Portfolio Optimization Strategies

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Overview

Project Overview

Our goal is to explore various portfolio optimization strategies:

- Mean-Variance Optimization
- Sparse Portfolio Construction
- Bonds
- Factor Models

Mean-Variance Optimization[1]

$$\min_{x} \quad x^T \Sigma x + \tau ||x||_1$$

$$s.t. \quad \hat{r}^T x \geq \mu,$$

$$x \in \mathcal{X}$$

- r is the return vector, which is a random variable
- $\hat{r} = E(r)$
- x is the weights
- $\Sigma = E(r - \hat{r})(r - \hat{r})^T$
- μ is a pre-determined minimum return
- $\mathcal{X}$ is a convex set of all feasible x
- τ is a parameter to promote sparsity[2]

Analytical Approach

- Monthly returns for 556 stocks listed on NYSE and NASDAQ.
- Train model on five years, test on next year, move one year forward, repeat, etc...
- Evaluation period: January 1991 - December 2015.
- Compare out of sample performance to equal-weight portfolio.
- Long only portfolios.
- Python package `CVXPY`[4] for optimization.

Mean-Variance Results

Period	Optimized			Equal Weight		
	m	σ	S	m	σ	S
01/91-12/15	.012	.027	.43	.014	.043	.324
01/91-12/95	.012	.019	.645	.020	.028	.711
01/96-12/00	.014	.031	.444	.015	.039	.388
01/01-12/05	.018	.029	.627	.015	.041	.366
01/06-12/10	.004	.034	.104	.010	.062	.165
01/11-12/15	.013	.026	.461	.010	.038	.257

Table: Mean-Variance Returns

m - average monthly returns
σ - std deviation
S - Sharpe Ratio, Ratio between returns and risk

Approach for Bonds and Mean Variance

- Increasing the number of the assets in the optimization does not give significant improvements, if any.
- Trying to maximize returns is far too unreliable to be used in practice.
- When using Bonds, we start with 20% invested in Bonds.
- We'll use a Bond Ladder, equally divided amongst Bonds with different Maturities, and repurchase new ones with 10 year Maturities when they mature.

Results for Mean-Variance with Bonds

Period	Bond Ladder			No Bonds			Equal Weight		
	m	σ	S	m	σ	S	m	σ	S
01/91-12/15	.011	.025	.443	.012	.027	.43	.014	.043	.324
01/91-12/95	.011	.016	.713	.012	.019	.645	.020	.028	.711
01/96-12/00	.013	.027	.465	.014	.031	.444	.015	.039	.388
01/01-12/05	.017	.027	.644	.018	.029	.627	.015	.041	.366
01/06-12/10	.003	.032	.109	.004	.034	.104	.010	.062	.165
01/11-12/15	.012	.025	.465	.013	.026	.461	.010	.038	.257

Table: Mean-Variance Returns with Bonds

m - average monthly returns

σ - std deviation

S - Sharpe Ratio, Ratio between returns and risk

Problems with Markowitz-type methods

- Main problem is lack of information (finite sample).
- Using sample mean to estimate mean return vector is notoriously bad, people actually dropped the sample mean constraint[6].
- The covariance matrix of 500 equities has 125,000 unique elements, whereas the historic return data of 5 years/60 months have only 30,000 elements.
- Efforts to reduce the estimation error by methods like shrinkage method, doesn't improve the performance[8].
- The out-of-sample performance of a certain robust method doesn't improve[9].

Analytical Approach for Fama and French Model

Step 1: Fit a time series regression for each stock:

$$r_i \sim \alpha_i + \beta_i(r_m - r_f) + \beta_{SMB,i} SMB + \beta_{HML,i} HML$$

- SMB: Returns of Small market capitalization companies Minus Big.
- HML: Returns of High value companies Minus Growth.
- $r_m - r_f$: Excess returns.

Data was downloaded from the Fama and French website.

Analytical Approach for Fama and French Model

Step 2: Minimize the variance across weighted combinations of the factors

$$\min_{x} \quad f^T \tilde{\Sigma} f + x^T D x$$

$$s.t. \quad \hat{r}^T x \geq \mu,$$

$$x \in \mathcal{X}$$

- $\tilde{\Sigma} \in R^{4x4}$ is the factor covariance matrix.
- $f = F^T x \in R^4$ are the factor exposures.
- $F \in R^{nx4}$ is factor loading matrix with the previously estimated β values.
- D is idiosyncratic risk which stores the SSE in diagonal matrix.

Results for Fama and French 3 Factor Model

Period	Optimal			Equal Weight		
	m	σ	S	m	σ	S
01/91-12/15	0.012	0.037	0.32	0.014	0.043	0.324
01/91-12/95	0.015	0.024	0.643	0.020	0.028	0.711
01/96-12/00	0.014	0.038	0.383	0.015	0.039	0.388
01/01-12/05	0.012	0.033	0.370	0.015	0.041	0.366
01/06-12/10	0.008	0.053	0.148	0.010	0.062	0.165
01/11-12/15	0.010	0.033	0.311	0.010	0.038	0.257

Table: Mean-Variance Returns with Factors

Analytical Approach for Cross-sectional 3-Factor Models[5]

- Models the weights of a portfolio directly using 3 cross-sectional factors: the market equity (me), the book-to-market ratio (btm), and the compounded monthly return (r_{comp})

$$x_{i,T+1} = y_{i,T} + \frac{1}{N_T}(\theta_1^{T+1} me_{i,T} + \theta_2^{T+1} btm_{i,T} + \theta_3^{T+1} r_{comp,i,T}) \quad (1)$$

- $y_{i,T}$ is the weight of the *ith* equity of a value equally distributed portfolio
- Calculate 3 numbers for each test year! (θ^{T+1} are the same for training and test periods)

Analytical Approach for Cross-sectional 3-Factor Models[5]

How to solve for $\theta_1^{T+1}, \theta_2^{T+1}, \theta_3^{T+1}$?

Maximizes the CRRA utility using 5 years' training data:

$$\max_{\theta} \quad \sum_{t=0}^{T-1} u(r_{p,t+1}) = \sum_{t=0}^{T-1} u(\sum_{i=1}^{N_t} x_{i,t+1} r_{i,t+1}) =$$

$$\sum_{t=0}^{T-1} u(\sum_{i=1}^{N_t} (y_{i,t} + \frac{1}{N_t}(\theta_1^{T+1} me_{i,t} + \theta_2^{T+1} btm_{i,t} + \theta_3^{T+1} r_{comp,i,t})) r_{i,t+1})) \tag{2}$$

Results for Cross-sectional 3-Factor Models

Period	Optimal			Equal Weight		
	m	σ	S	m	σ	S
01/91-12/15	.017	.047	.369	.014	.043	.324
01/91-12/95	.029	.040	.721	.020	.028	.711
01/96-12/00	.017	.045	.372	.015	.039	.388
01/01-12/05	.019	.045	.418	.015	.041	.366
01/06-12/10	.012	.061	.193	.010	.062	.165
01/11-12/15	.010	.036	.275	.010	.038	.257

- Superiority decayed vs. time
- People became aware of certain "anomalies", and then they were not any more "anomalies" once market reached equilibrium
- People are looking for new "anomalies" crazily

Conclusions and Next Steps

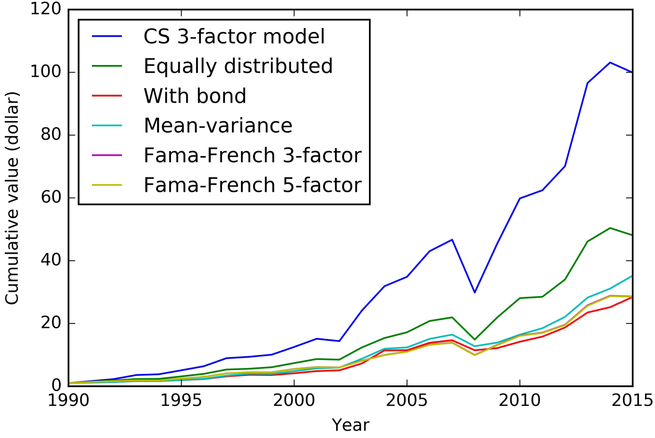

Figure: Cumulative value of different portfolios from 1991 - 2015

Conclusions and Next Steps

- Mean-variance is good at controlling risk, but it is difficult to achieve superior returns out-of-sample.
- While Fama and French factors explain results in-sample, it is still difficult to achieve superior returns out-of-sample.
- Cross-sectional factor models appear to provide good results in the short term, however their effectiveness decreases over time.

References

1. G. Calafiore and L. El Ghaoui, Optimization Models. Control systems and optimization series, Cambridge University Press, October 2014.
2. J. Brodie, I. Daubechies, C. De Mol, D. Giannone, and I. Loris, "Sparse and stable Markowitz portfolios," Proceedings of the National Academy of Science, vol. 106, pp. 12267-12272, July 2009.
3. S. D. S. Boyd and J. Park, "Convex optimization short course."
4. S. Diamond and S. Boyd, "CVXPY: A Python-embedded modeling language for convex optimization," Journal of Machine Learning Research, vol. 17, no. 83, pp. 1-5, 2016.
5. Michael W. Brandt, Pedro Santa-Clara, and Rossen Valkanov. Parametric portfolio policies: Exploiting characteristics in the cross-section of equity returns.The Review of Financial Studies, 22(9):3411, 2009.
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References

6. Tze Leung Lai, Haipeng Xing, and Zehao Chen. Meanvariance portfolio optimization when means and covariances are unknown.The Annals of Applied Statistics, 5(2A):798823, 2011.
7. Victor DeMiguel, Lorenzo Garlappi, and Raman Uppal. Optimal versus naive diversification: How inefficient is the 1/n portfolio strategy?Reviewof Financial Studies, 22(5):19151953, 2009.
8. Michael W. Brandt. Portfolio choice problems. In Handbook of Financial Econometrics, forthcoming. North-Holland, 2004.
9. Andre Santos. The out-of-sample performance of robust portfolio optimization.Brazilian Review of Finance, 8(2):141166, 2010.