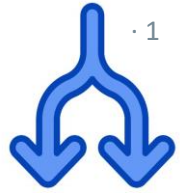
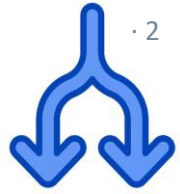


Practical Concurrent and Parallel Programming V

Performance Measurements

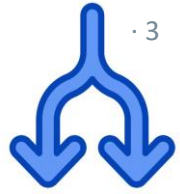
Jørgen Staunstrup

Agenda



- Performance measurements: motivation and introduction
- Pitfalls (and avoiding them)
- Calculating means and variance (efficiently)
- Measurements of thread overhead
- Algorithms for parallel computing

Agenda



- **Performance measurements: motivation and introduction**
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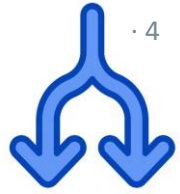
Motivations for Concurrency



From Week01

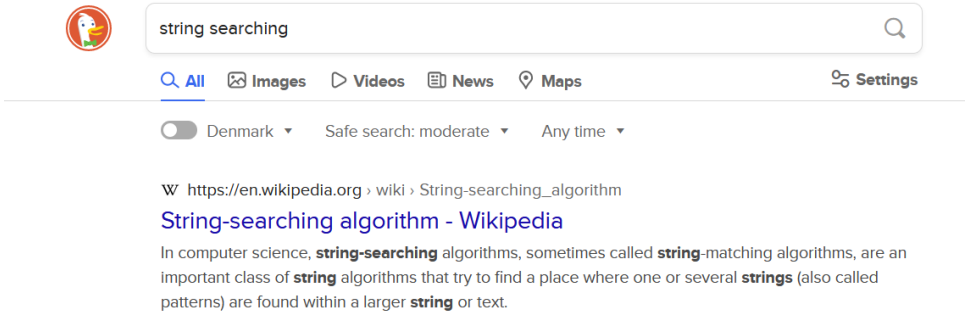
Inherent: User interfaces and other kinds of input/output

Exploitation: Hardware capable of simultaneously executing multiple streams of statements

Hidden: Enabling several programs to share some resources in a manner where each can act as if they had sole ownership

Motivation 1: Time consuming computations

Searching in a (large) text



https://www.geeksforgeeks.org/applications-of-string-matching-algorithms/

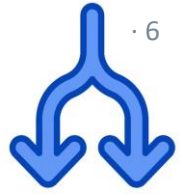
Computing prime numbers

```
2, 3, 5, 7, 11, 13, 17, 19, 23,
29, 31, 37, 41, 43, 47, 53, 59,
61, 67, 71, 73, 79, 83, 89, 97,
...
```

Cornerstone of all computer security

https://science.howstuffworks.com/math-concepts/prime-numbers.htm

Motivation 2: Analyzing code



Thread creation is expensive ?

The Java tutorials say that creating a Thread is expensive. But why exactly is it expensive? What exactly is happening when a Java Thread is created that makes its creation expensive? I'm taking the statement as true, but I'm just interested in mechanics of Thread creation in JVM.

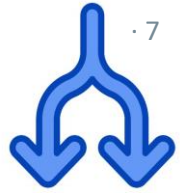
> *Thread lifecycle overhead. Thread creation and teardown are not free. The actual overhead*

But how expensive ?
~ 600 ns to create (on this laptop)
~ 20 times more time than creating a simple object

40000 ns to start a thread !!! (on this laptop)

Today: How to get such numbers !

(Performance) Measurements



Key in many sciences (experiments, observations, predictions, ...)

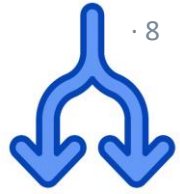
A bit of statistics
A bit of numerical analysis
A bit of computer architecture (cores, caches, number representation,)
Code for measuring execution time

Based on Microbenchmarks in Java and C# by Peter Sestoft (see benchmarkingNotes.pdf in material for this week)

All numbers in these slides were measured in August 2021 on a:

Intel Core i5-1035G4 CPU @ 1.10GHz, 4 Core(s), 8 Logical Processor(s)

Agenda



- Performance measurements: motivation and introduction
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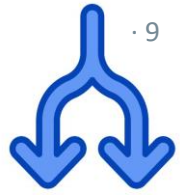
Example: measuring a (simple) function



```
private static double multiply(int i) {
    double x = 1.1 * (double)(i & 0xFF);
    return x * x * x * x * x * x * x * x * x * x * x
    * x * x * x * x * x * x * x * x * x * x * x;
  }
```

```
cd code-exercises/week05exercises
gradle -PmainClass=exercises05.Measurement
```

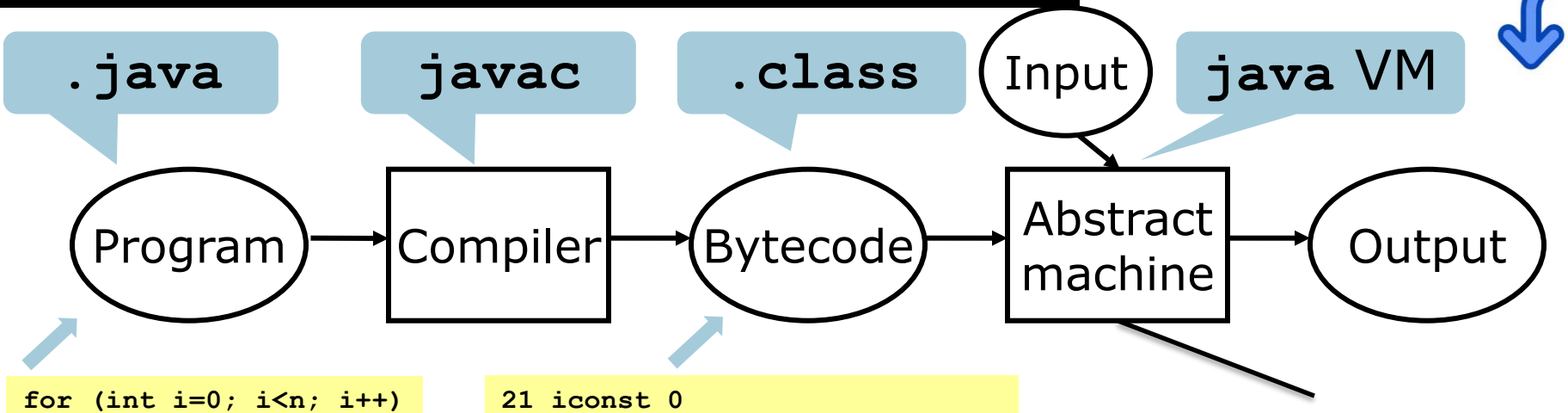
Try to do this, what result do you get?

```
start= System.nanoTime();
multiply(126465);
end= System.nanoTime();

System.out.println(end-start+" ns");
```

```
3600 ns
1400 ns
1500 ns

~ 2 - 5 ns
```

What is going on?

Java compiler and virtual machine




```
for (int i=0; i<n; i++)
  sum += sqrt(arr[i]);
```

```
21 iconst_0
22 istore 5
24 iload 5
26 iload 2
27 if_icmpge     46
30 dload 3
31 aload 1
32 iload 5
34 daload
35 invokestatic Math.sqrt:(D)D
38 dadd
39 dstore 3
40 iinc 5, 1
43 goto 24
```

JVM

JIT (Just In Time)

```
19 xorl %ebx,%ebx
1b jmp 3a
1d leal 0x00(%ebp),%ebp
20 fldl 0xec(%ebp)
23 cmpl %ebx,0x0c(%edi)
26 jbe 49
2c leal
0x10(%edi,%ebx,8),%eax
…
```

x86

Benchmarking note

Microbenchmarks in Java and C#

Peter Sestoft (sestoft@itu.dk)

IT University of Copenhagen, Denmark

Version 0.8.0 of 2015-09-16

Abstract: Sometimes one wants to measure the speed of software, for instance, to measure whether a

A goldmine of good advice

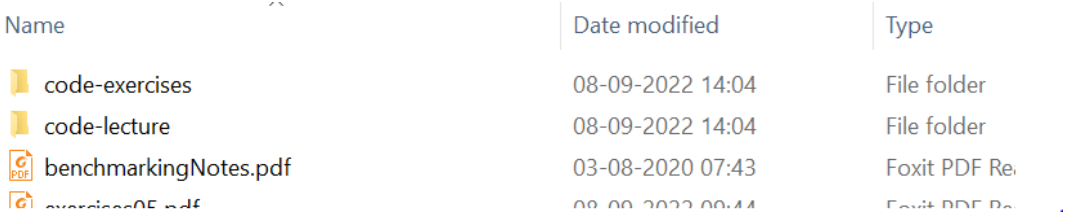
Accompanying code: **Benchmark.java**

On PCPP GitHub (week05)

```
class Benchmark {
  public static void main(String[] args) { new Benchmark(); }

  public Benchmark() {
    // SystemInfo();A
    // Mark0();
    // Mark1();
    ...
    Mark6("multiply", i -> multiply(i));
    ...
    // SortingBenchmarks();
    ...
```

How to use Benchmark

Go to the directory for Week 5



What do you see?

```
cd code-exercises/week05exercises
gradle -PmainClass=exercises05.Benchmark run

# OS:   Windows 10; 10.0; amd64
# JVM:  Oracle Corporation; 1.8.0_181
# CPU:  Intel64 Family 6 Model 126 Stepping 5, GenuineIntel; …
# Date: 2022-09-08T14:10:56+0200
   ...
```

Example: measuring a simple function

```
private static double multiply(int i) {
  double x = 1.1 * (double)(i & 0xFF);
  return x * x * x * x * x * x * x * x * x * x * x
    * x * x * x * x * x * x * x * x * x * x * x;
}
```

```
public static double Mark2() {
  Timer t = new Timer();
  int count = 100_000_000;
  double dummy = 0.0;
  for (int i=0; i<count; i++)
    dummy += multiply(i);
  double time = t.check() * 1e9 / count;
  System.out.printf("%6.1f ns%n", time);
  return dummy;
}
```

```
# OS:   Windows 10; 10.0; amd64
# JVM:  Oracle Corporation; 17.0.2
# CPU:  Intel64 Family 6 Model 126 Stepping 5, GenuineIntel; 8 "cores"
# Date: 2023-09-04T08:45:32+0200
  11.9 ns
```

The Timer class (in Benchmark.java)

A simple Timer class for Java

Works on all platforms (Linux, MacOS, Windows)

```
public class Timer {
  private long start, spent = 0;
  public Timer() { play(); }
  public double check()
  { return (System.nanoTime()-start+spent)/1e9; }
  public void pause() { spent += System.nanoTime()-start; }
  public void play() { start = System.nanoTime(); }
}
```

What does 1e9 mean?

In what time unit do we get the results?

Automating multiple runs (Mark3)

Results will usually vary

```
public static double Mark3() {
  int n = 10;
  int count = 100_000_000;
  double dummy = 0.0;
  for (int j=0; j<n; j++) {
    Timer t = new Timer();
    for (int i=0; i<count; i++)
    dummy += multiply(i);
    double time = t.check() * 1e9 / count;
    System.out.printf("%6.1f ns%n", time);
  }
  return dummy;
}
```

```
24.6 ns
24.6 ns
24.5 ns
24.6 ns
24.4 ns
24.3 ns
24.5 ns
24.4 ns
24.7 ns
24.6 ns
```

What is the running time?

What should you report as the result, when the observations are:

30.7 ns 30.3 ns 30.1 ns 30.7 ns 30.5 ns 30.4 ns 30.9 ns 30.3 ns 30.5 ns 30.8 ns ?

Mean: 30.4 ns

What if they are:

30.7 ns 100.2 ns 30.1 ns 30.7 ns 20.2 ns 30.4 ns 2.0 ns 30.3 ns 30.5 ns 5.4 ns ??

Mean: 31.0 ns

Agenda

- Performance measurements: motivation and introduction
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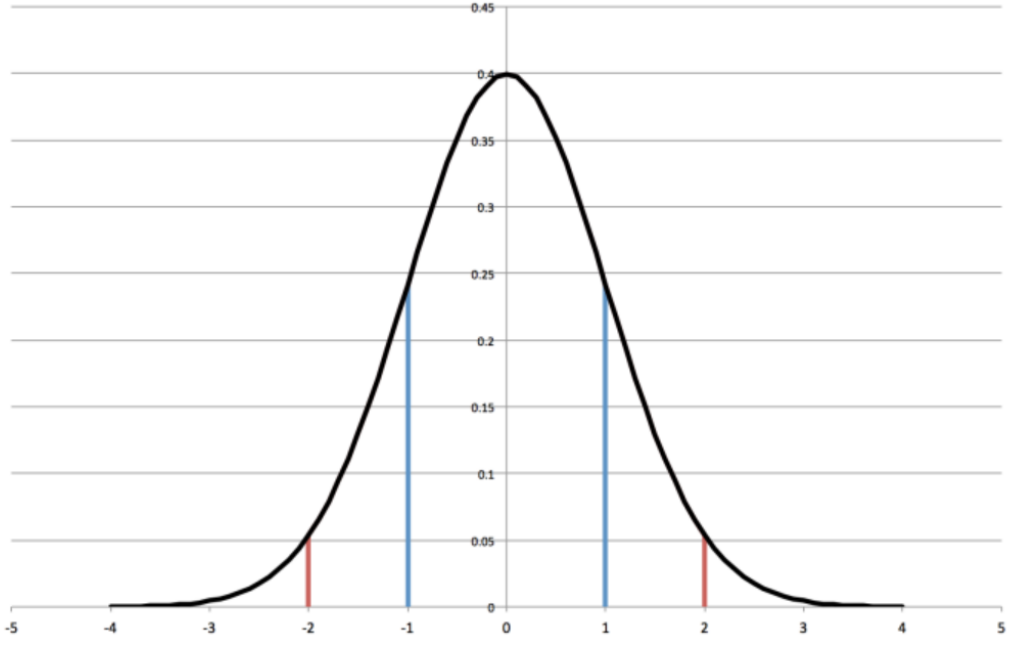
Normal distribution



Measuring physical properties

Your exam grades

Course evaluations

Fabrication faults

Running time of Java code

...

Mark5 - computes mean and variance

```
public static double Mark5() {
  int n = 10, count = 1, totalCount = 0;
  double dummy = 0.0, runningTime = 0.0, st = 0.0, sst = 0.0;
  do {
    count *= 2;
    st = sst = 0.0;
    for (int j=0; j<n; j++) {
      Timer t = new Timer();
      for (int i=0; i<count; i++) dummy += multiply(i);
      runningTime = t.check();
      double time = runningTime * 1e9 / count;
      st += time;
      sst += time * time;
      totalCount += count;
    }
    double mean = st/n, sdev = Math.sqrt((sst - mean*mean*n)/(n-1));
    System.out.printf("%6.1f ns +/- %8.2f %10d%n", mean, sdev, count);
  } while (runningTime < 0.25 && count < Integer.MAX_VALUE/2);
  return dummy / totalCount;
}
```

Mark5 - computes mean and variance

```
public static double Mark5() {
  int n = 10, count = 1, totalCount = 0;
  double dummy = 0.0, runningTime = 0.0, st = 0.0, sst = 0.0;
  do {
    count *= 2;
    st = sst = 0.0;
    for (int j=0; j<n; j++) {
      Timer t = new Timer();
      for (int i=0; i<count; i++) dummy += multiply(i);
      runningTime = t.check();
      double time = runningTime * 1e9 / count;
      st += time;
      sst += time * time;
      totalCount += count;
    }
    double mean = st/n, sdev = Math.sqrt((sst - mean*mean*n)/(n-1));
    System.out.printf("%6.1f ns +/- %8.2f %10d%n", mean, sdev, count);
  } while (runningTime < 0.25 && count < Integer.MAX_VALUE/2);
  return dummy / totalCount;
}
```

Can we give the function to be measured as a parameter?

```
public static double Mark5() {
  int n = 10, count = 1, totalCount = 0;
  double dummy = 0.0, runningTime = 0.0, st = 0.0, sst = 0.0;
  do {
    count *= 2;
    st = sst = 0.0;
    for (int j=0; j<n; j++) {
      Timer t = new Timer();
      for (int i=0; i<count; i++) dummy += multiply(i);
      runningTime = t.check();
      double time = runningTime * 1e9 / count;
      st += time;
      sst += time * time;
      totalCount += count;
    }
    double mean = st/n, sdev = Math.sqrt((sst - mean*mean*n)/(n-1));
    System.out.printf("%6.1f ns +/- %8.2f %10d%n", mean, sdev, count);
  } while (runningTime < 0.25 && count < Integer.MAX_VALUE/2);
  return dummy / totalCount;
}
```

Parameterizing function to be measured

```
private static double multiply(int i) {
. . .
}
```

Yes?

Java: `multiply(i)` **is a number**

Java: `i -> multiply(i)` **is a function**

https://docs.oracle.com/javase/tutorial/java/javaOO/lambdaexpressions.html

```
Mark6( . . . , i -> multiply(i));
```

Mark6 - introduce a functional argument

```
public static double Mark6(String msg, IntToDoubleFunction f) {
  int n = 10, count = 1, totalCount = 0;
  double dummy = 0.0, runningTime = 0.0, st = 0.0, sst = 0.0;
  do {
    count *= 2;
    st = sst = 0.0;
    for (int j=0; j<n; j++) {
      Timer t = new Timer();
      for (int i=0; i<count; i++) dummy += f.applyAsDouble(i);
      runningTime = t.check();
      double time = runningTime * 1e9 / count;
      st += time;  sst += time * time; totalCount += count;
    }
    double mean = st/n, sdev = Math.sqrt((sst - mean*mean*n)/(n-1));
    System.out.printf("%-25s %15.1f ns %10.2f %10d%n", msg, mean, sdev, count);
  } while (runningTime < 0.25 && count < Integer.MAX_VALUE/2);
  return dummy / totalCount;
}
public interface IntToDoubleFunction { double applyAsDouble(int i); }

Mark6("multiply", i -> multiply(i));
```

The function f is benchmarked

lambda

Mark6 - introduce a functional argument

```
public static double Mark6(String msg, IntToDoubleFunction f) {
  int n = 10, count = 1, totalCount = 0;
  double dummy = 0.0, runningTime = 0.0, st = 0.0, sst = 0.0;
  do {
    count *= 2;
    st = sst = 0.0;
    for (int j=0; j<n; j++) {
      Timer t = new Timer();
      for (int i=0; i<count; i++) dummy += f.applyAsDouble(i);
      runningTime = t.check();
      double time = runningTime * 1e9 / count;
      st += time;  sst += time * time; totalCount += count;
    }
    double mean = st/n, sdev = Math.sqrt((sst - mean*mean*n)/(n-1));
    System.out.printf("%-25s %15.1f ns %10.2f %10d%n", msg, mean, sdev, count);
  } while (runningTime < 0.25 && count < Integer.MAX_VALUE/2);
  return dummy / totalCount;
}
public interface IntToDoubleFunction { double applyAsDouble(int i); }

Mark6("multiply", i -> multiply(i));
```

Example use of Mark6

```
Mark6("multiply", i -> multiply(i));
```

```
multiply                595.0 ns    1407.81          2
multiply                147.5 ns      90.10          4
multiply                212.5 ns     152.53          8
multiply                170.6 ns      59.44         16
multiply                201.9 ns     157.69         32
multiply                 60.8 ns      34.55         64
multiply                 65.1 ns      59.83        128
multiply                 54.3 ns      14.85        256
...
multiply                 24.6 ns       0.75     524288
multiply                 24.6 ns       0.88    1048576
multiply                 24.9 ns       2.71    2097152
multiply                 24.3 ns       0.85    4194304
multiply                 24.2 ns       0.72    8388608
multiply                 25.0 ns       1.38   16777216
```

Mark7 - printing only final values

```
public static double Mark7(String msg, IntToDoubleFunction f) {
  ...
  do {
  ...
  } while (runningTime < 0.25 && count < Integer.MAX_VALUE/2);
  double mean = st/n, sdev = Math.sqrt((sst - mean*mean*n)/(n-1));
  System.out.printf("%-25s %15.1f %10.2f %10d%n", msg, mean, sdev, count);
  return dummy / totalCount;
}
```

Timing prime calculation

```
private static boolean isPrimeS(int n) {
  int k = 2;
  while (k < n && n % k != 0) k++;
  return n >= 2 && k >= n;
}

private static boolean isPrime(int n) {
  int k = 2;
  while (k * k <= n && n % k != 0) k++;
  return n >= 2 && k * k > n;
}
```

Does it matter which one we use?

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- **Measurements of thread overhead**
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Thread creation

```
Mark7("Thread create",
  i -> {
        Thread t = new Thread(() -> {
          for (int j=0; j<1000; j++)
            ai.getAndIncrement();
        });
  return t.hashCode();  // to confuse compiler to not optimize
});
```

Takes 700 ns

What are we really measuring?

Slow or fast?

Creating an object

A thread is an object, so let us start finding the cost of creating a simple object.

```
class Point {
  public final int x, y;
  public Point(int x, int y) { this.x = x; this.y = y; }
}

Mark7("hashCode()", i -> myPoint.hashCode());

Mark7("Point creation",
      i -> {
        Point p = new Point(i, i);
        return p.hashCode();
      });
```

hashCode() 3 ns

Point creation 50 ns

So object creation is: ~ 47 ns

Thread creation ~ 650ns

Thread create + start

```
Mark6("Thread create start",
      i -> {
        Thread t = new Thread(() -> {
          for (int j=0; j<1000; j++)
            ai.getAndIncrement();
        });
        t.start();
        return t.hashCode();
      });
```

What are we really measuring?

Thread create + start

```
Mark6("Thread create start",
      i -> {
        Thread t = new Thread(() -> {
          for (int j=0; j<1000; j++)
            ai.getAndIncrement();
        });
        t.start();
        return t.hashCode();
      });
```

For loop not included, why?

Thread create + start

```
Mark6("Thread create start",
      i -> {
        Thread t = new Thread(() -> {
          for (int j=0; j<1000; j++)  //most iterations not done
            ai.getAndIncrement();     // Why?
        });
        t.start();
        return t.hashCode();
      });
```

Takes ~ 47000 ns

- So, a lot of work goes into starting a thread
- Even after creating it
- Note: does not include executing the loop

Never create threads for small computations !!!

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Algorithms for parallel computing

Quicksort: https://www.chrislaux.com/quicksort.html

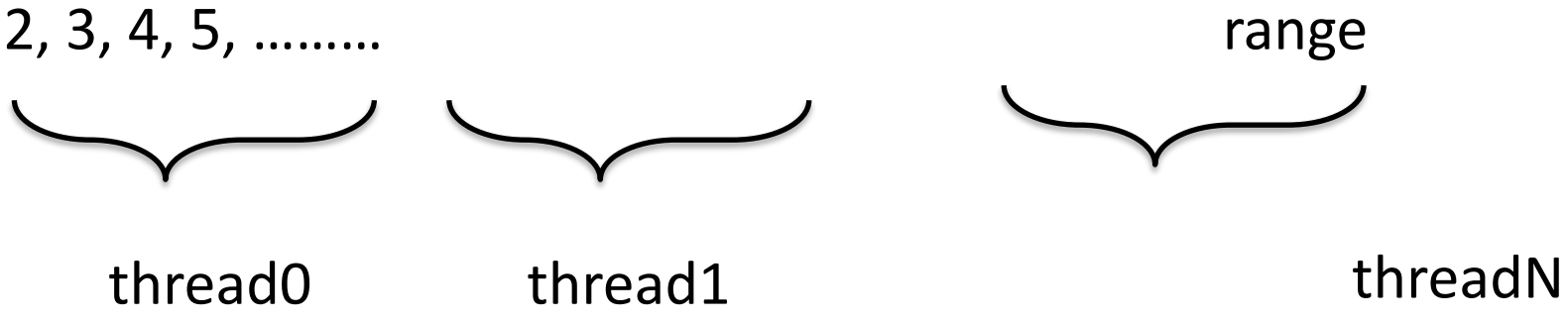
```
private static void qsort(int[] arr, int a, int b) {
  if (a < b) {
    int i = a, j = b;
    int x = arr[(i+j) / 2];
    do {
      while (arr[i] < x) i++;
      while (arr[j] > x) j--;
      if (i <= j) { swap(arr, i, j); i++; j--; }
    } while (i <= j);
    qsort(arr, a, j); qsort(arr, i, b);
  }
}
```

see SearchAndSort.java in week 05 material

Prime counting: https://www.dcode.fr/prime-number-pi-count

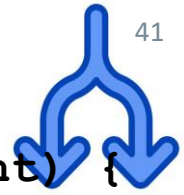
```
long count = 0;
final int from = 0, to = range;
for (int i=from; i<to; i++) if (isPrime(i))  count++;
```

Multithreaded version of CountPrimes



Code for exercises week05: TestCountPrimesThreads.java

Java code for TestCountTimesThreads



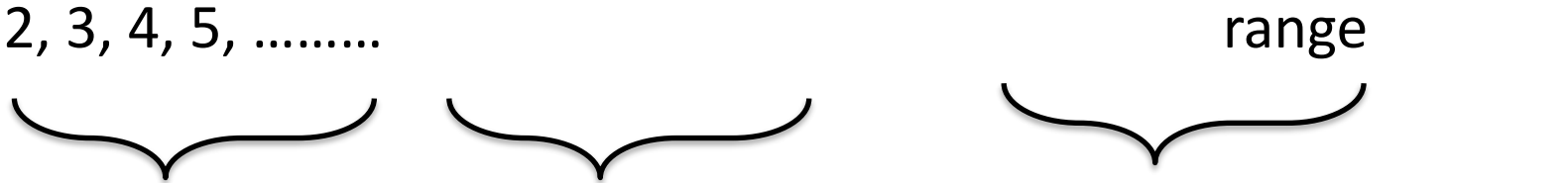
```
private static long countParallelN(int range, int threadCount) {
  final int perThread= range / threadCount;
  final LongCounter lc= new LongCounter();
  Thread[] threads= new Thread[threadCount];
  for (int t=0; t<threadCount; t++) {
    final int from= perThread * t,
          to= (t+1==threadCount) ? range : perThread * (t+1);
           threads[t]= new Thread(()
-                              > {for (int i=from; i<to; i++)
                                   if (isPrime(i)) lc.increment();
                            });
    }
    for (int t=0; t<threadCount; t++) threads[t].start();
    try { for (int t=0; t<threadCount; t++)  threads[t].join();
        } catch (InterruptedException exn) { }
    return lc.get();
  }
```

TestCountPrimesThreads

```
countSequential          5922958.0 ns  289879.33
countParallel  1         7107236.6 ns  448417.55
countParallel  2         6069944.7 ns  802224.61
countParallel  3         3621185.5 ns  152693.03
countParallel  4         3124067.0 ns  640480.51
countParallel  5         3699514.7 ns  364428.77
countParallel  6         4114074.2 ns  642562.19
countParallel  7         2049595.7 ns   26888.15
countParallel  8         1801465.6 ns   12532.85
countParallel  9         1793099.1 ns   11017.57
countParallel 10         1798921.4 ns   11541.43
countParallel 11         1807408.3 ns    9763.61
```

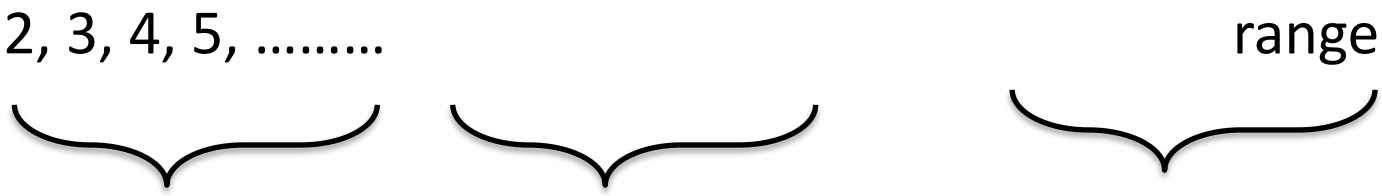
Good or bad?

```
countParallel   1          7107236.6 ns   448417.55
countParallel   2          6069944.7 ns   802224.61
countParallel   3          3621185.5 ns   152693.03
countParallel   4          3124067.0 ns   640480.51
...
```



Is this good or bad, and why?

Breaking the task into smaller pieces/tasks



When a thread is done with one task, it gets a new task

until all tasks are done

Java Executors

Goetz 6.2

```
new Thread(runnable1).start();
...
new Thread(runnable2).start();
...
new Thread(runnable3).start();
```

Threads are expensive !

```
ExecutorService pool;

pool.execute(runnable1);
...
pool.execute(runnable2);
...
pool.execute(runnable2);
```

Reuse of threads

https://howtodoinjava.com/java/multi-threading/java-fixed-size-thread-pool-executor-example/

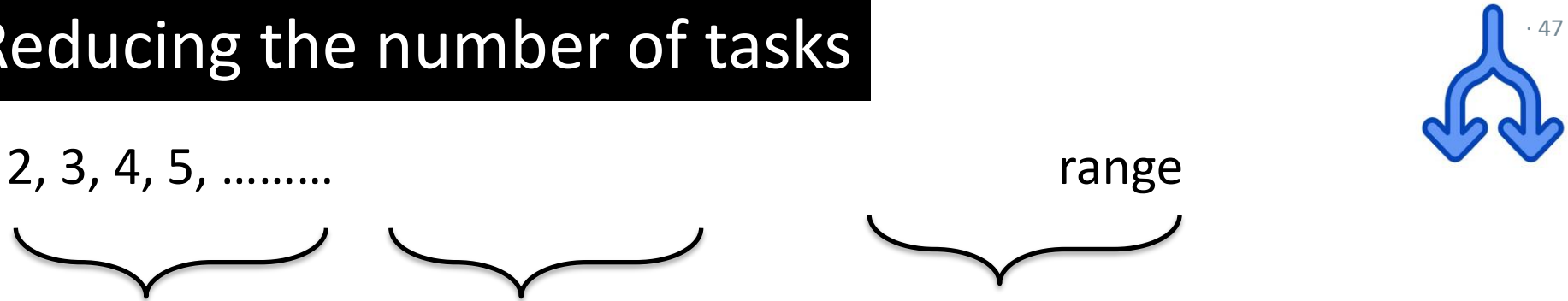
Prime counter task (skeleton)

```
public class countPrimesTask implements Runnable {
  private final int low;
  private final int high;
  private final ExecutorService pool;

  @Override public void run() {

    int mid= low+(high-low)/2;
    pool.submit( new countPrimesTask(low, mid, pool) );
    pool.submit( new countPrimesTask(mid+1, high, pool) );

  }
}
```

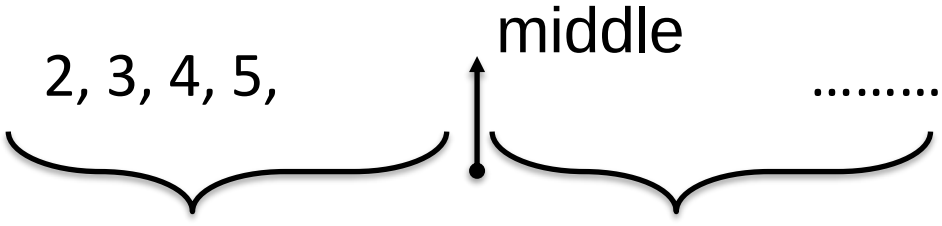
Shortcomings:

1. How to stop?
2. Will create too many "small" tasks
3. Returning result (# primes)

Reducing the number of tasks



range > threshold: split task



range <= threshold: count the number of primes

Splitting tasks

```
@Override
  public void run() {
  if ((high-low) < threshold) {
      for (int i=low; i<=high; i++)  if (isPrime(i)) lc.increment();
  } else {
      int mid= low+(high-low)/2;
      pool.submit(new countPrimesTask(lc, low, mid, pool, threshold) );
      pool.submit(new countPrimesTask(lc, mid+1, high, pool, threshold) );
  }
}
```

Shortcomings:

1. How to stop?
2. ~~Will create too many "small" tasks~~
3. Returning result (# primes)

More next week

Counting the primes

```
public class countPrimesExecutor {
..
  private final LongCounter lc;  // Global shared variable  !!!
...                              // requires atomicity

}

    public class countPrimesTask implements Runnable {
      ...
      public void run() {
        ...
          for (int i=low; i<=high; i++)
            if (isPrime(i)) lc.increment();
        ...
      }
    }
```

Could `lc` become a bottleneck?

Counting the primes

```
public class countPrimesExecutor {
..
  private final LongCounter lc;  // Global shared variable  !!!
...                              // requires atomicity

}

    public class countPrimesTask implements Runnable {
      ...
      public void run() {
        ...
          for (int i=low; i<=high; i++)
            if (isPrime(i)) lc.increment();
        ...
      }
    }
```

Exercise 5.2 ?

Thread vs Executor

Counting primes in the range 2..1_000_000

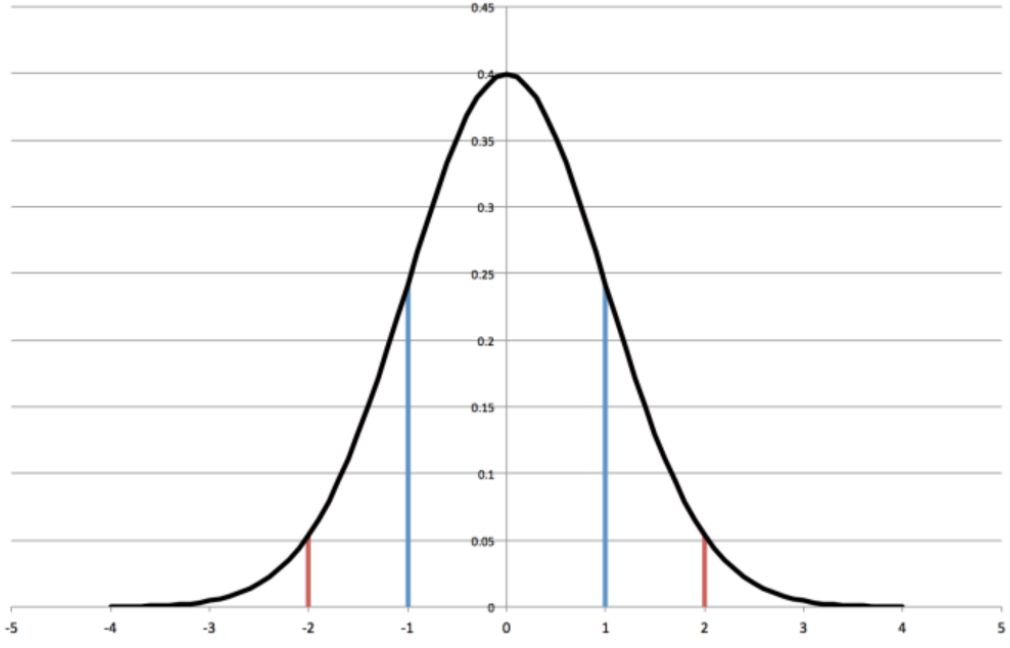
Sequential	1.2 Sec
Threads (4)	0.5 Sec
Executor	0.4 Sec

More on executors next week

Agenda

- Performance measurements: motivation and introduction
- Pitfalls (and avoiding them)
- **Calculating means and variance (efficiently)**
- Measurements of thread overhead
- Algorithms for parallel computing

Normal distribution



Measuring physical properties

Your exam grades

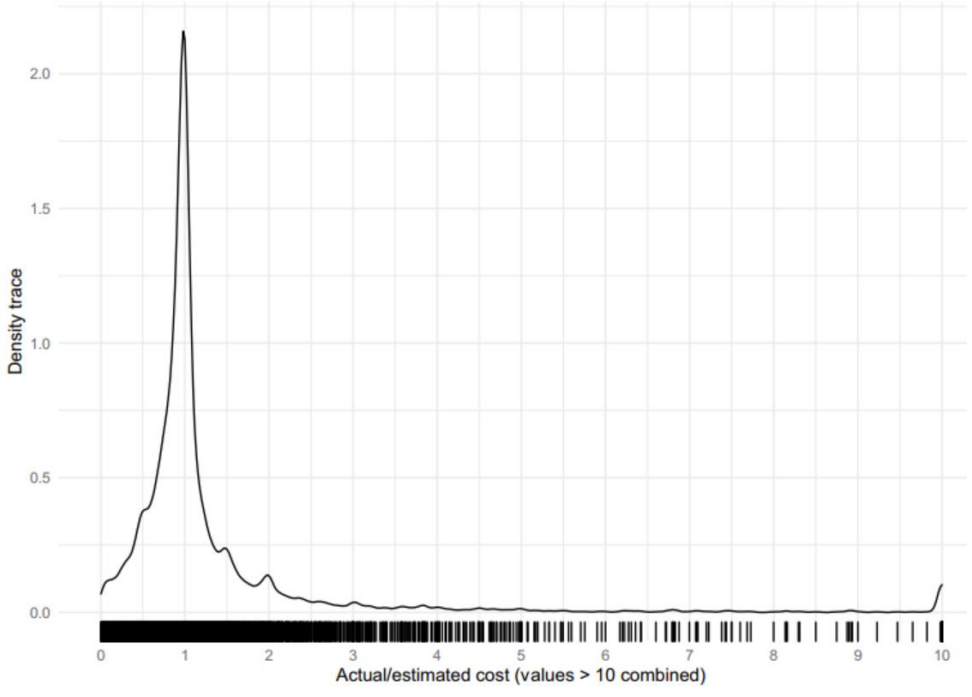
Course evaluations

Fabrication faults

Running time of Java code

...

But there are exceptions



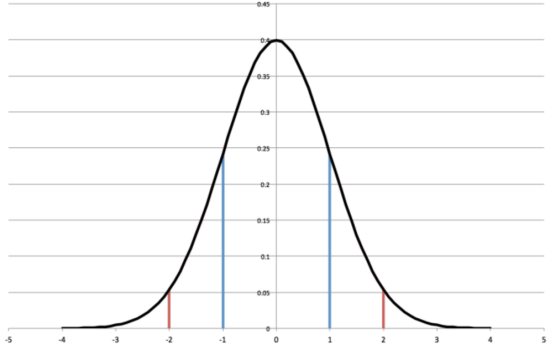
Source: Bent Flyvbjerg, Alexander Budzier, Jong Seok Lee, Mark Keil, Daniel Lunn & Dirk W. Bester (2022) The Empirical Reality of IT Project Cost Overruns: Discovering A Power-Law Distribution, Journal of Management Information Systems, 39:3, 607-639, DOI: 10.1080/07421222.2022.2096544

Standard deviation/variance

$$\mu = \frac{1}{n}\sum_{j=1}^{n} t_j$$

Mean

Benchmark note p6



Standard deviation/variance

$$\mu = \frac{1}{n}\sum_{j=1}^{n} t_j$$

Mean

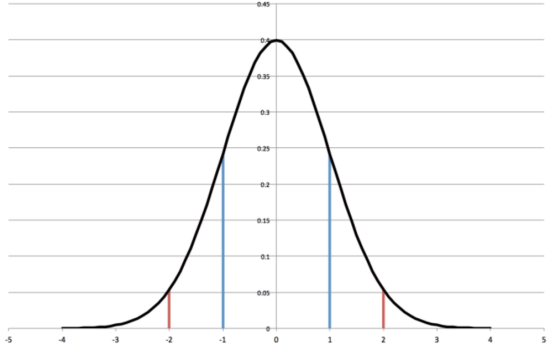
$$\sigma = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(t_j - \mu)^2}$$

Standard deviation

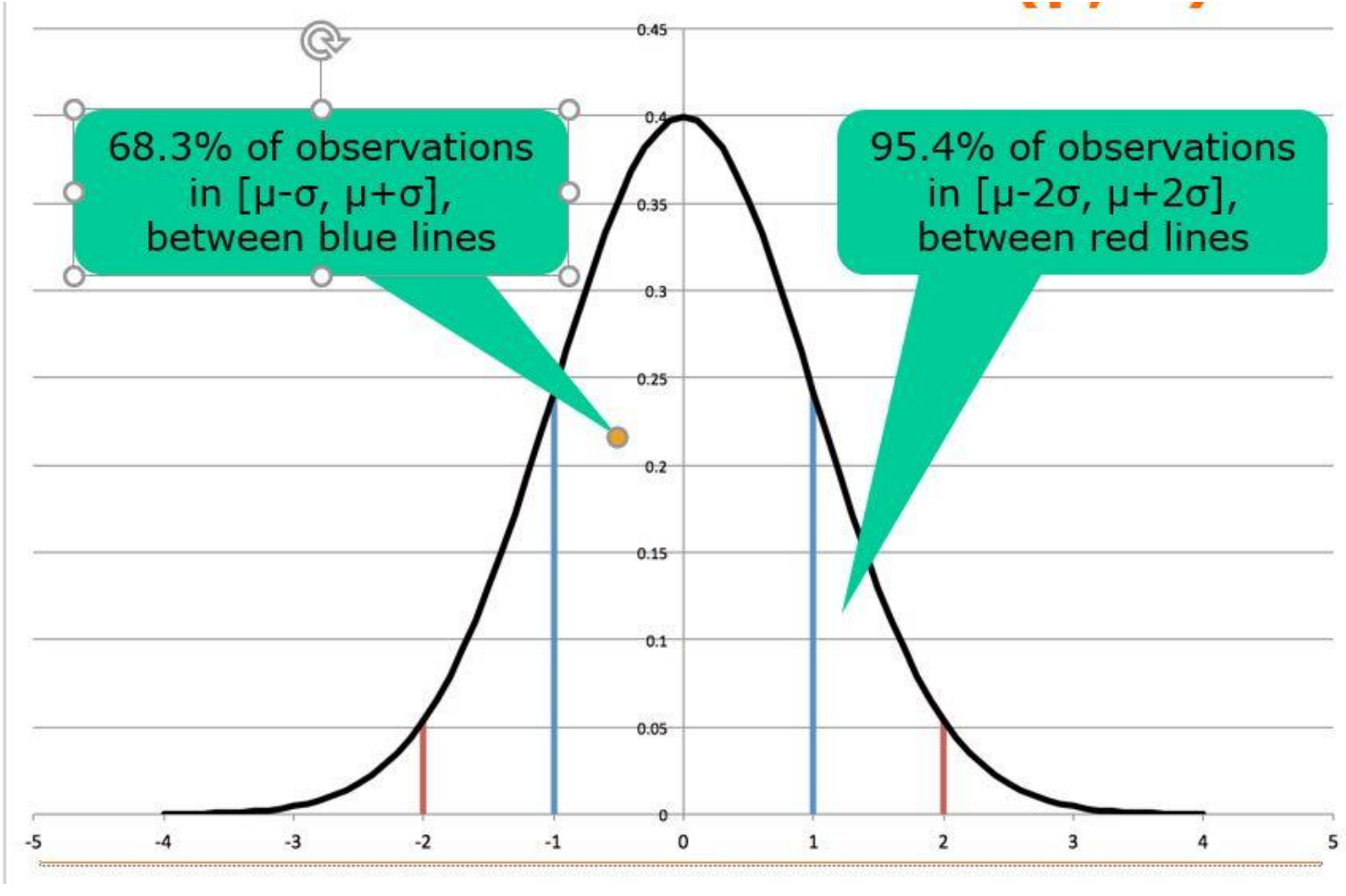
Benchmark note p6

30.7 ns 30.3 ns 30.1 ns 30.7 ns 50.2 ns 30.4 ns 30.9 ns 30.3 ns 30.5 ns 30.8 ns ??

Mean: 32.5 ns Standard deviation: 6.2



Normal distribution



Outliers

What should you report as the result, when the observations are:

30.7 ns 30.3 ns 30.1 ns 30.7 ns 50.2 ns 30.4 ns 30.9 ns 30.3 ns 30.5 ns 30.8 ns ??

Mean: 32.5 ns Standard deviation: 6.2

50.2 is an outlier

because there is a probability of less than 4.6 % that 50.2 is a correct observation

Computing the variance

$$\mu = \frac{1}{n}\sum_{j=1}^{n} t_j$$

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(t_j - \mu)^2}$$

Requires two passes through the data

$$\sigma^2 = \frac{1}{n(n-1)}\left(n\sum_{j=1}^{n} t_j^2 - \left(\frac{1}{n}\sum_{j=1}^{n} t_j\right)^2\right)$$

Can be done in one pass (on-line alg.)

```
for (int j=0; j<n; j++) {
   Timer t = new Timer();
   for (int i=0; i<count; i++)
         ...
   double time = t.check() * 1e9 / count;
   st += time;
   sst += time * time;
 }
 double mean = st/n, sdev = Math.sqrt((sst - mean*mean*n)/(n-1));
 System.out.printf("%6.1f ns +/- %6.3f%n", mean, sdev);
```

The two formulas give the same result

$$\mu = \frac{1}{n}\sum_{j=1}^{n} t_j$$

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(t_j-\mu)^2}$$

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(t_j^2+\mu^2-2t_j\mu)}$$

$$\sigma^2 = \frac{1}{n-1}\sum_{j=1}^{n}(t_j^2+\mu^2-2t_j\mu)$$

$$\sigma^2 = \frac{1}{n-1}(\sum_{j=1}^{n} t_j^2+\sum_{j=1}^{n}(\mu^2-2t_j\mu))$$

$$\sigma^2 = \frac{1}{n-1}(\sum_{j=1}^{n} t_j^2+n\mu^2-2\mu\sum_{j=1}^{n} t_j)$$

$$\sigma^2 = \frac{1}{n-1}(\sum_{j=1}^{n} t_j^2+n\mu^2-2\mu n\mu)$$

$$\sigma^2 = \frac{1}{n-1}(\sum_{j=1}^{n} t_j^2-n\mu^2)$$

$$\sigma^2 = \frac{1}{n(n-1)}(n\sum_{j=1}^{n} t_j^2-\mu^2)$$

$$\sigma^2 = \frac{1}{n(n-1)}(n\sum_{j=1}^{n} t_j^2-(\frac{1}{n}\sum_{j=1}^{n} t_j)^2)$$

Formula in Benchmark note

See exercises05.pdf

also https://en.wikipedia.org/wiki/Algorithms_for_calculating_variance

Formula used in code (one pass algorithm)

Warning

$$\sigma^2 = \frac{1}{n(n-1)}\left(n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2\right)$$

```
int n = 10;
...
for (int j=0; j<n; j++) {
  Timer t = new Timer();
  for (int i=0; i<count; i++)
        ...
  double time = t.check() * 1e9 / count;
  st += time;
  sst += time * time;
}
double mean = st/n, sdev = Math.sqrt((sst - mean*mean*n)/(n-1));
System.out.printf("%6.1f ns +/- %6.3f%n", mean, sdev);
```

Beware: sst - mean * mean * n can be a very small number

Digit loss

Beware of cancellation when subtracting numbers that are close to each other:

28 significant digits

```
 10101010000101101100011101 01.111
-10101010000101101100011100 01.100
 ------------------------------
 00000000000000000000000000 100.011
```

3 significant digits

https://blog.demofox.org/2017/11/21/floating-point-precision

Digit loss

Beware of cancellation when subtracting numbers that are close to each other:

28 significant digits

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 10101010000101101100011101 01.111
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