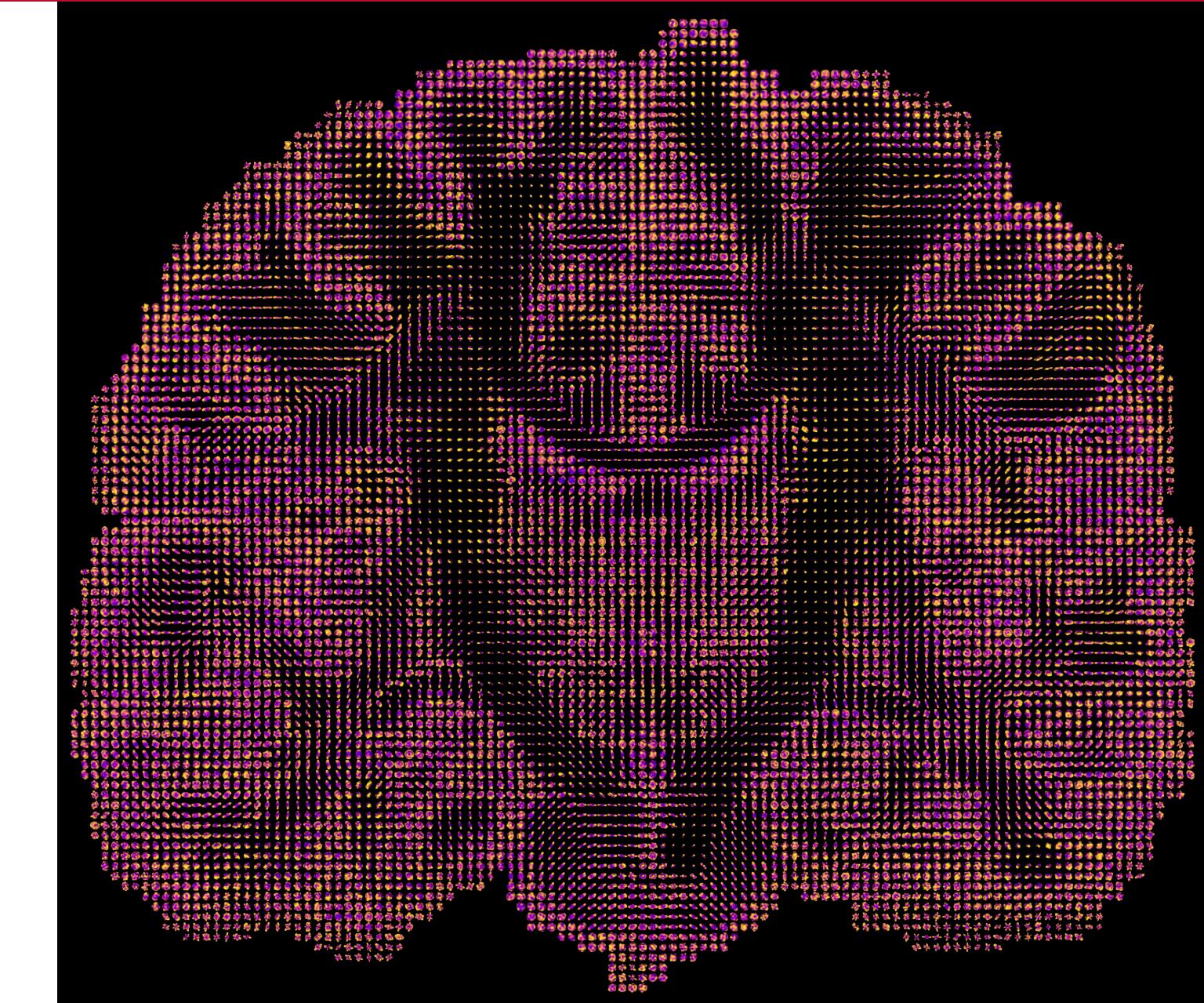
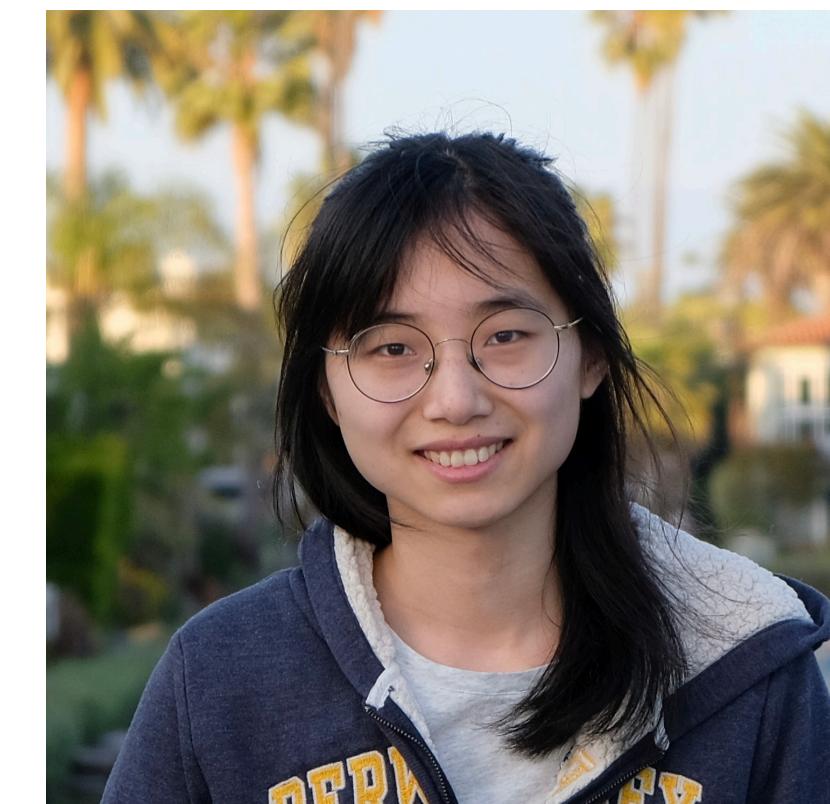
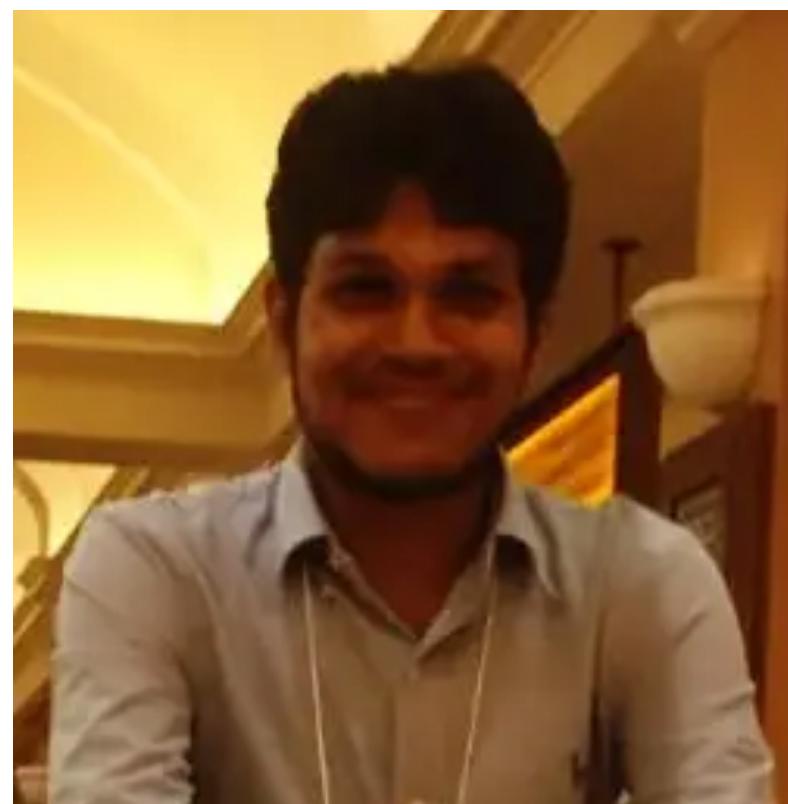




# Flow-based Generative Models For Learning Manifold to Manifold Mappings



Xingjian Zhen, Rudrasis Chakraborty, Liu Yang, Vikas Singh

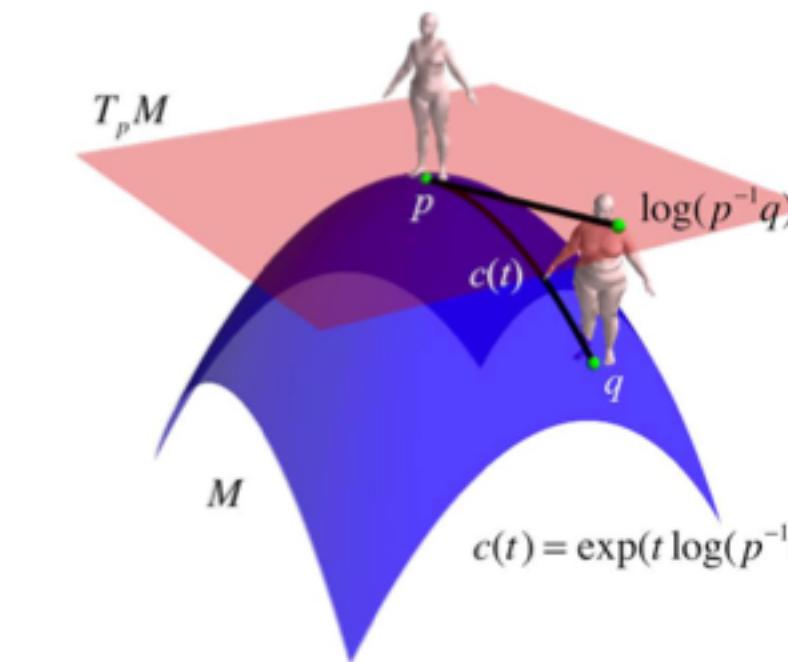




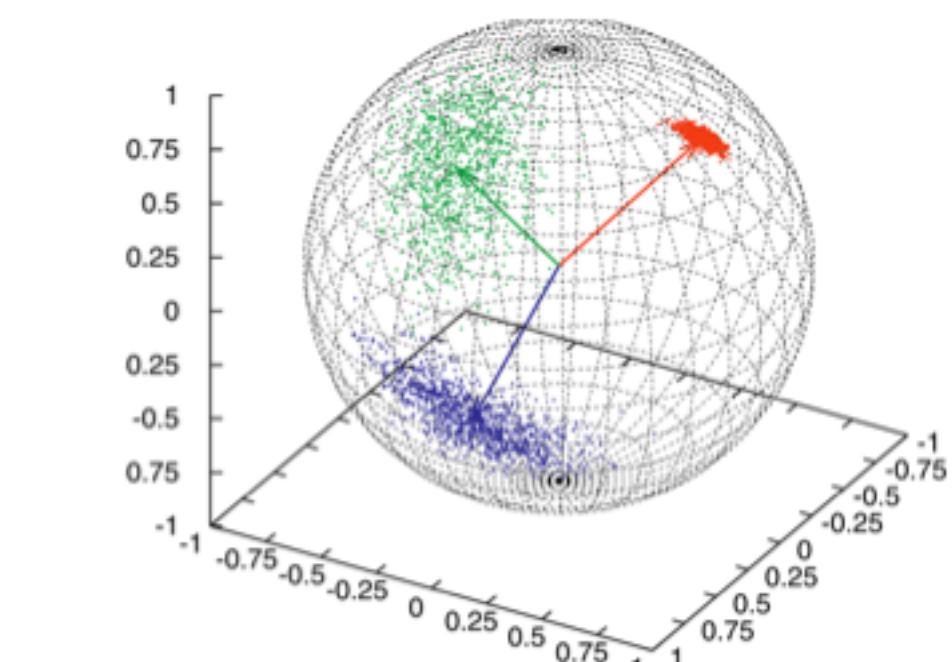
# Manifold data arises in many application



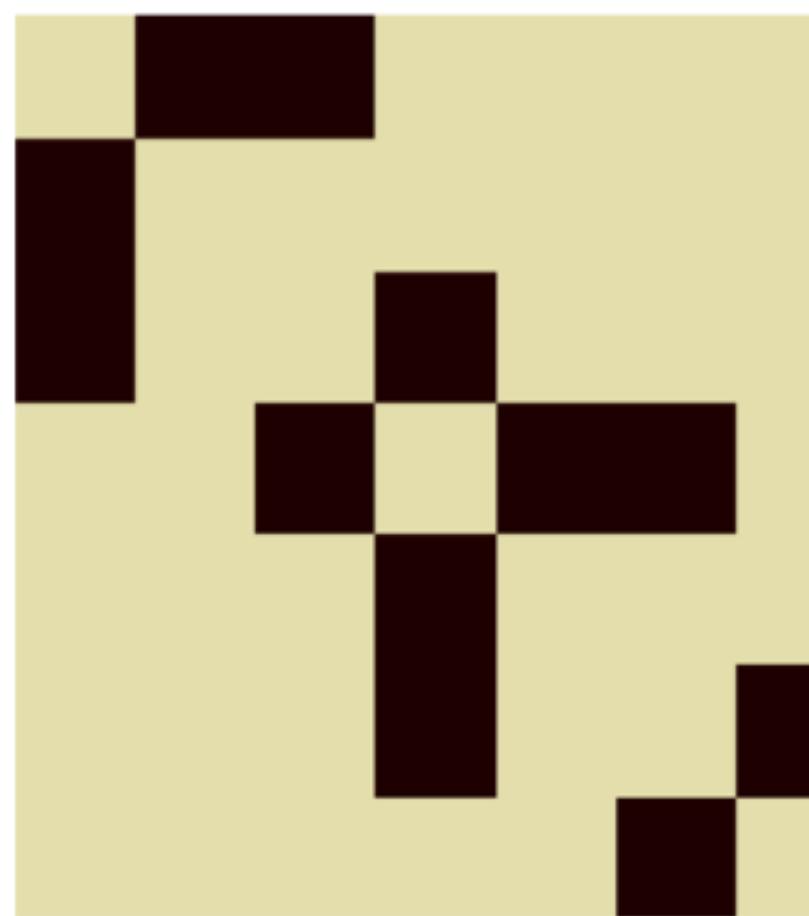
(a) Shapes



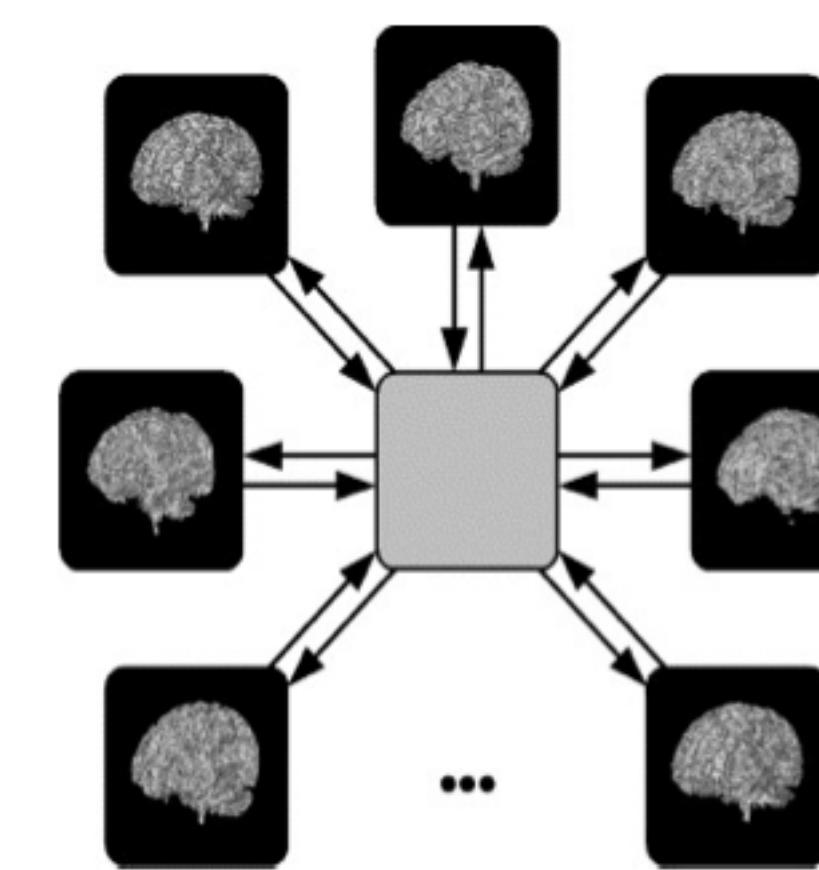
(b) Lie body manifolds



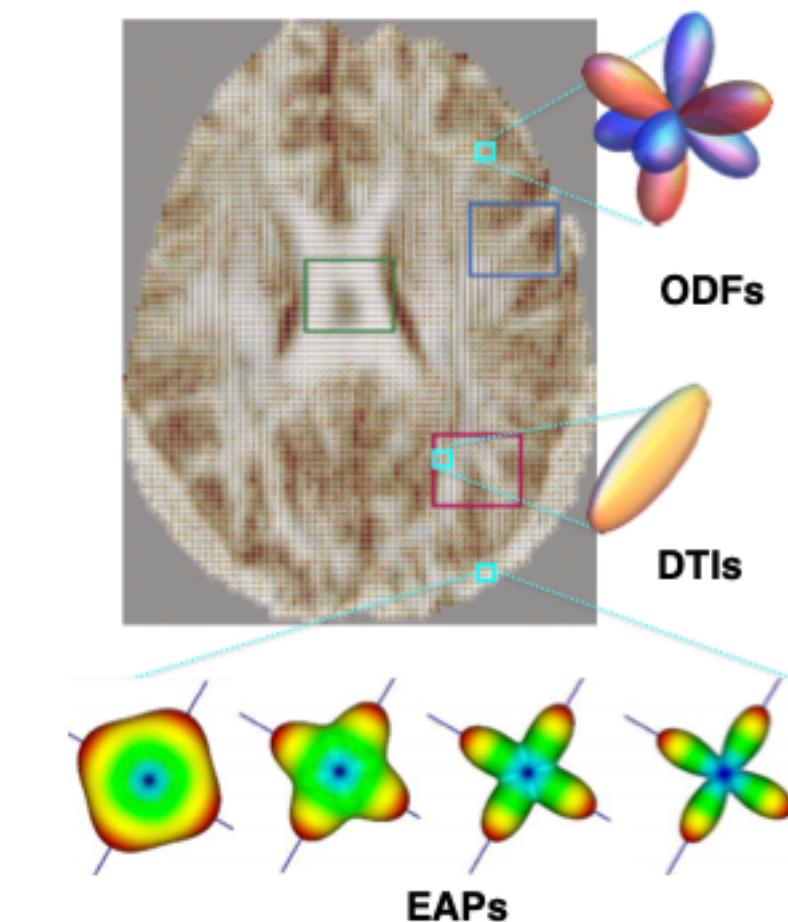
(c) Directional data



(d) Covariance matrix



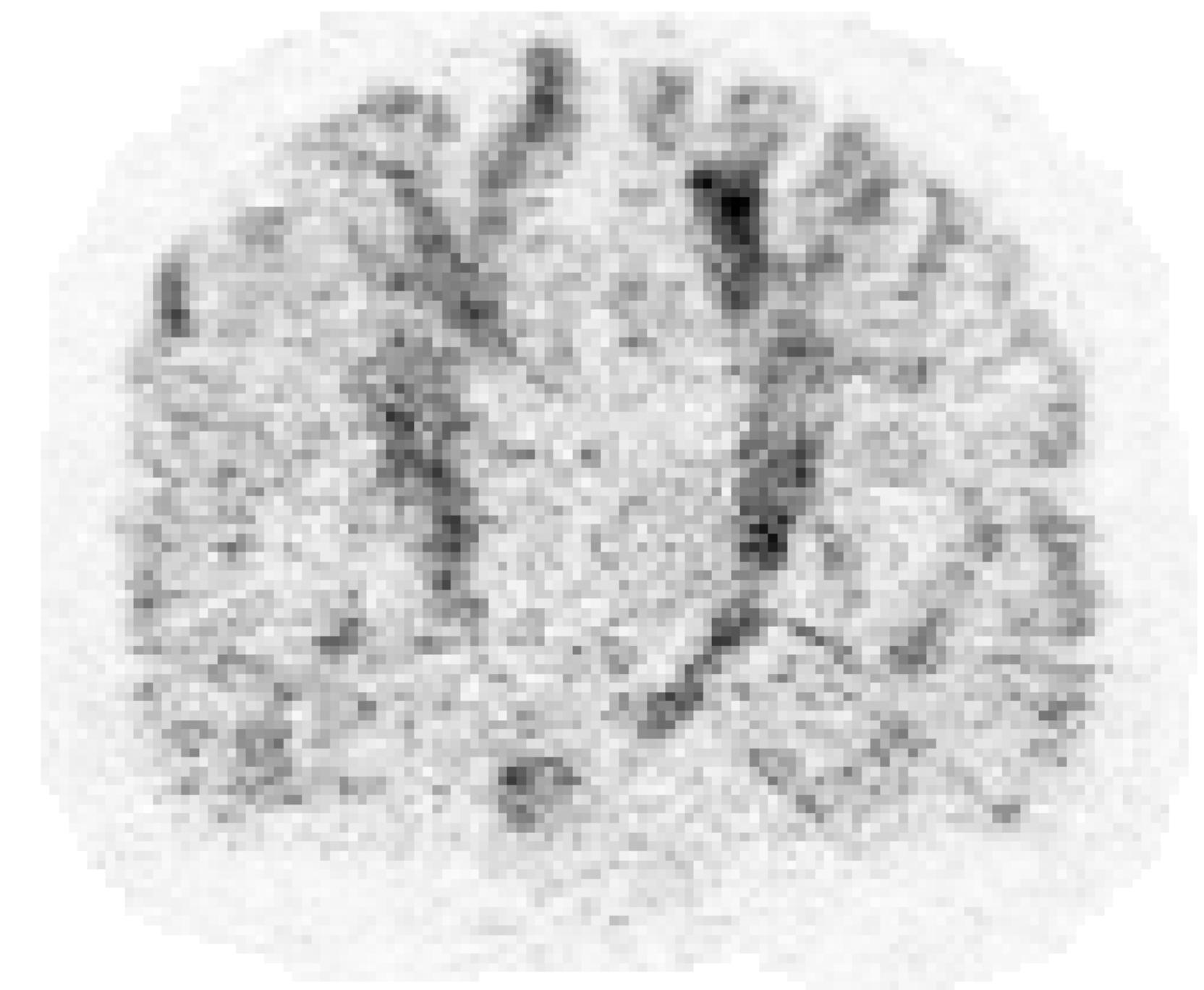
(e) Brain atlas



(f) Diffusion Measurements



# Brain connectivity and Diffusion MRI

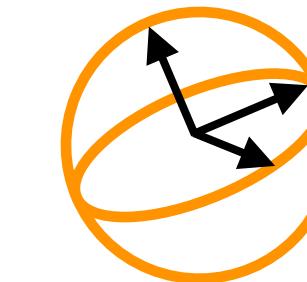
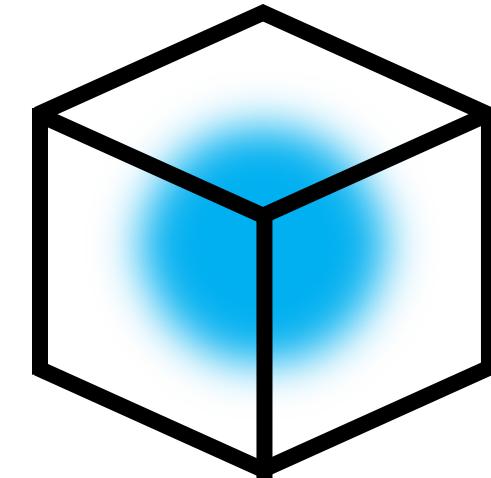


Diffusion MRI

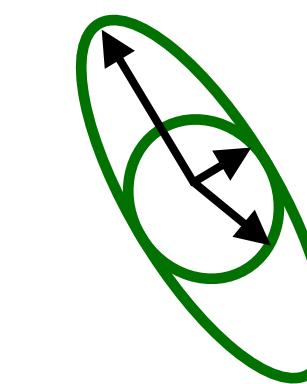
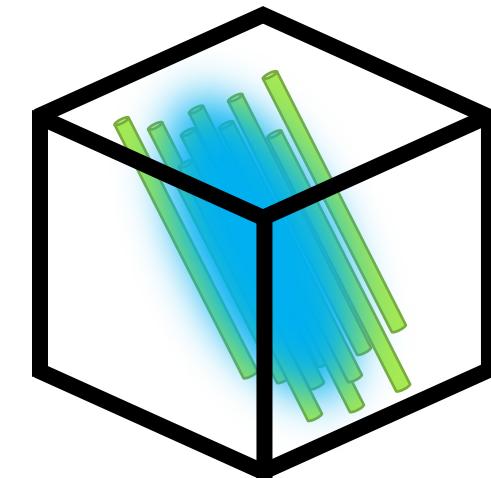


# Assessing tissue microstructure using dMRI

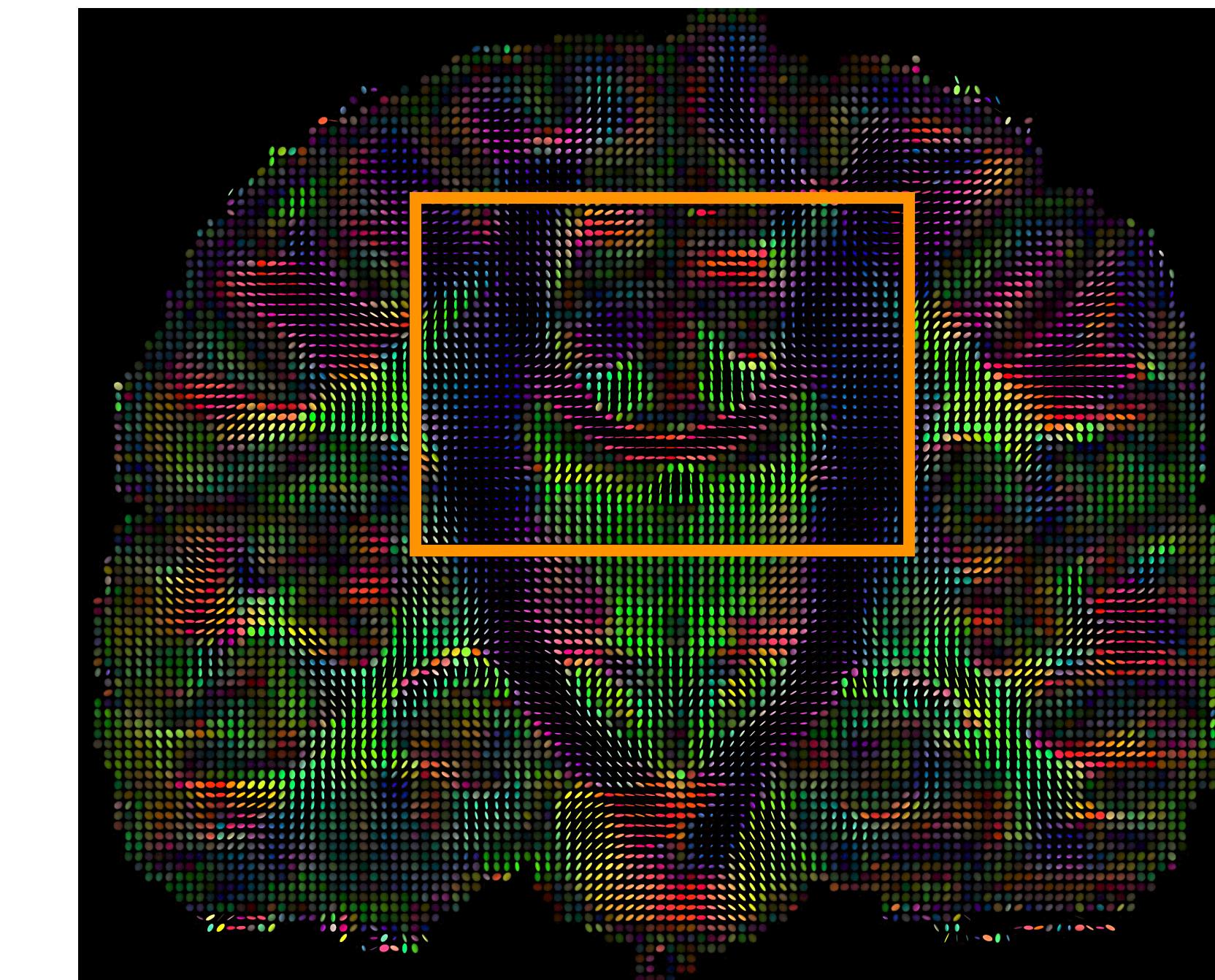
**Isotropic diffusion**



**Anisotropic diffusion**



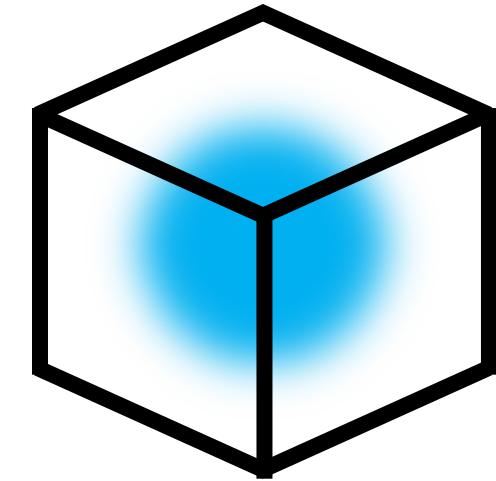
**Diffusion tensor imaging (DTI)**



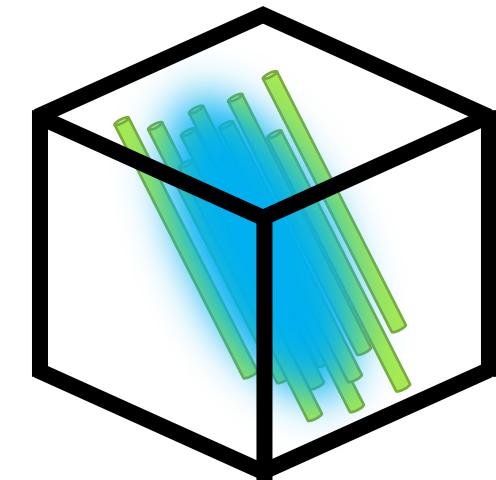


# Assessing tissue microstructure using dMRI

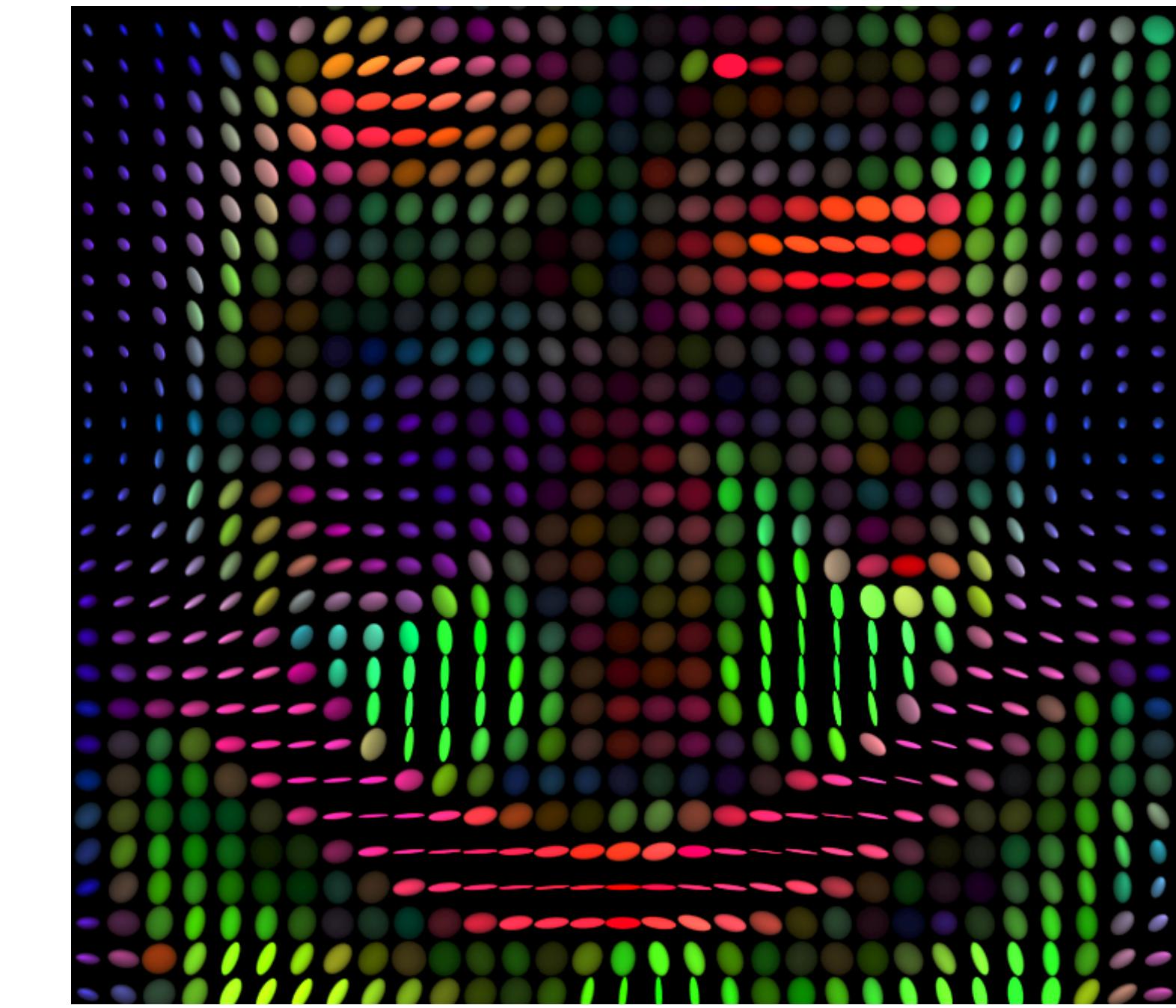
**Isotropic diffusion**



**Anisotropic diffusion**



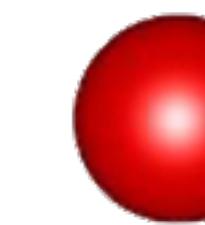
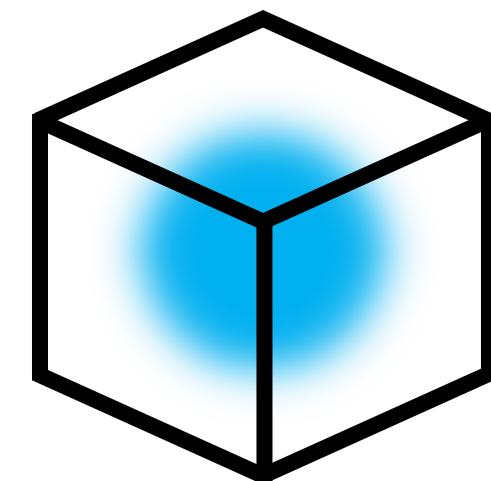
**Diffusion tensor imaging (DTI)**



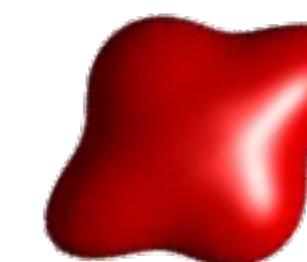
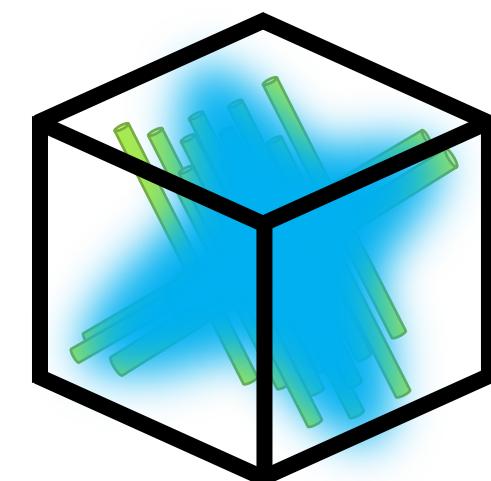


# Assessing tissue microstructure using dMRI

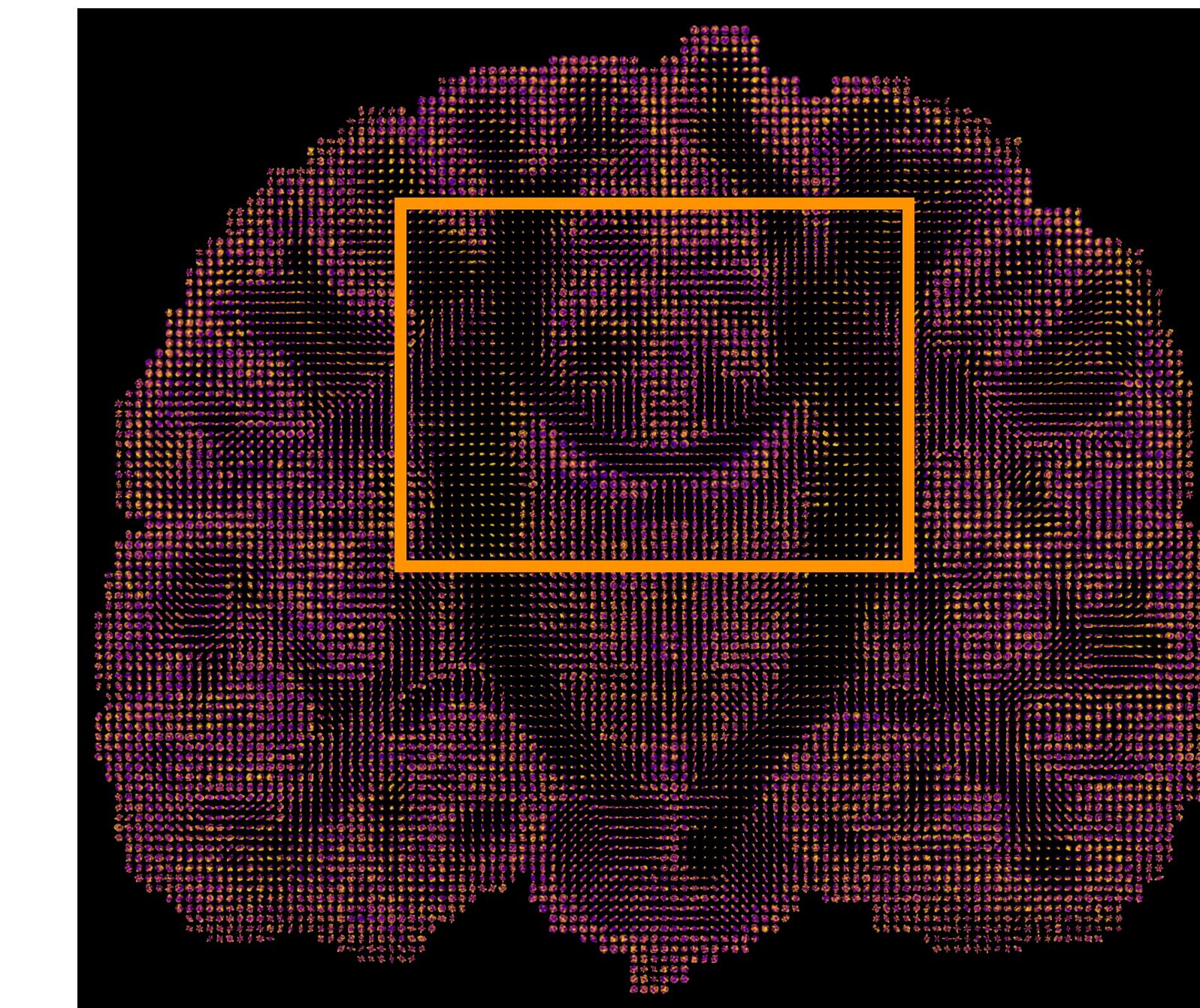
**Isotropic diffusion**



**Anisotropic diffusion**



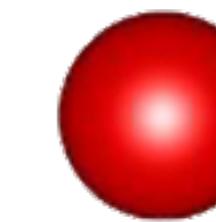
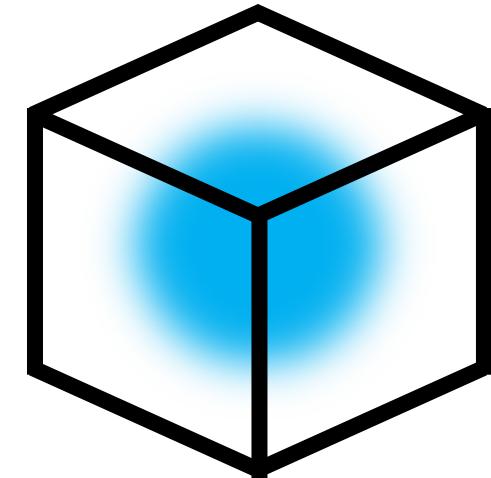
**Orientation distribution function (ODF)**



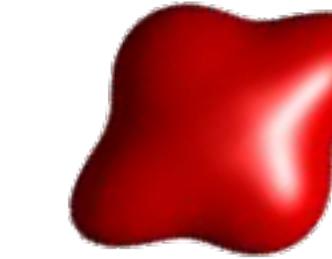
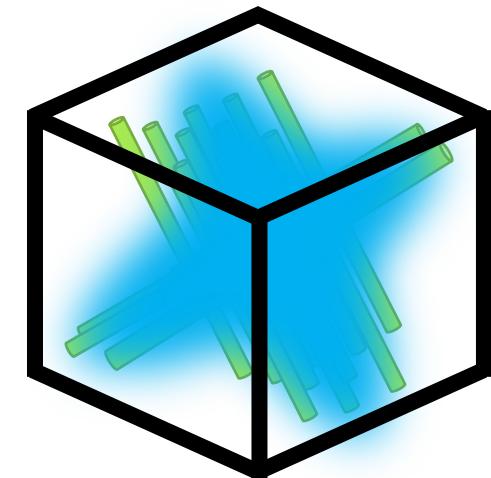


# Assessing tissue microstructure using dMRI

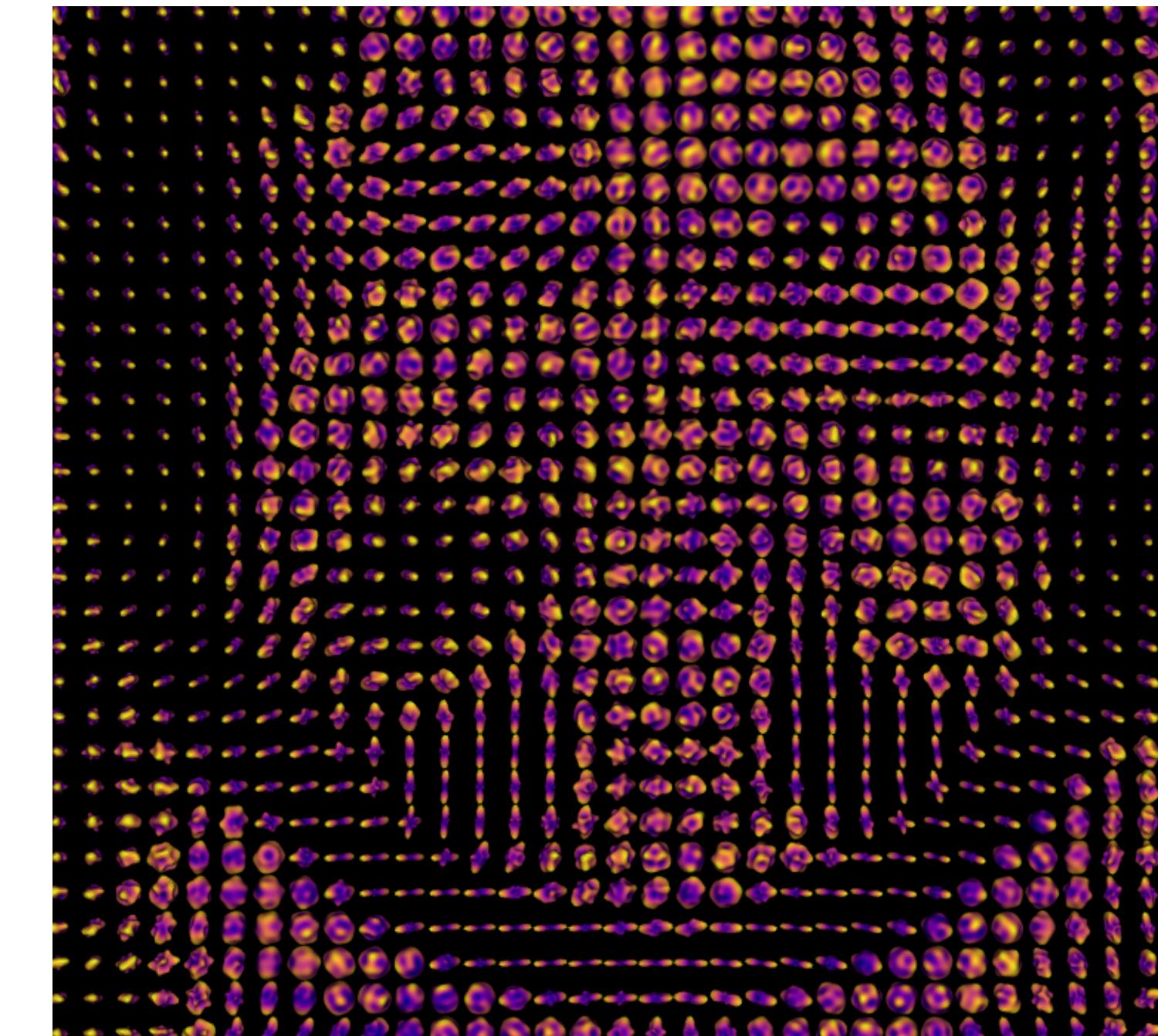
**Isotropic diffusion**



**Anisotropic diffusion**



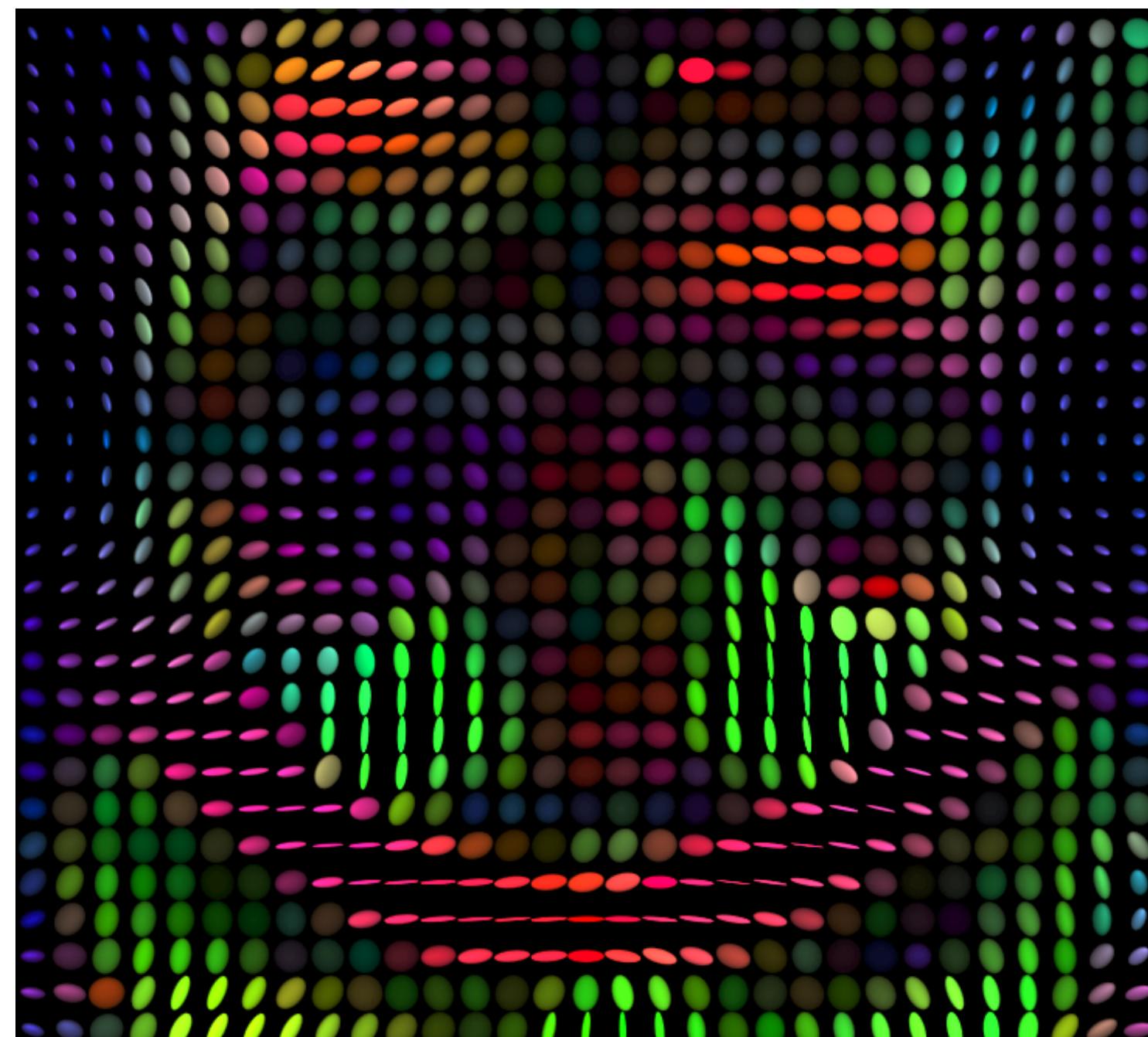
**Orientation distribution function (ODF)**



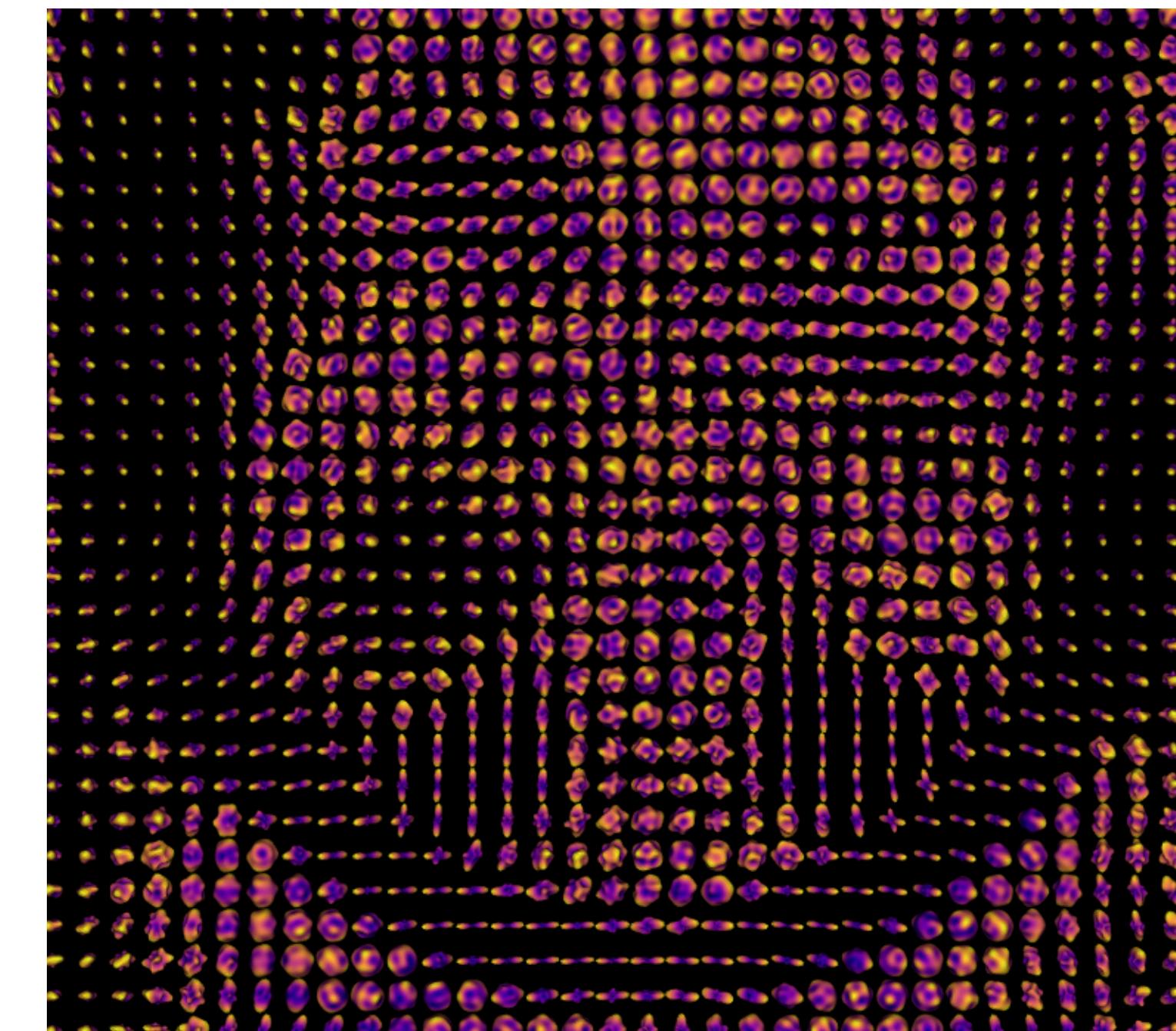
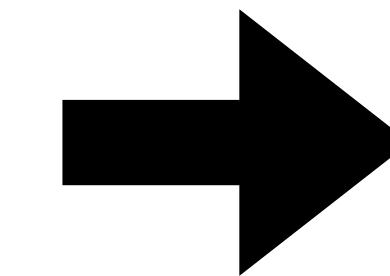


# Assessing tissue microstructure using dMRI

Diffusion tensor imaging (DTI)

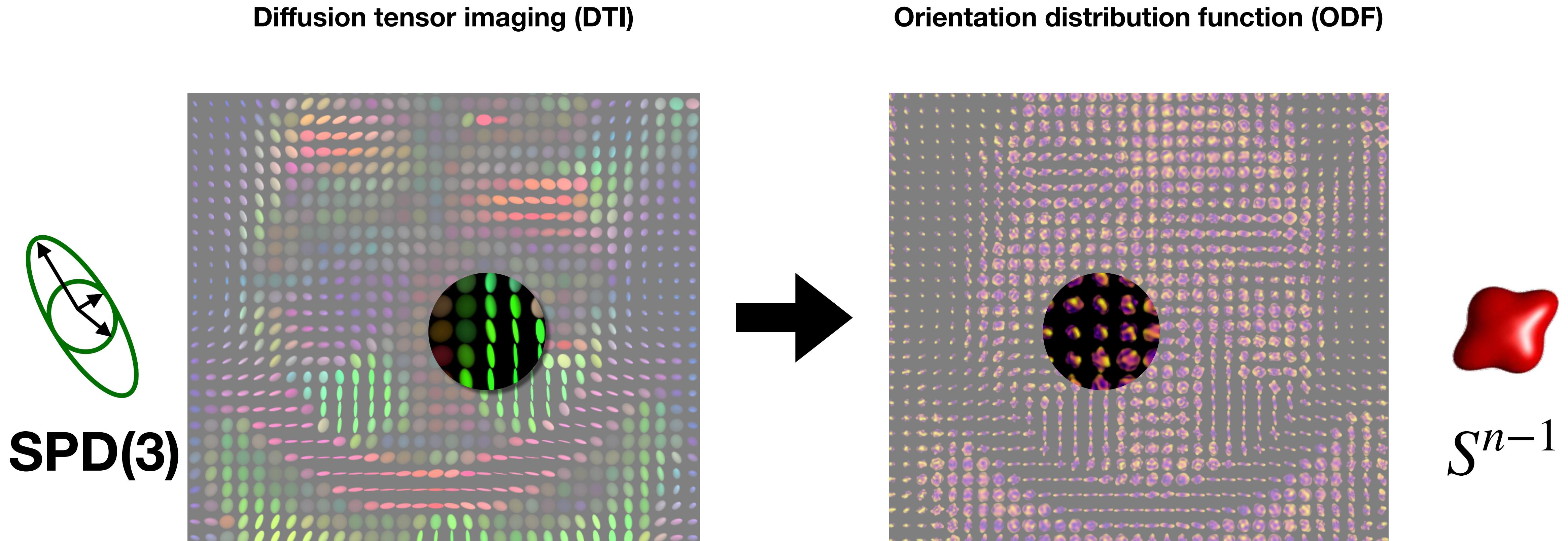


Orientation distribution function (ODF)





# Assessing tissue microstructure using dMRI





# First attempt - Vectorize

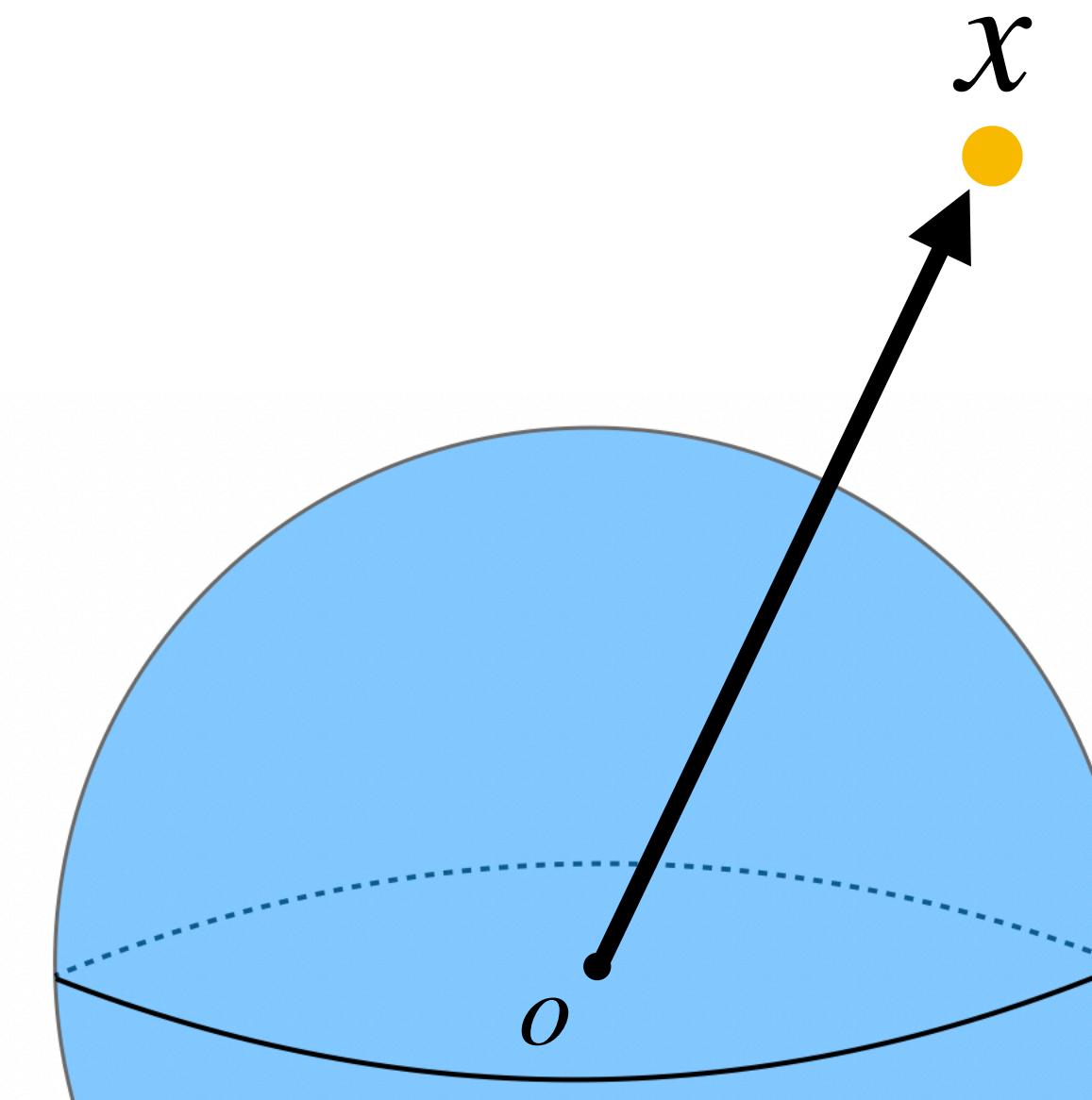
2	1	0
1	2	1
0	1	2

$\in SPD$



# First attempt - Vectorize

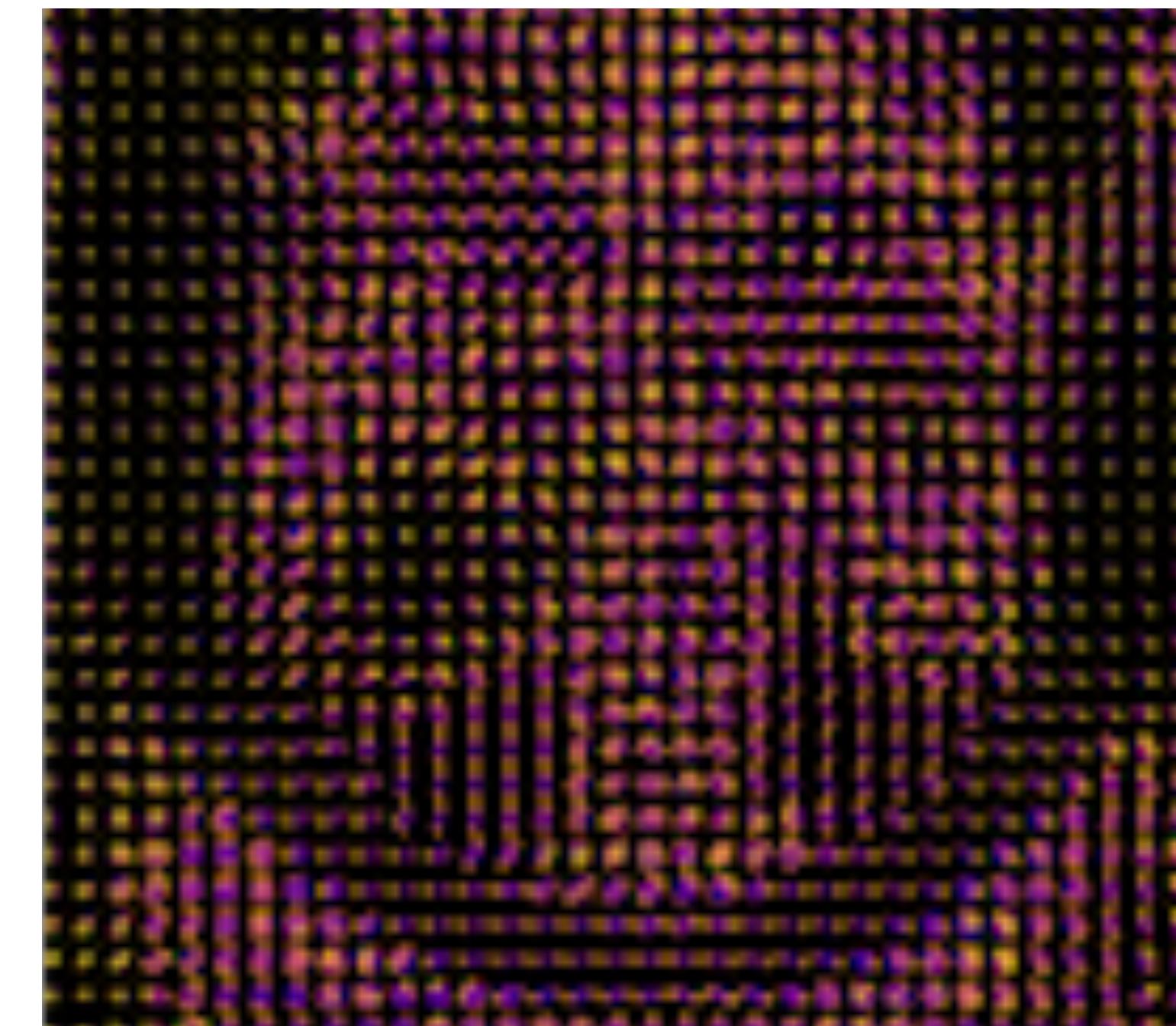
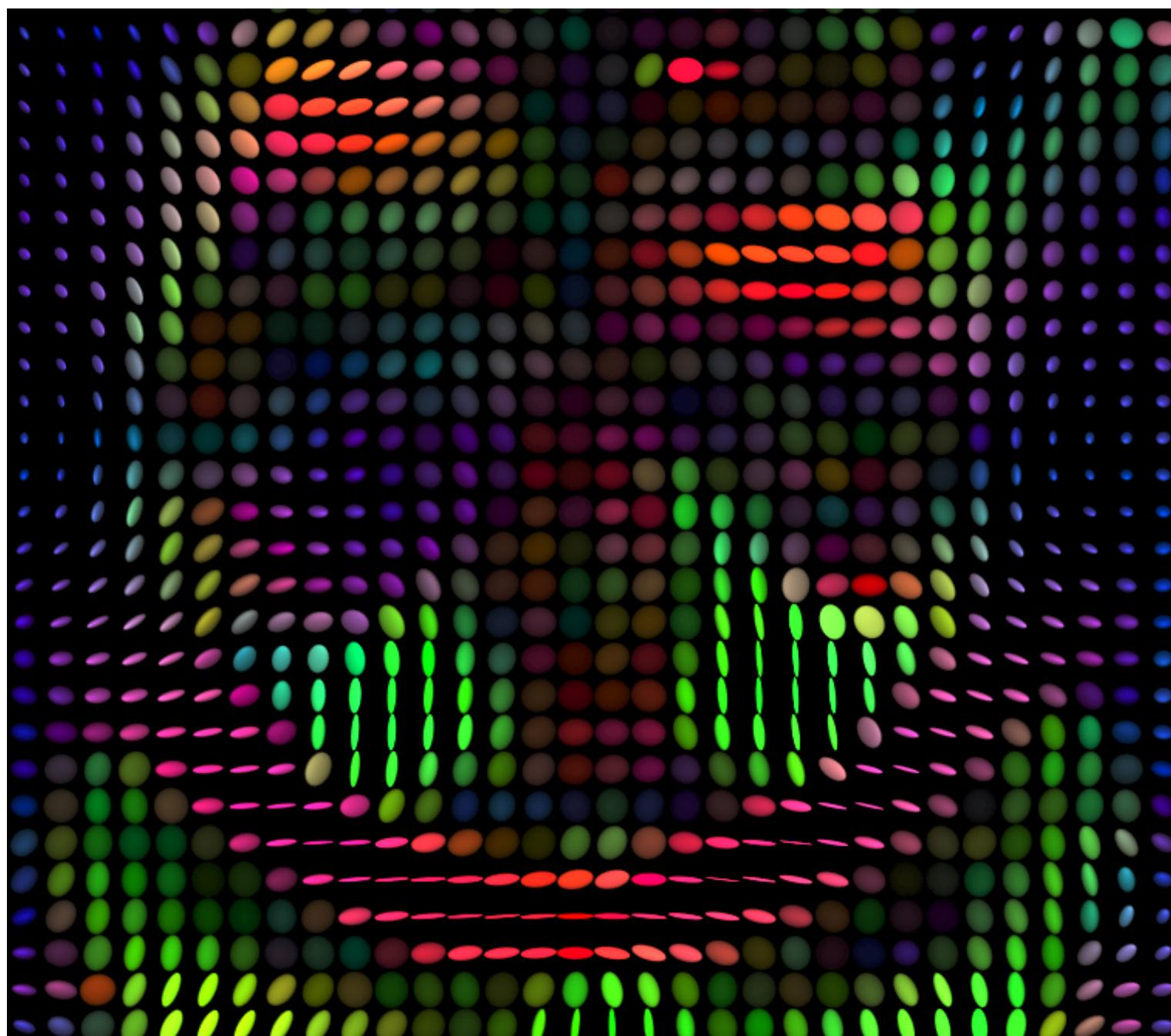
0.2
-0.1
0.02
0.1
0.9
-0.2
0.2
0.3
-0.9
-0.5
-0.8
0.1
0.2
0.1



$$\notin S^{n-1}$$



# First attempt - Convolution





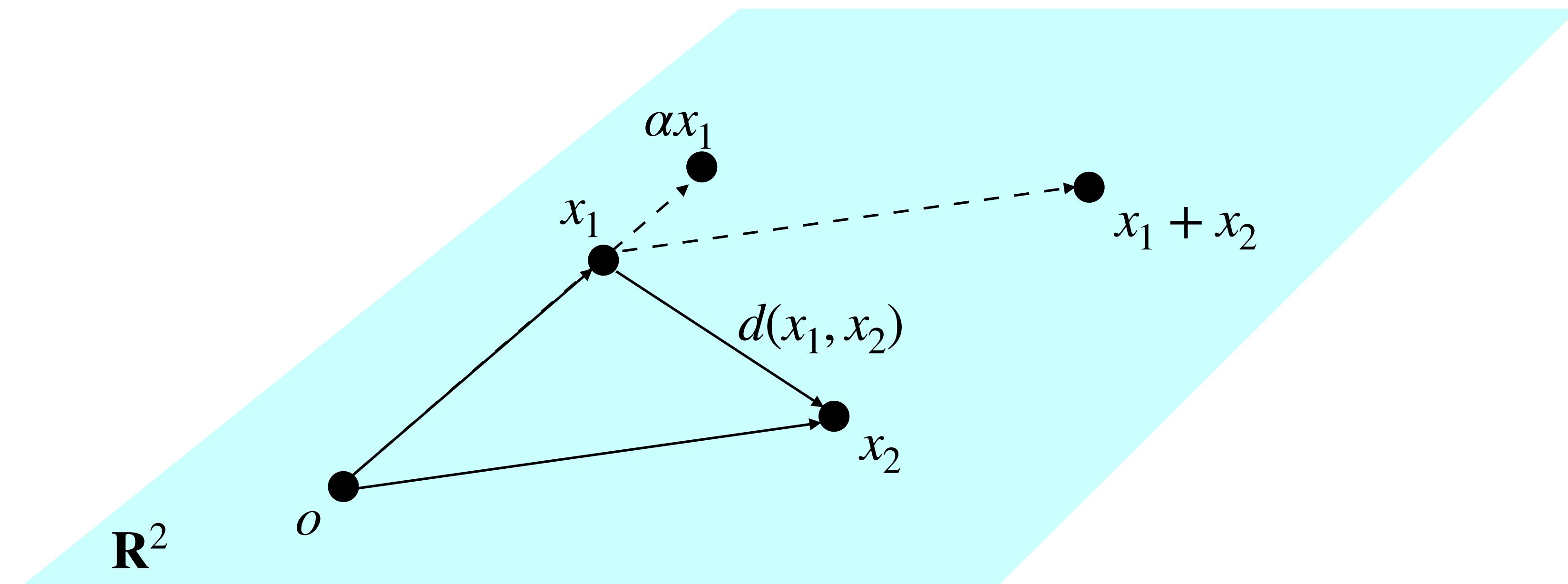
# What we need - DTI to ODF

**Operators** on manifold such that it preserves  
manifold constraint

A method that can **transfer** modalities between  
manifolds

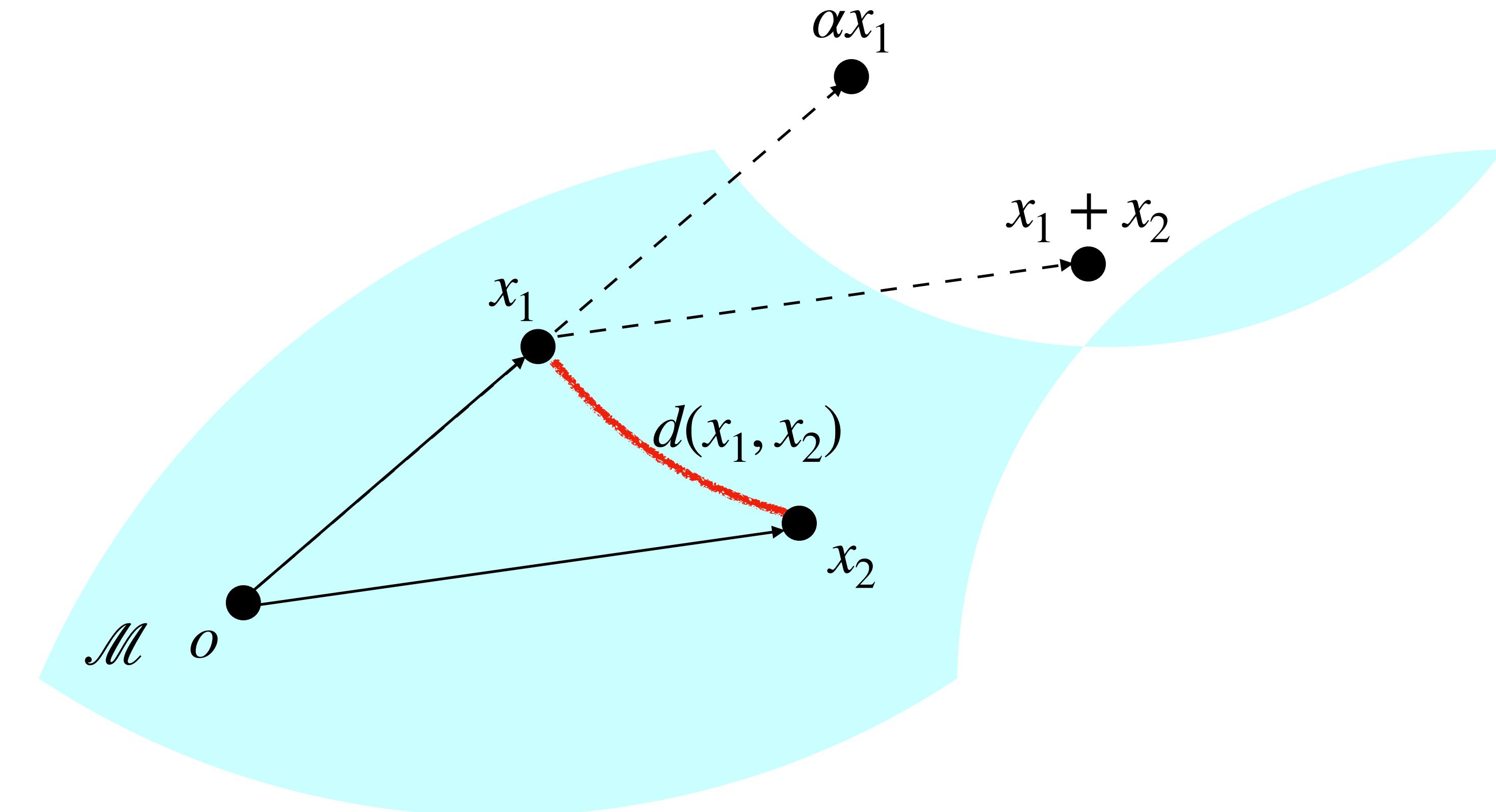


# Difference between Euclidean and Manifold





# Difference between Euclidean and Manifold





# From Euclidean to Manifold

What is **preserved** in Manifold from Euclidean space?

How to make minimum **modifications** on  
unpreserved parts from Euclidean space to Manifold?

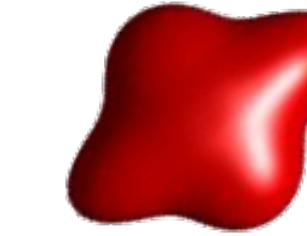


# The operators needed for Manifold

Main calculation in deep learning

$$y = \sigma(Wx + b)$$

Where  $x, y$  are



Multiplication  $\times$

Addition  $+$

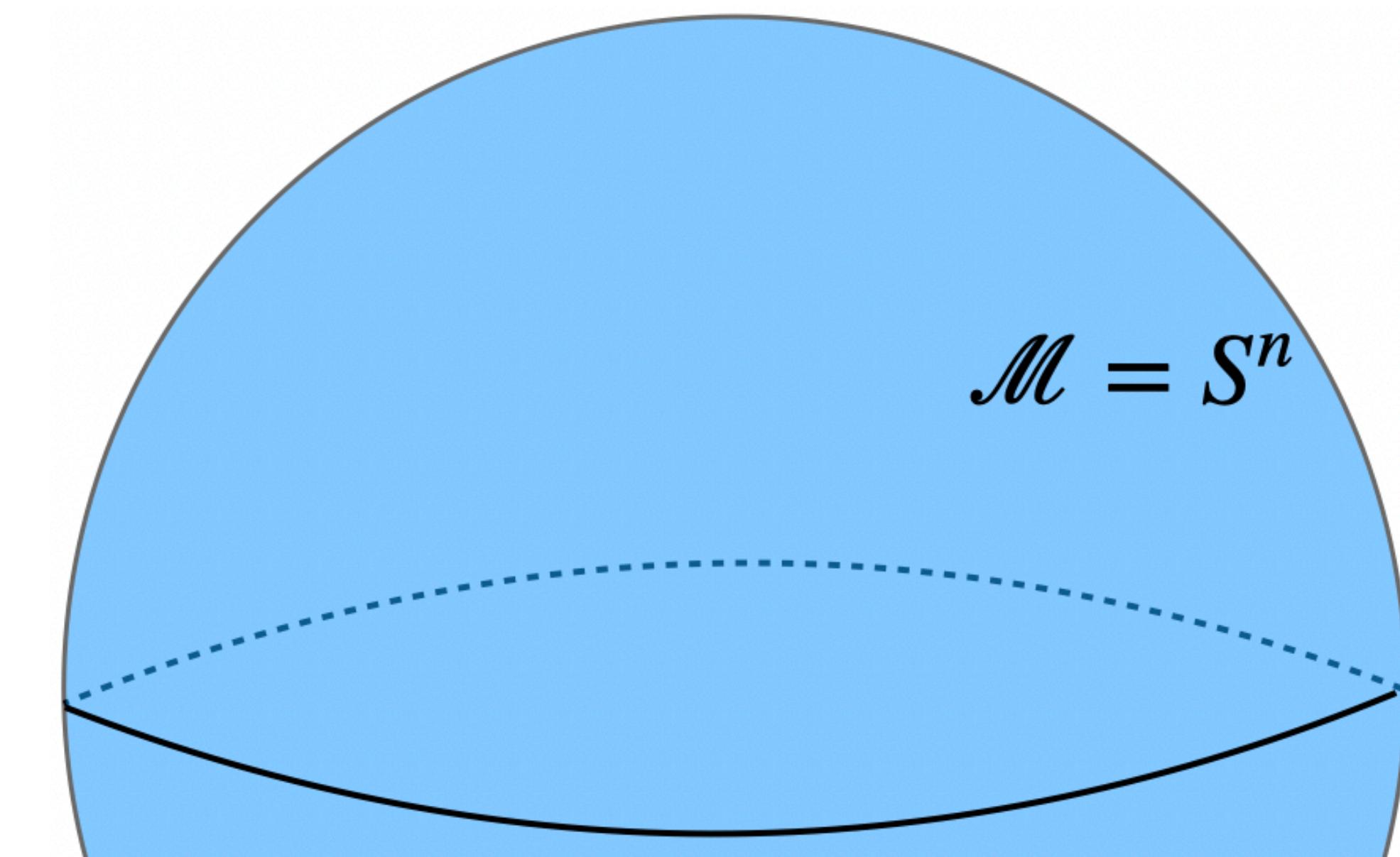
Activation



# The operators needed for Manifold

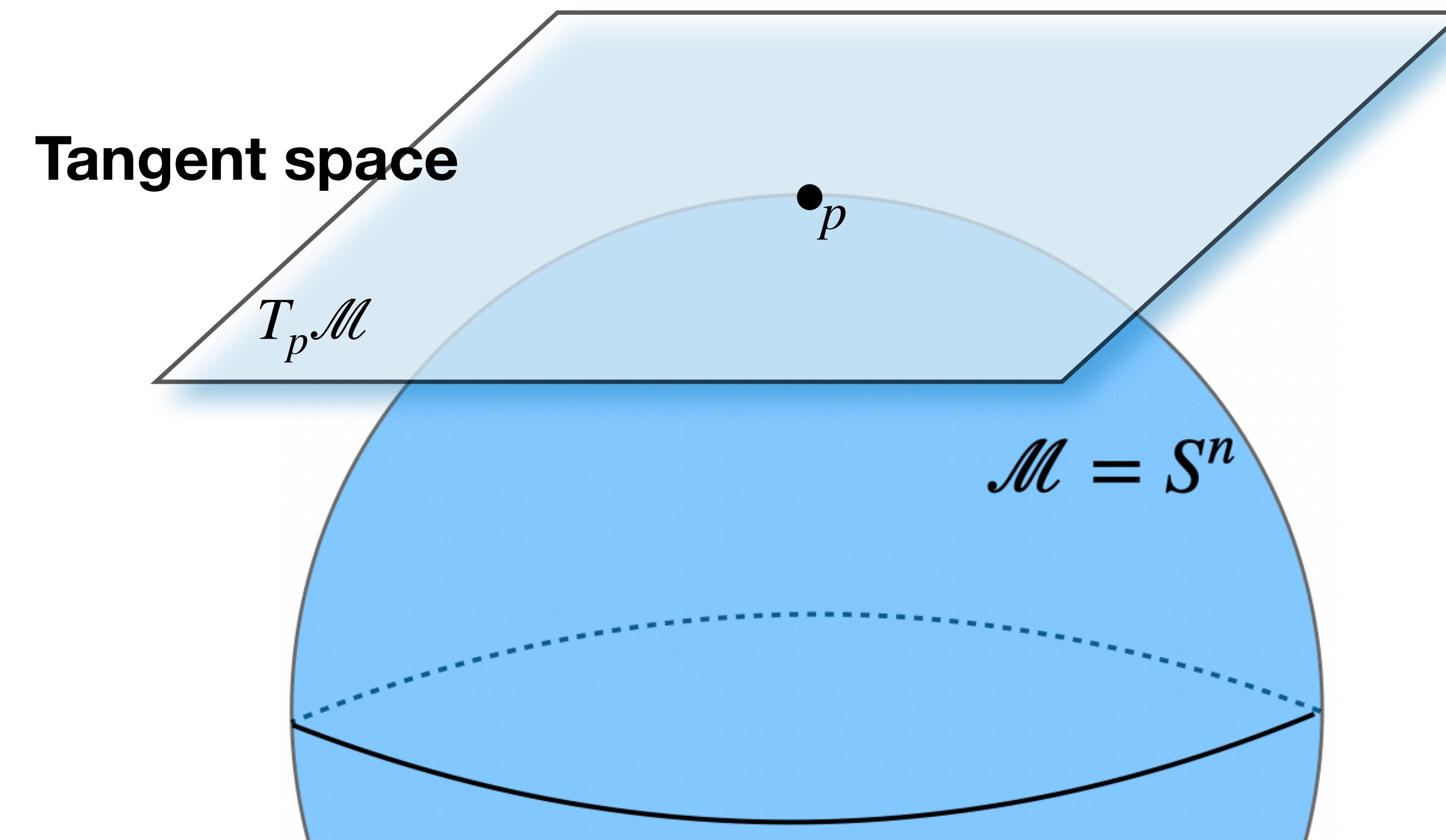


# The operators needed for Manifold



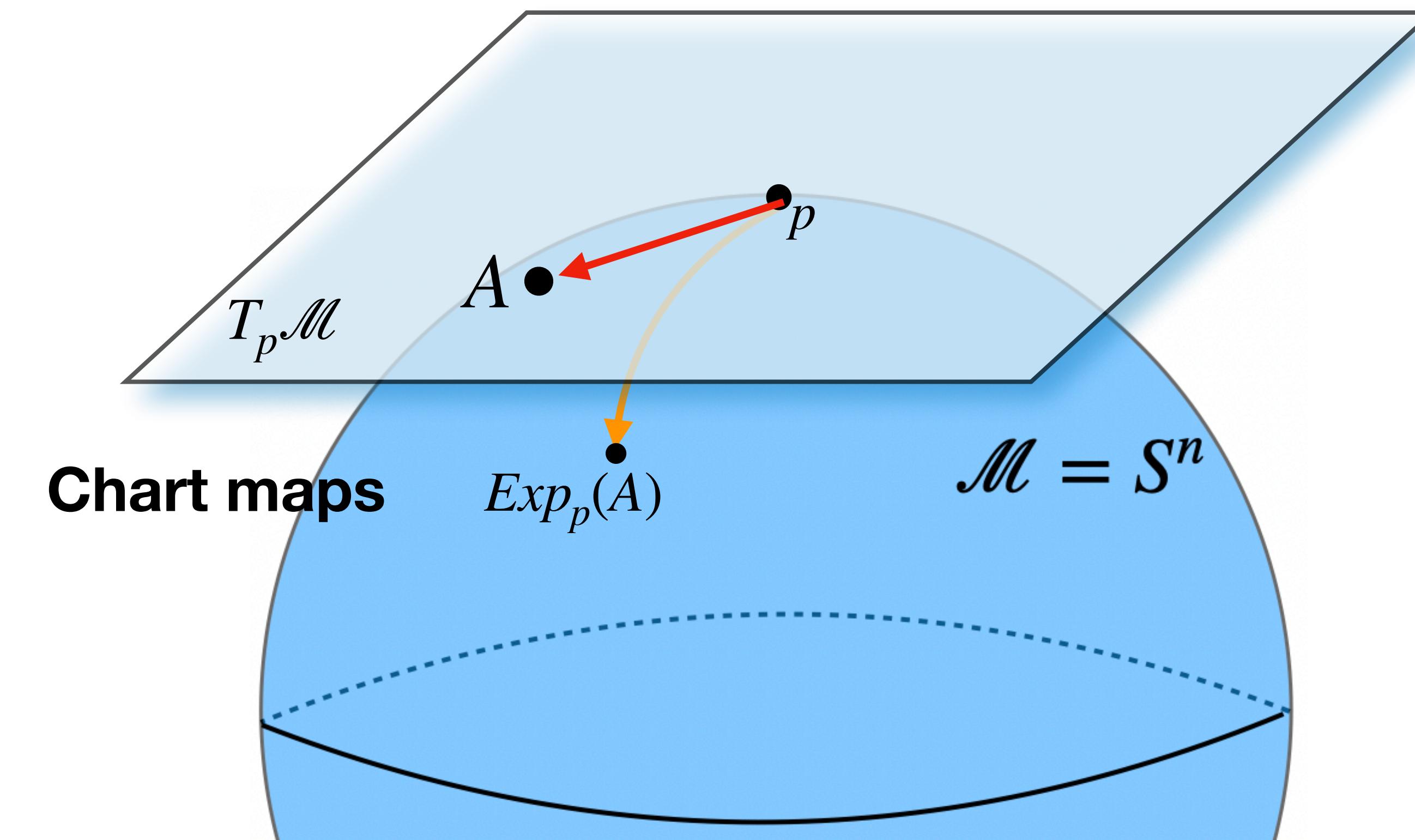


# The operators needed for Manifold





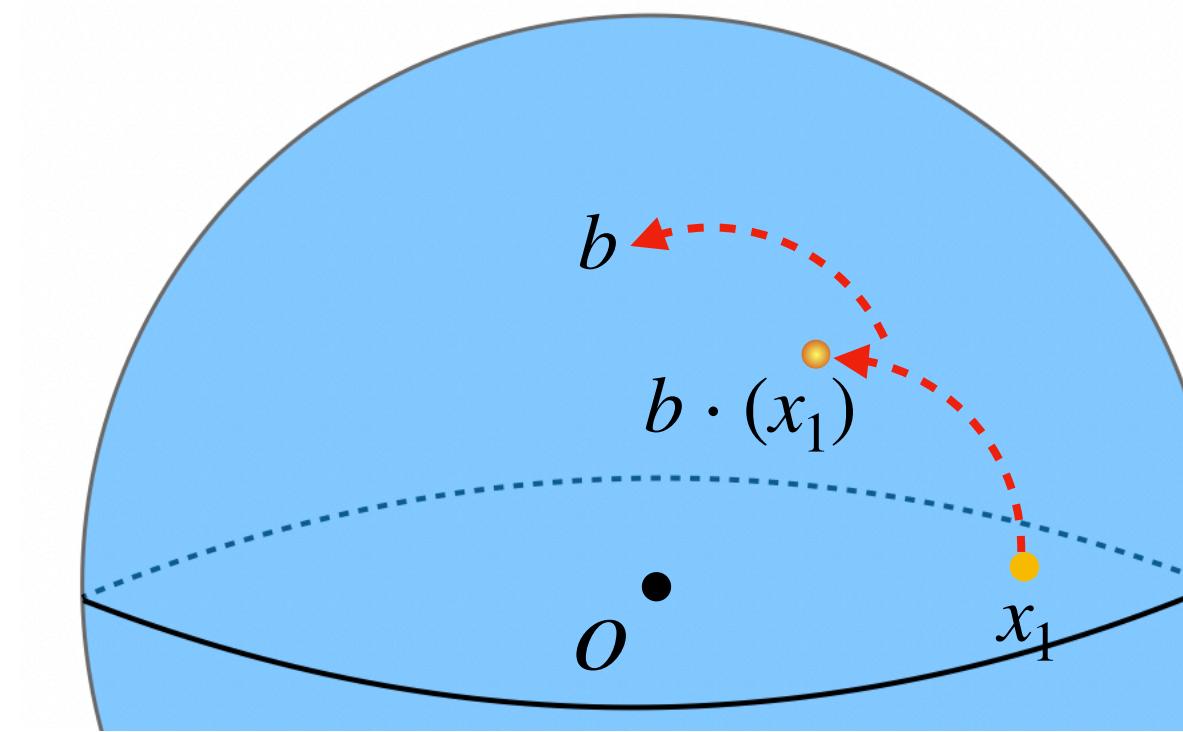
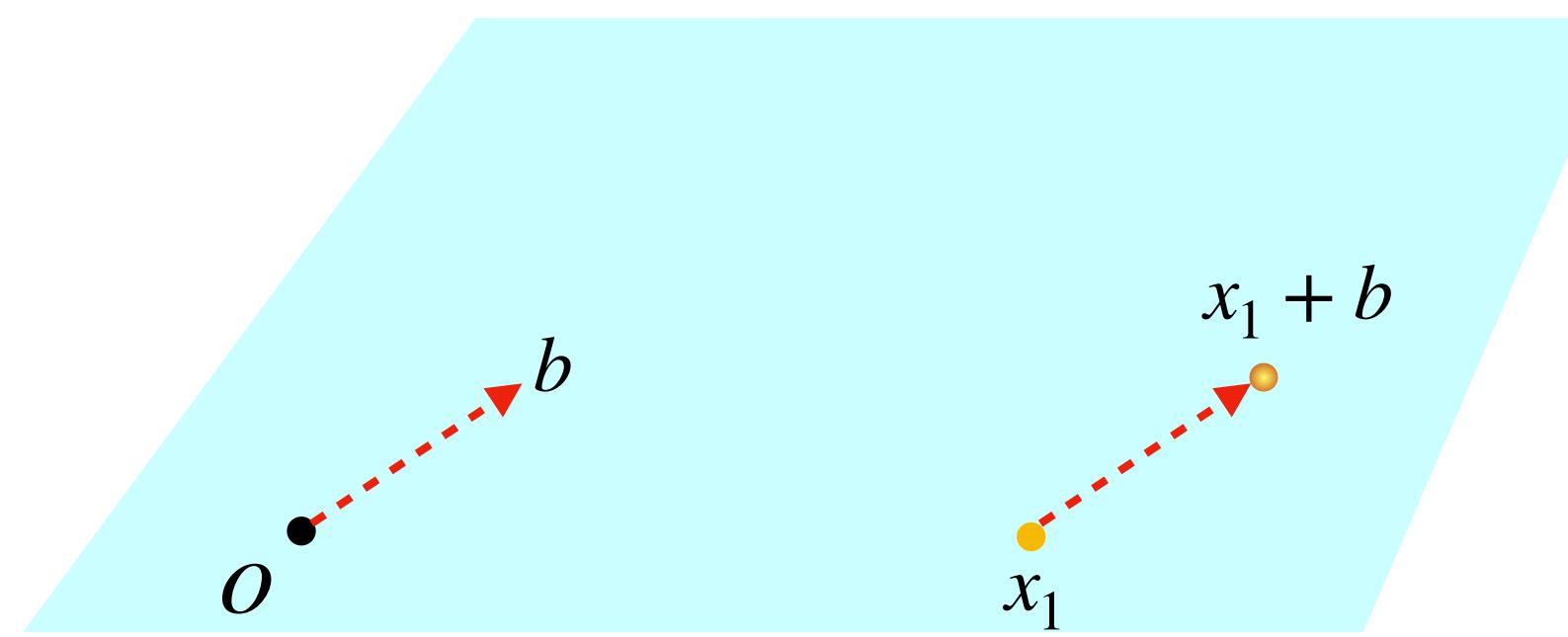
# The operators needed for Manifold





# The operators needed for Manifold

Group operations: A diffeomorphism  $\phi$ , if  $d(\phi(x), \phi(y)) = d(x, y)$





# What we need - DTI to ODF

Operators on manifold such that it preserves manifold constraint

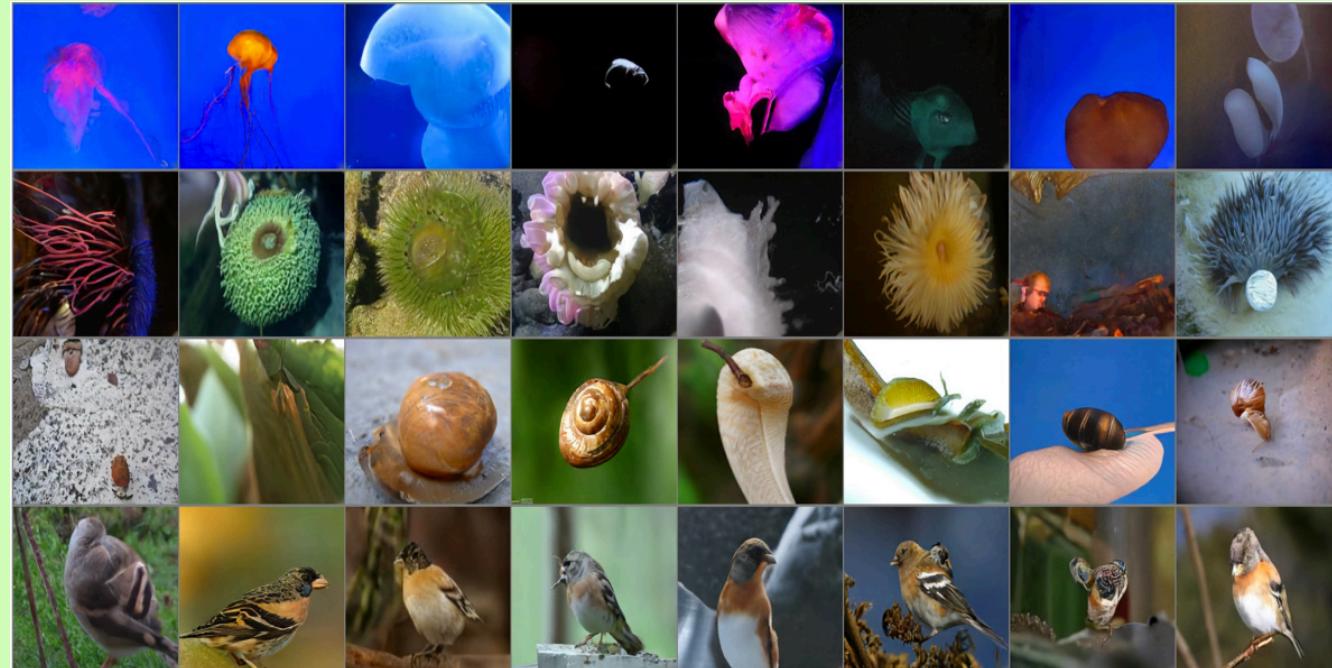
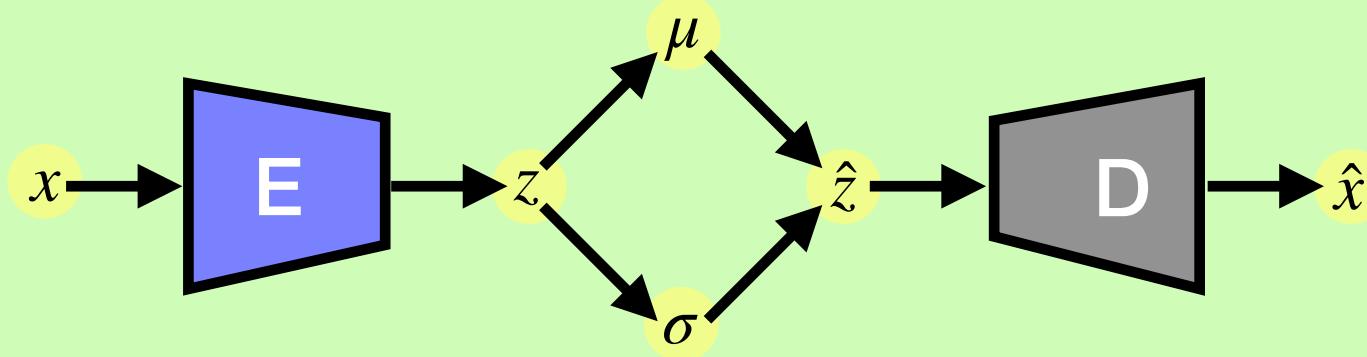
A method that can **transfer** modalities between manifolds



# Generative model

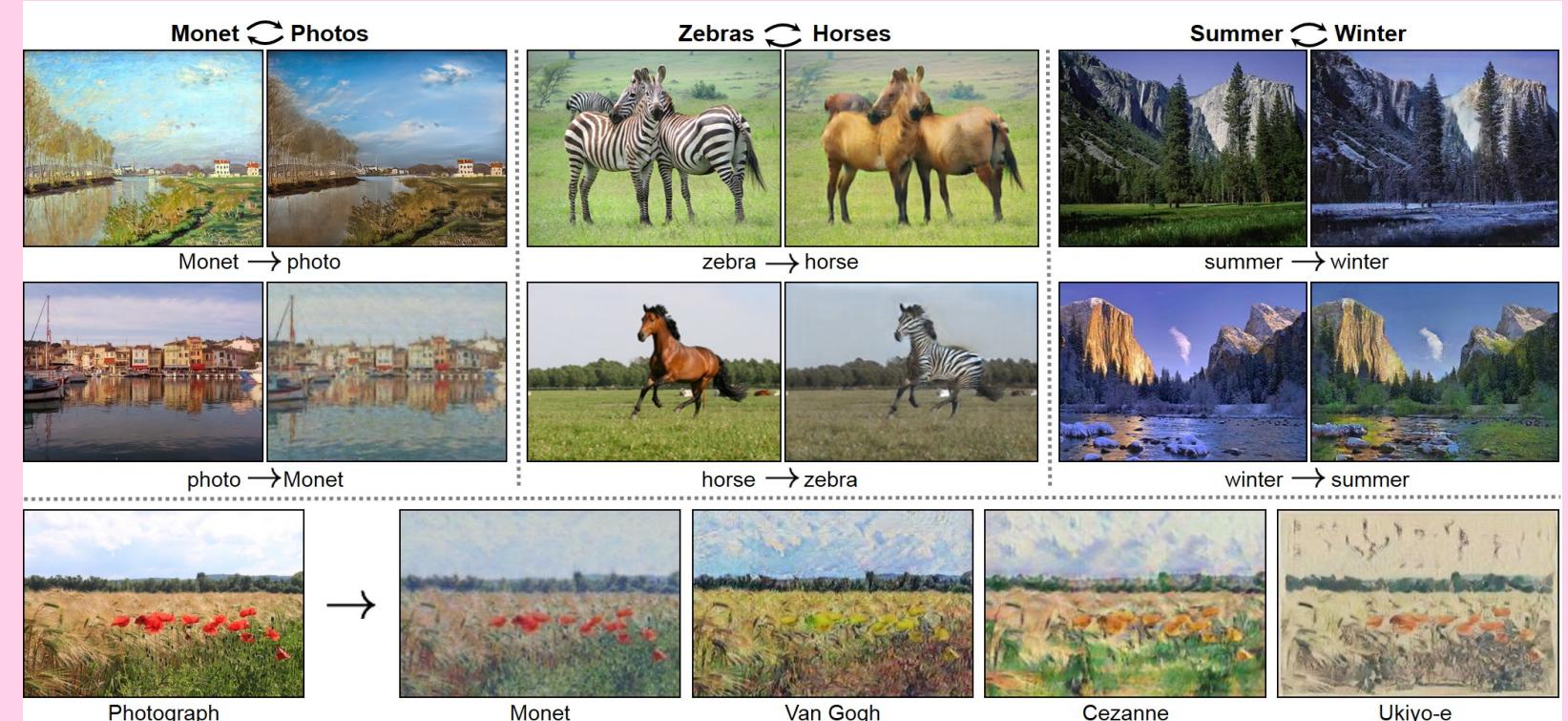
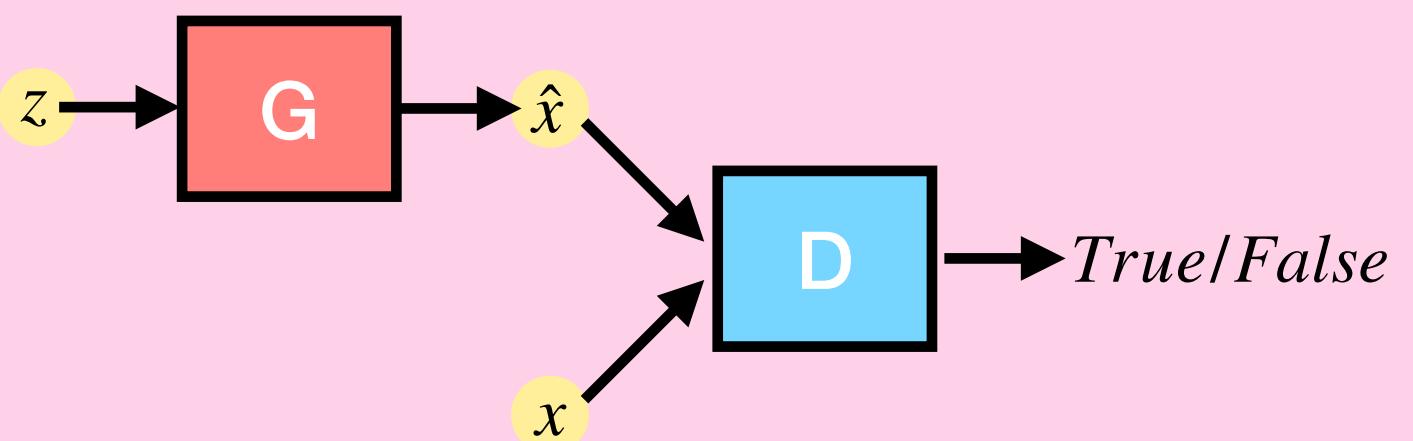
## Variational autoencoder (VAE)

$$l_i(\theta, \phi) = -\mathbf{E}_{z \sim q_{\theta}(z|x_i)}[\log p_{\phi}(x_i|z)] + \mathbf{KL}(q_{\theta}(z|x_i)||p(z))$$



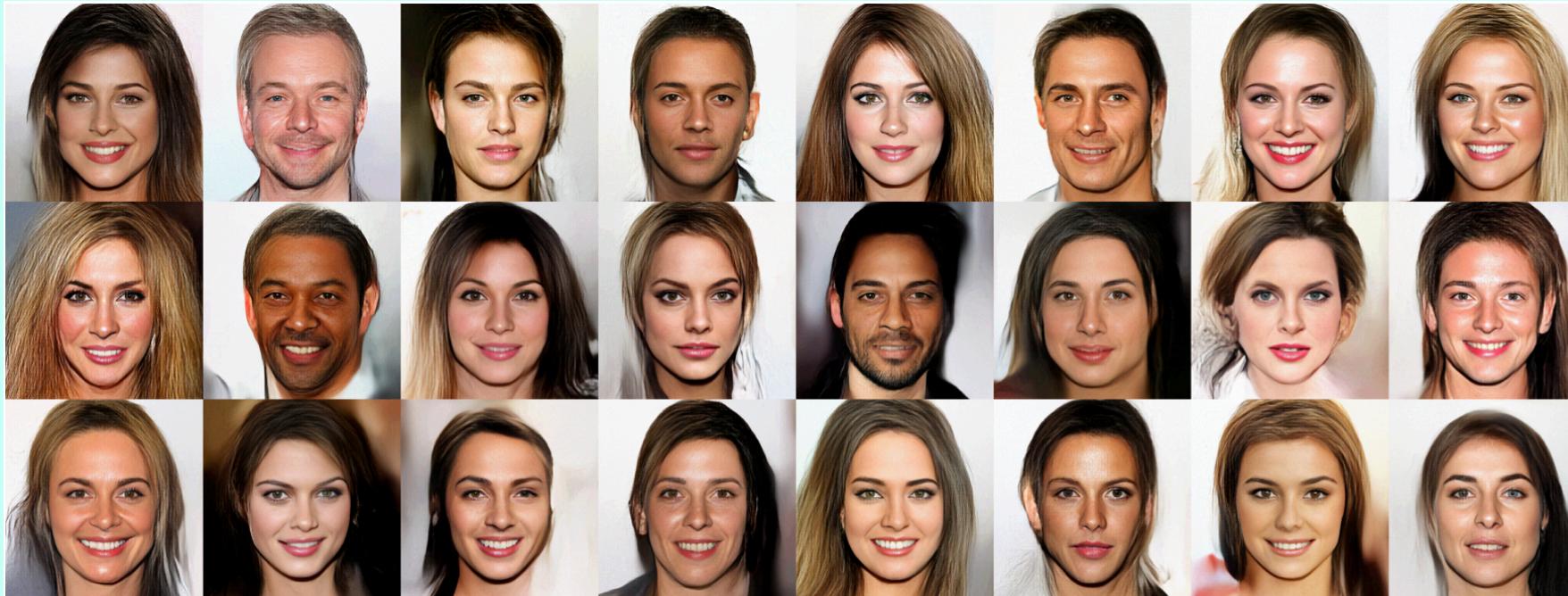
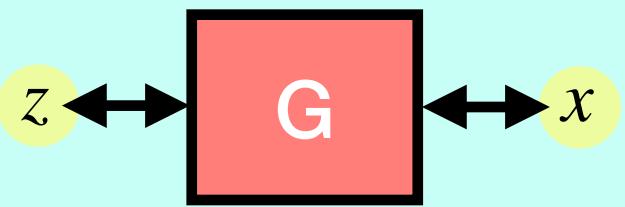
## Generative adversarial network (GAN)

$$\min_G \max_D V(D, G) = \mathbf{E}_{x \sim p_{data}(x)}[\log D(x)] + \mathbf{E}_{z \sim p_z(z)}[\log(1 - D(G(z)))]$$



## Normalizing flows

$$\min l(\mathcal{D}) = \frac{1}{N} \sum_{i=1}^N -\log p_{\theta}(x^{(i)}) , z \sim p_{\theta}(z), x = g_{\theta}(z), z = g_{\theta}^{-1}(x)$$



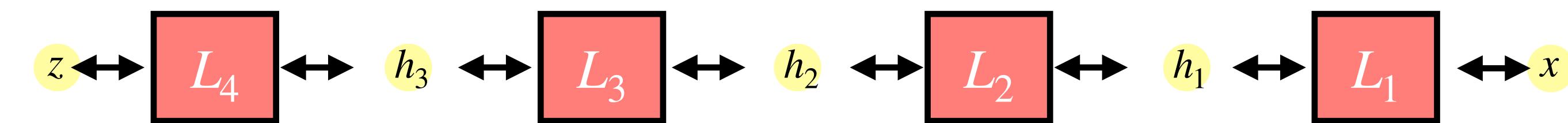


# Normalizing Flow

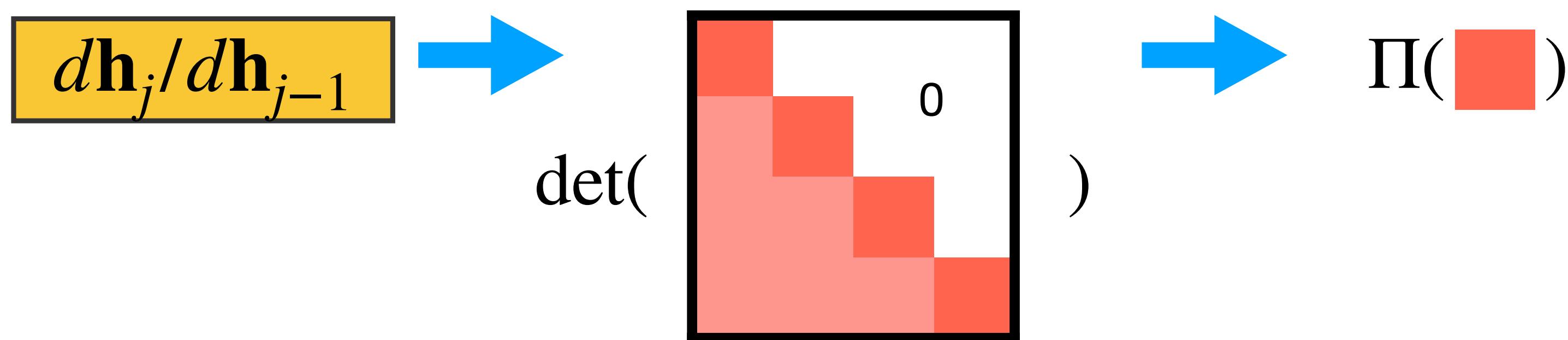




# Normalizing Flow



$$\log p_{\theta}(\mathbf{x}) = \log p(\mathbf{z}) + \log |\det(d\mathbf{z}/d\mathbf{x})| = \log p_{\theta}(\mathbf{z}) + \sum_{j=0}^K \log |\det(d\mathbf{h}_j/d\mathbf{h}_{j-1})|$$





# Normalizing Flow - Basic Structures

Actnorm

$$Y = \frac{1}{\sigma} \odot (X - \mu)$$

$1 \times 1$  convolution

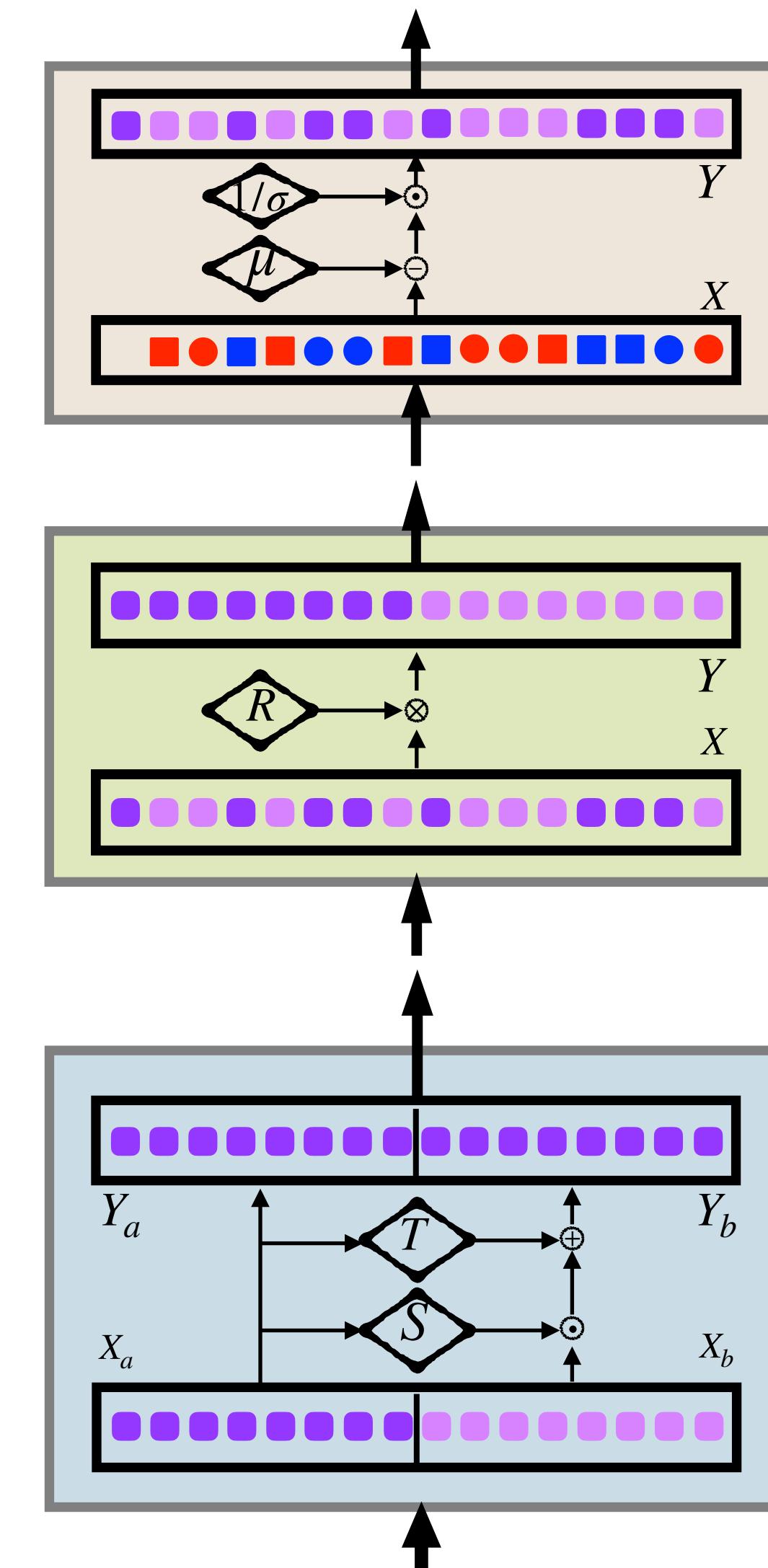
$$Y = R \times X$$

Affine Coupling

$$S, T = \text{NN}(X_a)$$

$$Y_b = S \odot X_b + T$$

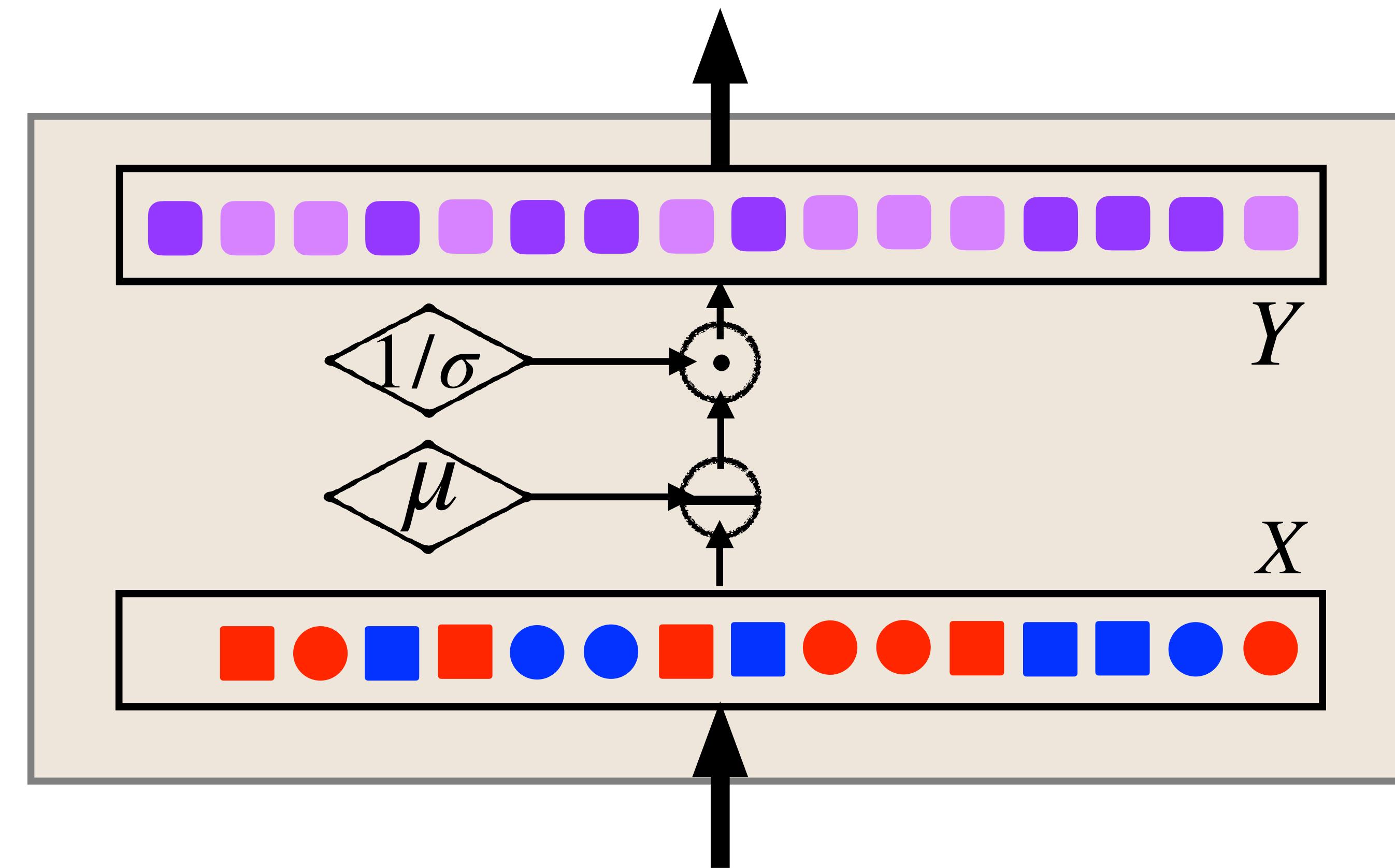
$$Y_a = X_a$$





# Actnorm

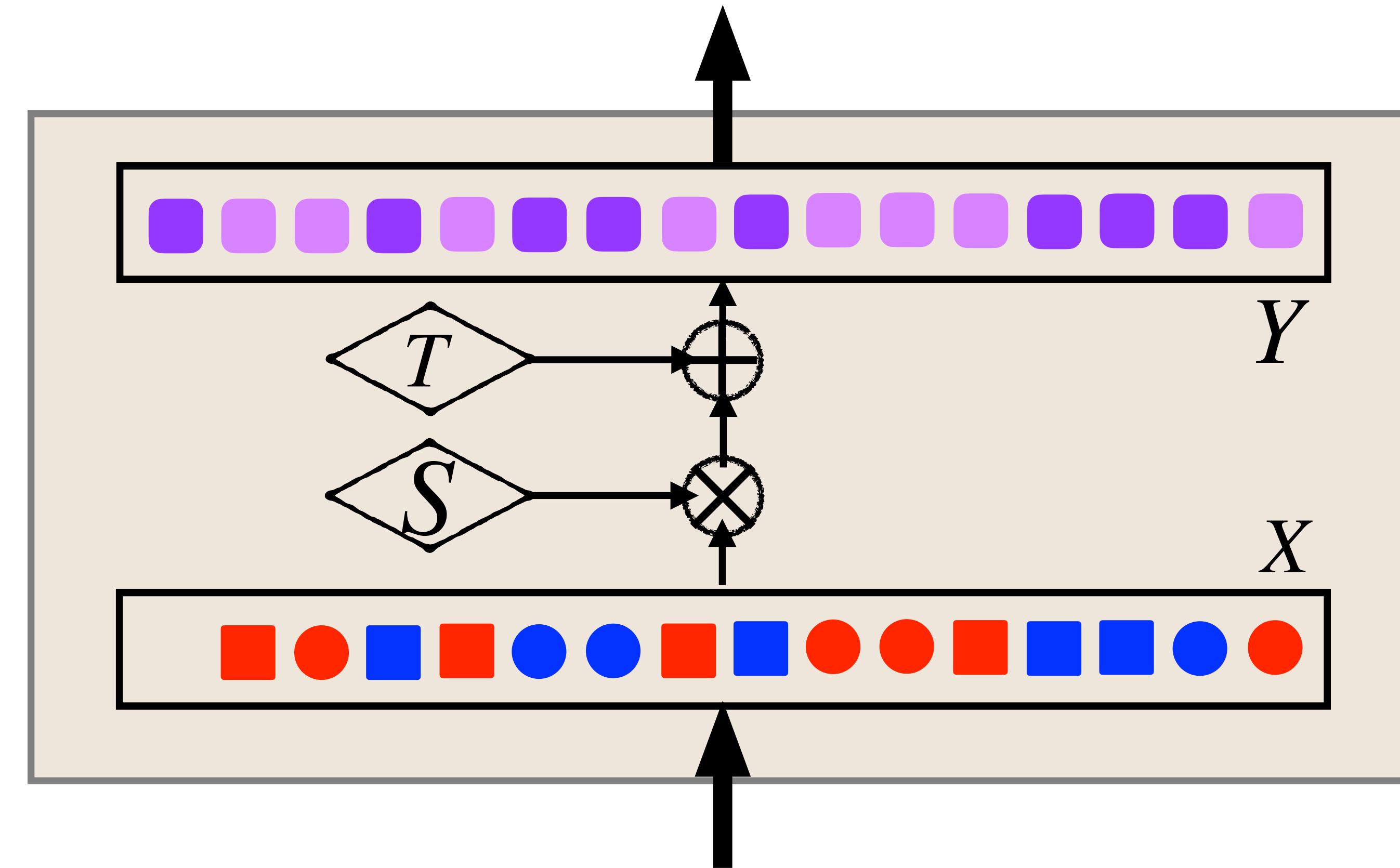
$$Y = \frac{1}{\sigma} \odot (X - \mu)$$





# Actnorm

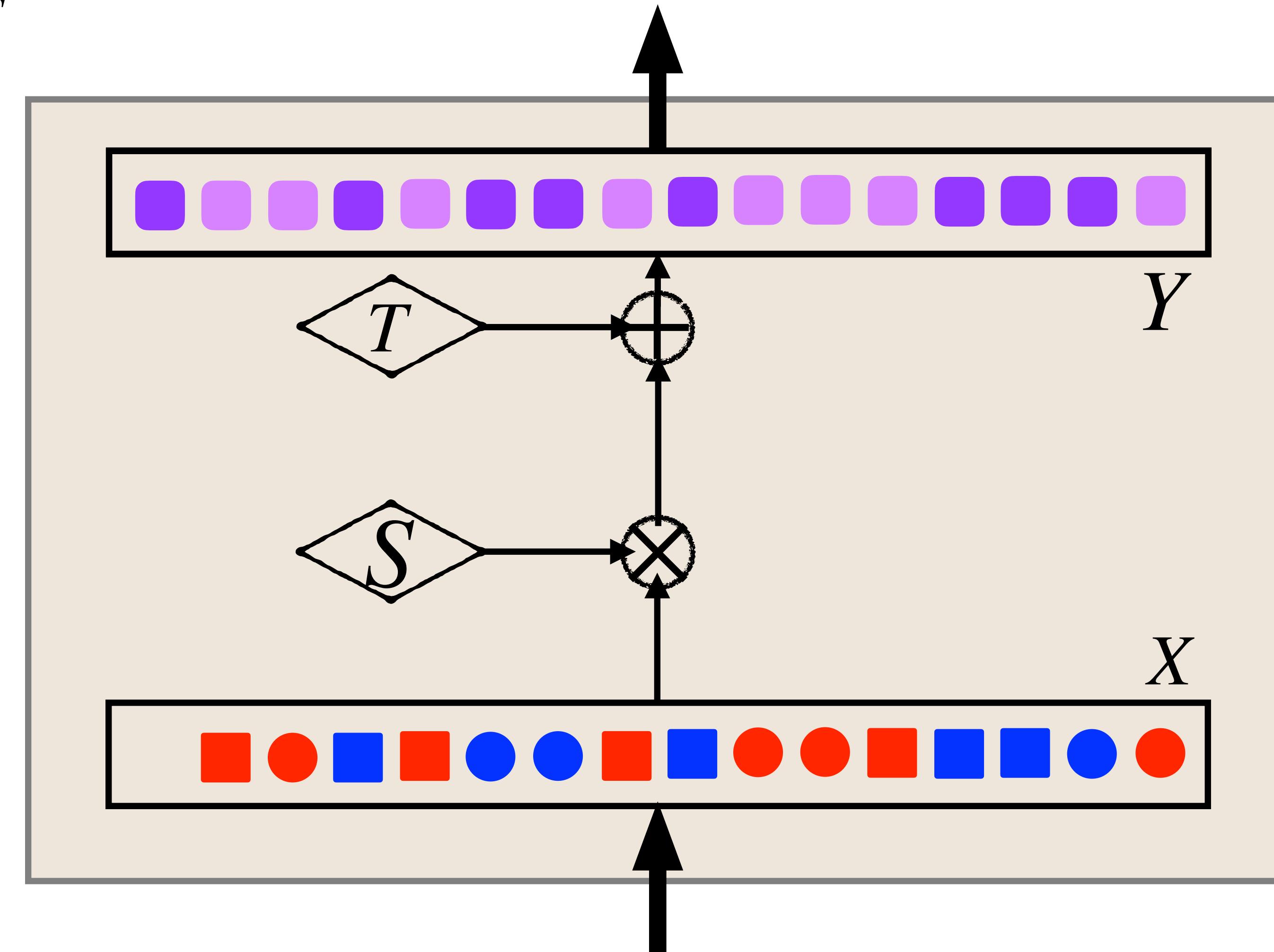
$$Y = \frac{1}{\sigma} \odot (X - \mu) = S \times X + T$$





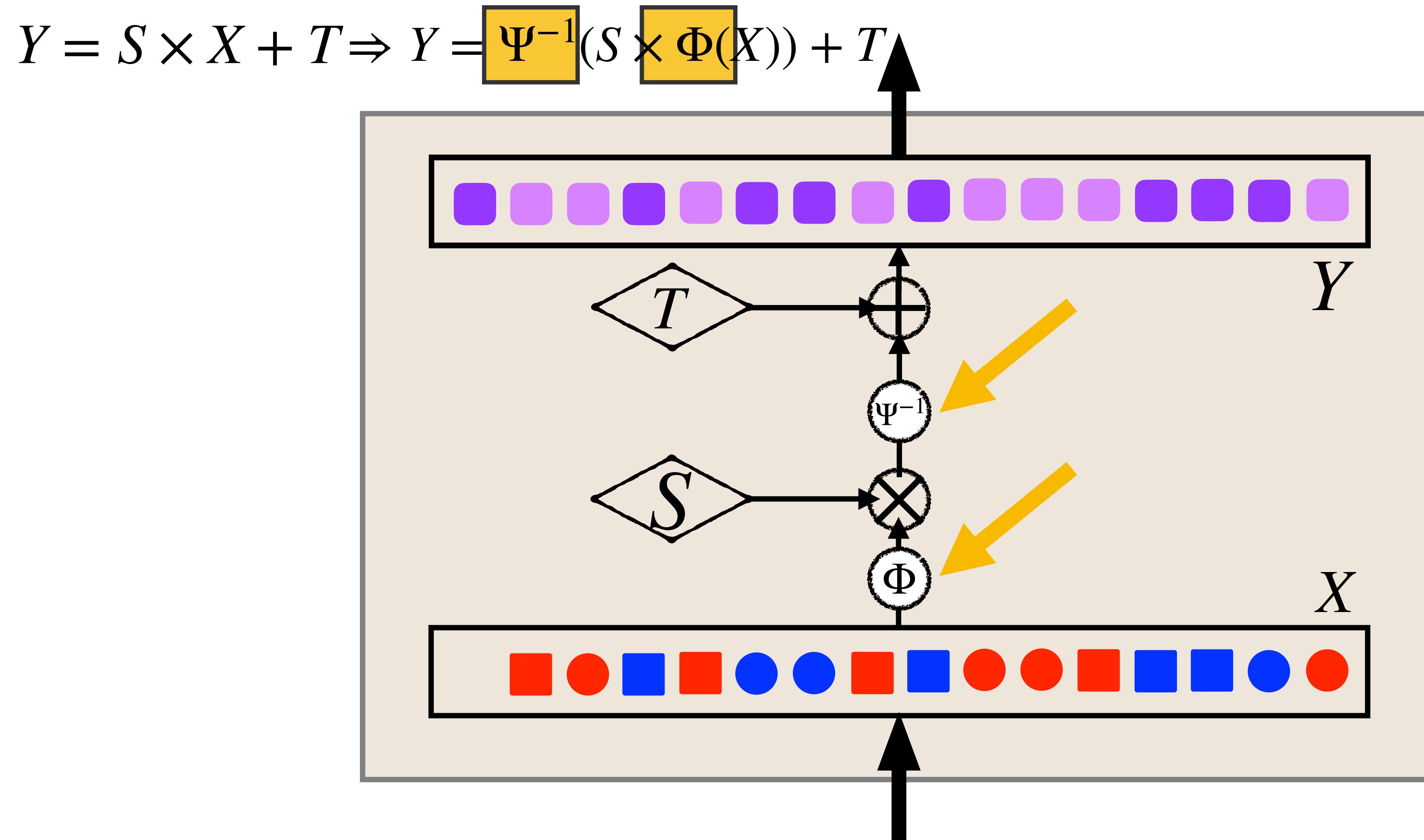
# Actnorm

$$Y = S \times X + T$$





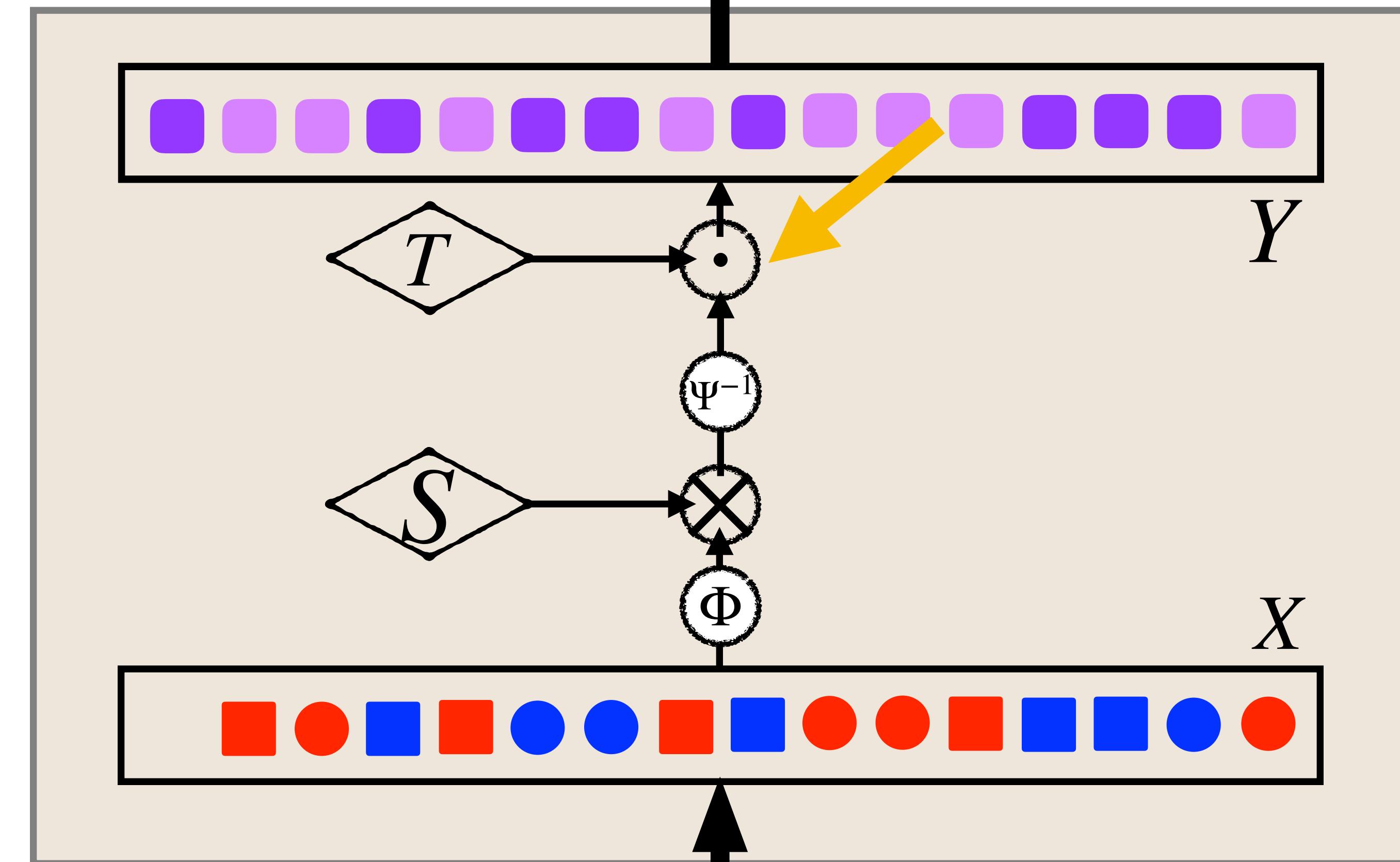
# Actnorm





# Actnorm

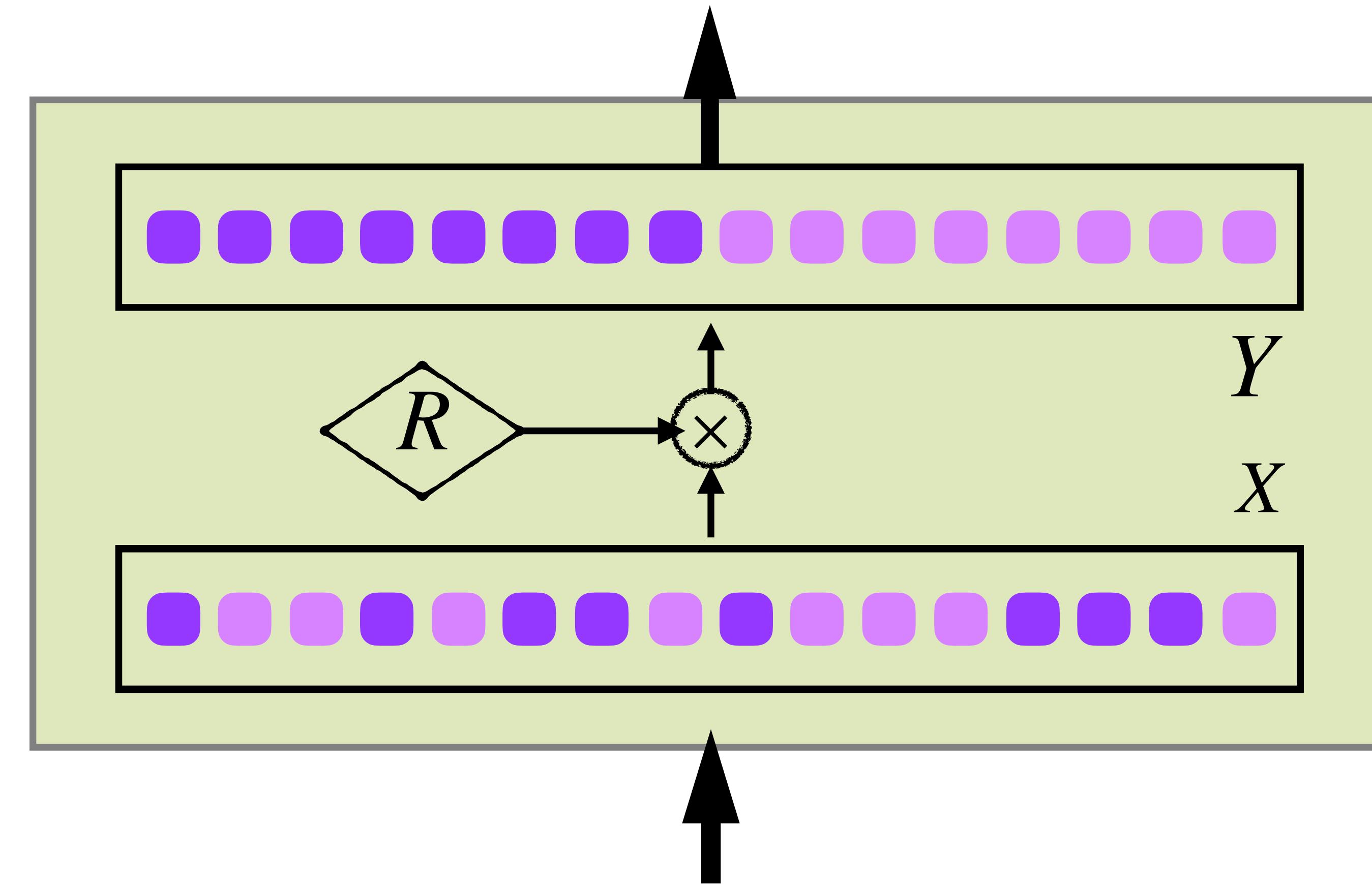
$$Y = S \times X + T \Rightarrow Y = T \cdot \Psi^{-1}(S \times \Phi(X))$$





# $1 \times 1$ convolution

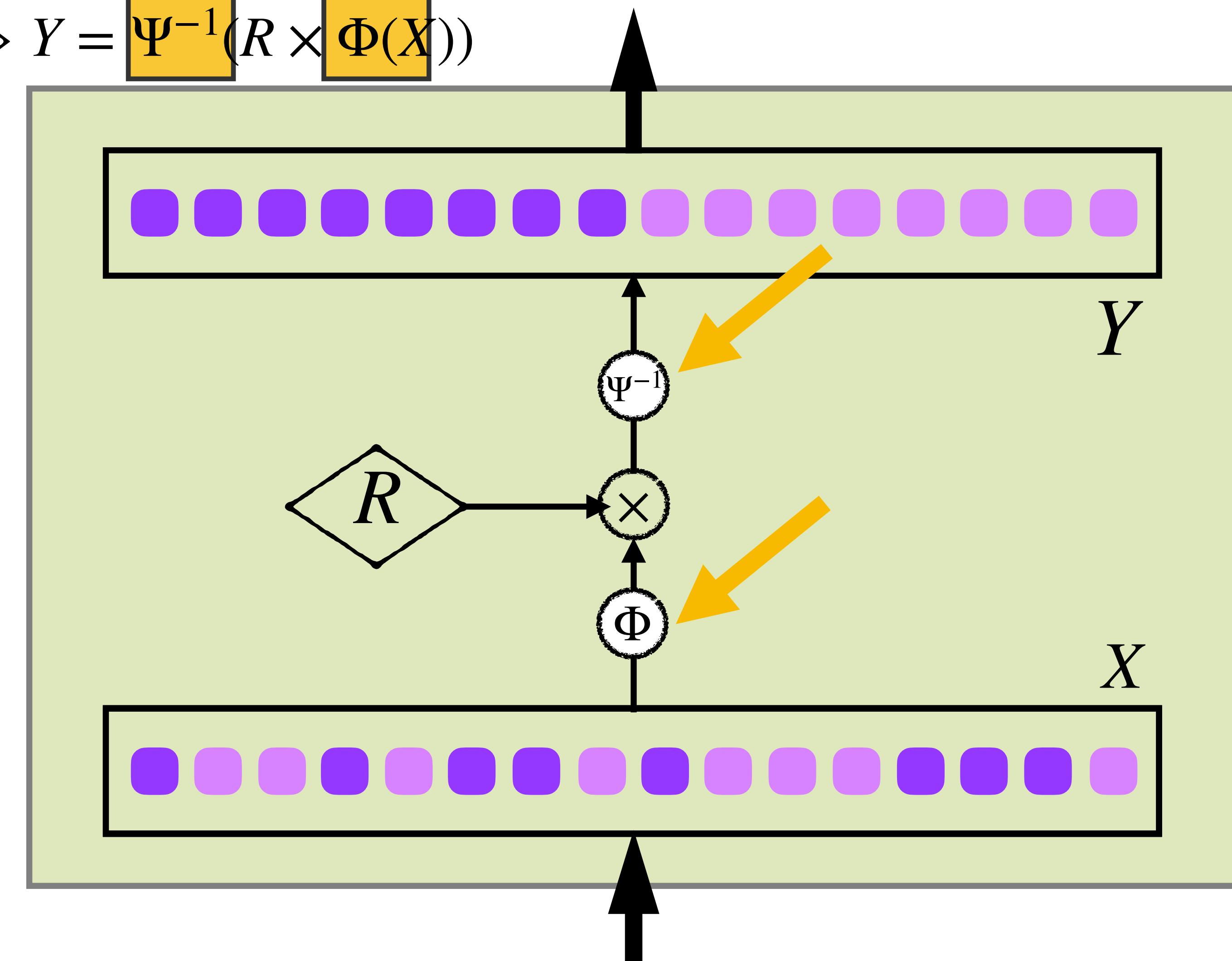
$$Y = R \times X$$





# $1 \times 1$ convolution

$$Y = R \times X \Rightarrow Y = \Psi^{-1}(R \times \Phi(X))$$



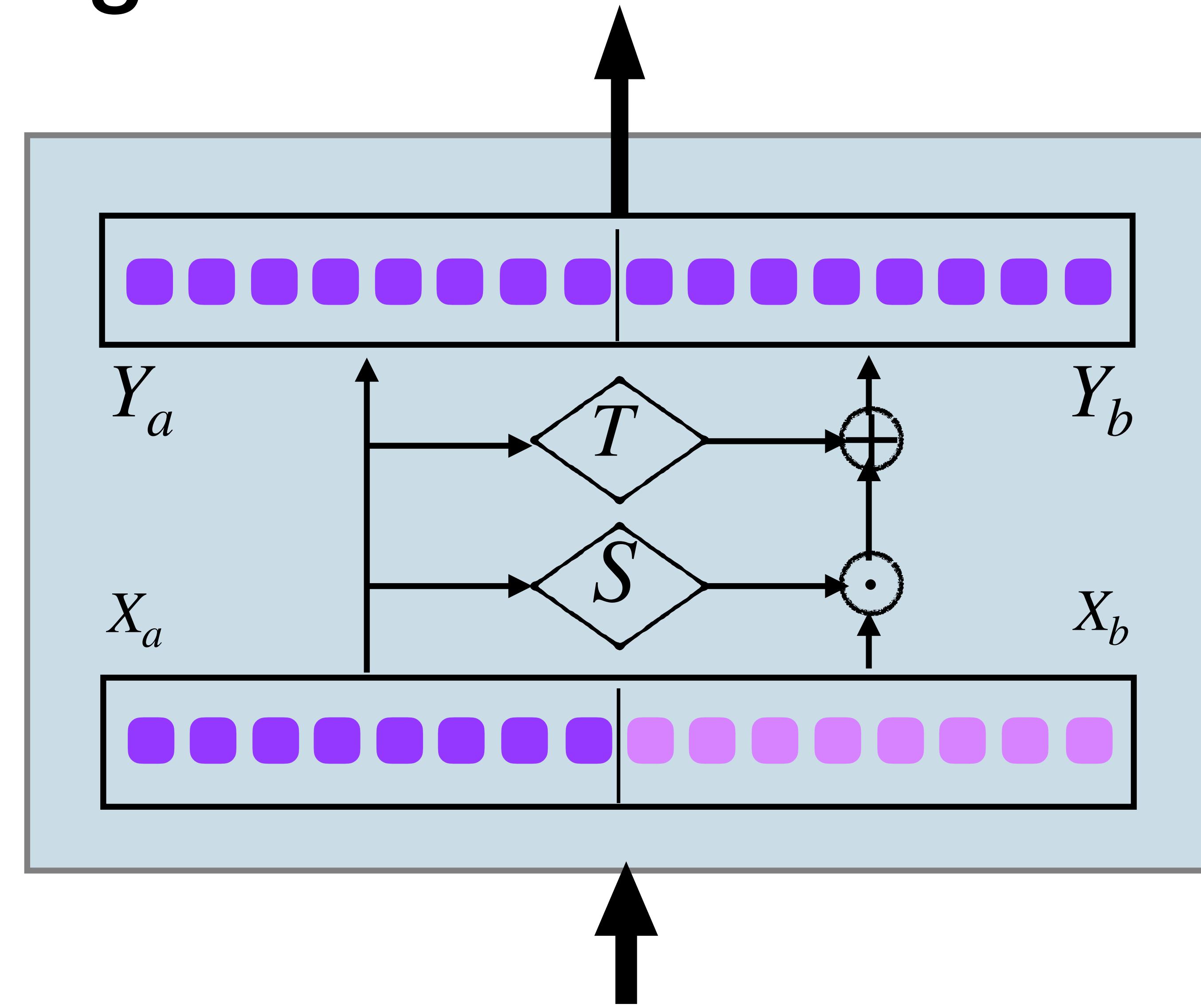


# Affine Coupling

$$S, T = \text{NN}(X_a)$$

$$Y_b = S \odot X_b + T$$

$$Y_a = X_a$$



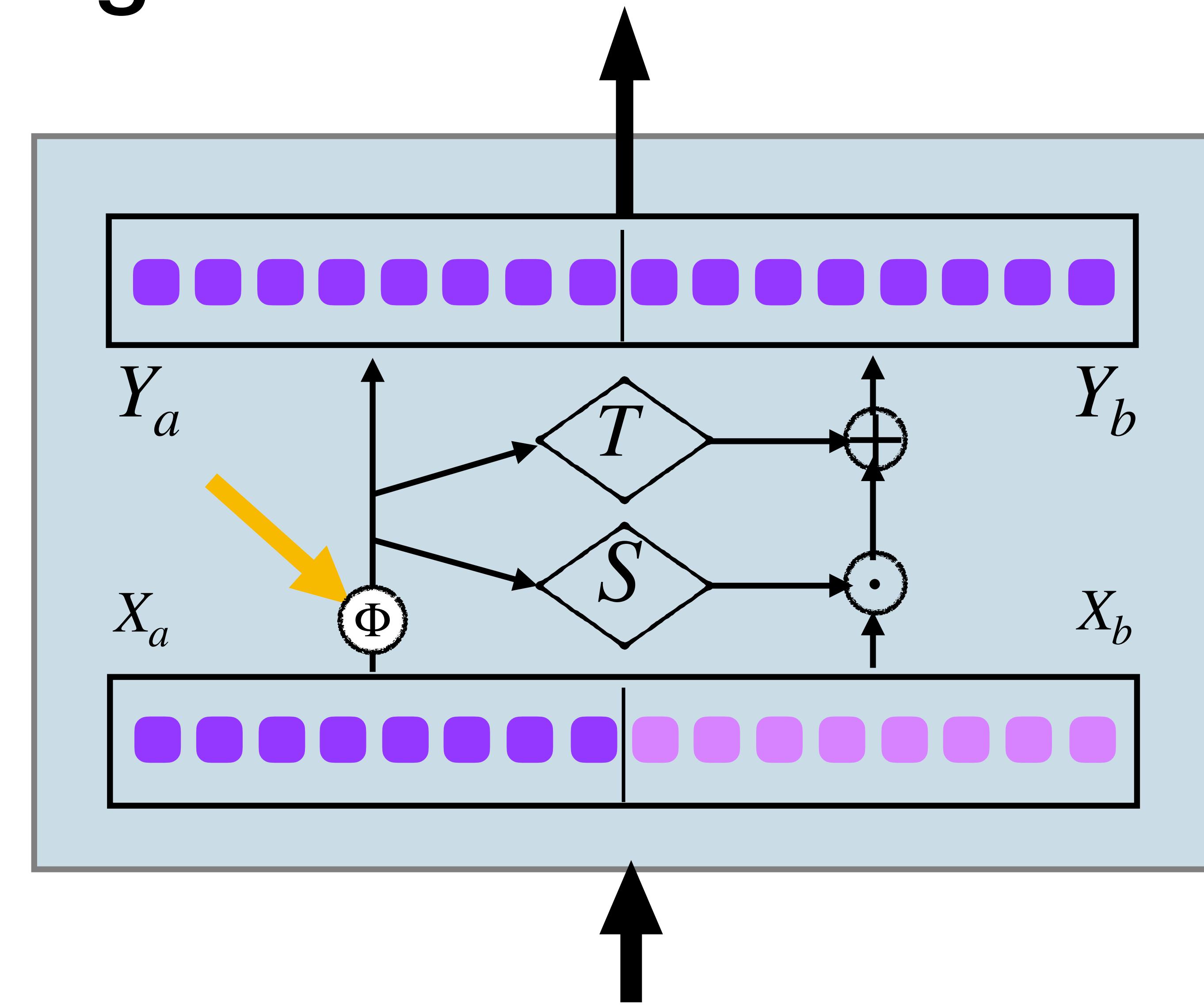


# Affine Coupling

$$S, T = \text{NN}(\Phi(X_a))$$

$$Y_b = S \odot X_b + T$$

$$Y_a = X_a$$



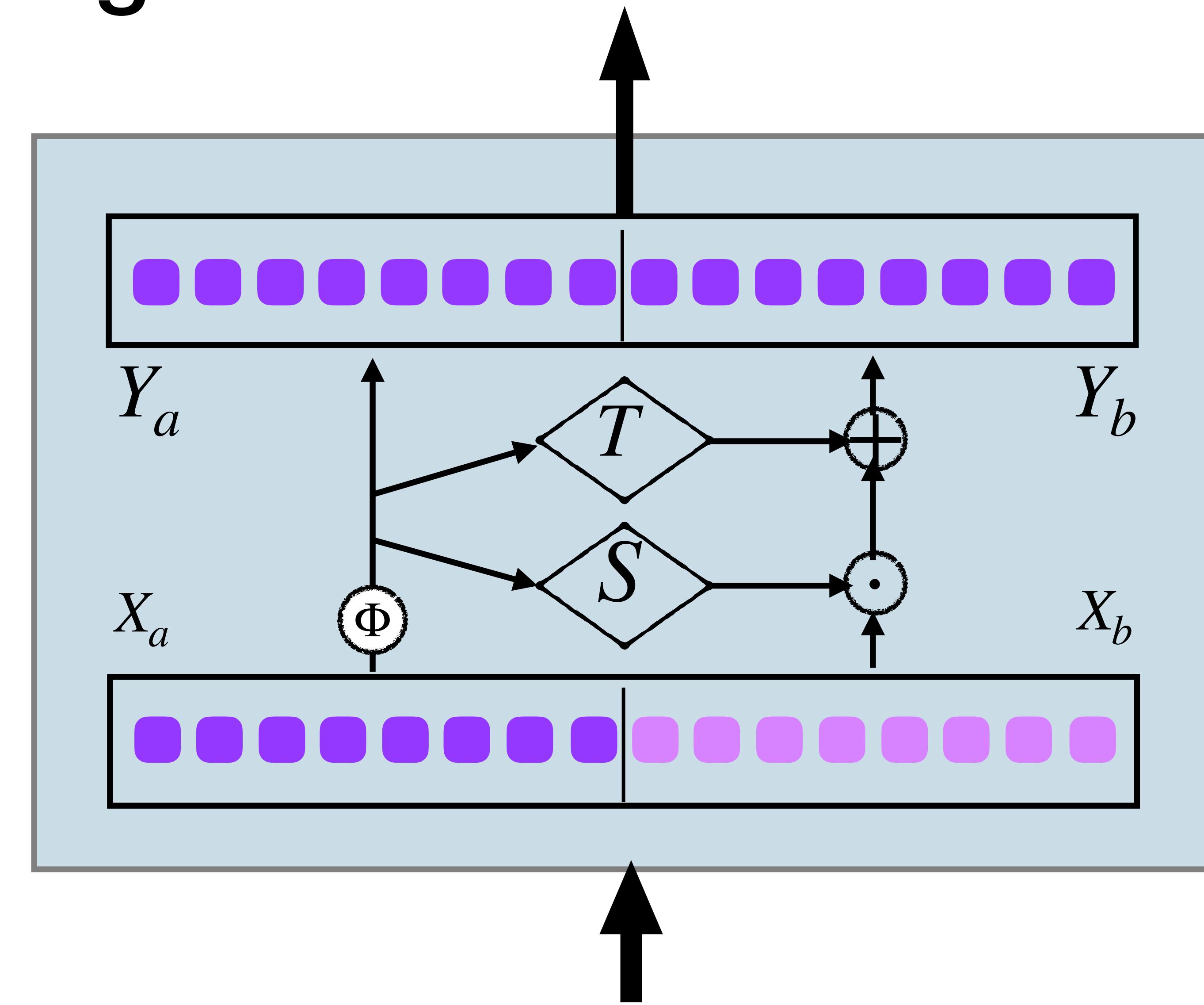


# Affine Coupling

$$S, T = \text{NN}(\Phi(X_a))$$

$$Y_b = S \odot X_b + T$$

$$Y_a = X_a$$



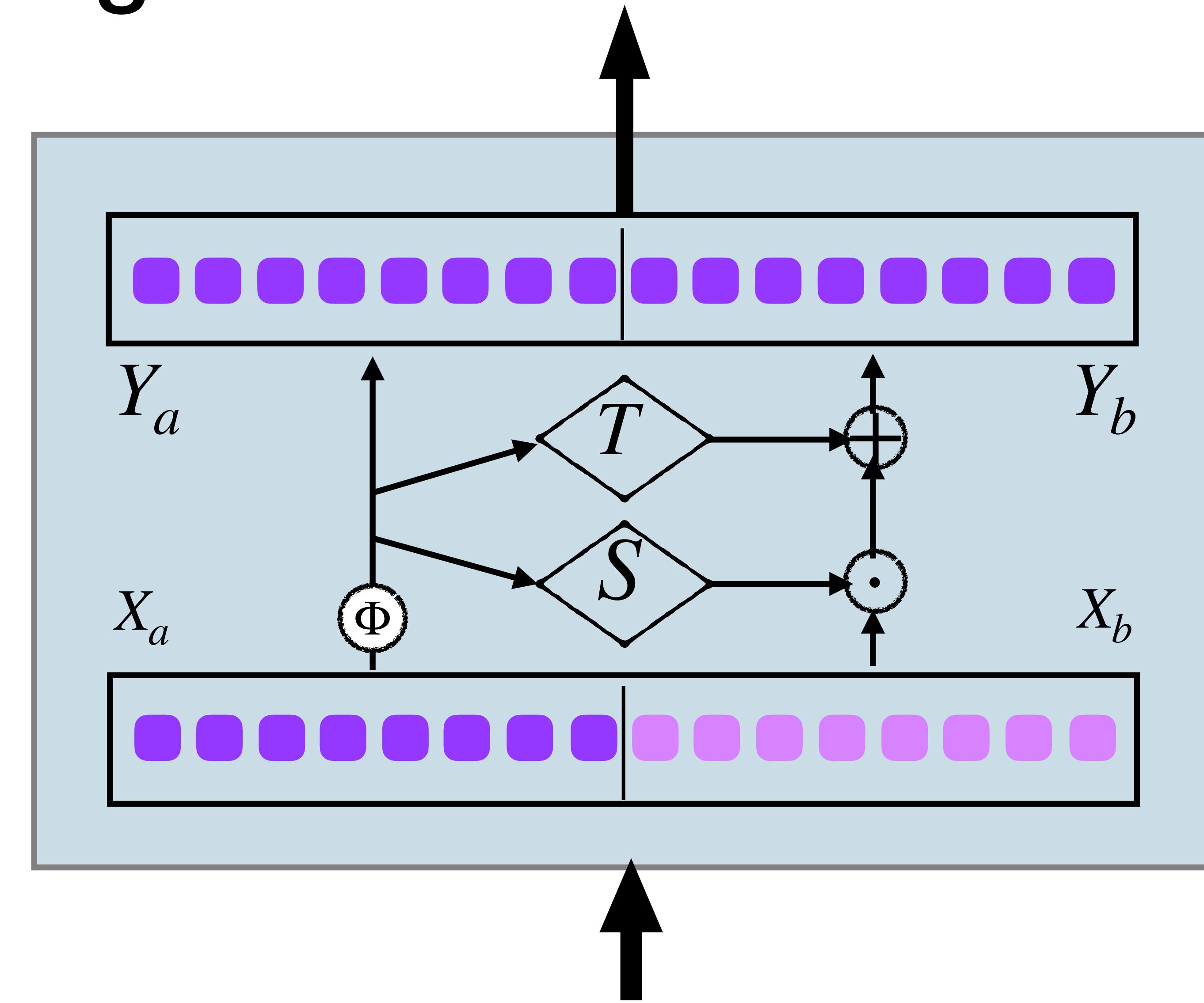


# Affine Coupling

$$S, T = \text{NN}(\Phi(X_a))$$

$$Y_b = T \cdot (S \times \Phi(X_b))$$

$$Y_a = X_a$$



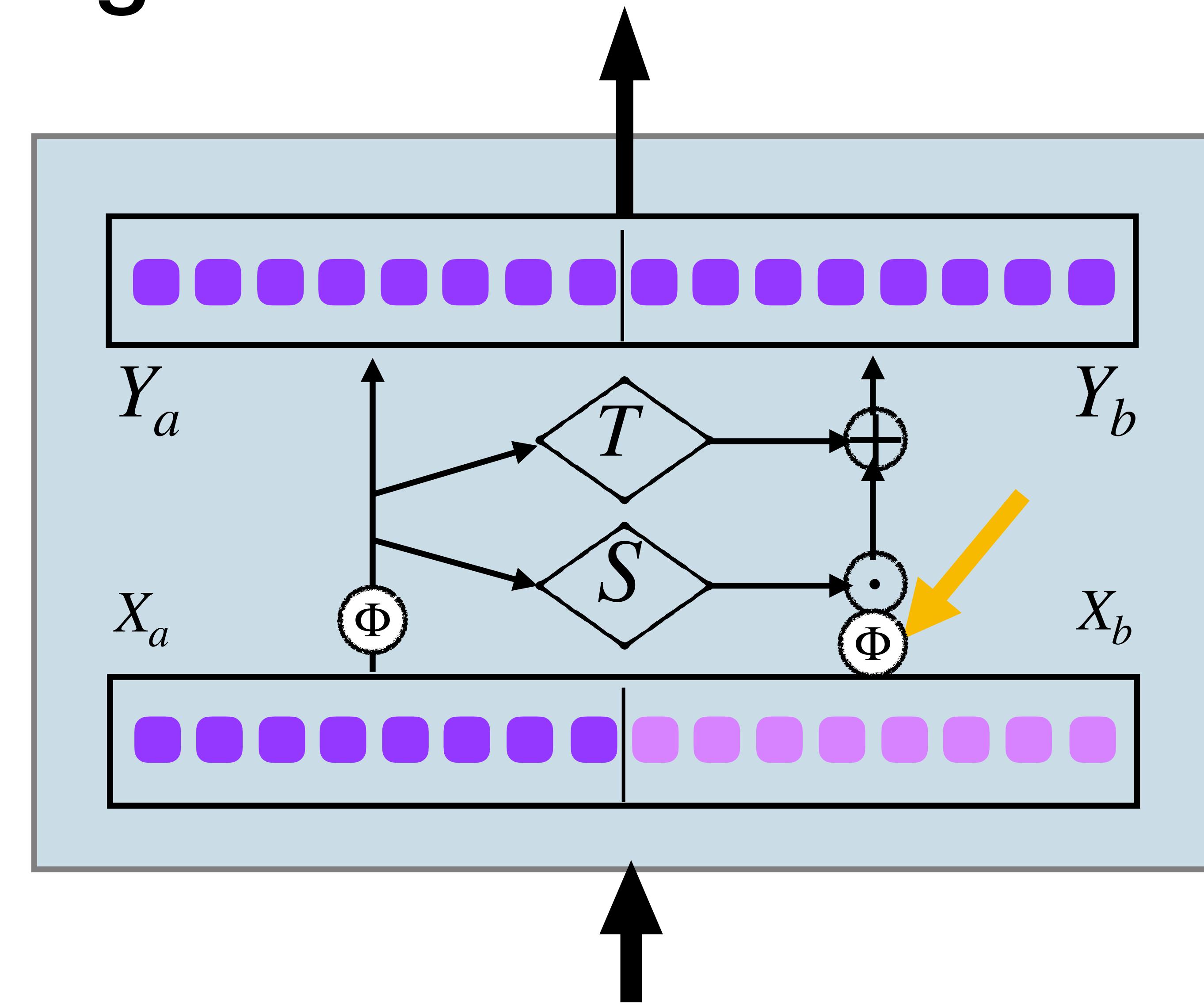


# Affine Coupling

$$S, T = \text{NN}(\Phi(X_a))$$

$$Y_b = T \cdot (S \times \Phi(X_b))$$

$$Y_a = X_a$$



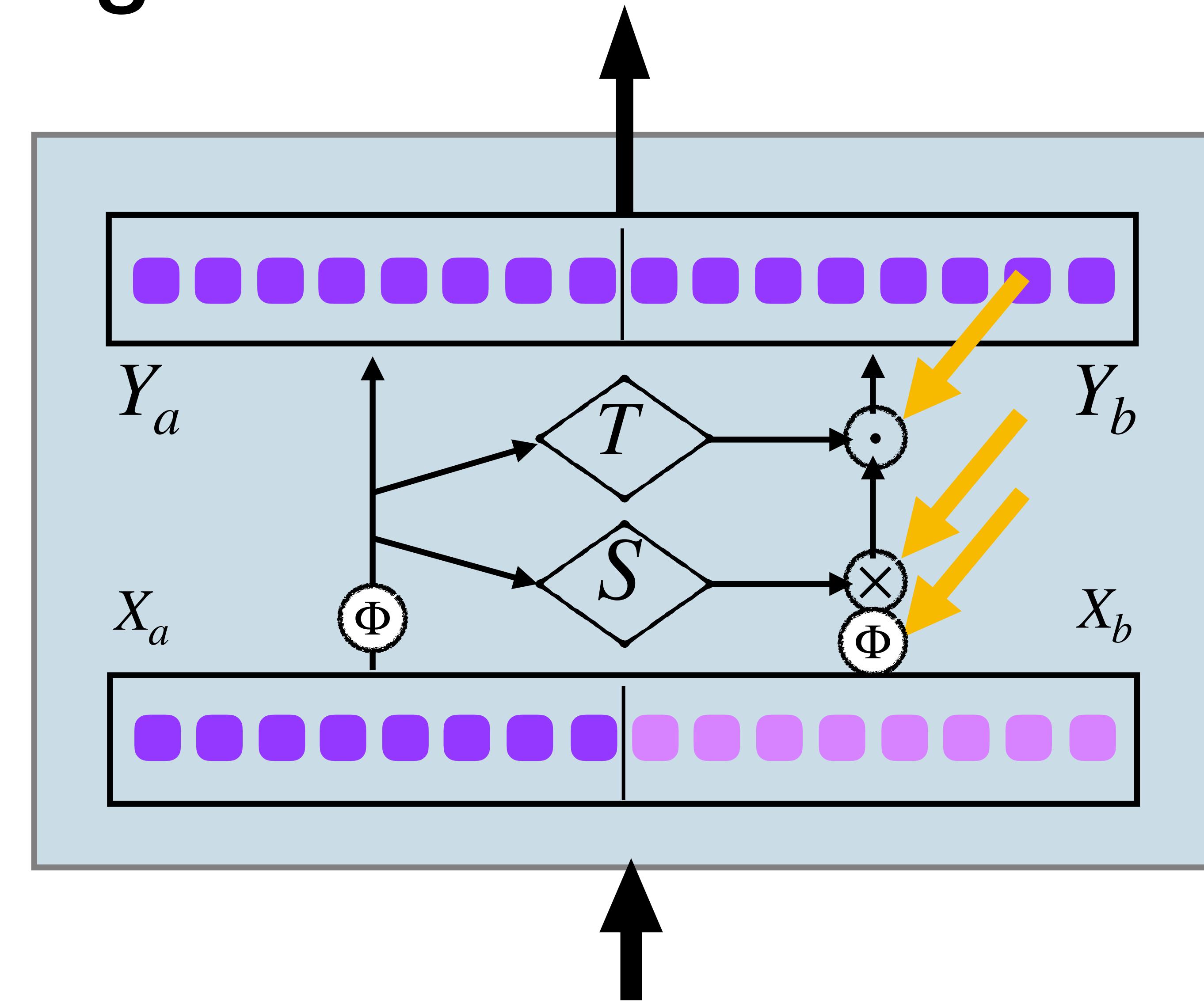


# Affine Coupling

$$S, T = \text{NN}(\Phi(X_a))$$

$$Y_b = T \cdot (S \times \Phi(X_b))$$

$$Y_a = X_a$$



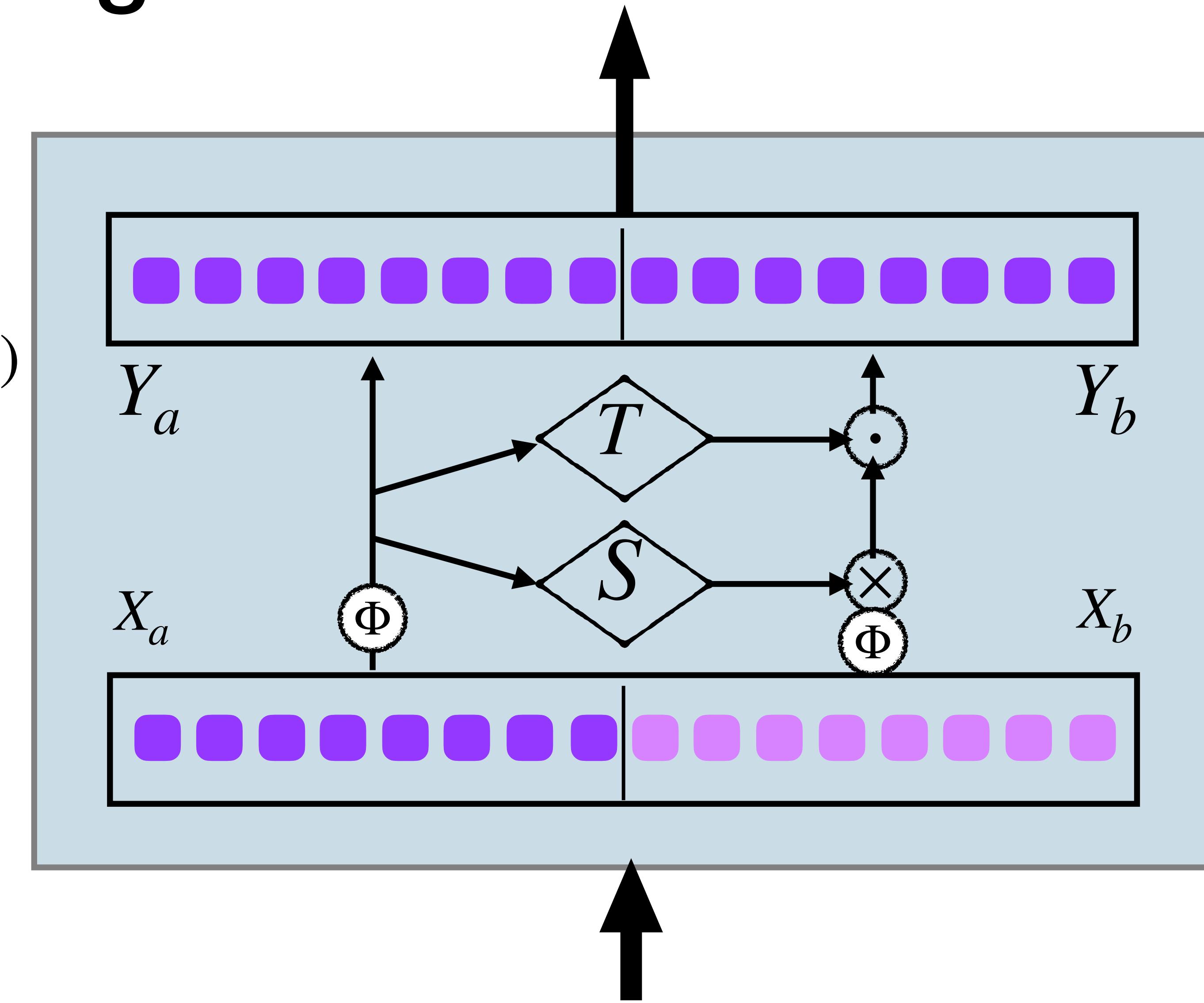


# Affine Coupling

$$S, T = \text{NN}(\Phi(X_a))$$

$$Y_b = T \cdot (S \times \Phi(X_b))$$

$$Y_a = X_a = \Phi^{-1}(\Phi(X_a))$$



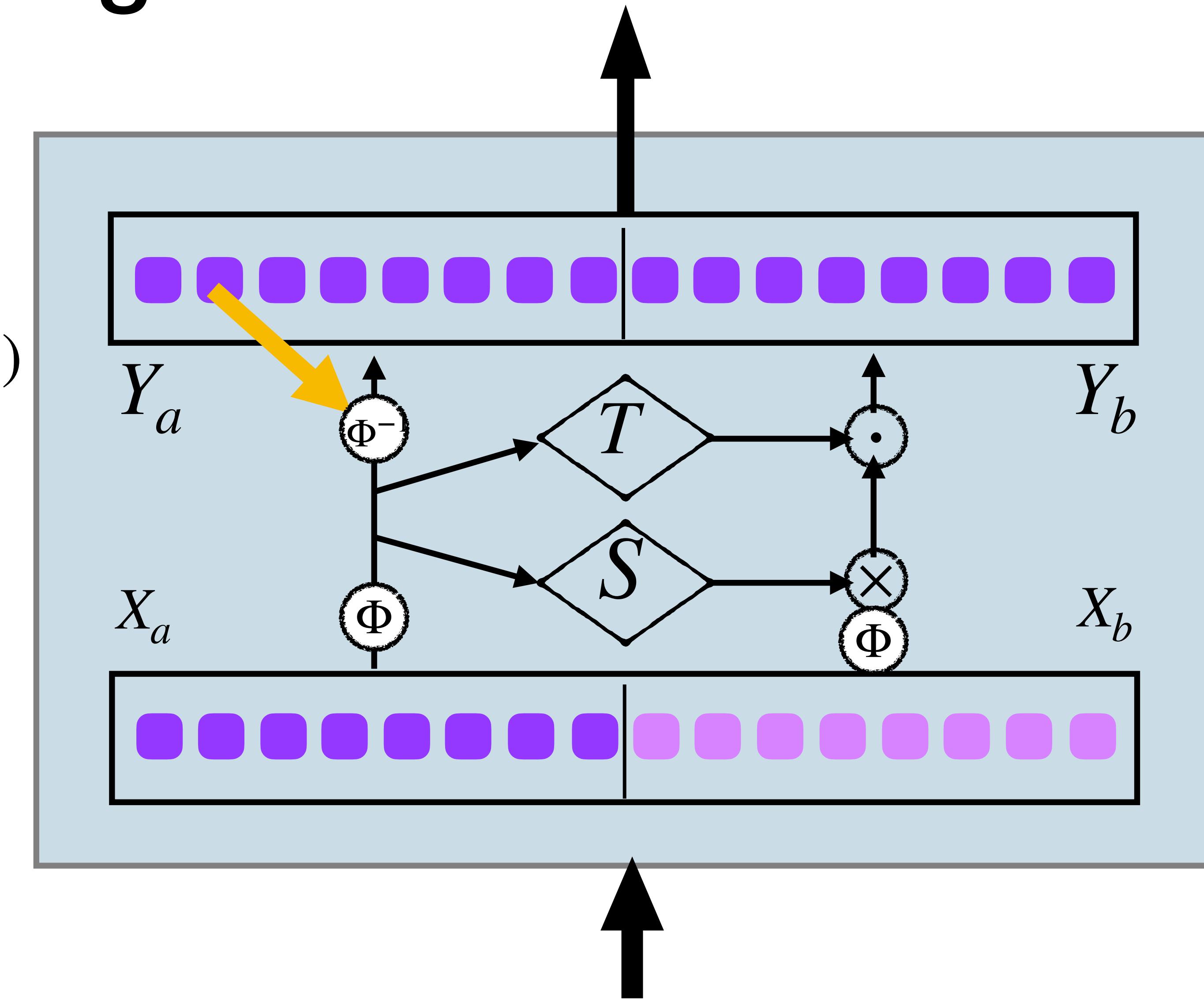


# Affine Coupling

$$S, T = \text{NN}(\Phi(X_a))$$

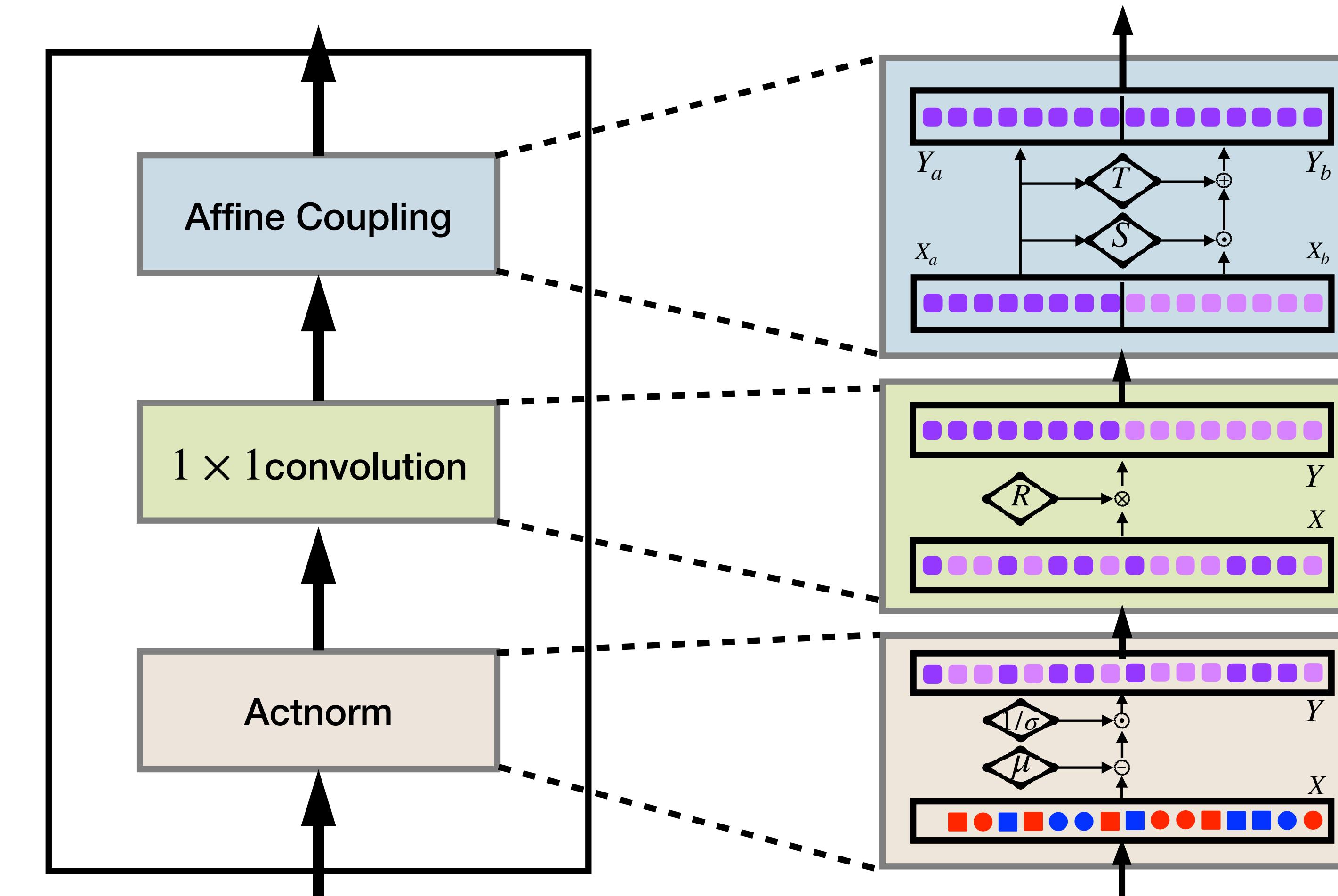
$$Y_b = T \cdot (S \times \Phi(X_b))$$

$$Y_a = X_a = \Phi^{-1}(\Phi(X_a))$$



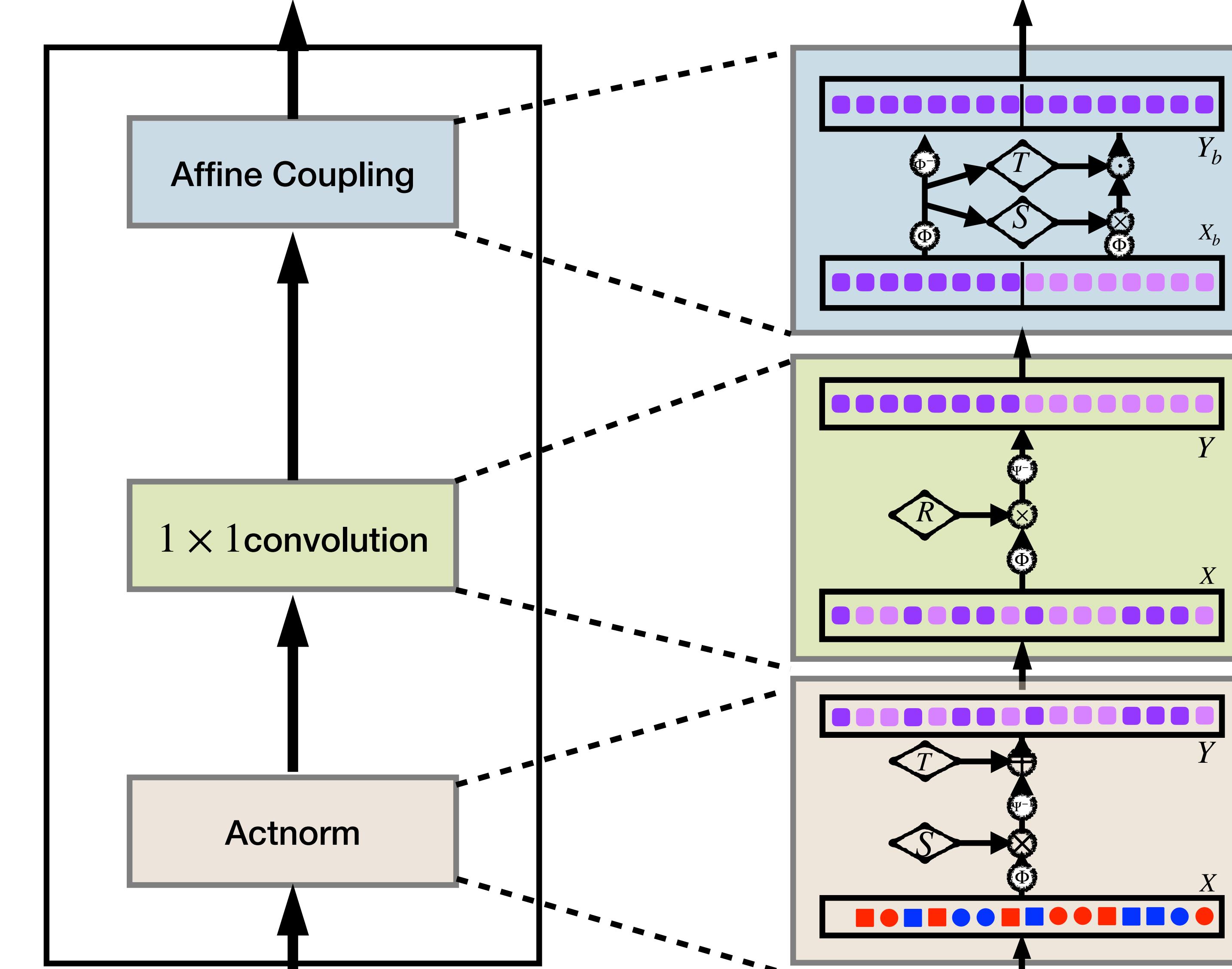


# A basic block for Euclidean data



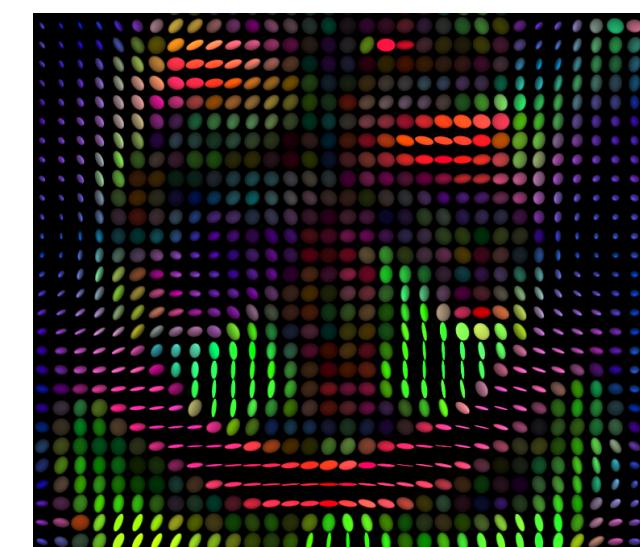


# A basic block for Manifold data

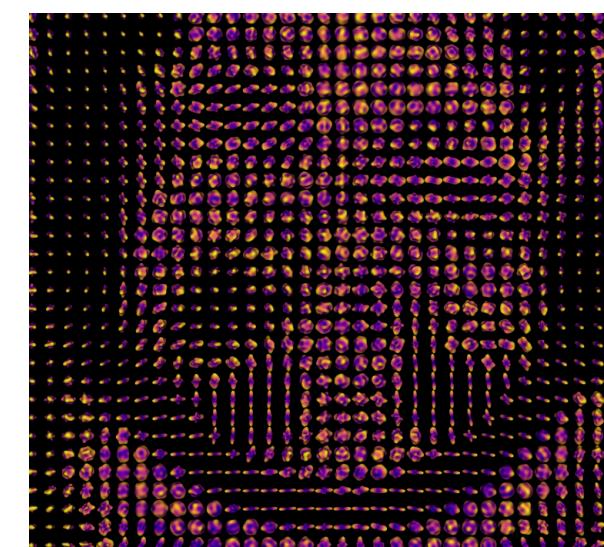
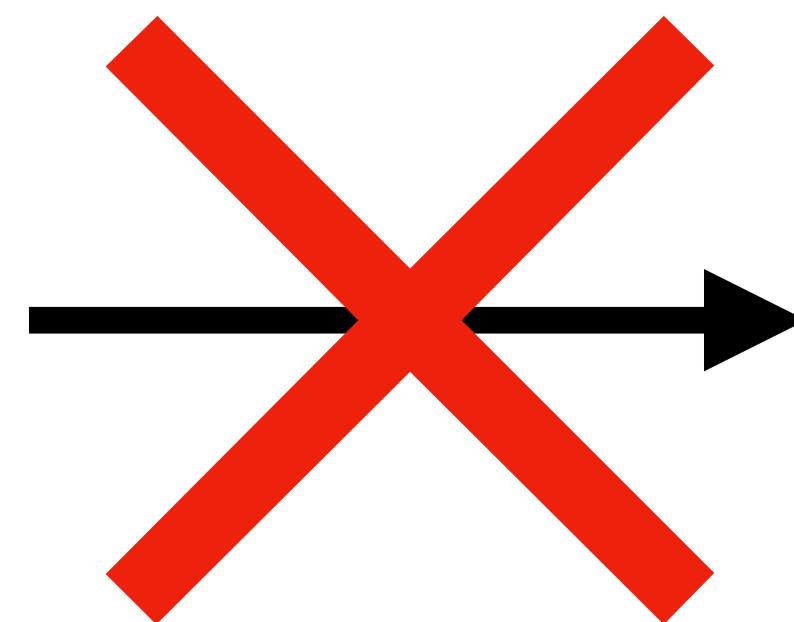




# From DTI to ODF



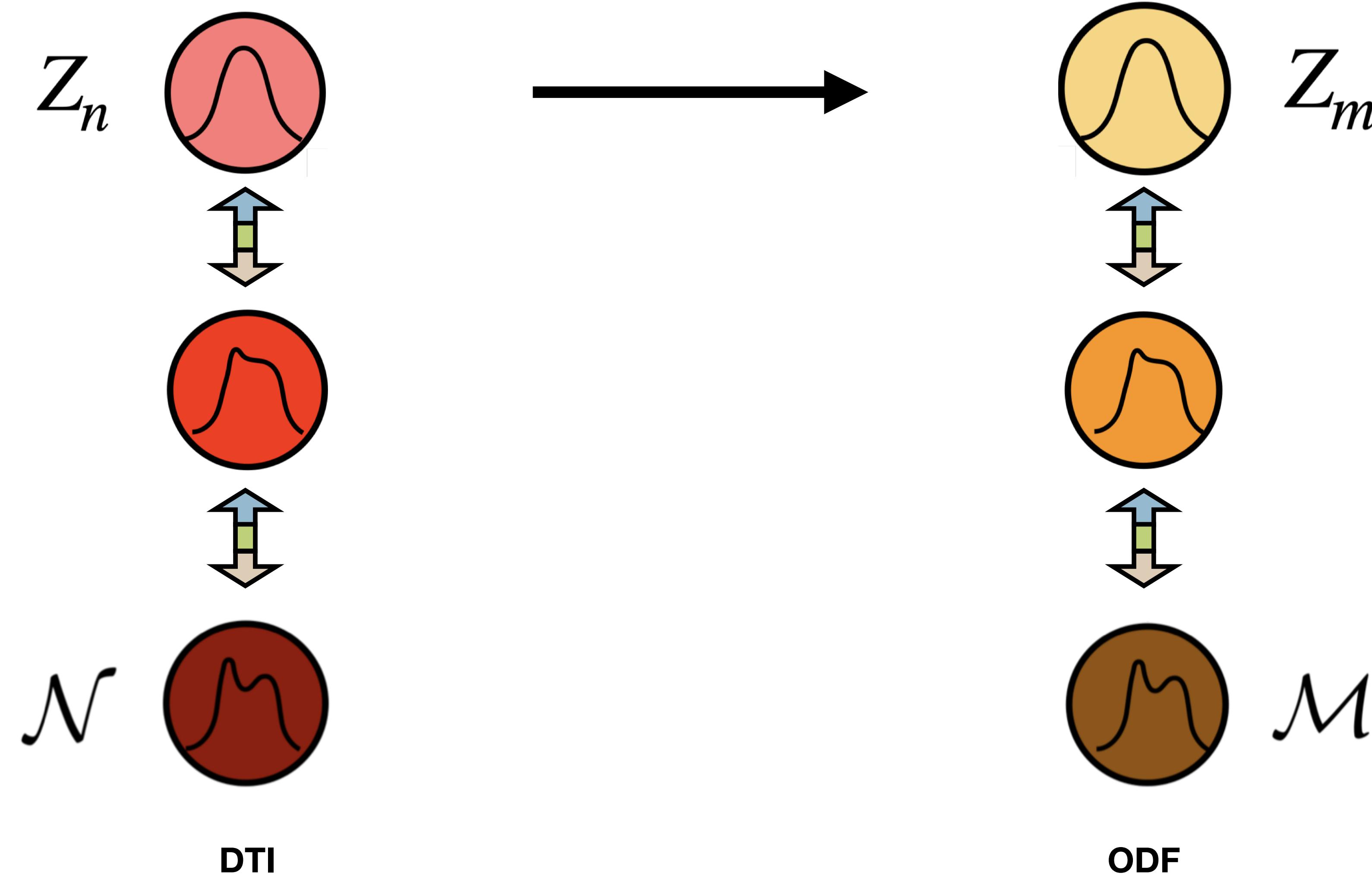
DTI



ODF



# From DTI to ODF





# Experiment - HCP dataset



**WU-Minn HCP Data - 1200 Subjects**

[Open Dataset](#) [Explore Subjects](#) [Download Image Data](#)

This HCP data release includes high-resolution 3T MR scans from young healthy adult twins and non-twin siblings (ages 22-35) using four imaging modalities: structural images (T1w and T2w), resting-state fMRI (rfMRI), task-fMRI (tfMRI), and high angular resolution diffusion imaging (dMRI). Behavioral and other individual subject measure data (both NIH Toolbox and non-Toolbox measures) is available on all subjects. MEG data and 7T MR data is available for a subset of subjects (twin pairs). The Open Access Dataset includes imaging data and most behavioral data. To protect subject privacy, some of the data (e.g., which subjects are twins) are part of a Restricted Access dataset.

Last Updated: April, 2018

ACCESS: ✓ [Open Access Terms Accepted](#)

Data Available on Amazon S3

**1113** SUBJECTS WITH MRI DATA

**95** SUBJECTS WITH MEG DATA

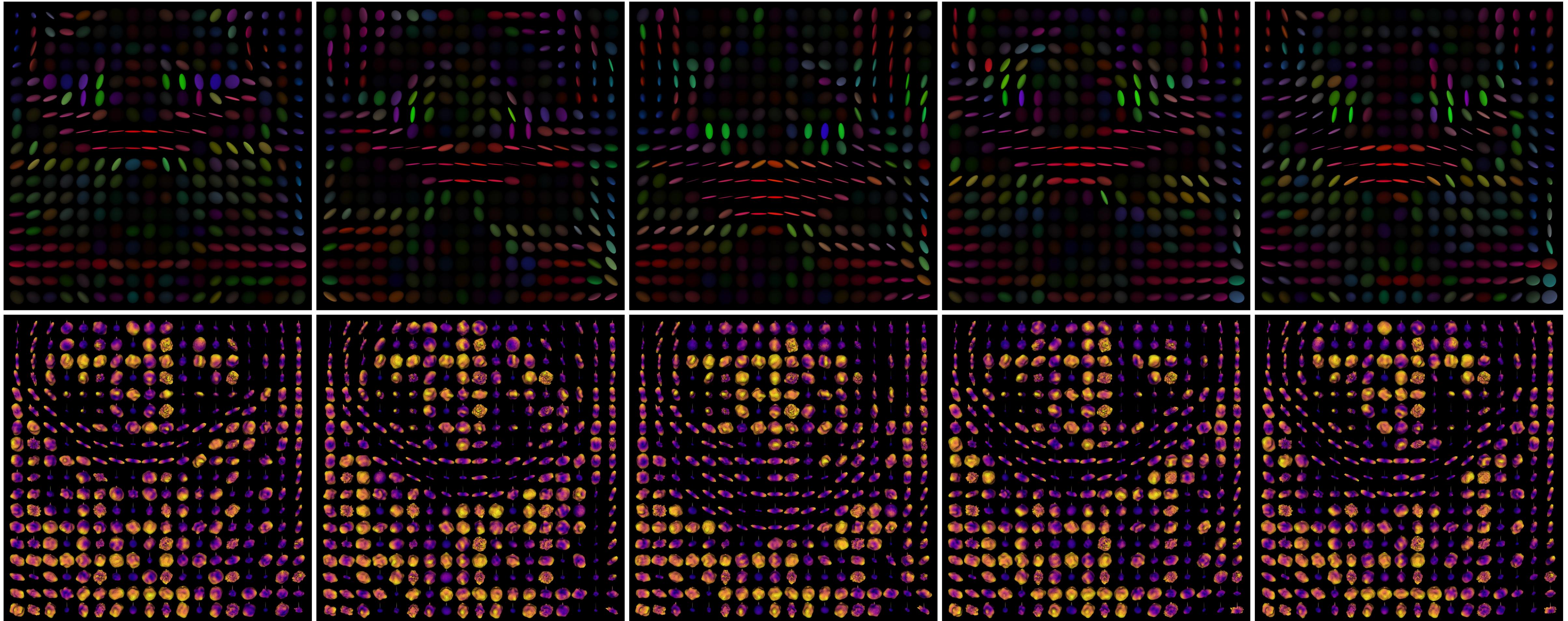
**184** SUBJECTS WITH 7T DATA

**1206** SUBJECTS WITH BEHAVIORAL DATA

**KEYWORDS:** HCP, MRI, CONNECTOME, MEG, RESTING STATE, DIFFUSION, RFMRI, DMRI, FMRI, RETEST DATA

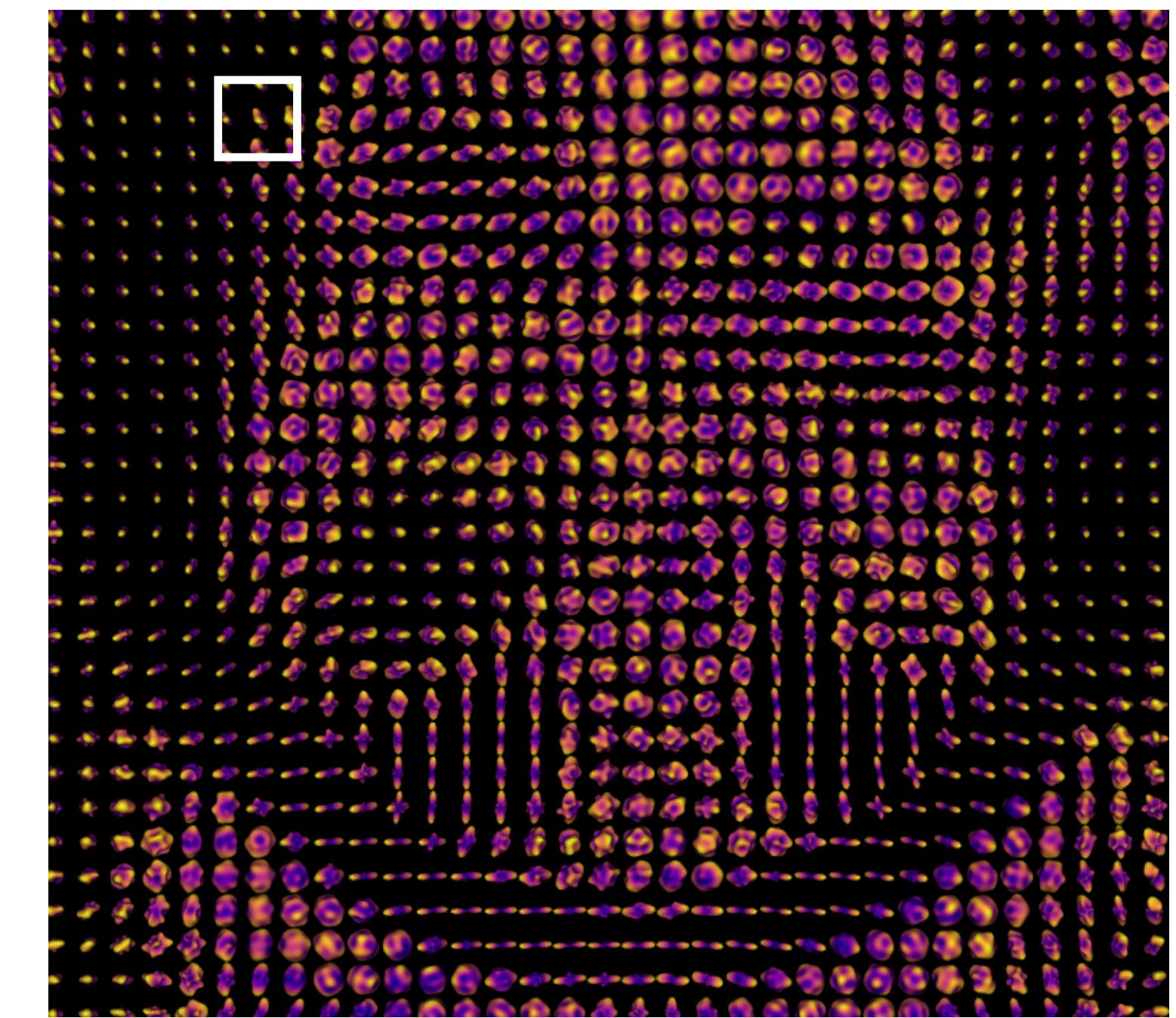
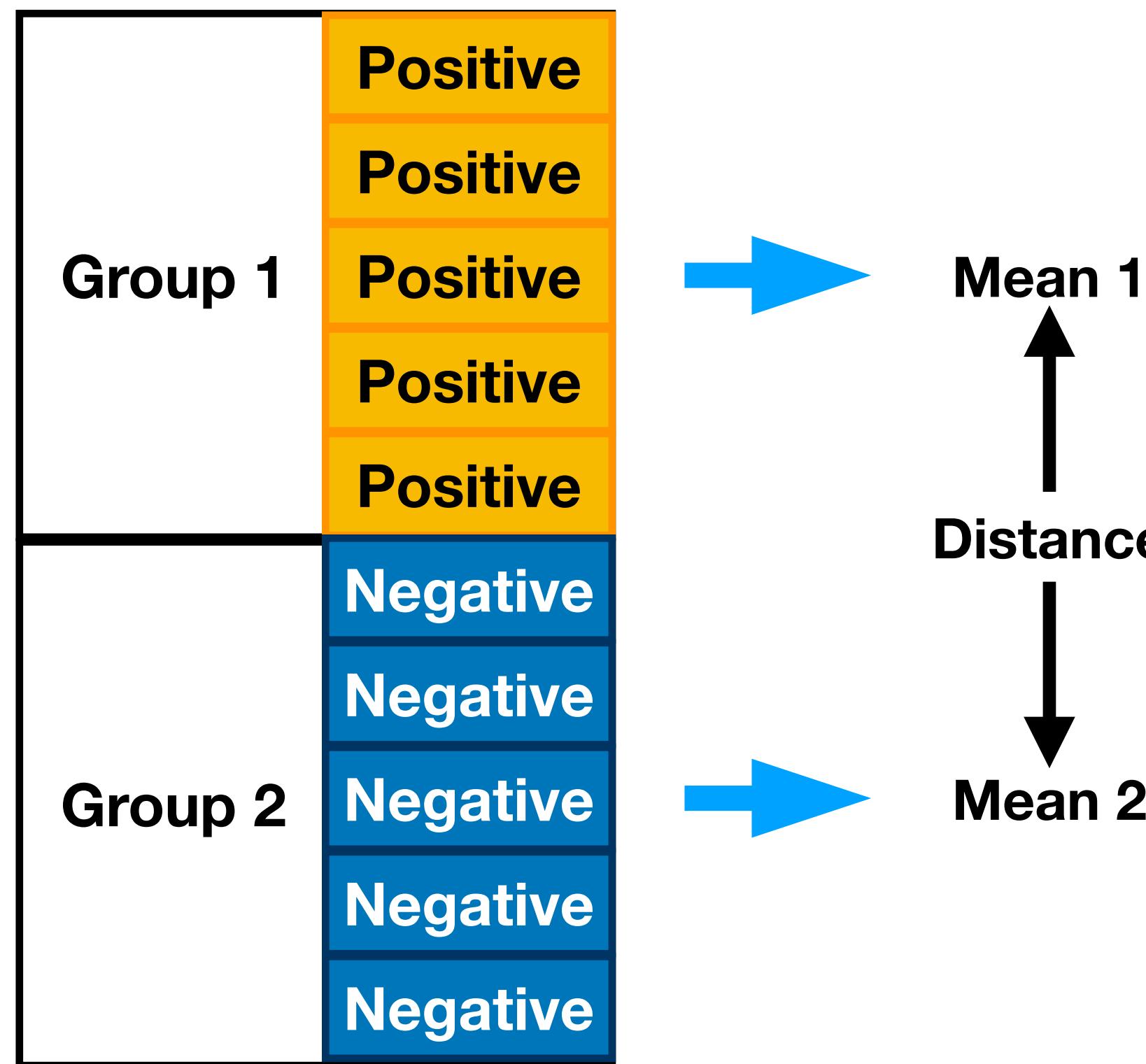


# Experiment - Some generated examples



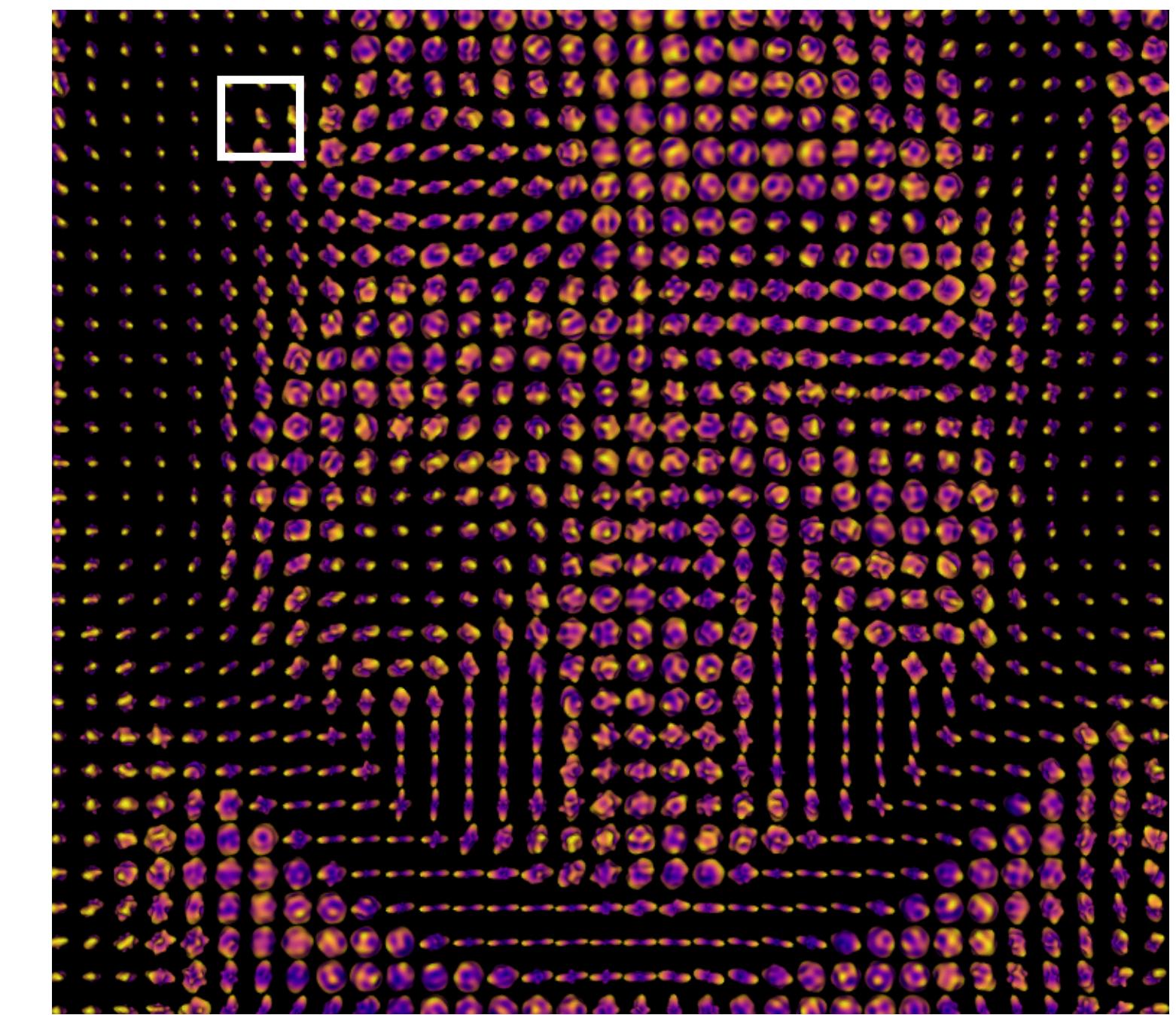
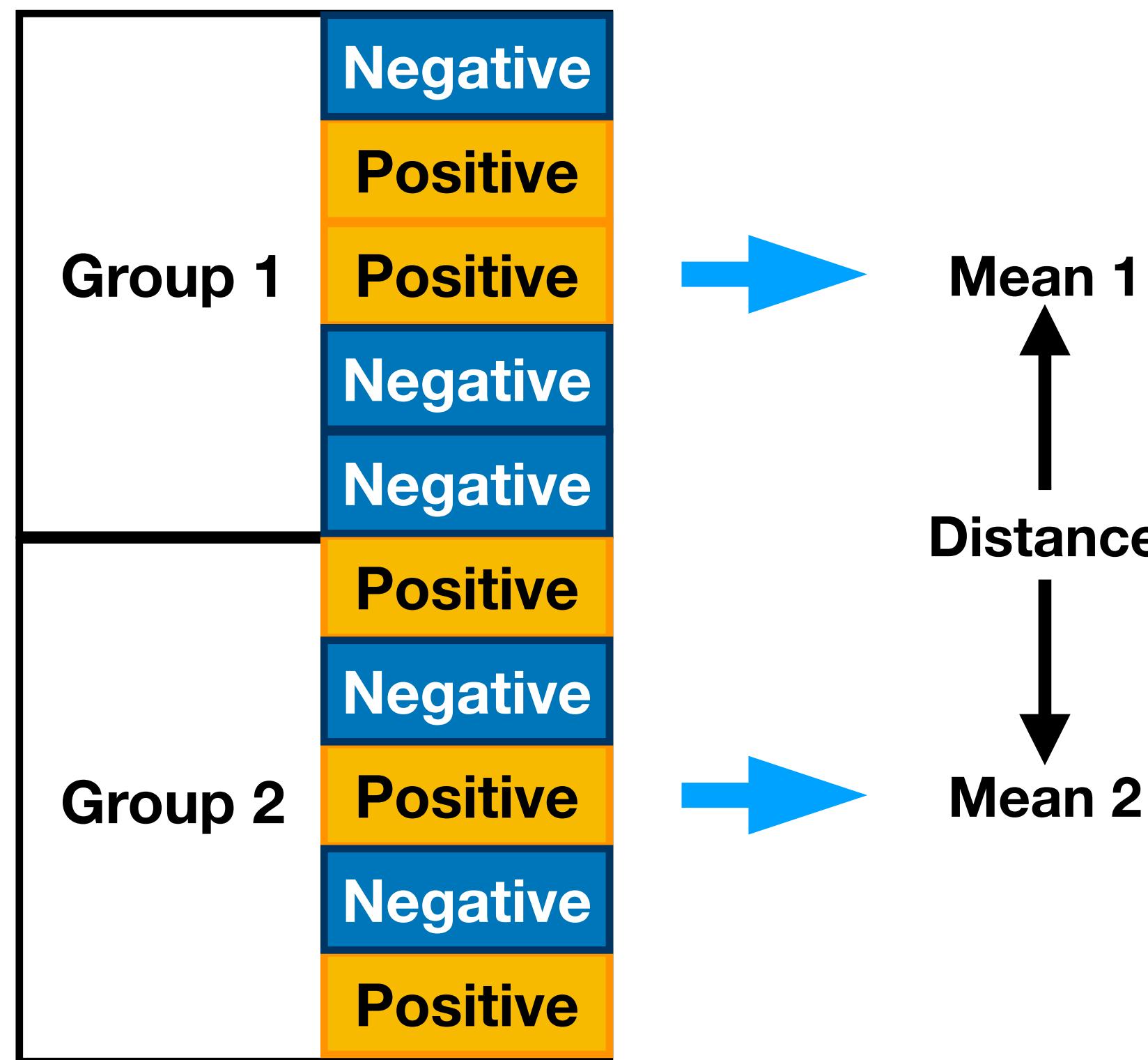


# Experiment - Group difference analysis



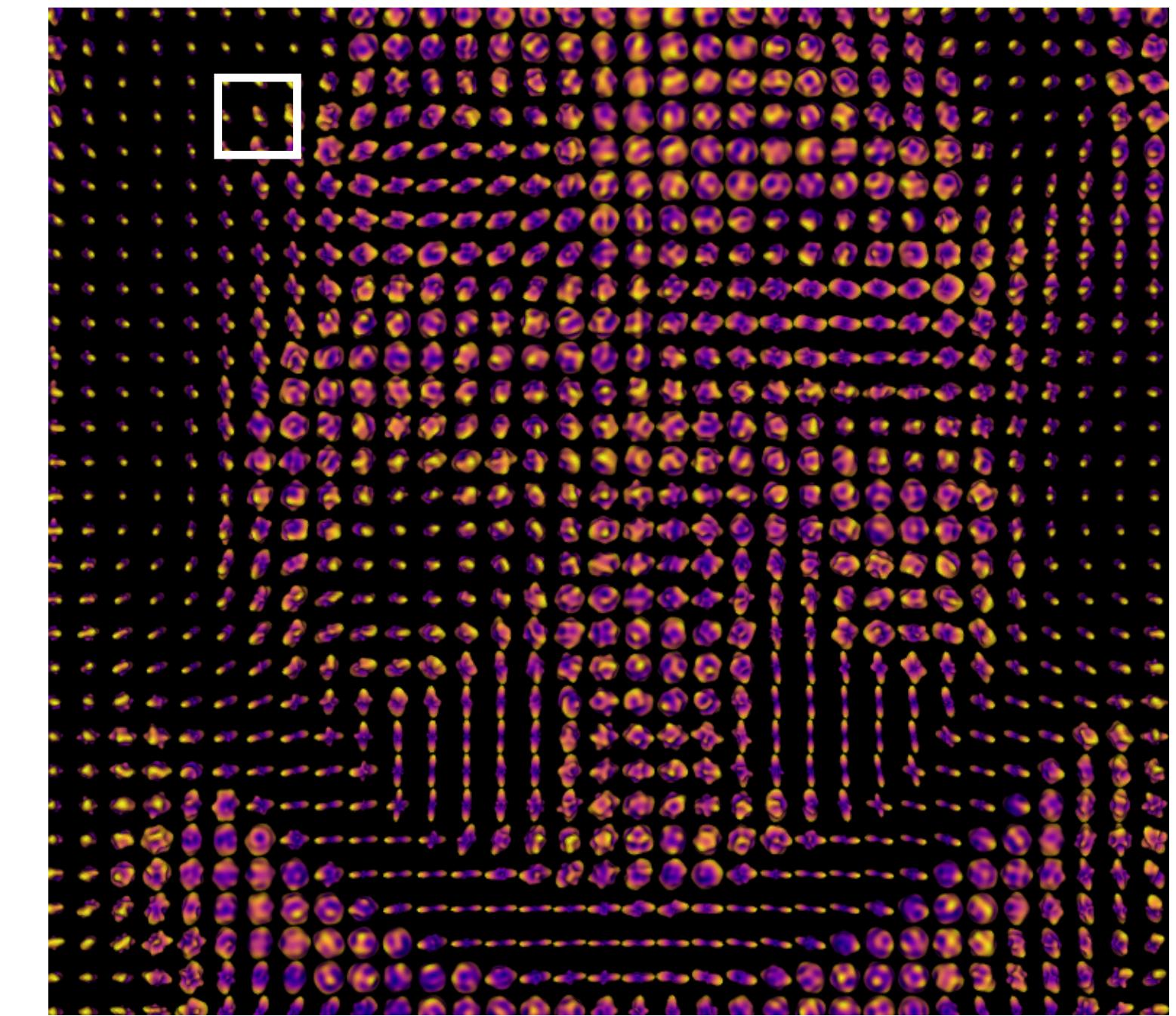
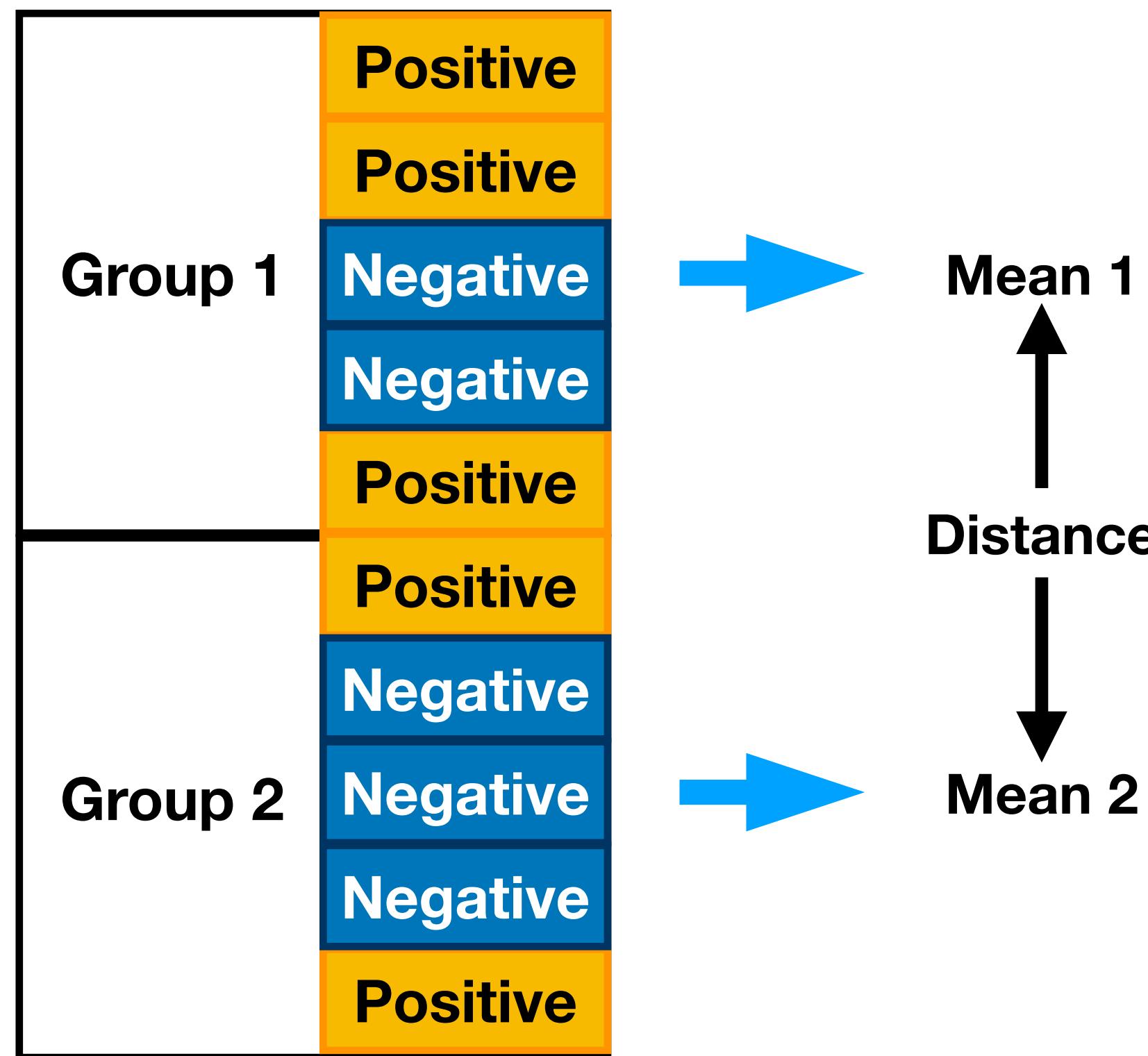


# Experiment - Group difference analysis





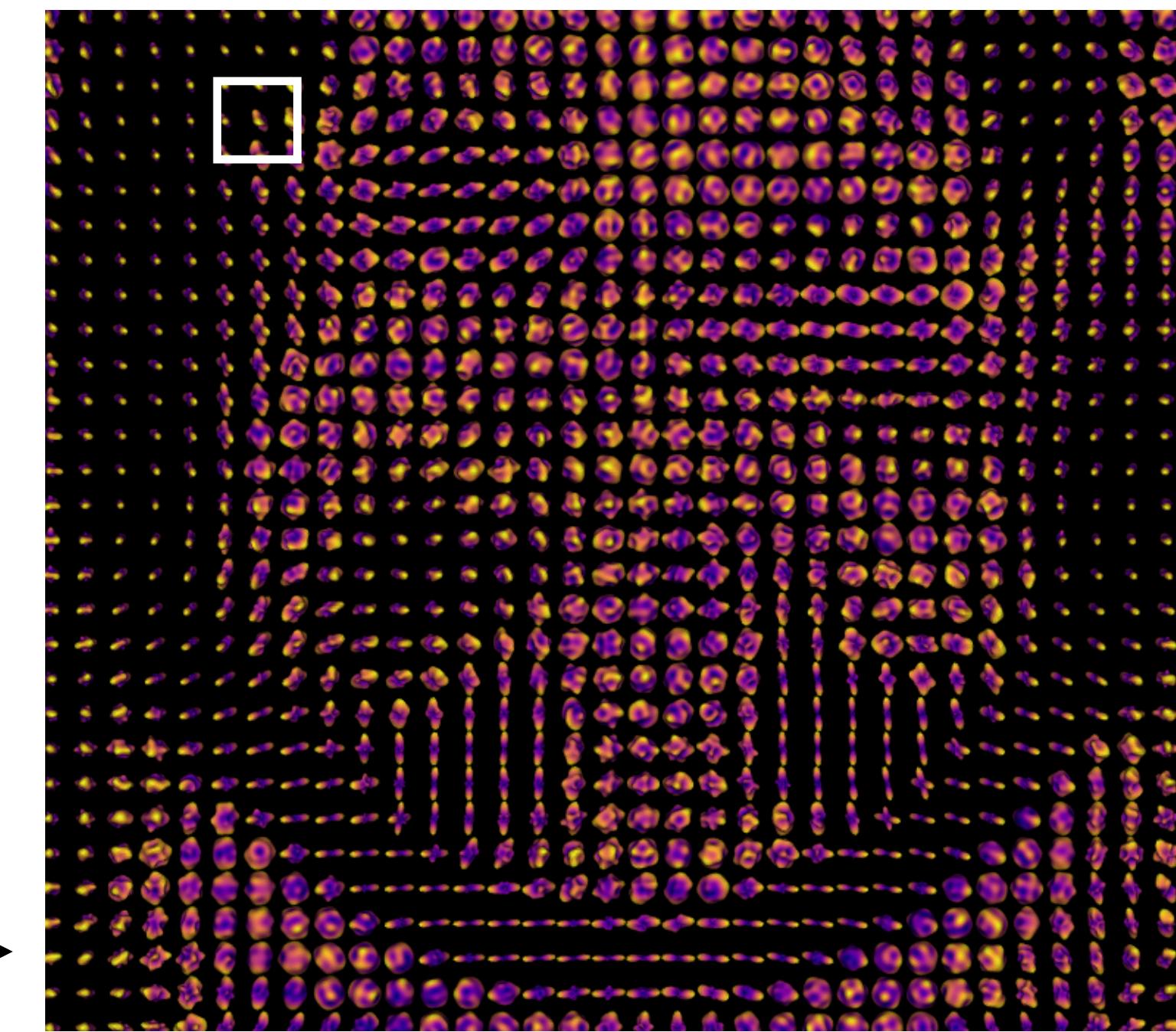
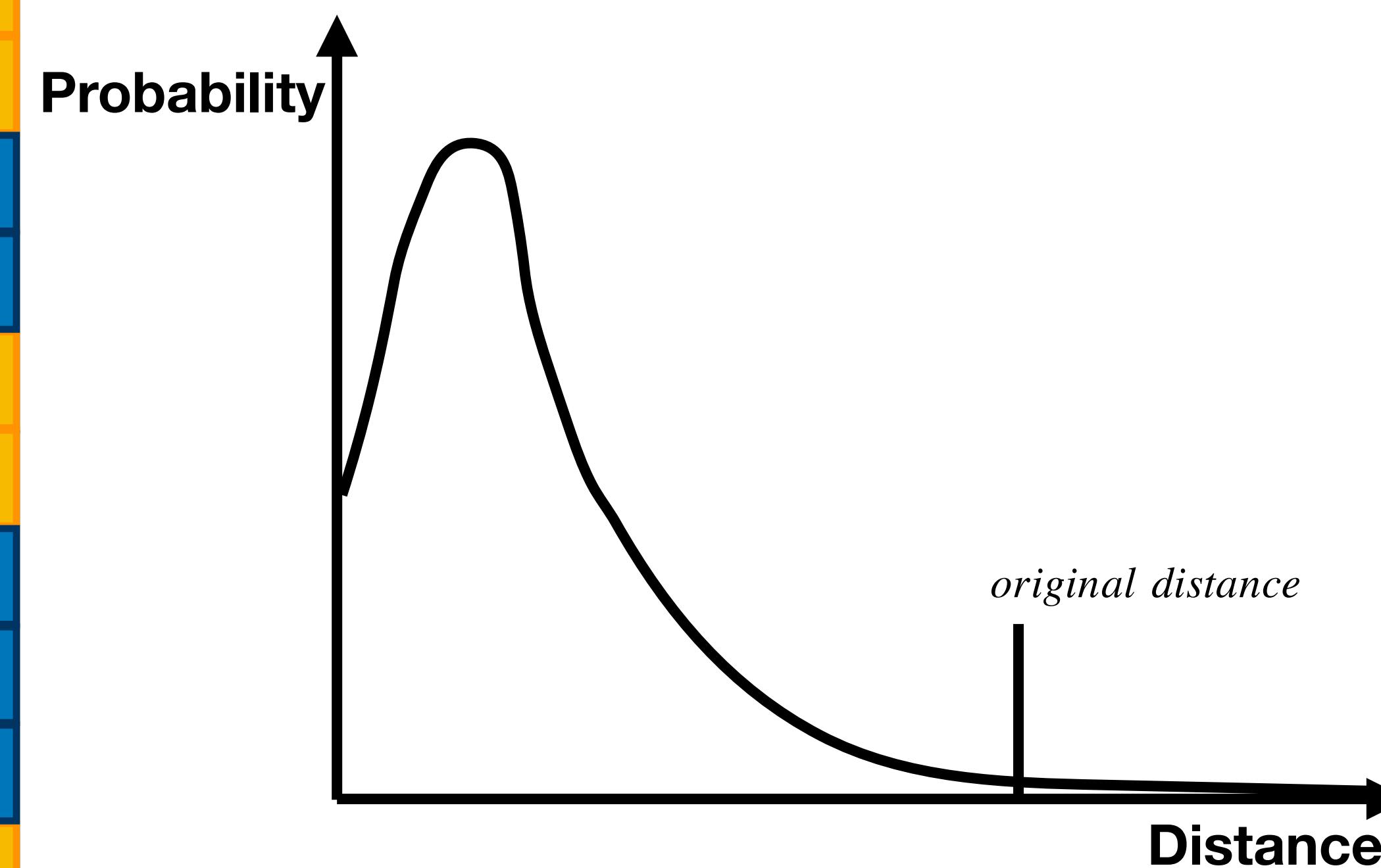
# Experiment - Group difference analysis





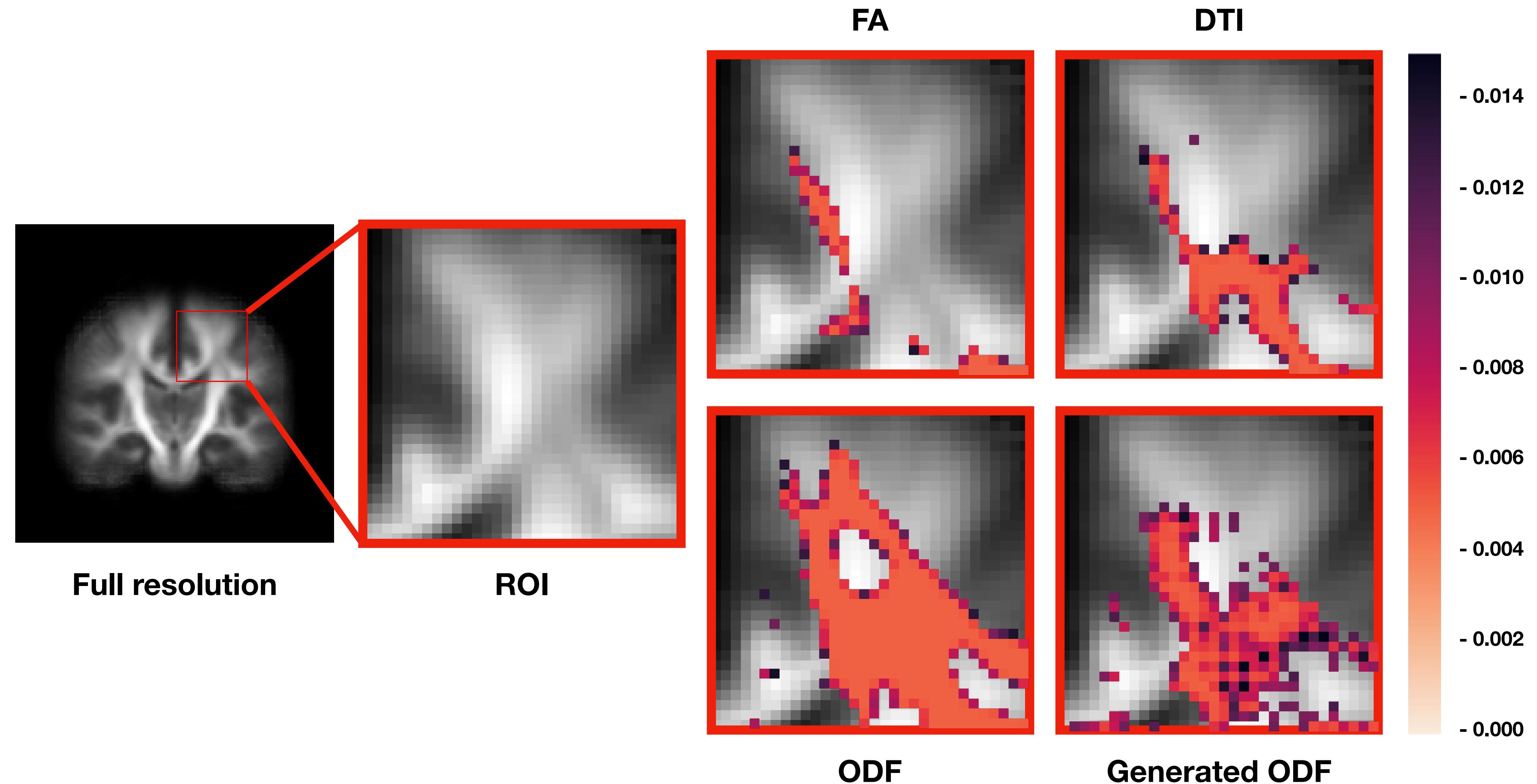
# Experiment - Group difference analysis

Group 1	Positive
	Positive
	Negative
	Negative
	Positive
	Positive
Group 2	Negative
	Negative
	Negative
	Positive





# Experiment - Group difference analysis





# Our main contribution

