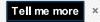
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probability puzzle, expected payoff for game

This was a question at a programming contest that finished recently at interviewstreet:

Alice and Bob play a game. The operations at round i ($i \ge 1$) is as follows:

- Alice pays Bob 2i 1 dollars,
- Alice tosses a biased coin,
- If the result of the coin was heads for k consecutive rounds, the game stops, otherwise the game continues.

Given k and the probablity p that the outcome of a toss is heads, your program should find the expected number of dollars Alice pays Bob, and also the expected number of rounds played.

I had gotten the first part of the problem, i.e the expected number of rounds of the game. I got it with the following code

```
if __name__ == '__main__':
    t = int(raw_input())
    while t :
        t -= 1
        temp = str(raw_input())
        p,k = temp.split(' ')
        p = float(p)
        k = int(k)
        num = k * (p**k)
        den = 1
        q = 1.0 - p
        for N in range(1,k+1):
            den = den - ((p**(N-1))*q)
            num = num + (N*(p**(N-1))*q)
            #print (N*(q**N))
```

But the second part of the problem is still puzzling me, i.e the expected number of dollars Alice pays bob. How can expected payoff be calculated?

(probability)

edited May 5 '12 at 21:18

MJD

20.2k 4 32 4 105



1 Answer

Call N the number of rounds played, then the number of dollars paid by Alice is \mathbb{N}^2 . The usual one-step recursion yields

$$\mathrm{E}_{i}(s^{N}) = s(p\mathrm{E}_{i+1}(s^{N}) + (1-p)\mathrm{E}_{0}(s^{N})),$$

for every $0 \le i \le k-1$, with $E_k(s^N) = 1$. One is after $E_0(N)$ and $E_0(N^2)$ and the linear system above yields $E_0(s^N) = u(s)$ with

$$u(s) = rac{{{{(1 - ps)}{{(ps)}^k}}}}{{1 - s + {{(1 - p)}s{(ps)}^k}}} = 1 - {{(1 - s)}}rac{{1 - {{(ps)}^k}}}{{1 - s + {{(1 - p)}s{(ps)}^k}}},$$

hence $u'(1) = E_0(N)$, that is,

$$\mathrm{E}_0(N) = rac{1-p^k}{(1-p)p^k}\,.$$

Similarly, $u''(1) = E_0(N(N-1))$. This yields a slightly more complicated, but explicit, formula for $E_0(N^2)$.

edited May 7 '12 at 16:10

answered May 6 '12 at 13:53



Thank you for your answer. Idon't quite understand, some of the notation, what is s, and how did you calculate u(s)? - rohanag May 9 '12 at 11:25

The argument s can be any number such that $|s|\leqslant 1$. One computes u(s) by considering, for each such s, the affine system the vector $(\mathbf{E}_i(s^N))_{0\leqslant i\leqslant k}$ solves. — Did May 9 '12 at 13:23