

CIS 425 Assignment 2

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1. Example of a variable occurs both bound and free.

$$\lambda x.(\lambda y.xy)y$$

In above lambda expression, the variable y occurs both bound and free. The first y is the binding occurrence, the second y is bound because it is in the scope of λy , the second y is free because it is not in the scope of λy .

2. Give the free variables and bound variables of there terms:

(a). $\lambda x.(\lambda y.xy)y$

Bound variable: x, y

Free variable: y .

The second x is in the scope of λx , the second y is in the scope of λy . So they are bound variable.

The third y is not in the scope of λy , it is free variable. So y is both free and bound variable.

(b). $\lambda k.k(\lambda f.hf)(qf)$

Bound variable: k, f

Free variable: f

The variable k is in the scope of λk , the second f is in the scope of λf , so they are bound variable.

The third variable f is not in the scope of λf , it is free variable. So f is both free and bound variable.

3. Give the result of performing the following substitutions:

$$\begin{aligned} \text{a. } & [(\lambda y.xy)/x](x(\lambda x.yx)) \\ & = ((\lambda y.xy)(\lambda x.yx)) \end{aligned}$$

The "x" in $\lambda x.yx$ is a bounded variable to λx (inside expression), so we can not replace it when doing the outside expression substitution.

$$\begin{aligned} \text{b. } & [(\lambda x.xy)/x](\lambda y.x(\lambda x.x)) \\ & = (\lambda y.(\lambda x.xy)(\lambda x.x)) \end{aligned}$$