Assignment II

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1 Problem 1.3

设取到一个苹果的概率为 P(A)，则有：

$$\begin{aligned}
P(A) &= P(r) \cdot P(A|r) + P(b) \cdot P(A|b) + P(g) \cdot P(A|g) \\
&= 0.2 \times \frac{3}{10} + 0.2 \times \frac{1}{2} + 0.6 \times \frac{3}{10} \\
&= 0.06 + 0.1 + 0.18 \\
&= 0.34
\end{aligned}$$

设取到一个橘子的概率为 P(B)，则有：

$$\begin{aligned}
P(B) &= P(r) \cdot P(B|r) + P(b) \cdot P(B|b) + P(g) \cdot P(B|g) \\
&= 0.2 \times \frac{4}{10} + 0.2 \times \frac{1}{2} + 0.6 \times \frac{3}{10} \\
&= 0.36
\end{aligned}$$

由贝叶斯公式：

$$\begin{aligned}
P(g|B) &= \frac{P(B|g) \cdot P(g)}{P(B)} \\
&= \frac{0.6 \times \frac{3}{10}}{0.36} \\
&= 0.5
\end{aligned}$$

2 Problem 1.6

因为 x, y 相互独立，所以有：

$$p(x, y) = p_x(x) \cdot p_y(y)$$

$$\begin{aligned}
\implies \iint xyp(x, y)\,\mathrm{d}x\mathrm{d}y &= \iint xyp_x(x) \cdot p_y(y)\,\mathrm{d}x\mathrm{d}y \\
&= \left(\int xp_x(x)\,\mathrm{d}x\right)\left(\int yp_y(y)\,\mathrm{d}y\right) \\
\implies E_{x,y}(xy) &= E(x) \cdot E(y) \\
\implies cov(x, y) &= E_{x,y}(xy) - E(x) \cdot E(y) = 0
\end{aligned}$$

3 Problem 1.8

$$
\begin{aligned}
E(x) &= \int_{-\infty}^{+\infty} x \frac{1}{\sqrt{2\pi\sigma^2}} exp\left\{-\frac{1}{2\sigma^2}(x-\mu)^2\right\} \mathrm{d}x \\
&\xlongequal{令 \frac{x-\mu}{\sqrt{2\sigma^2}}=t} \frac{\sqrt{2}\sigma}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} (\sqrt{2}\sigma t + \mu) exp\{-t^2\} \, \mathrm{d}t \\
&= \frac{1}{\sqrt{\pi}} \sqrt{\pi}\mu \\
&= \mu
\end{aligned}
$$

$$
\begin{aligned}
var(x) &= \int_{-\infty}^{+\infty} \big(x - E(x)\big) \frac{1}{\sqrt{2\pi\sigma^2}} exp\left\{-\frac{1}{2\sigma^2}(x-\mu)^2\right\} \mathrm{d}x \\
&\xlongequal{令 \frac{x-\mu}{\sqrt{2\sigma^2}}=t} \frac{4\sigma^2}{\sqrt{\pi}} \int_{0}^{+\infty} t^2 \mathrm{e}^{-t^2} \, \mathrm{d}t \\
&\xlongequal{令 t^2=u} \frac{2\sigma^2}{\sqrt{\pi}} \int_{0}^{+\infty} u^{\frac{1}{2}} \mathrm{e}^{-u} \, \mathrm{d}u \\
&= \frac{2\sigma^2}{\sqrt{\pi}} \Gamma(\frac{3}{2}) \\
&= \sigma^2
\end{aligned}
$$

$$
\begin{aligned}
\implies E(x^2) &= var(x) + [E(x)]^2 \\
&= \mu^2 + \sigma^2
\end{aligned}
$$