Stochastic Summer Seminar Series 2025 (Updated in May 23, 2025)

Date	Time	Speaker	Title
July 1, 2025	10:00-11:50	Oleg Butkovsky	Mini-course: Stochastic sewing with applications (Part 1)
July 2, 2025	10:00-11:50	Oleg Butkovsky	Mini-course: Stochastic sewing with applications (Part 2)
July 3, 2025	10:00-10:50	Oleg Butkovsky	Weak uniqueness for singular stochastic equations driven by fractional Brownian motion
July 3, 2025	11:00-11:50	Zimo Hao	Supercritical SDEs driven by fractional Brownian motion with divergence free drifts
July 3, 2025	14:00-14:50	Guohuan Zhao	TBA
July 3, 2025	15:00-15:50	Guopeng Li	TBA
July 3, 2025	16:00-17:50	Zhenyao Sun	Wright-Fisher stochastic heat equations with irregular drifts
July 4, 2025	10:00-10:50	Panki Kim	Potential theory of Dirichlet forms with jump kernels blowing up at the boundary
July 4, 2025	11:00-11:50	Renming Song	TBA
July 4, 2025	14:00-14:50	Soobin Cho	Approximate factorizations for non-symmetric jump processes
July 4, 2025	15:00-15:50	Haojie Hou	TBA

Location: School of Mathematics and Statistics at Beijing Institute of Technology–Liangxiang Campus

Mini-course: Stochastic sewing with applications

Speaker: Oleg Butkovsky (Weierstrass Institute)
Time: 2025 July 1st 10:00-11:50 & 2025 July 2nd 10:00-11:50
Location: TBA

Abstract: The sewing lemma of Gubinelli [Gub04] is a key tool in Lyons' theory of rough paths, as well as in Hairer's theory of regularity structures. Recently, a stochastic extension of this result (the stochastic sewing lemma by Lê) has been obtained [Lê20], which has already become a powerful tool for many applications. In this mini-course, we will discuss stochastic sewing ideas and show how they can help us study problems in the following research directions:

- *Regularization by noise*, [ABLM24, BM24]. The deterministic equation

$$dX_t = b(X_t)\,dt,$$

might have no solution or infinitely many solutions if the drift b is not smooth enough. However, its stochastic counterpart

$$dX_t = b(X_t)\,dt + dW_t, \tag{$*$}$$

where W is the standard Brownian motion, has a unique strong solution if the drift b is just a bounded measurable function, without any further assumptions on the regularity of b. Why does this phenomenon occur? What happens if the noise is a different random process (say, fractional Brownian motion)? Will regularization by noise take place for PDEs?
- *Numerical algorithms for SDEs and SPDEs*, [BDG21, BDG23]. If the drift b in the SDE ($*$) is smooth, it is well-known that the standard Euler scheme will converge to the solution of this equation with rate 1. Will the Euler scheme converge to the solution if the drift is non-smooth? What will be the convergence rate? Will it deteriorate as the drift becomes just a bounded measurable function?

- *Slow-fast systems of stochastic equations*, [HL20]. Consider a two-scale (fast/slow) stochastic evolution:

$$dX_t^\epsilon = f(X_t^\epsilon, Y_t^\epsilon)\,dt + w(\epsilon)g(X_t^\epsilon, Y_t^\epsilon)\,dB_t,$$

  where Y_t^ϵ is a stochastic process representing the “fast component” (with fluctuations of higher order in ϵ), f and g are smooth functions, w is a weight, and B represents the noise. If the noise B is a Brownian motion, $w(\epsilon) \equiv 1$, and the fast component is driven by an SDE, then Khasminskii [Kha68] showed that the slow component X^ϵ converges weakly as $\epsilon \to 0$ to the solution of the averaged SDE. We will discuss what obstacles appear if the slow process X^ϵ is driven by a fractional Brownian motion rather than by a Brownian motion and how they were resolved by Hairer and Li with the help of stochastic sewing [HL20].

References

[ABLM24] Siva Athreya, Oleg Butkovsky, Khoa Lê, and Leonid Mytnik. Well-posedness of stochastic heat equation with distributional drift and skew stochastic heat equation. *Communications on Pure and Applied Mathematics*, 77(5):2708–2777, 2024.

[BDG21] Oleg Butkovsky, Konstantinos Dareiotis, and Máté Gerencsér. Approximation of SDEs: a stochastic sewing approach. *Probab. Theory Related Fields*, 181(4):975–1034, 2021.

[BDG23] Oleg Butkovsky, Konstantinos Dareiotis, and Máté Gerencsér. Optimal rate of convergence for approximations of SPDEs with nonregular drift. *SIAM J. Numer. Anal.*, 61(2):1103–1137, 2023.

[BM24] Oleg Butkovsky and Leonid Mytnik. Weak uniqueness for singular stochastic equations. *arXiv preprint arXiv:2405.13780*, 2024.

[Gub04] M. Gubinelli. Controlling rough paths. *J. Funct. Anal.*, 216(1):86–140, 2004.

[HL20] Martin Hairer and Xue-Mei Li. Averaging dynamics driven by fractional Brownian motion. *Ann. Probab.*, 48(4):1826–1860, 2020.

[Kha68] R. Z. Khasminskii. On the principle of averaging the Itô’s stochastic differential equations. *Kybernetika (Prague)*, 4:260–279, 1968.

[Lê20] Khoa Lê. A stochastic sewing lemma and applications. *Electron. J. Probab.*, 25:Paper No. 38, 55, 2020.

Weak uniqueness for singular stochastic equations driven by fractional Brownian motion

Speaker: Oleg Butkovsky (Weierstrass Institute)
Time: 2025 July 3rd 10:00-10:50
Location: TBA

Abstract: Based on joint works with Leonid Mytnik (Technion - Israel Institute of Technology) and Konstantinos Dareiotis (University of Leeds). We consider the stochastic differential equation

$$dX_t = b(X_t)dt + dB_t^H,$$

where the drift b is a Schwartz distribution in the space $\mathcal{C}^\alpha$, $\alpha < 0$, and B^H is a fractional Brownian motion of Hurst index $H \in (0, 1/2]$. If $H = 1/2$, both weak and strong uniqueness theories for this SDE have been developed. However, the situation is much more complicated if $H < 1/2$, as the main tool, the Zvonkin transformation, becomes unavailable in this setting. The breakthroughs by Catellier and Gubinelli, and later by Le, established strong well-posedness of this SDE via sewing/stochastic sewing arguments. However, weak uniqueness for this SDE remained a challenge for quite some time, since a direct application of stochastic sewing alone does not seem very fruitful. I will explain how a combination of stochastic sewing with certain arguments from ergodic theory allows to show weak uniqueness in the whole regime where weak existence is known, that is, $\alpha > 1/2 - 1/(2H)$. If time permits, we will discuss weak uniqueness for rough SDEs

$$dX_t = \sigma(X_t)dB_t^H,$$

where σ is a Hölder continuous (but not necessarily Lipschitz!) function.

Supercritical SDEs driven by fractional Brownian motion with divergence free drifts

Speaker: Zimo Hao (Bielefeld University)
Time: 2025 July 3rd 11:00-11:50
Location: TBA

Abstract: We study stochastic differential equations (SDEs) driven by fractional Brownian motion, where the drift coefficient is divergence-free and supercritical with respect to scaling. Under the assumption that the drift belongs to $L_t^1 L_{loc}^1$ and has linear growth, we establish the existence of weak solutions for Lebesgue almost everywhere initial data. Furthermore, when the Hurst parameter $H \in (0, 1/2]$ and the drift lies in $L_t^{1/(1-H)} L_{loc}^{1/(1-H)}$, we give weak uniqueness. We also obtain the stability of the solution's law with respect to the drift. These results, in particular, allow us to treat McKean–Vlasov SDEs. This work is part of an ongoing collaboration with Lucio Galeati.

TBA

Speaker: Guohuan Zhao (Academy of Mathematics and Systems Science)
Time: 2025 July 3rd 14:00-14:50
Location: TBA

Abstract: TBA

TBA

Speaker: Guopeng Li (Beijing Institute of Technology)
Time: 2025 July 3rd 15:00-15:50
Location: TBA

Abstract: TBA

Wright-Fisher stochastic heat equations with irregular drifts

Speaker: Zhenyao Sun (Beijing Institute of Technology)
Time: 2025 July 3rd 16:00-16:50
Location: TBA

Abstract: Consider $[0,1]$-valued random field solution $(u_t(x))_{t\geq 0, x\in\mathbb{R}}$ to the one-dimensional stochastic heat equation

$$\partial_t u_t = \frac{1}{2}\Delta u_t + b(u_t) + \sqrt{u_t(1-u_t)}\dot{W}$$

where $b(1) \leq 0 \leq b(0)$ and $\dot{W}$ is a space-time white noise. In this talk, we present the weak existence and uniqueness of the above equation for a class of drifts $b(u)$ that may be irregular at the points where the noise is degenerate, that is, at $u = 0$ or $u = 1$. This class of drifts includes non-Lipschitz drifts like $b(u) = u^q(1-u)$ for every $q \in (0,1)$, and some discontinuous drifts like $b(u) = \mathbf{1}_{(0,1]}(u) - u$. This demonstrates a regularization effect of the multiplicative space-time white noise without assuming the standard assumption that the noise coefficient is Lipschitz and non-degenerate.

The method we apply is a further development of a moment duality technique that uses branching-coalescing Brownian motions as the dual particle system. To handle an irregular drift in the above equation, particles in the dual system are allowed to have a number of offspring with infinite expectation, even an infinite number of offspring with positive probability. We show that, even though the branching mechanism with infinite number of offspring causes explosions in finite time, immediately after each explosion the total population comes down from infinity due to the coalescing mechanism. Our results on this dual particle system are of independent interest.

This is based on a joint work with Clayton Barnes and Leonid Mytnik.

Potential theory of Dirichlet forms with jump kernels blowing up at the boundary

Speaker: Panki Kim (Seoul National University.)
Time: 2025 July 4rd 10:00-10:50
Location: TBA

Abstract: In this talk, we discuss some potential theory of Dirichlet forms on the half-space defined by the jump kernel $J(x,y) = |x-y|^{-d-\alpha}B(x,y)$ and the killing potential $\kappa x_d^{-\alpha}$, where $\alpha \in (0,2)$ and $B(x,y)$ can blow up to infinity at the boundary. The jump kernel and the killing potential depend on several parameters. For all admissible values of the parameters involved, we prove that the boundary Harnack principle holds, and establish sharp two-sided estimates on the Green functions of these processes. This is a joint work with Renming Song and Zoran Vondracek.

TBA

Speaker: Renming Song (University of Illinois Urbana-Champaign)
Time: 2025 July 4rd 11:00-11:50
Location: TBA

Abstract: TBA

Approximate factorizations for non-symmetric jump processes

Speaker: Soobin Cho (University of Illinois Urbana-Champaign)
Time: 2025 July 4rd 14:00-14:50
Location: TBA

Abstract: In this talk, we first discuss approximate factorizations of heat kernels and Green functions for purely discontinuous Markov processes. Under natural conditions, we show that the approximate factorization of the heat kernel is equivalent to that of the Green function. In the second part, we will discuss applications of these factorizations to derive two-sided heat kernel estimates for three classes of processes: stable-like processes with critical killing in $C^{1,Dini}$ open sets; killed stable-like processes with low regularity coefficients; and non-symmetric stable processes in $C^{1,2-Dini}$ open sets. In particular, we obtain sharp, explicit two-sided estimates for the killed and censored stable processes in $C^{1,Dini}$ open sets. This is based on joint work with Professor Renming Song (UIUC).

TBA

Speaker: Haojie Hou (Beijing Institute of Technology)
Time: 2025 July 4rd 15:00-15:50
Location: TBA

Abstract: TBA