CS 665 – Final Project Report

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1 Introduction

The purpose of this project is to evaluate the Stochastic Multiresolution Persistent Homology Kernel (SMURPH) [1] This kernel tries to capture the persistence homology information of point cloud data.

In order to evaluation SMURPH kernel, I used it to calculate kernel PCA. If the kernel is capables of what the paper claims, we should see from the 2D kernel PCA that point clouds with similar persistence homology should form a cluster.

I conducted experiments on three different datasets. The first dataset is the Kitchen Utensil Dataset, which is the dataset used in the original paper. The main reasion to use this dataset is to validate my implementation. If my implementation is correct, then the kernel PCA result should be at least similar to the result presented in the paper. The other two datasets are synthetic datasets with different features. One has different number of holes and the other has different scale of holes. Using synthetic datasets makes it easier to analyze and explain the results.

For each dataset, I also compared the kernel with two other kernels. The first is a simple linear kernel. This kernel serves as a base line. Since a linear kernel shouldn't make any sense considering persistence homology of a point cloud, we should expect the kernel PCA result don't form any clusters. When comparing with it, we should be able to see if a kernel is capturing the persistence homology features. The second kernel is proposed by my self, trying to come up with a simple but meaning kernel. Basically this kernel calculate a histogram of distances (HOD) between each pair of points. Then this histogram is used as a feature vector to calculate the inner product.

Detailed descriptions about these kernel will be presented in the following section.

2 Tested Kernels

2.1 SMURPH Kernel

The detailed algorithm for calculating SMURPH kernel can be found in Algorithm 1 in [1]. Here I just summerize the main idea of it. Given a point cloud, SMURPH kernel generate multiple samples at different scale: $[s_0, s_1, s_2, ..., s_n]$. Then for each s_i, SMURPH build a Vietoris-Rips filtration on it and calculate the persistence diagram. Next, the persistence diagram is converted to persistence landscape (PL) function, which becomes the representation r_i for the sample s_i. At last, each point cloud is represented by an array of PL functions: $[r_0, r_1, r_2..., r_n]$. The inner product of two different point clouds becomes the inner product of two array of PL functions, which could be calculated by taking integrals.

2.2 Linear Kernel

A simple linear kernel is used as baseline for comparison. The linear kernel generate same-sized samples from given point clouds. Then the inner product is defined as the sum of inner products of points from each sample.

2.3 Histogram of Distances Kernel

In order to have a meaningful yet still simple kernel for comparison, I proposed a new kernel: Histogram of Distances (HOD). The kernel generate a histogram of distances between each pair of points in a point cloud. Then a histogram of the distances is calculated. I use the normalized histogram as the vector representation of the point cloud. So the inner product of two point clouds becomes the inner product of two vectors. The intuition of this kernel is that point clouds don't have any holes tend to have a smooth and dense histogram of distances, while point clouds have many holes don't.

3 Datasets

3.1 Kitchen Utensil Dataset

This dataset [2] consists of 41 point clouds, generated by 3D scanning of kitchen utensils. Figure 1 shows two samples of this dataset.

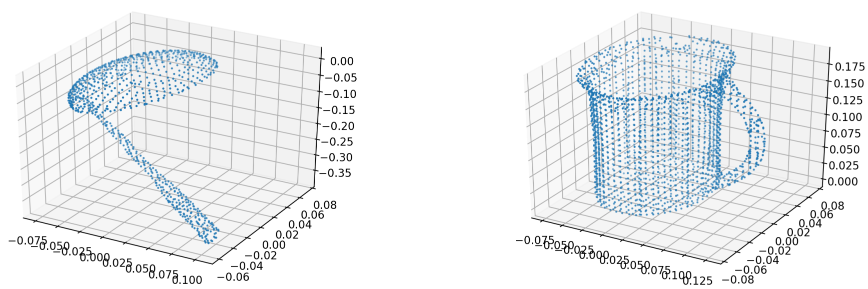


Figure 1: Samples from Kitchen Utensil Dataset

3.2 Synthetic Dataset – Multiple Holes

For synthetic dataset, I only generate 2D point clouds so that it's simple and fast to calculate, and also easy to understand.

The first synthetic dataset contains 2D point clouds with different number of holes. Figure 2 shows some samples from this dataset. Basically, this dataset starts from a disk-shaped point mesh. The distance between two nearest points is 1. Then I used different number of small holes (also have different size) to erode the disk. The number of holes are $0, 1, 2, 3$. The radius of holes are $2, 3, 4, 5$. There are totally 16 different point clouds in this dataset.

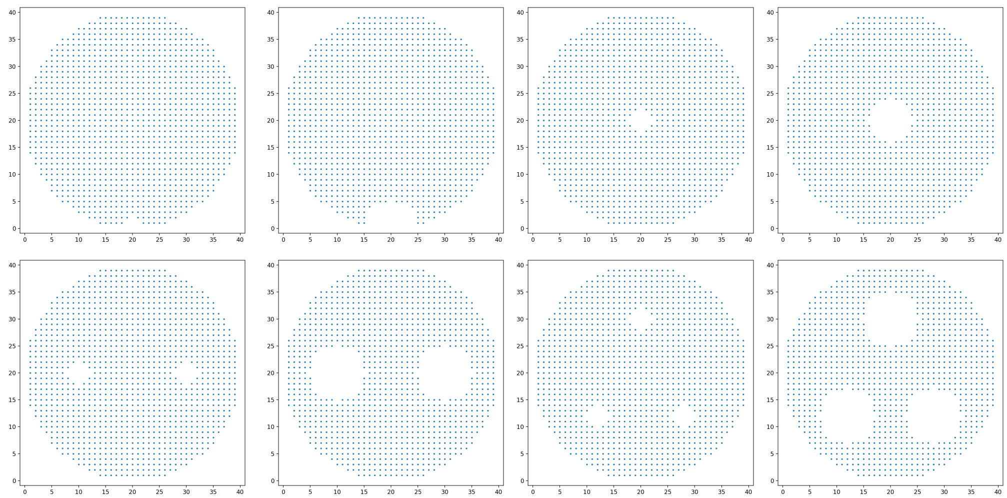


Figure 2: Synthetic Dataset: Different Number of Holes

3.3 Synthetic Dataset – Multiscale

The SMURPH kernel claims to be capable of doing multiresolution analysis. So I designed this dataset to evaluate this ability. Figure 3 shows a few samples. The dataset basically have two shapes of data: O-shaped and ∞-shaped. Each shape could be consist of solid ribbons, or ribbons with small holes. These four point clouds, as Figure 3 shows, forms a set. There are 3 sets in this dataset with size 40×40, 25×25, and 8×8.

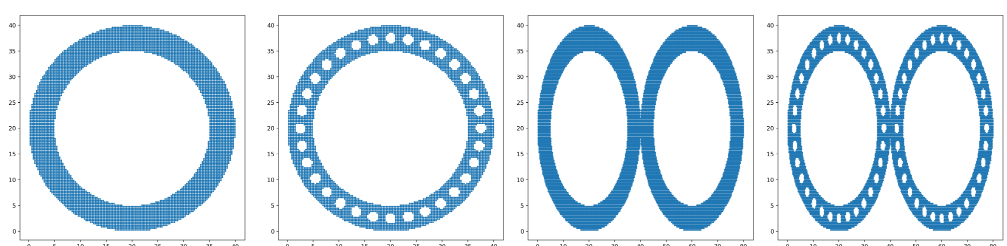
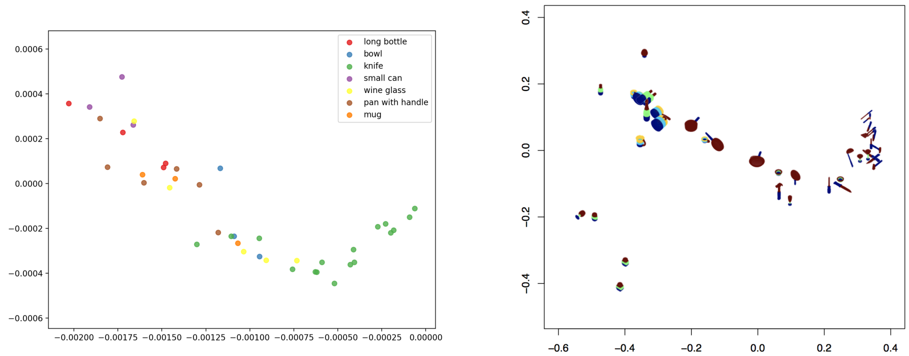


Figure 3: Synthetic Dataset: Different Scale of Holes

4 Experimental Results

4.1 Kitchen Utensil Dataset

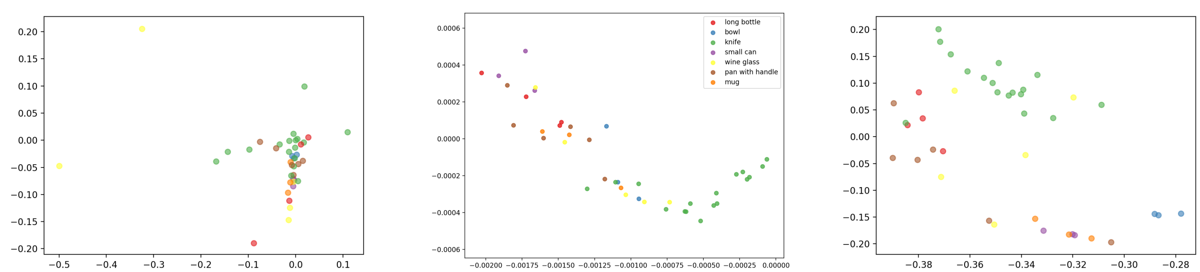
First, in order to validate my implementation of SMURPH kernel, I compared the kernel PCA result with the result given in the original paper. One thing to notice is that the parameters I used in my implementation is slightly different. This is due to the computation limitation. Specifically, in the original paper, they used a radius of $r = 0.1$, $m = 20$ centers per point cloud, $s = 1$ samples per center, and a budget of $b = 350$ points per sample. In my experiment, I used a radius of $r = 0.1$, $m = 10$ centers per point cloud, $s = 1$ samples per center, and a budget of $b = 100$ points per sample. The comparison is shown in Figure 4. As we can see, the overall distribution of my implementation is very close to the result from original paper. The only noticable difference is that the small cans don't form a cluster in my implementation. This probably because the sample size is only 100 points compared to the original 350 points. The smaller sample size could capture the local structure but failed to capture the overall topological structure. So small cans and large pans are all considered as cylinders though their overall shapes are different.



(a) My implementation (b) Result from original paper

Figure 4: SMURPH kernel PCA results for Kitchen Utensil Dataset

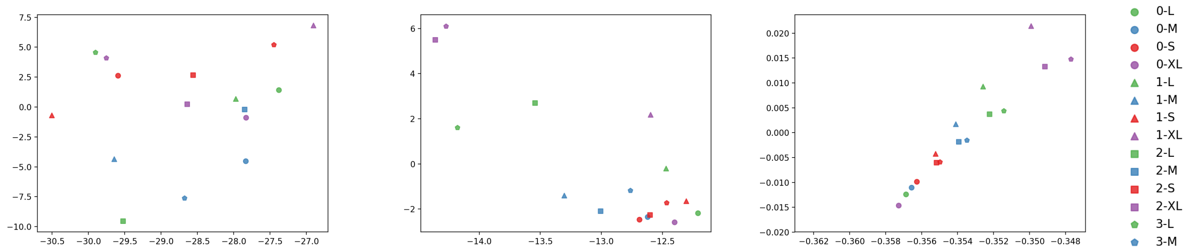
I also calculated the linear kernel ($s = 100$ for sample size) and HOD kernel ($b = 10$ for number of bins in histogram) of this dataset. The kernel PCA using these three different kernels are shown in Figure 5 First of all, we can easily see that the linear kernel don't perform very well. We can hardly see any meaningful structure from figure. The HOD kernel performs pretty well comparing with the other two. There are clearly four clusters using HOD kernel: {knife}, {pan with handle, long bottle}, {small can, mug}, and {bowl}. It seems HOD kernel could also capture topological feature of a point cloud. However, since the four clusters not only varies in topology but also varies in shape, it's also possible HOD kernel is capturing the overall shape (e.g. long and thin vs. short and round).



(a) Linear Kernel (b) SMURPH Kernel (c) HOD Kernel

Figure 5: Kernel PCA results for Kitchen Utensil Dataset

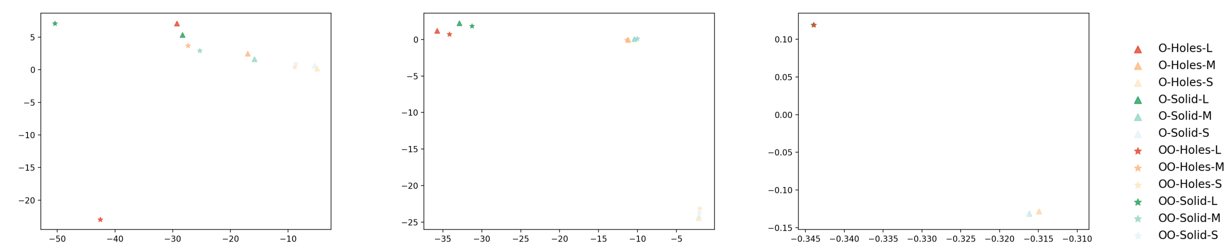
4.2 Synthetic Dataset – Multiple Holes



(a) Linear Kernel (b) SMURPH Kernel (c) HOD Kernel

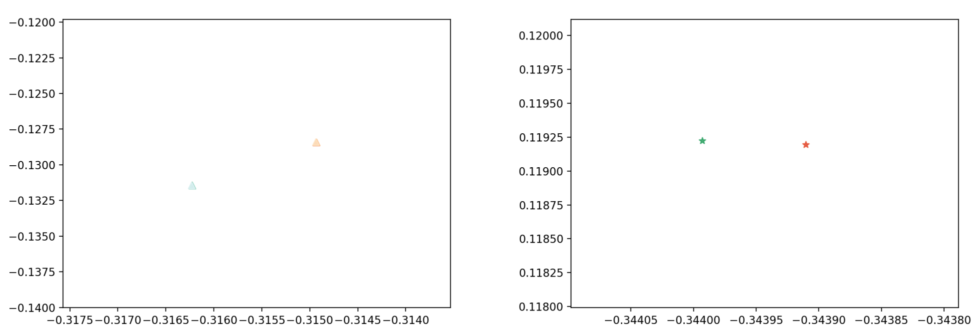
Figure 6: Kernel PCA results for Synthetic Multiholes Dataset. The legend X-Y means the point cloud has X number of holes and the size of holes is Y (S=Small, M=Medium, L=Large, XL=Extra Large).

4.3 Synthetic Dataset – Multiscale



(a) Linear Kernel (b) SMURPH Kernel (c) HOD Kernel

Figure 7: Kernel PCA results for Synthetic Multiscale Dataset. The legend X-Y-Z means the point cloud is X-shaped, formed with Y kind of ribbon, and size of the point cloud is Z (S=Small, M=Medium, L=Large, XL=Extra Large).



(a) Bottom-right area (b) Top-left area

Figure 8: HOD result zoom in

5 Discussion

References

[1] Xiaojin Zhu, Ara Vartanian, Manish Bansal, Duy Nguyen, and Luke Brandl. Stochastic multiresolution persistent homology kernel. In *International Joint Conference on Artificial Intelligence*, 2016.

[2] M. Neumann, P. Moreno, L. Antanas, R. Garnett, and K. Kersting. Graph Kernels for Object Category Prediction in Task-Dependent Robot Grasping. In *Proceedings of the Eleventh Workshop on Mining and Learning with Graphs (MLG–2013)*, Chicago, US, 2013.