Stochastic Summer Seminar Series 2025 (Updated in July 1, 2025)

Date	Time	Speaker	Title
July 1	10:00-11:50	Oleg Butkovsky	Mini-course: Stochastic sewing with applications (Part 1)
July 1	14:00-14:50	Zhenyao Sun	Wright-Fisher stochastic heat equations with irregular drifts
July 1	15:00-15:50	Guopeng Li	Regularization by noise phenomena in nonlinear PDEs with modulated dispersion
July 2	10:00-11:50	Oleg Butkovsky	Mini-course: Stochastic sewing with applications (Part 2)
July 2	14:00-14:50	Yuzhao Wang	Construction and phase transitions of focusing Gibbs measures with harmonic confinement
July 2	15:00-15:50	Shukun Wu	On almost everywhere convergence of planar Bochner Riesz means
July 3	10:00-10:50	Oleg Butkovsky	Weak uniqueness for singular stochastic equations driven by fractional Brownian motion
July 3	11:00-11:50	Zimo Hao	Supercritical SDEs driven by fractional Brownian motion with divergence free drifts
July 3	14:00-14:50	Guohuan Zhao	Some results on McKean–Vlasov equations with subcritical and critical interaction kernels
July 3	15:00-15:50	Haojie Hou	Heat kernel estimates for nonlocal kinetic operators
July 4	10:00-10:50	Panki Kim	Potential theory of Dirichlet forms with jump kernels blowing up at the boundary
July 4	11:00-11:50	Renming Song	Heat kernel estimates for (fractional) Laplacians with supercritical killings
July 4	14:00-14:50	Soobin Cho	Approximate factorizations for non-symmetric jump processes
July 4	15:00-15:50	Johnny Yang	SPDEs on metric measure spaces

Location: Wencui Buiding E 207
Organizer: Zhenyao Sun

Mini-course: Stochastic sewing with applications

Speaker: Oleg Butkovsky (Weierstrass Institute)
Time: 2025 July 1st 10:00-11:50 & 2025 July 2nd 10:00-11:50
Location: Wencui Buiding E 207

Abstract: The sewing lemma of Gubinelli [Gub04] is a key tool in Lyons' theory of rough paths, as well as in Hairer's theory of regularity structures. Recently, a stochastic extension of this result (the stochastic sewing lemma by Lê) has been obtained [Lê20], which has already become a powerful tool for many applications. In this mini-course, we will discuss stochastic sewing ideas and show how they can help us study problems in the following research directions:

- *Regularization by noise*, [ABLM24, BM24]. The deterministic equation

$$dX_t = b(X_t)\,dt,$$

  might have no solution or infinitely many solutions if the drift b is not smooth enough. However, its stochastic counterpart

$$dX_t = b(X_t)\,dt + dW_t, \tag{$*$}$$

  where W is the standard Brownian motion, has a unique strong solution if the drift b is just a bounded measurable function, without any further assumptions on the regularity of b. Why does this phenomenon occur? What happens if the noise is a different random process (say, fractional Brownian motion)? Will regularization by noise take place for PDEs?
- *Numerical algorithms for SDEs and SPDEs*, [BDG21, BDG23]. If the drift b in the SDE ($*$) is smooth, it is well-known that the standard Euler scheme will converge to the solution of this equation with rate 1. Will the Euler scheme converge to the solution if the drift is non-smooth? What will be the convergence rate? Will it deteriorate as the drift becomes just a bounded measurable function?
- *Slow-fast systems of stochastic equations*, [HL20]. Consider a two-scale (fast/slow) stochastic evolution:

$$dX_t^\epsilon = f(X_t^\epsilon, Y_t^\epsilon)\,dt + w(\epsilon) g(X_t^\epsilon, Y_t^\epsilon)\,dB_t,$$

  where Y_t^ϵ is a stochastic process representing the "fast component" (with fluctuations of higher order in ϵ), f and g are smooth functions, w is a weight, and B represents the noise. If the noise B is a Brownian motion, $w(\epsilon) \equiv 1$, and the fast component is driven by an SDE, then Khasminskii [Kha68] showed that the slow component X^ϵ converges weakly as $\epsilon \to 0$ to the solution of the averaged SDE. We will discuss what obstacles appear if the slow process X^ϵ is driven by a fractional Brownian motion rather than by a Brownian motion and how they were resolved by Hairer and Li with the help of stochastic sewing [HL20].

References

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[BM24] Oleg Butkovsky and Leonid Mytnik. Weak uniqueness for singular stochastic equations. *arXiv preprint arXiv:2405.13780*, 2024.

[Gub04] M. Gubinelli. Controlling rough paths. *J. Funct. Anal.*, 216(1):86–140, 2004.

[HL20] Martin Hairer and Xue-Mei Li. Averaging dynamics driven by fractional Brownian motion. *Ann. Probab.*, 48(4):1826–1860, 2020.

[Kha68] R. Z. Khasminskii. On the principle of averaging the Itô's stochastic differential equations. *Kybernetika (Prague)*, 4:260–279, 1968.

[Lê20] Khoa Lê. A stochastic sewing lemma and applications. *Electron. J. Probab.*, 25:Paper No. 38, 55, 2020.

Wright-Fisher stochastic heat equations with irregular drifts

Speaker: Zhenyao Sun (Beijing Institute of Technology)
Time: 2025 July 1st 14:00-14:50
Location: Wencui Building E 207

Abstract: Consider $[0,1]$-valued random field solution $(u_t(x))_{t\geq 0, x\in\mathbb{R}}$ to the one-dimensional stochastic heat equation

$$\partial_t u_t = \frac{1}{2}\Delta u_t + b(u_t) + \sqrt{u_t(1-u_t)}\dot{W}$$

where $b(1) \leq 0 \leq b(0)$ and $\dot{W}$ is a space-time white noise. In this talk, we present the weak existence and uniqueness of the above equation for a class of drifts $b(u)$ that may be irregular at the points where the noise is degenerate, that is, at $u=0$ or $u=1$. This class of drifts includes non-Lipschitz drifts like $b(u) = u^q(1-u)$ for every $q \in (0,1)$, and some discontinuous drifts like $b(u) = \mathbf{1}_{(0,1]}(u) - u$. This demonstrates a regularization effect of the multiplicative space-time white noise without assuming the standard assumption that the noise coefficient is Lipschitz and non-degenerate.

The method we apply is a further development of a moment duality technique that uses branching-coalescing Brownian motions as the dual particle system. To handle an irregular drift in the above equation, particles in the dual system are allowed to have a number of offspring with infinite expectation, even an infinite number of offspring with positive probability. We show that, even though the branching mechanism with infinite number of offspring causes explosions in finite time, immediately after each explosion the total population comes down from infinity due to the coalescing mechanism. Our results on this dual particle system are of independent interest.

This is based on a joint work with Clayton Barnes and Leonid Mytnik.

Regularization by noise phenomena in nonlinear PDEs with modulated dispersion

Speaker: Guopeng Li (Beijing Institute of Technology)
Time: 2025 July 1st 15:00-15:50
Location: Wencui Building E 207

Abstract: In this talk, we consider the Korteweg–de Vries equation (KdV) and related models such as the Benjamin–Ono equation (BO) with modulated dispersion. In particular, the modulation may be given, for example, by a sample path of a fractional Brownian motion. We demonstrate regularization-by-noise phenomena in the following three ways.

(1) Well-posedness beyond the classical threshold: we establish the well-posedness of the modulated KdV in regimes where the (unmodulated) KdV is ill-posed. More interestingly, we show that the modulated KdV on the circle is locally well-posed in Sobolev spaces of arbitrarily low regularity, provided that the modulation is sufficiently irregular.

(2) Semilinearization of the quasilinear equation: while BO exhibits quasilinear behavior, we show that irregular modulations "semilinearize" the equation, providing local Lipschitz continuity of its solution map.

(3) Nonlinear smoothing effects: we establish nonlinear smoothing for these modulated equations, showing that the nonlinear part gains regularity, and this gain can be arbitrarily large for more irregular modulations.

If time permits, I will also discuss more recent results on the stochastic modulated KdV on the circle with multiplicative noise. In this setting, we observe a new type of regularization-by-noise phenomenon arising from the stochastic term.

This talk is based on joint works with Khalil Chouk (formerly Edinburgh), Massimiliano Gubinelli (Oxford), Tadahiro Oh & Jiawei Li (both Edinburgh), and Andreia Chapouto (Versailles).

Construction and phase transitions of focusing Gibbs measures with harmonic confinement

Speaker: Yuzhao Wang (University of Birmingham)
Time: 2025 July 2nd 14:00-14:50
Location: Wencui Building E 207

Abstract: This talk presents a complete resolution to the problem of constructing Gibbs measures for the focusing nonlinear Schr¨odinger equation (NLS) with harmonic potential on Euclidean spaces. By employing the Barashkov-Gubinelli variational framework - specifically through a stochastic optimization reformulation using the Bou´e-Dupuis variational formula - we establish a sharp dichotomy for measure normalizability in the following cases: (i) Full classification in one spatial dimension; (ii) Complete characterization under radial symmetry in higher dimensions. Our work provides the final piece to research programs of constructing focusing Gibbs measure with harmonic potential initiated by Burq-Thomann-Tzvetkov (2005) for 1D case, and Deng (2013) for 2D radial case.

On almost everywhere convergence of planar Bochner Riesz means

Speaker: Shukun Wu (Indiana University)
Time: 2025 July 2nd 15:00-15:50
Location: Wencui Building E 207

Abstract: The Bochner Riesz means were introduced in the 30s, aiming to study radial convergence of Fourier series. It was later realized that this object is closed related to other problems in Fourier analysis, for example, Stein's Fourier restriction conjecture. In this talk, I will discuss a recent result with Xiaochun Li that proves the planar Bochner Riesz means converge almost everywhere for any L^p function in the optimal range, for $5/3 < p < 2$. Our approach is based on weighted L^2 estimates.

Weak uniqueness for singular stochastic equations driven by fractional Brownian motion

Speaker: Oleg Butkovsky (Weierstrass Institute)
Time: 2025 July 3rd 10:00-10:50
Location: Wencui Building E 207

Abstract: Based on joint works with Leonid Mytnik (Technion - Israel Institute of Technology) and Konstantinos Dareiotis (University of Leeds). We consider the stochastic differential equation

$$dX_t = b(X_t)dt + dB_t^H,$$

where the drift b is a Schwartz distribution in the space $\mathcal{C}^\alpha$, $\alpha < 0$, and B^H is a fractional Brownian motion of Hurst index $H \in (0, 1/2]$. If $H = 1/2$, both weak and strong uniqueness theories for this SDE have been developed. However, the situation is much more complicated if $H < 1/2$, as the main tool, the Zvonkin transformation, becomes unavailable in this setting. The breakthroughs by Catellier and Gubinelli, and later by Le, established strong well-posedness of this SDE via sewing/stochastic sewing arguments. However, weak uniqueness for this SDE remained a challenge for quite some time, since a direct application of stochastic sewing alone does not seem very fruitful. I will explain how a combination of stochastic sewing with certain arguments from ergodic theory allows to show weak uniqueness in the whole regime where weak existence is known, that is, $\alpha > 1/2 - 1/(2H)$. If time permits, we will discuss weak uniqueness for rough SDEs

$$dX_t = \sigma(X_t)dB_t^H,$$

where σ is a Hölder continuous (but not necessarily Lipschitz!) function.

Supercritical SDEs driven by fractional Brownian motion with divergence free drifts

Speaker: Zimo Hao (Bielefeld University)
Time: 2025 July 3rd 11:00-11:50
Location: Wencui Building E 207

Abstract: We study stochastic differential equations (SDEs) driven by fractional Brownian motion, where the drift coefficient is divergence-free and supercritical with respect to scaling. Under the assumption that the drift belongs to $L_t^1 L_{loc}^1$ and has linear growth, we establish the existence of weak solutions for Lebesgue almost everywhere initial data. Furthermore, when the Hurst parameter $H \in (0, 1/2]$ and the drift lies in $L_t^{1/(1-H)} L_{loc}^{1/(1-H)}$, we give weak uniqueness. We also obtain the stability of the solution's law with respect to the drift. These results, in particular, allow us to treat McKean–Vlasov SDEs. This work is part of an ongoing collaboration with Lucio Galeati.

Some results on McKean–Vlasov equations with subcritical and critical interaction kernels

Speaker: Guohuan Zhao (Academy of Mathematics and Systems Science)
Time: 2025 July 3rd 14:00-14:50
Location: Wencui Building E 207

Abstract: In this talk, we will discuss well-posedness results for McKean–Vlasov equations with subcritical and critical interaction kernels. Additionally, we will present large deviation principles for the empirical measures of the corresponding particle systems.

Heat kernel estimates for nonlocal kinetic operators

Speaker: Haojie Hou (Beijing Institute of Technology)
Time: 2025 July 3rd 15:00-15:50
Location: Wencui Building E 207

Abstract: We employ probabilistic techniques to derive sharp, explicit two-sided estimates for the heat kernel of the nonlocal kinetic operator

$$\Delta_v^{\alpha/2} + v \cdot \nabla_x, \quad \alpha \in (0,2),\ (x,v) \in \mathbb{R}^d \times \mathbb{R}^d,$$

where $\Delta_v^{\alpha/2}$ represents the fractional Laplacian acting on the velocity variable v. Additionally, we establish logarithmic gradient estimates with respect to both the spatial variable x and the velocity variable v. In fact, the estimates are developed for more general non-symmetric stable-like operators, demonstrating explicit dependence on the lower and upper bounds of the kernel functions. These results, in particular, provide a solution to a fundamental problem in the study of *nonlocal* kinetic operators. This talk is based on a joint work with Xicheng Zhang.

Potential theory of Dirichlet forms with jump kernels blowing up at the boundary

Speaker: Panki Kim (Seoul National University.)
Time: 2025 July 4th 10:00-10:50
Location: Wencui Building E 207

Abstract: In this talk, we discuss some potential theory of Dirichlet forms on the half-space defined by the jump kernel $J(x, y) = |x - y|^{-d-\alpha}B(x, y)$ and the killing potential $\kappa x_d^{-\alpha}$, where $\alpha \in (0, 2)$ and $B(x, y)$ can blow up to infinity at the boundary. The jump kernel and the killing potential depend on several parameters. For all admissible values of the parameters involved, we prove that the boundary Harnack principle holds, and establish sharp two-sided estimates on the Green functions of these processes. This is a joint work with Renming Song and Zoran Vondracek.

Heat kernel estimates for (fractional) Laplacians with supercritical killings

Speaker: Renming Song (University of Illinois Urbana-Champaign)
Time: 2025 July 4th 11:00-11:50
Location: Wencui Building E 207

Abstract: In this talk, I will present some recent results on sharp two-sided estimates on the heat kernels of (fractional) Laplacians with supercritical killing potentials, that is, heat kernels of operators of the form

$$-(-\Delta)^{\alpha/2} - \kappa(x)$$

where $\alpha \in (0, 2]$ and κ belongs to a class of positive supercritical potentials including $\kappa(x) = c|x|^{-\beta}$ with $\beta > \alpha$. This talk is based on a joint paper with Soobin Cho, and a joint paper with Soobin Cho and Panki Kim.

Approximate factorizations for non-symmetric jump processes

Speaker: Soobin Cho (University of Illinois Urbana-Champaign)
Time: 2025 July 4th 14:00-14:50
Location: Wencui Building E 207

Abstract: In this talk, we first discuss approximate factorizations of heat kernels and Green functions for purely discontinuous Markov processes. Under natural conditions, we show that the approximate factorization of the heat kernel is equivalent to that of the Green function. In the second part, we will discuss applications of these factorizations to derive two-sided heat kernel estimates for three classes of processes: stable-like processes with critical killing in $C^{1,Dini}$ open sets; killed stable-like processes with low regularity coefficients; and non-symmetric stable processes in $C^{1,2-Dini}$ open sets. In particular, we obtain sharp, explicit two-sided estimates for the killed and censored stable processes in $C^{1,Dini}$ open sets. This is based on joint work with Professor Renming Song (UIUC).

SPDEs on metric measure spaces

Speaker: Johnny Yang (Indiana University)
Time: 2025 July 4th 15:00-15:50
Location: Wencui Building E 207

Abstract: This talk explores stochastic partial differential equations (SPDEs) on metric spaces with fractional dimensions, focusing on equations with random field solutions. A classical example of such spaces is the Sierpinski gasket. I will talk about some qualitative properties of solutions to a general class of SPDEs on those spaces focusing on various comparison principles. Lastly, I will discuss how some of the techniques from singular SPDEs on Euclidean settings transfer to general metric spaces settings with Phi4 model as the example.