

## リスト処理の例(その1)

胡 振江

## 例題1: 数をことばに

### ■ 問題:

0以上100万以下の数 → 通常の英語表現

例:

- 308000 → three hundred and eight thousand
- 369027 → three hundred and sixty-nine thousand and twenty-seven
- 369401 → three hundred and sixty-nine thousand four hundred and one

## 解決法

### ■ 簡単な問題から複雑問題へ

- $n < 100$  の数字を対象に
- $n < 1000$  の数字を対象に
- $n < 1000,000$  の数字を対象に

## 数の英語名: 文字列

units = [ "one", "two", "three", "four", "five",  
"six", "seven", "eight", "nine"]

teens = ["ten", "eleven", "twelve", "thirteen",  
"fourteen", "fifteen", "sixteen",  
"seventeen", "eighteen", "nineteen"]

tens = ["twenty", "thirty", "forty", "fifty", "sixty",  
"seventy", "eighty", "ninety"]

## $0 < n < 100$ の場合

convert2 n = combine2 (digits2 n)

digits2 n = (n `div` 10, n `mod` 10)

|                    |                                     |
|--------------------|-------------------------------------|
| combine2 (0,u+1)   | = units !! u                        |
| combine2 (1,u)     | = teens !! u                        |
| combine2 (t+2,0)   | = tens !! t                         |
| combine2 (t+2,u+1) | = tens !! t ++ "-" ++<br>units !! u |

## $0 < n < 1000$ の場合

convert3 n = combine3 (digits3 n)

digits3 n = (n `div` 100, n `mod` 100)

|                    |  |
|--------------------|--|
| combine3 (0,t+1)   | = convert2 (t+1)                                     |
| combine3 (h+1,0)   | = units !! h ++ " hundred"                           |
| combine3 (h+1,t+1) | = units !! h ++ " hundred<br>and " ++ convert2 (t+1) |

## 0<n<1000,000の場合

```
convert6 n = combine6 (digits6 n)
digits6 n = (n `div` 1000, n `mod` 1000)

combine6 (0,h+1) = convert3 (h+1)
combine6 (m+1,0) = convert3 (m+1) ++
  " thousand"
combine6 (m+1,h+1) = convert3 (m+1) ++
  " thousand" ++
  link (h+1) ++
  convert3 (h+1)

link h | h < 100 = " and "
| otherwise = " "
```

## 実行例

```
Convert> convert6 308000
"three hundred and eight thousand"
(985 reductions, 1350 cells)

Convert> convert6 369027
"three hundred and sixty-nine thousand and twenty-seven"
(1837 reductions, 2547 cells)

Convert> convert6 369401
"three hundred and sixty-nine thousand four hundred and one"
(1851 reductions, 2548 cells)
```

## 例題2: 可変長の算術演算

- 問題:  
任意の大きさの整数計算を行う関数パッケージを作る。
- 比較:  $[2,1,3,4] > [3]$
- 加算:  $[7,3,7] + [4,6,9] = [1,2,0,6]$
- 減算:  $[4,0,6] - [3,7,5] = [3,1]$
- 乗算:  $[1,2] * [1,5] = [1,8,0]$
- 除算:  $[1,7,8,4] \div [6,2] = [2,8] \dots [4,8]$

## 可変長整数の表現

- リストでの表現

```
type VInt = [Bigit]
type Bigit = Int
b = 10 :: Int
```
- 標準形

```
strep xs | ys == [] = [0]
| otherwise = ys
where ys = dropWhile (==0) xs

norm [3,-3,-2] = [2,6,8]

norm = strep . foldr carry [0]
where carry :: Bigit -> VInt -> VInt
      carry x (c:xs) = (x+c) `div` b : (x+c) `mod` b : xs
```

## 比較演算

```
vcompare :: (VInt->VInt->Bool) -> VInt ->
  VInt -> Bool
vcompare op xs ys = op us vs
  where (us,vs) = align xs ys

veq = vcompare (==)
vleq = vcompare (<=)
vless = vcompare (<)
```

## 加算

```
vadd :: VInt -> VInt -> VInt
vadd xs ys = norm (zipWith (+) us vs)
  where (us,vs) = align xs ys

例: vadd [7,3,7] [4,6,9] = [1,2,0,6]
```

## 減算

```
vsub :: VInt -> VInt -> VInt
vsub xs ys = norm (zipWith (-) us vs)
  where (us,vs) = align xs ys
```

例: `vsub [1,0,6] [3,7,5] = [-1,7,3,1]`

## 符号反転する関数

符号の判定:

`negative xs = head xs < 0`

符号の反転:

`vnegate = norm . map neg`

`neg x = -x`

例: `vnegate [-1,7,3,1] = [2,6,9]`

## 乗算

```
vmul xs ys = foldl1 oplus (psums xs ys)
  where psums xs ys = [norm (map (y*) xs) | y<-ys]
        xs `oplus` ys = vadd (xs++[0]) ys
```

例: `vmul [1,2,3] [4,5] = [5,5,3,5]`

## 除算: 商と余り

商と余りを求めるアルゴリズムは  
商の1桁を求められ、  
次の桁のための余りが計算される  
という計算段階を繰り返して行うものである



その結果: `[(q0,rs0),(q1,rs1),...,(qn,rsn)]`

- 商: `[q0,q1,...qn]`
- 余り: `rsn`

## divalg

■ 被除数 `xs = [x1,x2,...,xn]`

■ 除数 `ys = [y1,y2,...,ym]`



■ `(q0,rs0) = (0,[x1,...,x(m-1)])`

■ `(qk,rsk), ys, x(k+m-1) → q(k+1),rs(k+1)`  
  dstep

`divalg xs ys = scanl (dstep ys) (0,take m xs) (drop m xs)`

例: `divalg [1,7,8,4] [6,2]`  
      `= [(0,[1]),(0,[1,7]),(2,[5,4]),(8,[4,8])]`

## dstep

dstepの定義:

- 被除数`xs`の長さが除数`ys`の長さより短い
- または、等しいか
- または、それより長い

`dstep ys (q,rs) x`

| `length xs < length ys = astep xs ys`

| `length xs == length ys = bstep xs ys`

| `length xs == length ys + 1 = cstep xs ys`

**where** `xs = rs ++ [x]`

## astep, bstepの定義

- 1 被除数 $xs$ の長さが除数 $ys$ の長さより短い  
 $astep\ xs\ ys = (0, xs)$
- 2 被除数 $xs$ の長さが除数の長さと等しい  
 $bstep\ xs\ ys \mid negative\ zs = (0, xs)$   
 $\mid otherwise = (1, zs)$   
**where**  $zs = vsub\ xs\ ys$   
**条件:**  $head\ ys \geq b \div 2$

## cstepの定義

- 3 被除数 $xs$ の長さが除数 $ys$ の長さより長い  
 $q'-2 \leq q \leq q'$   
ここで、 $q' = \min((x0*b+x1) \div y1) (b-1)$   
 $cstep\ xs\ ys \mid vless\ rs0\ ys = (q, rs0)$   
 $\mid vless\ rs1\ ys = (q+1, rs1)$   
 $\mid otherwise = (q+2, rs2)$   
**where**  $rs0 = vsub\ xs\ (bmul\ ys\ q)$   
 $rs1 = vsub\ rs0\ ys$   
 $rs2 = vsub\ rs1\ ys$   
 $q = guess\ xs\ ys - 2$

## 条件を満たすように

- 2 被除数 $xs$ の長さが除数の長さと等しい  
 $bstep\ xs\ ys \mid negative\ zs = (0, xs)$   
 $\mid otherwise = (1, zs)$   
**where**  $zs = vsub\ xs\ ys$   
**条件:**  $head\ ys \geq b \div 2$   
  
 $vqrm\ xs\ ys = (strep\ qs, strep\ rs)$   
**where**  $qs = map\ fst\ ds$   
 $rs = bdiv\ (snd\ (last\ ds))\ d$   
 $ds = divalg\ (bmul\ xs\ d)\ (bmul\ ys\ d)$   
 $d = b \div (head\ ys + 1)$