Mathematical Structures in Programming

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プログラム構造論

- 講義内容
  - 構成的手法に基づくプログラミング方法論を扱う。
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Introduction

What is Programming?

- Programming is the art of designing efficient *algorithms* that meet their specification.
- Two factors by which algorithms may be judged:
  - Correctness: do they solve the right problems
  - Performance: how fast do they run and how much space do they use
- Classical way of judging the quality of an algorithm is
  - by tracing execution patterns,
  - by providing test inputs, or
  - by supplying formal proofs.

Verification of Algorithms

- Verification is the process of proving the correctness of an algorithms after it has been designed.
- Verification of algorithms is important:
  - bank systems
  - flight scheduling systems

  But it is often regarded as a waste of time and were largely rejected or neglected by the software community.
- Verification of algorithms is difficult, including
  - development of specification languages
  - tools supporting program verification

Could a program and its verification be constructed hand in hand, while making a posteriori program verification superfluous?

Calculational Style of Programming

- Developed by E.W. Dijkstra and others during 1970s.
- Programs are derived from their specification by formula manipulation.
  - The calculation that leads to the algorithm are carried out in small steps;
  - Each individual step is easily verified.
- In this way the design decision is manifest.
  - Program derivation is not mechanical; it is challenging activity and it requires creativity.

This calculational way of programming shows where creativity comes in. It is this method that will be explained and exemplified in this class!

Two Views of Programming

- A Common View: a program is a recipe, which
  - ▶ explains what steps have to be performed to achieve a certain goal.

```
first do this;
then apply that;
perform the following N times;
...
```

- Another View: a program together with its specification is a theorem, which
  - ▶ expresses that the program satisfy the specification.

  $\Rightarrow$ all programs require proofs (as theorems do).

We shall derive programs according to their specification in a constructive way, such that program development and correctness proof go hand in hand.

An Example of Specification and Program

- A Specification:

$\|[$	
$\mathbf{var}\ x, y : \mathrm{int}$	state space $\mathcal{Z} \times \mathcal{Z}$
$\{x = A \wedge y = B\}$	precondition
$maximum$	program under construct
$\{x = A\ \mathbf{max}\ B\}$	postcondition
$]\|$	

- A Program:

$$
\begin{array}{l}
|[\\
var\ x, y : \text{int} \\
\{x = A \wedge y = B\} \\
\textbf{if}\ x < y\ \rightarrow x := y\ [\!]\ x \geq y\ \rightarrow \text{skip}\ \textbf{fi} \\
\{x = A\ \textbf{max}\ B\} \\
]|
\end{array}
$$

The Textbook and References

- The Textbook
  - A. Kaldewaij, *Programming: The Derivation of Algorithms*, Pretence-Hall, 1990.
- References
  - First book on science of programming:
    - E.W. Dijkstra, *A Discipline of Programming*, Prentice-Hall, 1976.
  - Many notations and exercises follow the two books:
    - D. Gries, *The Science of Programming*, Springer Verlag, 1981.
    - E.W. Dijkstra and W.H.J. Feijen, *A Method of Programming*, Addison Wesley, 1988.
  - Other good books:
    - Spivey, *Programming from Specification*, Prentice-Hall, 1990.
    - R. Bird and O. de Moor, *Algebras of Programming*, Prentice-Hall, 1996.