



# Human Priors in Hierarchical Program Induction

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## Background

- Human problem-solving behavior is organized into **rich hierarchical structure** [1, 2].
- Inferring this structure is essential for interpreting behavior and anticipating how others will act.

## Research Goals

- Previous work cast hierarchy learning as an efficient coding problem and found people often generate shorter programs to solve problems [3].
- Here, we examine alternative **program features** that constrain how people **interpret** hierarchically organized behavior.

## Programs as Problem Solutions

- A program ( $\pi$ ) is a set of subprocesses,  $\sigma_i$ , which are sequences of **primitive actions** or **calls to other subprocesses**.

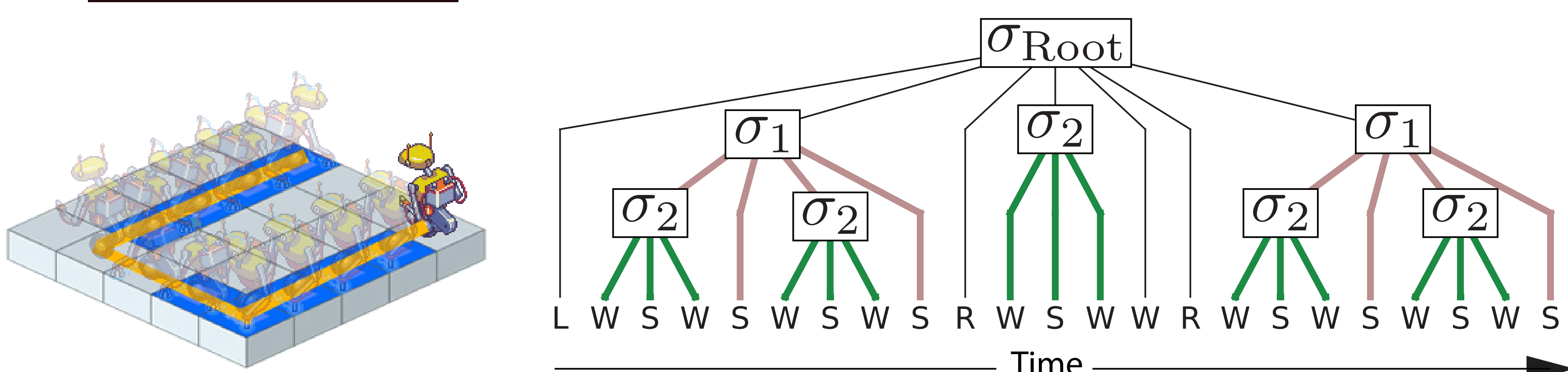
$$\pi = \{\sigma_{\text{Root}}, \sigma_1, \sigma_2\}$$

$$\sigma_{\text{Root}} = (\text{Left}, \sigma_1, \text{Right}, \sigma_2, \text{Walk}, \text{Right}, \sigma_1)$$

$$\sigma_1 = (\sigma_2, \text{Switch}, \sigma_2, \text{Switch})$$

$$\sigma_2 = (\text{Walk}, \text{Switch}, \text{Walk})$$

- Executing a program produces a **state-action trace** as well as an **execution tree**.



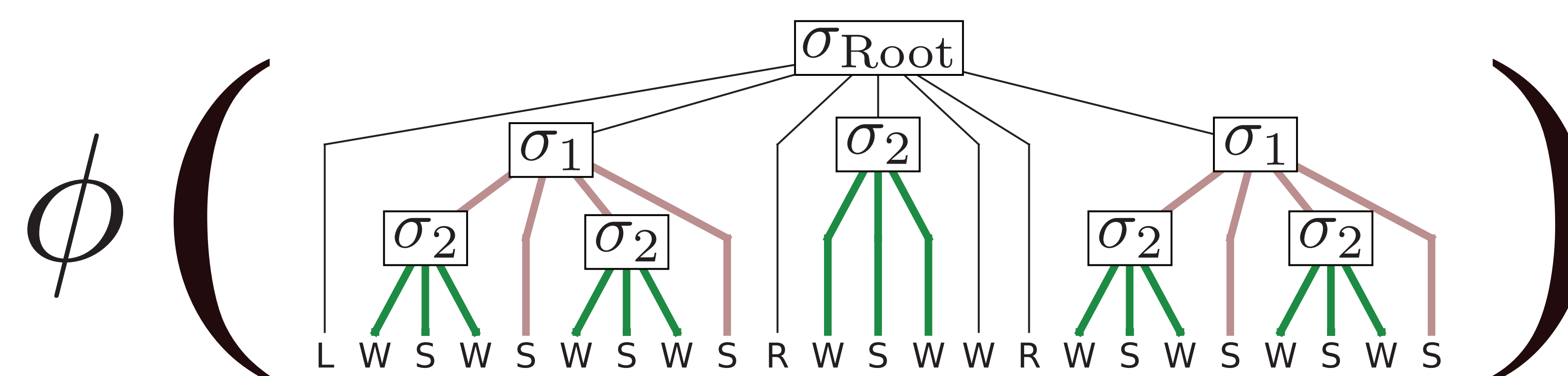
## Inferring Programs

- Inducing a program from a trace can be expressed as probabilistic inference:

$$p(\pi | \zeta) = \frac{p(\zeta | \pi)p(\pi)}{\sum_{\pi'} p(\zeta | \pi')p(\pi')}$$

- The program prior is a function of weighted program features.

$$p(\pi; \theta) \propto \exp\{\theta^\top \phi(\pi)\}$$



$$= \begin{bmatrix} 14 \\ 3 \\ 2 \\ 1.7 \\ 0.66 \\ 0.81 \\ 0.46 \\ 7 \\ 1.67 \\ 1.5 \\ 1.38 \\ 0.58 \end{bmatrix}$$

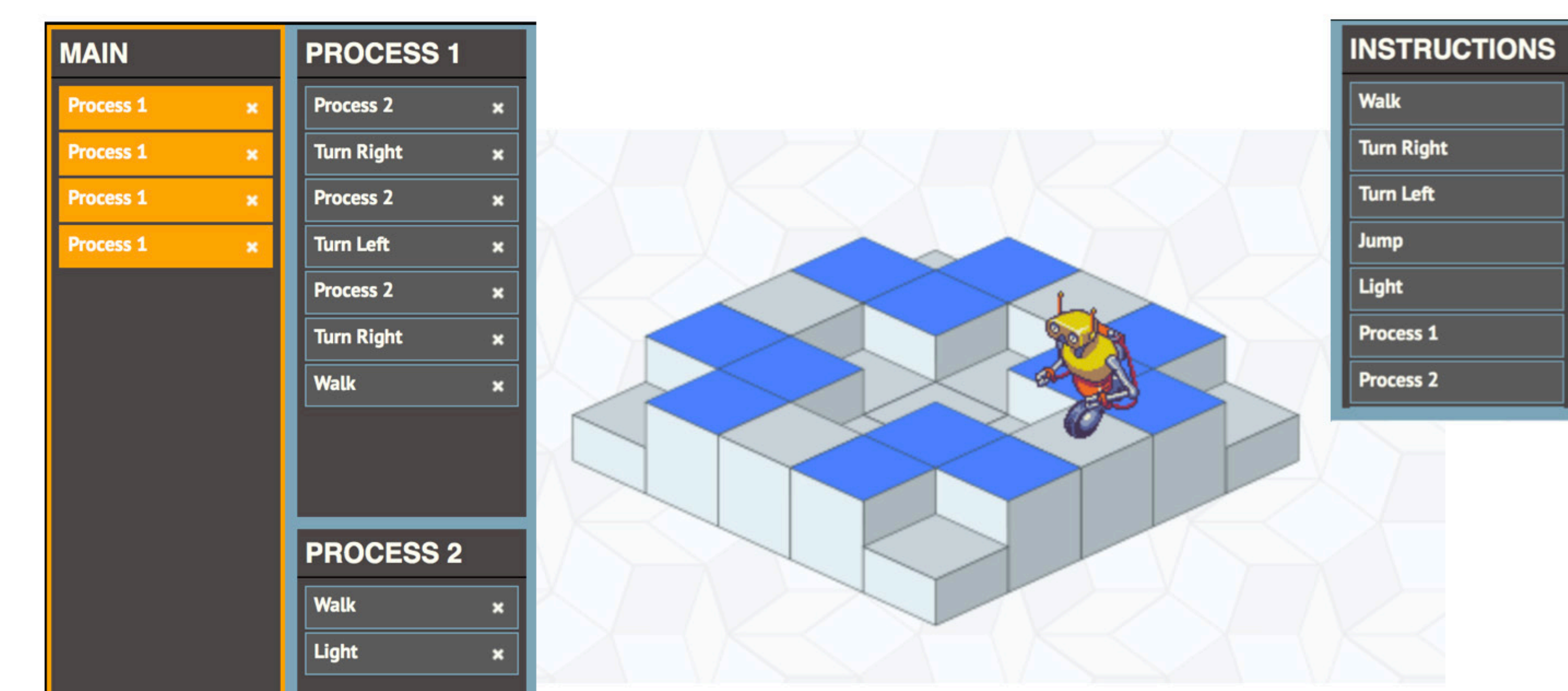
Total Program Length  
 Execution Tree Depth  
 Number of Subprocesses  
 Subprocess Length (S.D.)  
 Action/Subprocess Entropy (M)  
 Action-Call Entropy (M)  
 Process-Call Entropy (M)  
 Root Length  
 Children per Subprocess (M)  
 Parents per Subprocess (M)  
 Subprocess Entropy (M)  
 Subprocess to Action Ratio (M)

- Given features  $\phi$ , trace  $\zeta$ , and program  $\pi$ , we want to estimate the feature weights  $\theta$

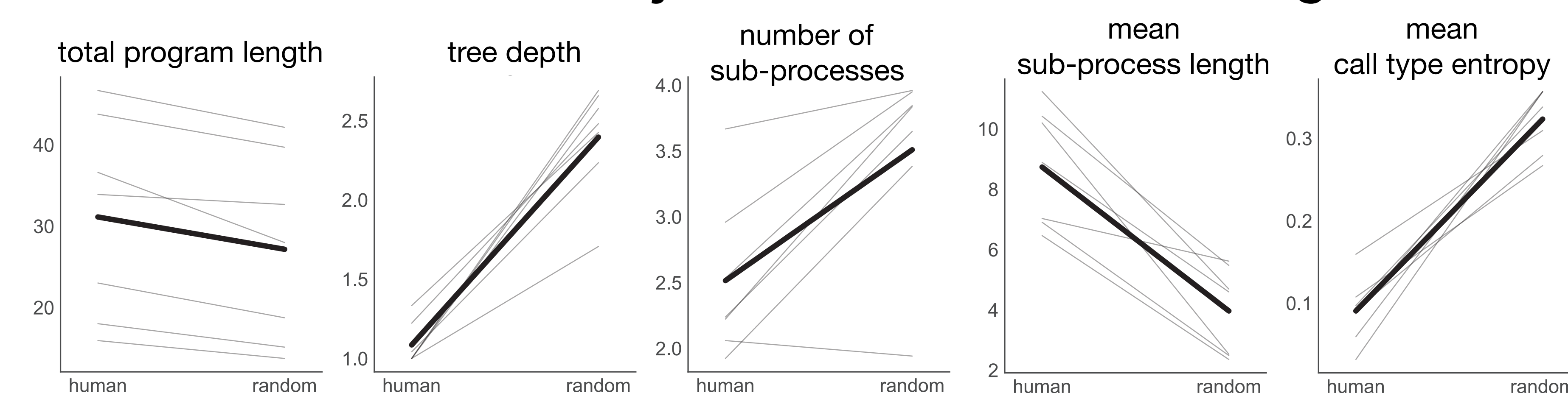
$$p(\theta | \pi, \zeta) \propto \frac{p(\zeta | \pi)p(\pi; \theta)}{p(\zeta; \theta)}p(\theta)$$

## Estimating Human Priors

- 77 participants observed human-generated solution traces in the Lightbot domain [1] and tried to reconstruct the original programs that generated the traces.



### Empirical Feature Weights for Humans vs. Randomly Generated Consistent Programs



## Conclusions

- Participants prefer programs that have shallower trees and use fewer, longer subprocesses.
- These findings may reflect working memory constraints that limit the structural complexity of induced programs.
- In ongoing work, we are investigating the relationships between these features and their individual contributions to program induction, program generation, and perceived program complexity.

### References

- [1] Simon, H. A. (1991) The Architecture of Complexity.
- [2] Solway, A., Diuk, C., Cordova, N., Yee, D., Barto, A., Niv, Y., & Botvinick, M. (in press). Optimal behavioral hierarchy. PLOS Computational Biology.
- [3] Sanborn, S., Bourgin, D., Chang, M., & Griffiths, T. (2018). Representational efficiency outweighs action efficiency in human program induction. In Proceedings of the 40th annual cognitive science society.