Reconstruction algorithm for DBT

1. **Introduction**

We are using separable quadratic surrogate (SQS) algorithm with Nesterov acceleration for the reconstruction.

For the SQS, please refer to

* Elbakri, I.A. and Fessler, J.A., 2002. Statistical image reconstruction for polyenergetic X-ray computed tomography. *IEEE transactions on medical imaging*, *21*(2), pp.89-99.

For Nesterov acceleration, we used it in our previous works:

* Wu, D., Kim, K., El Fakhri, G. and Li, Q., 2017. Iterative low-dose CT reconstruction with priors trained by artificial neural network. *IEEE transactions on medical imaging*, *36*(12), pp.2479-2486.
* Kim, K., El Fakhri, G. and Li, Q., 2017. Low‐dose CT reconstruction using spatially encoded nonlocal penalty. *Medical physics*, *44*(10), pp.e376-e390.

1. **Object Function**

The reconstruction object function is

where is the image to be reconstructed; is the system matrix constructed by distance driven; is the after-log projection; is the noise-weighting; is the hyperparameter to balance between data fidelity and the prior; and means the th and th pixel in ; is the 3×3×3 cube around including itself.

We constructed using branchless distance driven [1] in phase1A, phase1B, and pre-phase2, but it requires double precision when calculating the accumulated images, float32 will lead to overflow. We just changed it to the non-branchless version. There is only minor difference on the results between branchless and non-branchless DD invisible to human eyes, and no major difference in speed was observed.

For the in the data fidelity term, it is only meant to stabilize the choice of , when the changes significantly. Otherwise, the value of may go from to for different geometries. It was calculated using the power method by 10 iterations (forward and backprojection), and can be safely removed for the inference code, where the geometry is basically the same.

When generating the training dataset or testing using original projections, we set ; when testing using the forward projections, we set . was set to 1 for valid projections and 0 for the regions that were blocked by the collimators.

[1] Liu, R., Fu, L., De Man, B. and Yu, H., 2017. GPU-based branchless distance-driven projection and backprojection. *IEEE transactions on computational imaging*, *3*(4), pp.617-632.

1. **Separable Quadratic Surrogate (SQS)**

Please refer to Fessler’s paper on how SQS works and how to derive the optimization algorithm from the object function. The paper gave the derivation of the data fidelity part. We also give the derivation for the prior part in the appendix.

The optimization algorithm is

where is the 3×3×3 mean filter on ; is an all-one matrix sharing the same shape with .

1. **Nesterov Acceleration**

Nesterov acceleration will accelerate the convergence of the algorithm. Denote the base optimization algorithm as , Nesterov acceleration do the following steps for each iteration:

where is the acceleration factor which is set to 0.5 in our studies. When starting the algorithm, we used .

1. **Summary of the Parameters**

For reconstruction of the training dataset, we used , , 20 iterations.

For testing-time post iteration with forward projection, we used , , 10 iterations.

For testing-time post iteration with original projection, we used , , 10 iterations.

**Appendix. Derivation of SQS for Non-local weighted Gaussian prior**

Prior function is defined as:

Use convex splitting:

Equality holds at .

Surrogate:

First order derivative:

Second order derivative:

Derivatives at current position:

For most cases, we have

And the derivatives became:

The loss can be calculated as