7_均值-方差投资组合

可行集：资产组合的机会集合，即资产可构造出的所有组合的期望收益和方差。
有效组合：给定风险水平下的具有最高收益的组合或者给定收益水平下具有最小风险的组合。
每一个组合代表一个点。
有效集：它是有效组合的集合（点的连线）

两种风险资产构成组合的风险和收益

若已知两种资产的期望收益、方差和它们之间的相关系数，则可知两种资产构成的组合之期望收益和方差为

$$\overline{r}_p = w_1\overline{r}_1 + w_2\overline{r}_2$$

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12} = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{12}$$

由于 $w_1 + w_2 = 1$,

$$\overline{r}_p(w_1) = w_1\overline{r}_1 + (1-w_1)\overline{r}_2$$

$$\sigma_p(w_1) = \sqrt{w_1^2\sigma_1^2 + (1-w_1)^2\sigma_2^2 + 2w_1(1-w_1)\sigma_1\sigma_2\rho_{12}}$$

由此就构成了资产在给定条件下的可行集。

两种完全正相关资产的可行集

两种完全正相关资产构成的可行集是一条直线

完全正相关，即 $\rho_{12} = 1$，则有

$$\sigma_p(w_1) = \sqrt{w_1^2\sigma_1^2 + (1-w_1)^2\sigma_2^2 + 2w_1(1-w_1)\sigma_1\sigma_2}$$

即

$$\sigma_p = \sqrt{[w_1\sigma_1 + (1-w_1)\sigma_2]^2} = w_1\sigma_1 + (1-w_1)\sigma_2$$

所以

$$w_1 = \frac{\sigma_p - \sigma_2}{\sigma_1 - \sigma_2}$$

代入 $\overline{r}_p(w_1) = w_1\overline{r}_1 + (1-w_1)\overline{r}_2$ 得

$$\begin{aligned}\overline{r}_p(\sigma_p) \quad &= \frac{\sigma_p-\sigma_2}{\sigma_1-\sigma_2}\overline{r}_1 + (1 - \frac{\sigma_p-\sigma_2}{\sigma_1-\sigma_2})\overline{r}_2 \\ &= \overline{r}_2 - \frac{\overline{r}_1-\overline{r}_2}{\sigma_1-\sigma_2}\sigma_2 + \frac{\overline{r}_1-\overline{r}_2}{\sigma_1-\sigma_2}\sigma_p\end{aligned}$$

当权重 w_1 从 1 减少到 0 时，即为一条直线

两种完全负相关资产的可行集

完全负相关的两种资产构成的可行集是两条直线，其截距相同，斜率异号

完全正相关，即$\rho_{12} = -1$，则有

$$\sigma_p(w_1) = \sqrt{w_1^2\sigma_1^2 + (1-w_1)^2\sigma_2^2 - 2w_1(1-w_1)\sigma_1\sigma_2}$$

即

$$\sigma_p = \sqrt{[w_1\sigma_1 + (1-w_1)\sigma_2]^2} = w_1\sigma_1 + (1-w_1)\sigma_2$$

$w_1 = \frac{\sigma_2}{\sigma_1+\sigma_2}, \sigma_p = 0$

$w_1 \geq \frac{\sigma_2}{\sigma_1+\sigma_2}, \sigma_p(w_1) = w_1\sigma_1 - (1-w_1)\sigma_2$

$w_1 \leq \frac{\sigma_2}{\sigma_1+\sigma_2}, \sigma_p(w_1) = (1-w_1)\sigma_2 - w_1\sigma_1$

当$w_1 \geq \frac{\sigma_2}{\sigma_1+\sigma_2}, \sigma_p(w_1) = w_1\sigma_1 - (1-w_1)\sigma_2$，此时

$$w_1 = \frac{\sigma_p + \sigma_2}{\sigma_1 + \sigma_2}$$

$$\begin{aligned}
\overline{r}_p(\sigma_p) &= \frac{\sigma_p + \sigma_2}{\sigma_1 + \sigma_2}\overline{r}_1 + (1 - \frac{\sigma_p + \sigma_2}{\sigma_1 + \sigma_2})\overline{r}_2 \\
&= \frac{\overline{r}_1 - \overline{r}_2}{\sigma_1 + \sigma_2}\sigma_p + \frac{\overline{r}_1 - \overline{r}_2}{\sigma_1 + \sigma_2}\sigma_2 + \overline{r}_2
\end{aligned}$$

当$w_1 \leq \frac{\sigma_2}{\sigma_1+\sigma_2}, \sigma_p(w_1) = (1-w_1)\sigma_2 - w_1\sigma_1$，此时

$$w_1 = \frac{\sigma_p - \sigma_2}{\sigma_1 + \sigma_2}$$

$$\begin{aligned}
\overline{r}_p(\sigma_p) &= \frac{\sigma_p - \sigma_2}{\sigma_1 + \sigma_2}\overline{r}_1 + (1 - \frac{\sigma_p - \sigma_2}{\sigma_1 + \sigma_2})\overline{r}_2 \\
&= -\frac{\overline{r}_1 - \overline{r}_2}{\sigma_1 + \sigma_2}\sigma_p + \frac{\overline{r}_1 - \overline{r}_2}{\sigma_1 + \sigma_2}\sigma_2 + \overline{r}_2
\end{aligned}$$