Report on "QUASI-STATIONARY DISTRIBUTIONS FOR SUBCRITICAL SUPERPROCESSES"

by Rongli Liu, Yan-Xia Ren, Renming Song and Zhenyao Sun

In this manuscript, the authors discuss the quasi-stationary distribution for a class of subcritical superprocesses. Under some asymptotic conditions, they prove that the Yaglom limit exists. For the subcritical superprocesses, the extinction probability is 1, then it is natural to consider the limit of X_t conditioned on the event $\{\|X_t\| > 0\}$. The authors also identify all quasi-stationary distributions. Similar problems have been studied for subcritical Galton-Watson processes and branching Markov processes.

In this manuscript, the superprocesses they deal with are general: spatial motion of the superprocesses is a class of general Markov processes and the branching mechanism is of a spatially dependent general form. Overall, I think the results of this article is worthwhile and interesting. I am very happy to recommend it for publication in SPA. The following are some minor comments and remarks.

Other comments/suggestions:

(1) In the article, the authors use too many $C^{\cdots}_{\cdots}$ to denote different constants or functions. It is hard to distinguish these symbols, so the readers have to go back to their definitions frequently, which makes the paper not easy to read. And I think some of them are not necessary, for example, the C in (2.16) and (2.18),.....

(2) p1: Please note that the branching process Z is nontrivial ($p_1 < 1$), otherwise, the extinction probability is 0.

(3) p1, line-1: "(If a...) "delete ( ).

(4) p2, line 10: change "$t \geq 0$." to "$t \geq 0$,".

(5) p3, condition (H2): give more explanation of condition (H2), or give some sufficient condition for (H2).

(6) p7, Proposition 1.8: $e^{r\lambda}$ should be e^{rs}, that is

$$1 - e^{-\mathcal{L}_{\mathbf{Q}_{r^*}} V_s f} = e^{rs}(1 - e^{-\mathcal{L}_{\mathbf{Q}_{r^*}} f}), ...$$

(7) p7, Proposition 1.9: Idem.

(8) p8: in the last line of equation (2.3): $g \in \mathcal{B}(E, [0, \infty])$ should be $g \in L_1^+(\nu)$.

(9) p15, line 2: recall the definition of Γ_t.

(10) p19: the formula below (2.46), change $v_{(t_n)}$ to v_{t_n}.

(11) p21, line7: the law of Z should be

$$p(Z=n) = \frac{\gamma \prod_{k=1}^{n-1}(k-\gamma)}{n!} = \frac{\gamma(1-\gamma)\cdots(n-1-\gamma)}{n!}, \quad n \in \mathbb{Z}+.$$

(12) p21, line9: Idem.