

Assignment 2 for Math 2370

The due date for this assignment is Thursday September 19.

1. Suppose $\{x_1, x_2, \dots, x_n\}$ is a basis for the vector space X . Show there exists linear functions $\{e_1, e_2, \dots, e_n\}$ in the dual space X' satisfying

$$e_i(x_j) = \delta_{ij} = \begin{cases} 0 & \text{if } i \neq j, \\ 1 & \text{if } i = j. \end{cases}$$

Show that $\{e_1, e_2, \dots, e_n\}$ is a basis of X' , called the dual basis.

2. Let X be a finite dimensional linear space. Show that two nonzero linear functionals $T, S \in X'$ have the same null space if and only if there is a nonzero scalar λ such that $S = \lambda T$.
3. Let X, U be two linear spaces such that

$$\dim X = \dim U < \infty.$$

Show that a linear mapping $T \in L(X, U)$ is one-to-one iff it is onto.

4. Let X be a finite dimensional linear space and $T \in L(X) = L(X, X)$. Suppose

$$\dim(R_{T^2}) = \dim(R_T),$$

show that

$$R_T \cap N_T = \{0\}.$$

Here $T^2 = T \circ T$.