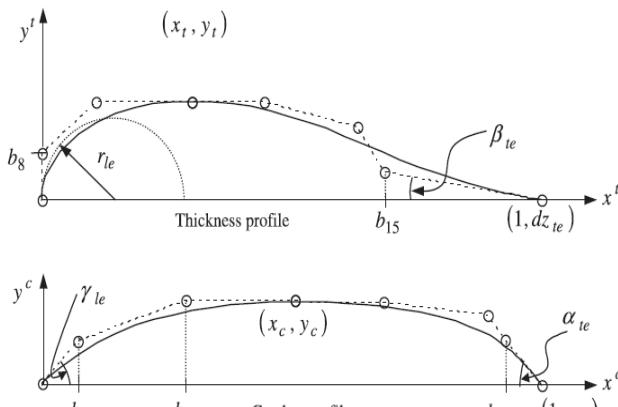
The control points are given by

$$\begin{aligned} x_0 &= x_c & y_0 &= y_c \\ x_1 &= x_c + \sqrt{2(r_c - y_c)/3k_c} & y_1 &= y_c \\ x_2 &= 1 + (Z_{te} - r_c)\cot(\alpha_{te}) & y_2 &= r_c \\ x_3 &= 1 & y_3 &= Z_{te} \end{aligned}$$

### BP 3434 parameterization

As name suggests, in BP 3434 parameterization both the leading edge curves have polynomials of degree three while both the trailing edge curves have the polynomials of degree four. The BP 3434 parameterization depends on

ten aerodynamic parameters, plus five Bezier parameters. These are shown in Figure 7.



**Figure 7.** BP 3434 airfoil geometry and Bezier control points defined by ten aerodynamic and five Bezier parameters

Parametrically a fourth degree Bezier curve is given by

$$\begin{aligned} x(u) = & x_0(1-u)^4 + 4x_1u(1-u)^3 + 6x_2u^2(1-u)^2 \\ & + 4x_3u^3(1-u) + x_4u^4 \end{aligned}$$

and

$$\begin{aligned} y(u) = & y_0(1-u)^4 + 4y_1u(1-u)^3 + 6y_2u^2(1-u)^2 \\ & + 4y_3u^3(1-u) + y_4u^4 \end{aligned} \quad (9)$$

#### Leading edge thickness curve

The control points are given by

$$\begin{aligned} x_0 &= 0 & y_0 &= 0 \\ x_1 &= 0 & y_1 &= b_8 \\ x_2 &= -3b_8^2/2r_{le} & y_2 &= y_t \\ x_3 &= x_t & y_3 &= y_t \end{aligned}$$

#### Trailing edge thickness curve

The control points are given by

$$\begin{aligned} x_0 &= x_t & y_0 &= y_t \\ x_1 &= (7x_t + 9b_8^2/2r_{le})/4 & y_1 &= y_t \\ x_2 &= 3x_t + 15b_8^2/4r_{le} & y_2 &= (y_t + b_8)/2 \\ x_3 &= b_{15} & y_3 &= dZ_{te} + (1-b_{15})\tan(\beta_{te}) \\ x_4 &= 1 & y_4 &= dZ_{te} \end{aligned}$$

#### Leading edge camber curve

The control points are given by

$$\begin{aligned} x_0 &= 0 & y_0 &= 0 \\ x_1 &= b_0 & y_1 &= b_0\tan(\gamma_{le}) \\ x_2 &= b_2 & y_2 &= y_c \end{aligned}$$

$x_3 = x_c$ <b>Trailing edge camber curve</b> The control points are given by $x_0 = x_c$ $x_1 = (3x_c - y_c \cot(\gamma_{le})) / 2$ $x_2 = (-8y_c \cot(\gamma_{le}) + 13x_c) / 6$ $x_3 = b_{17}$ $x_4 = 1$	$y_3 = y_c$ $y_0 = y_c$ $y_1 = y_c$ $y_2 = 5y_c / 6$ $y_3 = Z_{te} - (1-b_{17})\tan(\alpha_{te})$ $y_4 = Z_{te}$
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[1].

## 8. Conclusion

Bezier parameterization is a Well-known parameterization technique. PARSEC method parameterizes the airfoil with maximum aerodynamic parameters. Combination of Bezier and PARSEC techniques has proved to be very advantageous and covering a wide range of airfoils. Combination of these two techniques combines the advantages of both the techniques. BP 3333 technique converges speedily due to less number of parameters involved. But BP 3333 does not control trailing edge very sharply due to less number of parameters. BP 3434 technique overcomes the drawback of BP 3333 but convergence speed reduces due to greater number of variable. High speed digital computers can compensate the slow convergence rate of BP 3434. Hence BP 3434 can be effectively applied for optimization of airfoil.

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