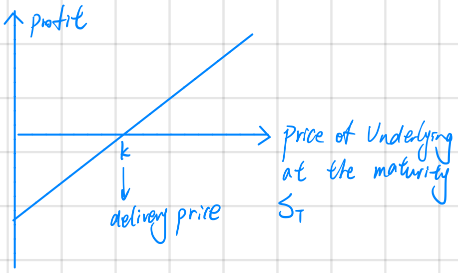
① pay off from a long position

当期价格 − 合约价格

$S_T - K$

S_T → spot price

K → delivery price



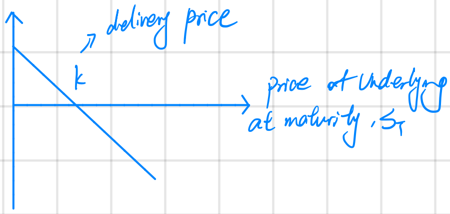
pay off from a short position

合约价格 − 当期价格

$K - S_T$

K → delivery price

S_T → spot price



② A资金被投资n年，以R利率，如利率在一年内以m次支付，则投资结束后会得到的金额为

$$A\left(1+\frac{R}{m}\right)^{mn}$$

如 $m \to$ 无穷大 (infinity)

$$Ae^{Rn}$$

③ R_c : 连续复合率 (continuously compounded rate)　m多无数次

R_m : 普通的分m次的利率 (same rate with compounding m times per year)

n : 年限

$$Ae^{R_c n} = A\left(1+\frac{R_m}{m}\right)^{mn} \Rightarrow R_c = m\ln\left(1+\frac{R_m}{m}\right)$$

$$\Rightarrow R_m = m\left(e^{\frac{R_c}{m}} - 1\right)$$

④

$$F_0 = S_0 e^{rT} + FV(\text{net storage cost})$$

F_0: Future or forward price today

S_0: spot price today

T: time until delivery date (以年算 eg: 6 months = 0.5 year) 此时 T=0.5

r: risk free interest rate for maturity T

5-6 asset 的 Forward

⑤ 远期价格 (forward price) P48

1) 当资产不提供 Income 时 $F_0 = S_0 e^{rT}$

2) 当资产提供 Income 时 $Income = I = 单位收益 \cdot e^{-r \cdot n_1} + 单位收益 \cdot e^{-r \cdot n_2}$ (以半年度的 coupon-payment 为例，因此要加一个，因为 $n_1 = 0.1$，$n_2 = 1$)

I: the PV of the income during life of a forward contract

$F_0 = (S_0 - I) e^{rT}$

3) 当资产提供已知收益率时 (a known Yield) $F_0 = S_0 e^{(r-q)T}$

q: average yield during the life of the contract (以连续复合表示)

投资于 stock Index 时也用该公式，此时 q: average divident yield

⑥ k = delivery price F_0 = forward price that would apply to the contract today

The value of a long forward contract $f = (F_0 - k) e^{-rT}$

The value of a short forward contract $f = (k - F_0) e^{-rT}$

foreign currency 的 forward 和 future

⑦ $F_0 = S_0 e^{(r-r_f)T}$ r_f: 外汇的 risk-free rate

8-9 Futures on Commodities P55

⑧ Storage cost - investment assets (投资型资产)

the forward price = $F_0 = (S_0 + U) e^{rT}$ $U = Storage\ cost \cdot e^{-r \cdot n}$

U: the present value of all storage cost

如果存储成本与商品价格成正比 (storage costs are proportional to the commodity price)

则 $F_0 = S_0 e^{(r+u)T}$

u: 每年存储成本占商品价格的比例

⑨ $F_0 \leq S_0 e^{(r+u)T}$ u: storage cost per unit time as a percent of the asset value

$F_0 \leq (S_0 + U) e^{rT}$ U: present value of the storage costs

⑩ The cost of carry

c : 对无分红的股票则为 r

对股指则为 $r-q$

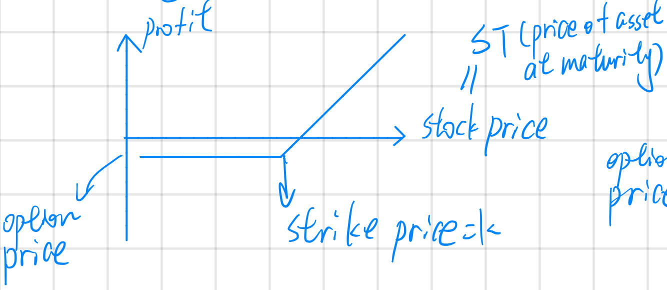
对货币则为 $r-r_f$

对于投资型资产 $F_0 = S_0 e^{cT}$

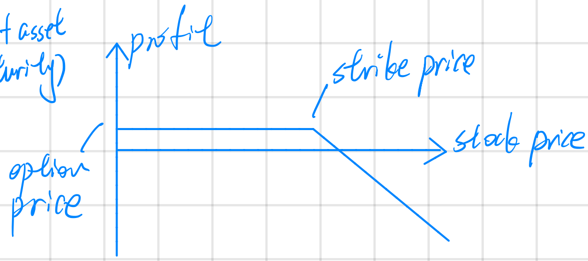
对于消费型资产 $F_0 \le S_0 e^{cT}$

消费型资产的便利收益 y $\Rightarrow F_0 = S_0 e^{(c-y)T}$

⑪ long call

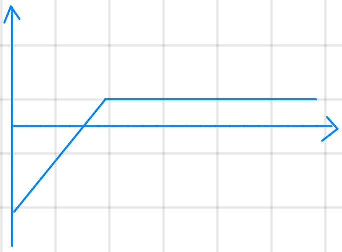


short call



long put

short put



⑫ call，无红利 欧式

$c \geqslant \max(\underline{S_0 - Ke^{-rT}}, 0)$

有红利

$c \geqslant \max(\underline{S_0 - D - Ke^{-rT}}, 0)$

c: price of call option

put，无红利，欧式

$p \geqslant \max(\underline{Ke^{-rT} - S_0}, 0)$

有红利

$p \geqslant \max(\underline{Ke^{-rT} + D - S_0}, 0)$

p: price of put option

D与Ke^{-rT}同号

⑬ put-call parity： $c + Ke^{-rT} = p + S_0$

Long stock + long put = long call + cash $L_2 P_{12}$

加上红利：European option：$D>0$ $\Rightarrow$ $C + D + Ke^{-rT} = p + S_0$

$D =$ 单位红利$\cdot e^{-rT_1}$ + 单位红利$\cdot e^{-rT_2}$ + ···

例题见 L_2 ③

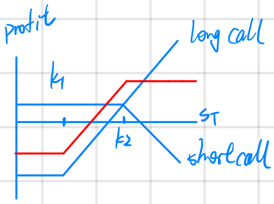
long
short

$K_2 > K_1$

(14) Bull call：买一 call 在 K_1
卖一 call 在 K_2

long 比 short 本金大

牛市买 call 贵

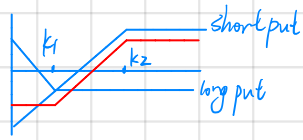


因为是call所以-K	long call	short call	total
$S_T \le K_1$	0	0	0
$K_1 < S_T < K_2$	$S_T - K_1$	0	$S_T - K_1$
$S_T > K_2$	$S_T - K_1$	$-(S_T - K_2)$	$K_2 - K_1$

因为是short 所以异号

Bull put：买一 put 在 K_1
卖一 put 在 K_2

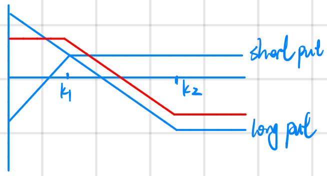
short 比 long 本金大



	long put	short put	total
$S_T \le K_1$	$K_1 - S_T$	$-(K_2 - S_T)$	$K_1 - K_2$
$K_1 < S_T < K_2$	0	$-(K_2 - S_T)$	$S_T - K_2$
$S_T \geq K_2$	0	0	0

Bear put：买一 put 在 K_2
卖一 put 在 K_1

熊市买 put 贵

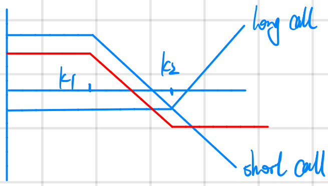


因为用put所以 $-S_T$

因为是short 所以异号

	long put	short put	total
$S_T \le K_1$	$K_2 - S_T$	$-(K_1 - S_T)$	$K_2 - K_1$
$K_1 < S_T < K_2$	$K_2 - S_T$	0	$K_2 - S_T$
$S_T > K_2$	0	0	0

Bear call：买一 call 在 K_2
卖一 call 在 K_1



	long call	short call	Total
$S_T \le K_1$	0	0	0
$K_1 < S_T < K_2$	0	$-(S_T - K_1)$	$K_1 - S_T$
$S_T \geq K_2$	$S_T - K_2$	$-(S_T - K_1)$	$K_1 - K_2$

(15) Box spread = a bull call + a bear put　　payoff 永远为 $K_2 - K_1$

16 Butterfly spread:

是spread，同call or 同put

2份 short合约

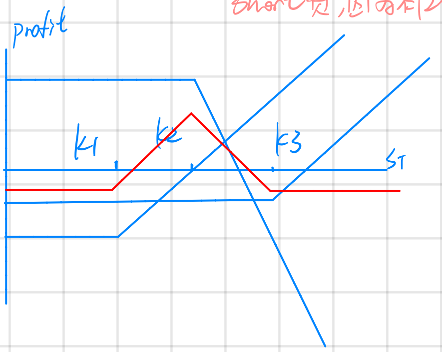
1) long Butterfly using call/put : 1 long call/put at k_1
2 short calls/puts at k_2
1 long call/put at k_3

call 和 put 一样操作
计算payoff时注意S_T和k的位置
注意! $k_3>k_2>k_1$
并且 $k_2=\frac{k_1+k_3}{2}$

short Butterfly using call/put : 1 short call/put at k_1
2 long calls/puts at k_2
1 short calls/puts at k_3

Long B using Calls

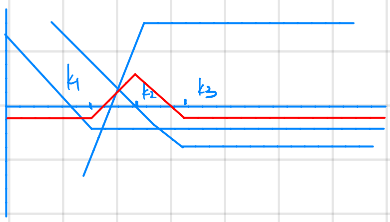
第一个long 过k_2
short贵，因为有2个short



因为用call 所以$-k$ · 因为是short 所以负号 · 因为有两份 所以要乘2 · 因为$k_2=\frac{k_1+k_3}{2}$

	1 long call	2 short calls	1 long call	total
$S_T \leq k_1$	0	0	0	0
$k_1<S_T<k_2$	S_T-k_1	0	0	S_T-k_1
$k_2<S_T<k_3$	S_T-k_1	$-2(S_T-k_2)$	0	k_3-S_T
$S_T \geq k_3$	S_T-k_1	$-2(S_T-k_2)$	S_T-k_3	0

long B using Puts



因为用put 所以$-S_T$

	1 long Put	2 short Puts	1 long put	total
$S_T \leq k_1$	k_1-S_T	$-2(k_2-S_T)$	k_3-S_T	0
$k_1<S_T<k_2$	0	$-2(k_2-S_T)$	k_3-S_T	S_T-k_1
$k_2<S_T<k_3$	0	0	k_3-S_T	k_3-S_T
$S_T \leq k_3$	0	0	0	0

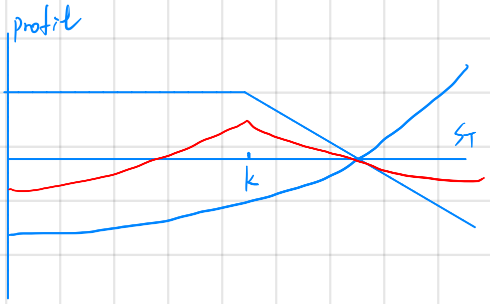
17 Calendar Spread Using Calls

同call 同put

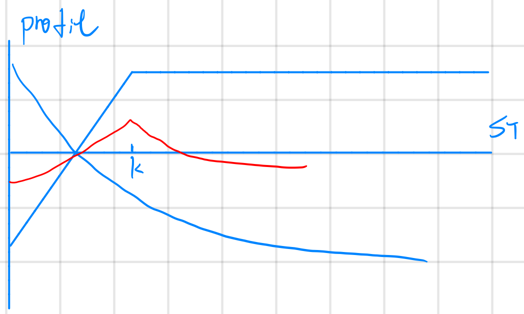
short a call/put at k and T_1
long a call/put at k and T_2 $\quad T_2>T_1$

Short a call/put with strike price k and maturity T_1
long a call/put with strike price k and maturity T_2, $T_2>T_1$

用call:

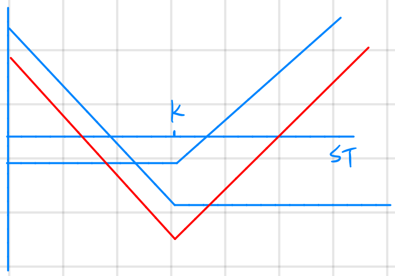


用 Put



(18) A straddle Combination   long call and put at k 买 call 便宜

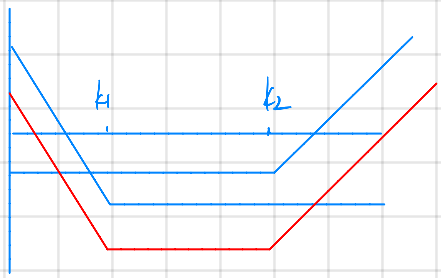
long a call and put with the same strike price and expiration date



	call	put	Total
$S_T \le k$	0	$k - S_T$	$k - S_T$
$S_T > k$	$S_T - k$	0	$S_T - k$

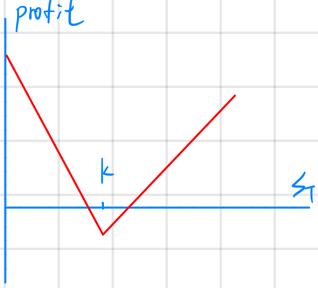
(19) A strangle Combination 买 call 便宜   long a call at k_2 long a put at k_1

long a call and a put with the same expiration date and different strike prices $(k_2 > k_1)$ $\left(\begin{array}{l}\text{long a call at } k_2 \\ \text{long a put at } k_1\end{array}\right)$

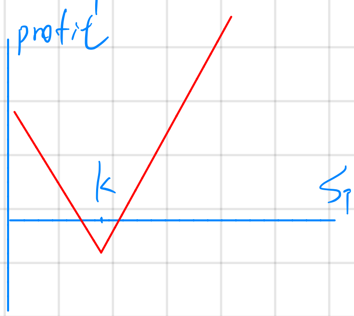


	long call at k_2	long put at k_1	total
$S_T \le k_1$	0	$k_1 - S_T$	$k_1 - S_T$
$k_1 < S_T < k_2$	0	0	0
$S_T \geq k_2$	$S_T - k_2$	0	$S_T - k_2$

(20) strip



Strap

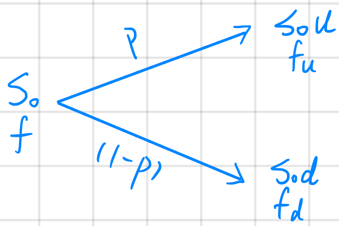


long 1 call with k and maturity T
long 2 puts with k and maturity T

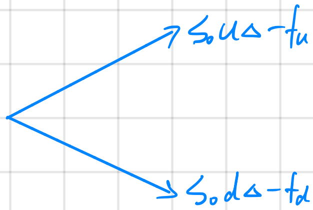
long 2 calls with k and maturity T
long 1 put with k and maturity T

L3 P13

㉑



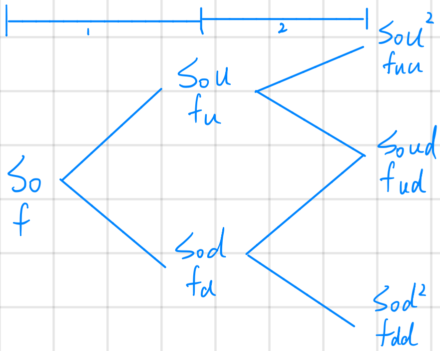
如果 long Δ shares
short in 1 option ⇒



$E(S_T) = S_0e^{rT} = pS_0u + (1-p)S_0d$ 这俩是可以替代的

$p = \frac{e^{rT}-d}{u-d}$

The value of the option $f = e^{-rT}[pf_u + (1-p)f_d]$



此时有 $f = e^{-r\Delta t}[pf_u + (1-p)f_d]$ 1)

$p = \frac{e^{r\Delta t}-d}{u-d}$ (every time step $=\Delta t$ years) - 阶段的日期, 如图中 1, 2

$f_u = e^{-r\Delta t}[pf_{uu} + (1-p)f_{ud}]$ 2)

$f_d = e^{-r\Delta t}[pf_{ud} + (1-p)f_{dd}]$ 3)

将 2) 3) 代入 1) 有 $f = e^{-2r\Delta t}[p^2f_{uu} + 2p(1-p)f_{ud} + (1-p)^2f_{dd}]$

㉒ Itô's Lemma $dG = a\,dt + b\,dz$ 几何布朗运动 geometric Brownian motion process

$dG = \left(\frac{\partial G}{\partial x}a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}b^2\right)dt + \frac{\partial G}{\partial x}b\,dz$

$dS = \mu S\,dt + \mu S\,dz$

㉓ Black-Scholes-Merton Model 方差(volatility)

$\ln S_T \sim \phi[\ln S_0 + (\mu - \frac{\sigma^2}{2})T, \sigma^2 T]$ 时间(以year算) ⇒ $\ln S_T \sim \phi[a, b]$

Initial price expected rate of return

$a -$ 题中 ΔSD of its mean × 标准差(σ) $< \ln S_T <$ $a +$ 题中 ΔSD of its mean × 标准差(σ) 符号不一样

⇒ $e^{a-\cdots(\sigma)} < S_T < e^{a+\cdots(\sigma)}$

方差 × π × $\sqrt{\Delta T}$ = SD of price change in Oneweek L4 P12

㉔ Black-Scholes-Merton partial differential equation

$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2S^2\frac{\partial^2 f}{\partial S^2} = rf$

Payoff is $S_T - K$ (long position)

Expected payoff in a risk-neutral world is $S_0e^{rT} - K$

Present value of expected payoff is $e^{-rT}[S_0e^{rT} - K) \Rightarrow S_0 - Ke^{-rT}$

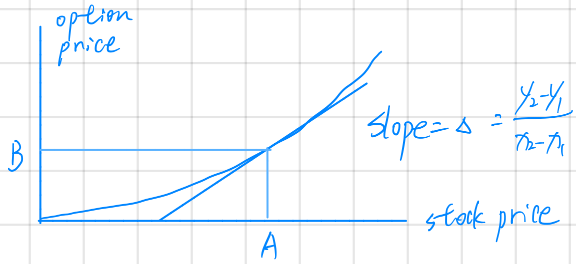
25. European Options on Assets Providing known Yield

We get the same probability distribution for the asset price at time T in each of the following cases:

1. The assets starts at price S_0 and provide a yield $= q$
2. The aset starts at price $S_0 e^{-qT}$ and provide no income

Delta

26. Δ of call is positive Δ of put is negetive
(call gain value when the price increase)



$0 \leq \Delta_c \leq 1$ $-1 \leq \Delta_p \leq 0$

都是 close 1 or -1 当 in-the-money, close 0 当 out-of-the-money

27. 总的Δ = 个体Δ相加

$$\Delta = \sum_{i=1}^{n} w_i \Delta_i$$

Δ_i → 个体Δ

w_i → 数量, 如n个call option

注意: short 和 put 的Δ是负的, 如果 short put, 则负负为正

Theta (反应变化率和时间的关系)

28. call 和 put 的Θ总是负的

$$\Theta_{call} = -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} - rke^{-rT}N(d_2)$$

$$N'(d_1) = \frac{1}{\sqrt{2\pi}} e^{-d_1^2/2}$$

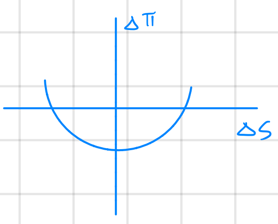
$$\Theta_{put} = -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} + rke^{-rT}N(-d_2)$$

Gamma (反映Δ和基础资产价格的关系) 即在Δ的基础上对价格(S)再求导

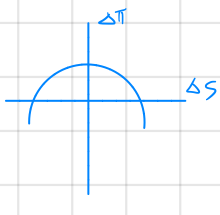
29. $$\gamma = \frac{\partial \Delta}{\partial S} = \frac{\partial}{\partial S}\left(\frac{\partial f}{\partial S}\right) \Rightarrow \gamma = \frac{\partial^2 f}{\partial S^2}$$

如果gamma绝对值很大, 则Δ很sensitive, 让delta neutral portfolio 不变会很risky

positive gamma



negetive gamma



For a delta neutral portfolio

$$\Delta \Upsilon \approx \Theta \Delta t + \frac{1}{2} \gamma \Delta S^2$$

(30) when Gamma is positive, theta is negative ($\gamma +\ \theta -$) 时
portfolio 减少价值当 S 没有变化，增加价值当 S 有大的正或负变化

当 Gamma is negative, theta is positive ($\gamma -\ \theta +$) 时
portfolio 增加价值，当 S 无变化，减少价值当 S 有大的正或负变化

(31) gamma neutral : $\frac{-\gamma}{\gamma_T} = x$, 在 call op'm

(32) 计算 gamma

$$\gamma = \frac{N'(d_1)}{S_0 \sigma \sqrt{T}} \qquad N'(d_1) = \frac{1}{\sqrt{2\pi}} e^{\frac{-d_1^2}{2}}$$

(33) Delta Δ Gamma γ Theta θ 之间的关系
portfolio 的 value 一定满足 BSM 公式

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf \qquad \Pi \text{ — value of portfolio}$$

$$\frac{\partial \Pi}{\partial t} + rS\frac{\partial \Pi}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 \Pi}{\partial S^2} = r\Pi$$

However, $\theta = \frac{\partial \Pi}{\partial t}$, $\Delta = \frac{\partial \Pi}{\partial S}$, $\gamma = \frac{\partial^2 \Pi}{\partial S^2} \Rightarrow \theta + rS\Delta + \frac{1}{2}\sigma^2 S^2 \gamma = r\Pi$

Vega 表示 Π (portfolio 的价值) 和 volatility of the underlying asset 的变化率

(34) $V = \frac{\partial \Pi}{\partial \sigma}$
如果 V 的绝对值很高 则 组合的价值对 波动的小变化敏感

对于 European call/put with non-dividend-paying stock, $V = S_0 \sqrt{T} N'(d_1)$

(35) Vega neutral : $\frac{-V}{V_T}$

Rho value 和 interest rate 的关系

(36) $Rho = \frac{\partial \Pi}{\partial r}$

对于 European call/put with non-dividend-paying stock, $Rho(call) = kTe^{-rT}N(d_2)$
$Rho(put) = -kTe^{-rT}N(-d_2)$