Assignment 1 for Math 2370

The due date for this assignment is Thursday September 5.

1. Let U, V, and W be subspaces of some finite-dimensional linear space X. Show that if $W \subset U$, then
$$U \cap (V + W) = U \cap V + W.$$

2. Denote by X the linear space of all polynomials $p(t)$ of degree less than n, and denote by Y the subset of X containing polynomials that are zero at distinct $t_1, t_2, \cdots t_m \in K$ where $m < n$.
(i) Show that Y is a subspace of X.
(ii) Determine $\dim Y$ and find a basis of Y.
(iii) Determine $\dim X/Y$ and find a basis of X/Y.
Justify your answers.

3. Let U, V, and W be subspaces of some finite-dimensional vector space X. Is it always true that
$$\begin{aligned}\dim(U + V + W) = {} & \dim(U) + \dim(V) + \dim(W) \\ & - \dim(U \cap V) - \dim(U \cap W) - \dim(V \cap W) \\ & + \dim(U \cap V \cap W)\end{aligned}$$
If true, prove it. If false, provide a counterexample.

4. Let $U_1, U_2, \cdots, U_k$ be subspaces of a finite-dimensional linear space X such that
$$\dim U_1 = \dim U_2 = \cdots = \dim U_k.$$
Then there is a subspace V of X for which
$$X = U_1 \oplus V = U_2 \oplus V = \cdots = U_k \oplus V.$$