Hoare 論理 (2)
- プログラム証明と構築のための手法と論理 東京大学計数工学科
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復習:代入文の公理
{Q[e/x]} x := e {Q}



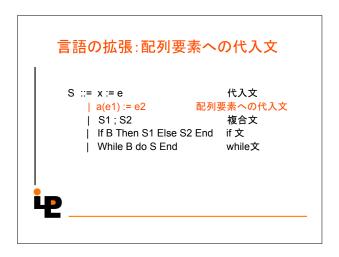
復習: if 文の規則

{P and B} S1 {Q} {P and not B} S2 {Q}

{P} If B Then S1 Else S2 End {Q}



## 



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問題点
配列変数を使うと、複雑になる。
{x(1)=1 and x(2)=3}
x(x(1)) := 2
{x(x(1))=2}
No!!!
```



```
(a(i)=a(j))[(if z=i then 1 else a(z) end)/a(z)]

←→

a(i)[(if z=i then 1 else a(z) end)/a(z)]

= a(j)[(if z=i then 1 else a(z) end)/a(z)]

←→

(if z=i then 1 else a(z) end)[i[(if z=i then 1 else a(z) end)/a(z)]/z]

= (if z=i then 1 else a(z) end)[i[(if z=i then 1 else a(z) end)/a(z)]/z]

←→

(if z=i then 1 else a(z) end)[i/z] = (if z=i then 1 else a(z) end)[i/z]
```

```
(if z=i then 1 else a(z) end)[i/z] = (if z=i then 1 else a(z) end)[j/z]

←→

(if i=i then 1 else a(i) end) = (if j=i then 1 else a(j) end)

←→

1 = (if j=i then 1 else a(j) end)

←→

(j=i => 1=1) and (j<>i => 1=a(j))

←

j=i or a(j)=1
```

```
何更2

配列要素への代入 a(a(2)):=1 に対する部分的正当性の表明
{ a(2)<>2 or a(1)=1 } a(a(2)) := 1 { a(a(2))=1 }
を示す。公理より、
{(a(a(2))=1)[(if z=a(2) then 1 else a(z) end)/a(z)]}
a(a(2)) := 1
{a(a(2))=1}
すると、
{ a(2)<>2 or a(1)=1 }
→ (a(a(2))=1)[(if z=a(2) then 1 else a(z) end)/a(z)]
を示せればよい。
```

(a(a(2))=1)[(if z=a(2) then 1 else a(z) end)/a(z)]

←→ a(a(2))[(if z=a(2) then 1 else a(z) end)/a(z)] = 1

←→ (if z=a(2) then 1 else a(z) end)

[a(2)[(if z=a(2) then 1 else a(z) end)/a(z)] /z] = 1

←→ (if z=a(2) then 1 else a(z) end)

[((if z=a(2) then 1 else a(z) end)

[2[(if z=a(2) then 2 else a(z) end)/a(z)]/z] /z] = 1

←→ (if z=a(2) then 1 else a(z) end)

[((if z=a(2) then 1 else a(z) end)[2/z]/z] = 1

## Dijkstra's WP

· Weakest Precondition (WP)



wp(S,Q):

the set of initial states that this guarantee termination of S in a state satisfying Q

$$\frac{P \Rightarrow wp(S,Q)}{\{P\} \ S \ \{Q\}}$$

wp(If B Then S1 Else S2 End, Q) = (B → wp(S1,Q)) and (not B → wp(S2,Q) wp(While B do S End, Q) = 3k:k>=0. Pk  $where P_0 = (not B) \text{ and } Q$   $P_k = B \text{ and } wp(S,P_{k-1})$ 

wp(S1;S2, Q) = wp(S1, wp(S2, Q))

WPの定義

wp(x:=e, Q) = Q(e/x)

```
wp(x:=x+1; y:=y+1, x=y)
= wp(x:=x+1, wp(y:=y+1, x=y))
= wp(x:=x+1, x=y+1)
= x+1=y+1
= x=y
```

```
wp(If i=j Then m:=k else j:=k End, k=j=m)
= (i=j → wp(m:=k,k=j=m)) and
(i/=j → wp(j:=k, k=j=m))
= (i=j → k=j=k) and (i/=j → k=k=m))
= (i=j → k=j) and (i/=j → k=m))
```

```
次のWPを求めよ.
wp(W, Q)
where W = While n<>m do S End
S = j:=j*i; k:=k+j; n:=n+1
Q = k=(i^(m+1)-1)/(i-1) and j=i^m
ただし、i<>0 and i<>1.
```

```
P2 = n=m-2 and k=(i^m-1)/(i-1) and j=i^n
...
Pr = n=m-r and k=(i^m-1)/(i-1) and j=i^n
従って:
wp(W,Q)
= ヨr:r>=0. Pr
= n=m-r and k=(i^m-1)/(i-1) and j=i^n
```

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演習問題3
次のWPを計算せよ.
wp(While i<>n do i:=i+1;s:=s+i End,
s=n*(n+1)/2)
```

## WP's Healthiness Conditions wp(S, Q and R) = wp(S,Q) and wp(S,R) wp(S, Q or R) = wp(S,Q) or wp(S,R) wp(S, not Q) = not wp(S,Q) wp(S, false) = false

wp(S, true) = all states that guarantee termination of S

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## 演習問題4

次のことを証明せよ. wp(S,Q→R) → (wp(S,Q) → wp(S,R))

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