Reply to Referee Report - DU12480/Pan

Zhen Pan

I thank the referee for careful reading of this manuscript and giving insightful comments. I summarize our corrections/clarifications as follows, and all important modifications in the manuscript are highlighted in bold font.

The author discusses the role of boundary conditions (at infinity and at the equator) in black hole forcefree magnetospheres. This clarifies a few aspects of the problem and is worthy of publication. However, I have some concerns I would like to first see addressed:

1) There should be more discussion of the physical meaning of the radiation condition (3). What is the relationship to radiation, and what does it mean?

Pan et al. proposed that the two eigenfunctions are not independent; instead, they are related by the radiation condition at infinity, which is formulated as $\hat{E}_{\theta} = \hat{B}_{\phi}$, with \hat{E}_{θ} and \hat{B}_{ϕ} being the θ component of electric field and ϕ component of the magnetic field measured by zero-angular-momentum-observers, respectively. As for the uniform field solution, the radiation condition is explicitly expressed as

$$I = 2\Omega A_{\phi}$$

which has been readily confirmed by recent high-accuracy FFE simulations (East2018).

- 2) The notion of a light surface (LS) is not defined anywhere, as far as I can tell. Also, it would help to state the meaning of the LS function (it vanishes at LSs) right when it is defined.
- 3) The argument for the LS intersecting the ergosphere boundary on the equator needs clarification. I found it hard to follow, and I was easily able to find a counter-example to the first statement (a=.1, Omega=.15; there are two zeros of K on the equator, both at r;2).

Finally, a suggestion: The proof that the inner light cylinder intersects the ergosphere boundary at the equator is a very nice result (assuming it is correct). A few groups have seen that the current sheet ends at the ergosphere in full simulations but people seem sheepish about making the claim because of numerical resolution. The analytical argument is a big help here, so the author might consider emphasizing the point more somehow.

We thank the referee for comments that we feel have .