Assignment 3 for Math 2370

The due date for this assignment is Thursday September 26.

1. If Y and Z are subspaces of a finite dimensional linear space, show that

$$(Y+Z)^{\perp} = Y^{\perp} \cap Z^{\perp} \text{ and } (Y \cap Z)^{\perp} = Y^{\perp} + Z^{\perp}.$$

2. Let X, Y be finite dimensional linear space and $T \in L(X,Y)$ be invertible. Show that T' is also invertible and

$$\left(T^{-1}\right)' = \left(T'\right)^{-1}.$$

3. Let X be an n-dimensional linear space and $T \in L(X,X)$. Show that there is a non-zero polynomial $p(t)$ of degree no more than n^2 such that $p(T) = 0$. (Use only the material we have learned so far)

4. Show that if U, V, W are finite dimensional vector spaces, and $T \in L(U,V)$, $S \in L(V,W)$, then

$$\dim(N_{ST}) \leq \dim(N_S) + \dim(N_T).$$

5. Let u, v be two linearly independent unit vectors in $\mathbb{R}^2$. Show that there exists a unique projection P such that $Pu = u$ and $Pv = 0$. Suppose $u = \left(\frac{3}{5}, \frac{4}{5}\right)$ and $v = \left(\frac{4}{5}, \frac{3}{5}\right)$, find the matrix representing P.