Computation Structures 2014. Yue Zhang

S1
- ☐ Course introduction
- ☐ The digital abstraction — why?
- ☐ Binary numbers and 2's complement form
- ☐ A little bit about information theory (ISTD)

- ## Course introduction

- What is this course about?
  - 2 how digital systems work
  - 1 how computers work

- Course website. 50-002. wikispaces.com

- Teaching: cohort sessions + labs + projects
  (Yue Zhang) (Oka Kurniawan)
  course calendar online.
  roughly { 1.5+1.5 (cohort) + 2 (lab) normal weeks
  1D/2D weeks

- Assessment.
  4 quizzes (30%) differentiating
  6 labs (25%)
  1D (25%)
  2D (10%)
  Mini hardware project (7%)
  } you get 100% if you finish it (some delay)
  Participation (3%) most get 100%

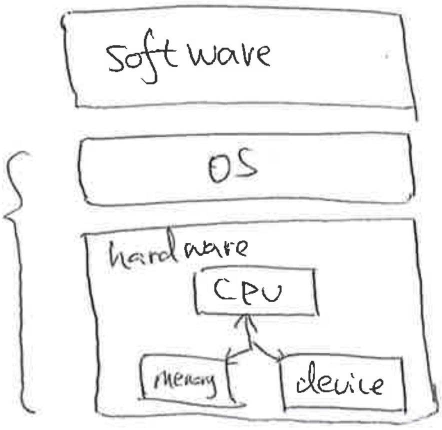
- Associated MIT course (6.004)
  useful to watch but absolutely not nessary
  (can get 100 if all cohort contents are mastered)

- how to learn? take notes + solve cohort questions + labs.

- Course content introduction

Digital systems
- $i \rightarrow \boxed{} \rightarrow o$ combinational logic
  - (both are numbers / integers)
  - e.g. adder $i1, i2 \rightarrow \boxed{} \rightarrow o = i1 + i2$
  - ROM $addr \rightarrow \boxed{} \rightarrow o$
- $i \rightarrow \boxed{s} \rightarrow o$ sequential logic
  - e.g. accumulator ($i = [1, 3, 5, 7, 9 \cdots]$ $o = [1, 4, 9, 16, 25, \cdots]$)

Computers — a special type of sequential logic

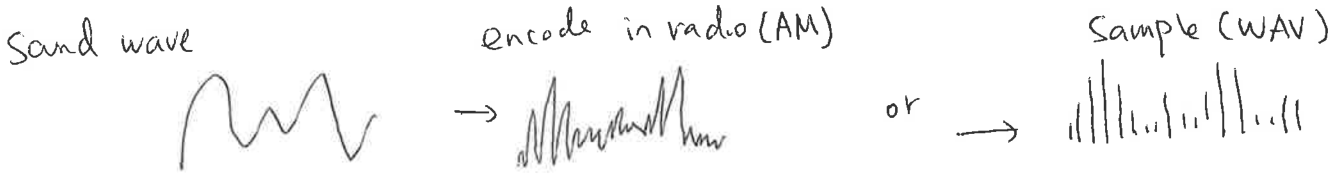


• The digital abstraction

- All inputs and outputs are ~~numbers~~ integers, with discrete values
  - └ represented by 0s and 1s !
- Why? what is continuous value? (real numbers)

Analogue systems vs Digital systems

TV. radio. cassette — digital TV. CD. internet



Analogue	Digital
can be more precise	can be as precise as you want
subject to noise in transfer (radio)	easy to guarantee
storage media can wear out (tape)	accurate transfer / storage

- Why is digital signal guaranteed more robust → noise?
 ↳(the integers) (real values)

Because analog signals try to keep an infinite amount of info, but digital signals try to keep only a finite amount. infinite real nums

(talk about binary number first, move to *)

- ↳What is information?

Introduce theory later; uncertainty resolving measure; $\boxed{\text{consider equal choices from a set of N numbers}}$

You can measure information by integers.

If the set size is N, then knowing one answer from the set is N informatn

e.g. I tell you "there is no final-exam" = 2 (amount of info)

A roll of a dice yields 4. = 6 (amount of info)

One card out of a deck of 54 = 54 (amount of info)

Think reversely, a binary choice holds info 2, a dice roll holds info 6, a card draw hold info 54.

What does an arbitrary integer hold?

How much does an integer from $[0, 256)$ hold?

How much does an arbitrary ~~float~~ real number hold?

- Now analog signals are represented by real numbers. A little fluctuation of media will change the information. (e.g. A photo can fade), digita signals are represented by bounded integers. There is safty guarantee of preventing information corrption

Hence

$$i \rightarrow \boxed{} \rightarrow o$$

$i \in [0, A)$, $o \in [0, B)$

- Information is measured by bits. 1 bit represents a binary choice. ∴ 2 bits represents 2 binary choices = 4 choices;

3 bits represents 3 binary choices = 8 choices;

⋮

N choices can be represented by $\log_2 N$ bits.

- <u>Binary numbers</u>

- How do we store ~~information~~ integers ?

  Bi-state storge medium easiest to find.

  $\boxed{0}$ / $\boxed{1}$

  Get two pieces ?

  $\boxed{0\,|\,0}$ $\boxed{0\,|\,1}$ $\boxed{1\,|\,0}$ $\boxed{1\,|\,1}$

  Get three pieces ?

  $\boxed{0\,|\,0\,|\,0}$ $\boxed{0\,|\,0\,|\,1}$ $\boxed{0\,|\,1\,|\,0}$ $\boxed{0\,|\,1\,|\,1}$ $\boxed{1\,|\,0\,|\,0}$ $\boxed{1\,|\,0\,|\,1}$ $\boxed{1\,|\,1\,|\,0}$ $\boxed{1\,|\,1\,|\,1}$

  (You can see that M pieces holds 2^M choices, or M bits of info)
  Therefore it is impossible to hold a real number with finite pieces.
  Recall round-off errors.

- This is also binary number

  $\boxed{1\,|\,1\,|\,1\,|\,1\,|\,0\,|\,0\,|\,1\,|\,0}$

  $2^7\ 2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

  $128+64+32+16+0+0+2+0= 242$

Q1: what is this? 1101
Q2: what is this? 1011
$\}\rightarrow$ n-ary number.

Q3: what is the binary form of 19 ?

$19 = 16+2+1 \quad \therefore 2^4+2^1+2^0 \quad \therefore 10011$

A formal way of doing this

$$
\begin{aligned}
19 &= 2\times 9 + 1\\
9 &= 2\times 4 + 1\\
4 &= 2\times 2 + 0\\
2 &= 2\times 1 + 0\\
1 &= 2\times \underset{\text{stop}}{\underline{\underline{0}}} + 1
\end{aligned}
\quad \uparrow \text{ read}
$$

why? can prove mathematically $\underbrace{2\times(\overbrace{2\times(\underbrace{2\times \cdots +0}_{4})+1}^{9})+1}_{19}$

2^3 2^1 2^2

- How to represent a negative number?

can add one bit in the front,

1101 → 01101

− 1101 → 11101

Hence we must assume that a number is 5 bit long.

First bit: sign bit, rest: absolute value.

A commonly used alternative: 2's complement

1101 → 01101

− 1101 → reverse 01101 = 10010 → plus 1 = 10011

The first bit is still sign bit here

The rest do not stand for absolute value directly.

However, the big advantage of 2's complement scheme

$$
\begin{array}{r}
01101\\
+)\ 10011\\
\hline
1\,00000
\end{array}
$$

A − B = A + (−B) !

- information theory

- Developed during 1940s—1950s

Channel

Send $\longrightarrow$ $\boxed{}$ $\longrightarrow$ receive

Security, noise etc.

- resolve uncertainty

	ambiguity to encode
action / no action ?	1 bit
attack A / B / C / D ?	2 bits

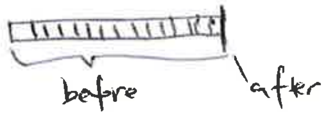
a random card — ambiguity (choices) 54

↓

a specific card — 1

↓

information ? $\log_2 54 - \log_2 1$ bits



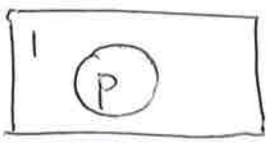
* - unequal choices

3 balls, red 1, red 2, blue.

know red $\log_2 3 - \log_2 2$

know blue $\log_2 3$

probability



- original choices 1
- known p
- information

$\log_2 1 - \log_2 p = \log_2 \frac{1}{p}$

- robustness (how can one make digital signals resistent noise)

2 bit 00 / 01 / 10 / 11

if 1 bit is wrong, the number of 0/1 changes.

add one bit, 1 even odd ?

$00\boxed{0}$ $01\boxed{1}$ $10\boxed{1}$ $11\boxed{0}$

if the 3 bits you receive is not in the above, send again.