

- S7
- ☐ Turing machines & computability
 - ☐ The von Neumann architecture
 - ☐ A rough sketch of CPU
 - ☐ Three types of beta instructions

• Turing machines & computability

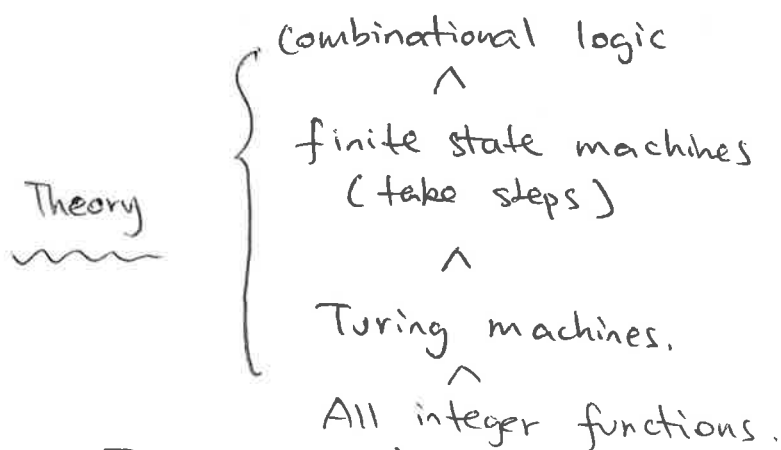
- mathematical functions

$+$ $-$ $*$ $/$ \log \cos factorial ...

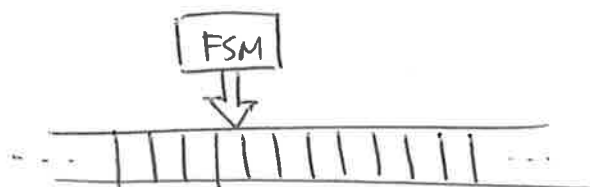
Map $X \mapsto Y$

- integer functions $X, Y = \mathbb{Z}$

- can all integer functions be computed? no.



- Turing machines.



infinite tape

i : data on tape

o : write; move.

Turing machines are more powerful because of infinite tape.

- It has been shown that Turing Machines have the same computation power as recursion and lambda calculus, another two famous computation tools 50s.
- It has been hypothesized that anything computable by a machine is computable by Turing machine.
(Church-Turing thesis)
- It has been proved that there are integer functions that Turing machines cannot compute.

(optional proof)

	I_1	I_2	...
FSM1	O_1	O_2	...
FSM2	O_3	Loop	...
...

Some inputs make Turing machines loop infinitely.

There are an infinite number of rows / columns

$T_k(j) \rightarrow$ FSM_k on I_j ; $\text{halt}(k,j)$ is computable.

{ Can build a Turing machine $T_N(x)$ that loops if $T_x(x)$ halts
halts if $T_x(x)$ loops
 T_N cannot exist, because $T_N(N)$ is dilemma.

Practice — no infinite tapes
therefore,

Turing machines with finite ~~can~~ must be FSM

All computers are FSMs.

- So is it only useful theoretically to study Turing machines?
not really, because of the concept being extended

- Universal Turing machines.

$\forall k, j \quad U(k, j) = T_k(j)$ is computable.
Useful!

One machine, simulates all machines.

k — algorithm (code)

j — data

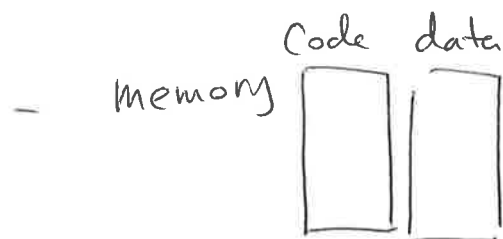
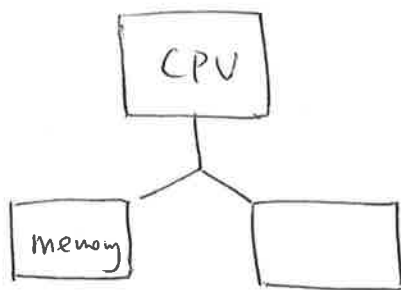
Prototype of computers.

• von Neumann machine

- The current computers are based on von Neumann machines.

- It models computers in a more engineering way.

- machine



- CPU repeatedly load code and execute, reading and writing data

- Devices interact with CPU via memory for data interaction,
and via OS for command interaction.

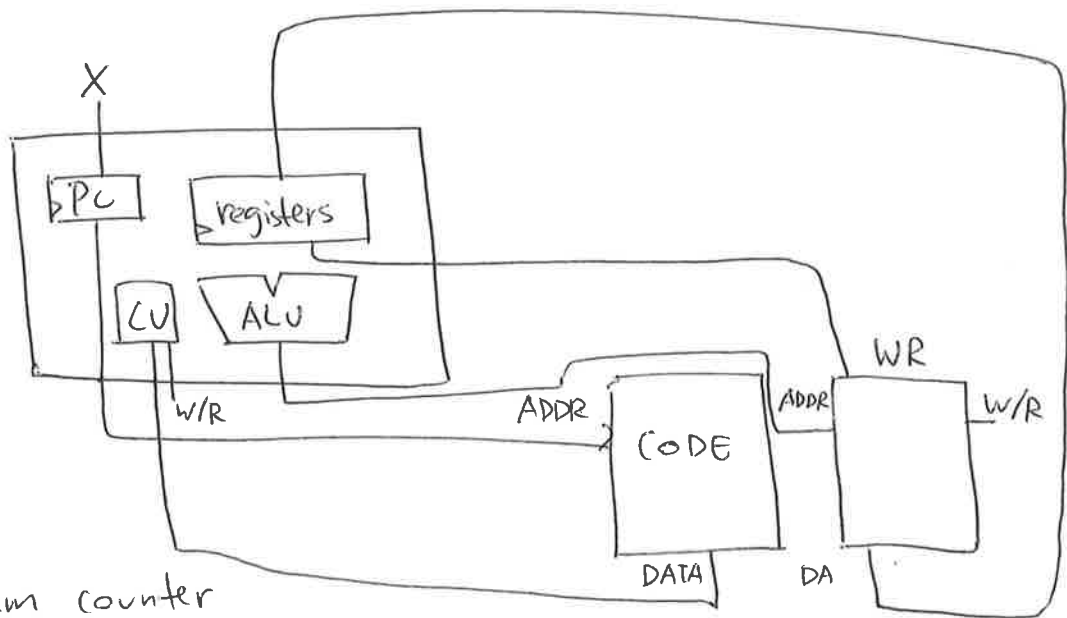
- Is that all?

Yes.

{ Showing to monitor is writing to monitor
controlling robots is writing to robots.
ready from the Internet is from URL

all is data/I/O.

• CPU



- program counter

points to the code section of the memory
loading the current instruction
automatically increment by 1

- control unit

interprets instructions and decides output signals

- what instructions are?

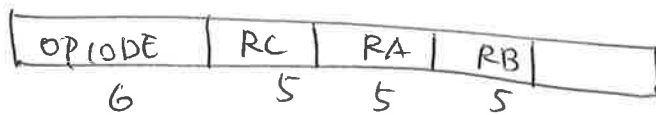
\$ a computer word just like other data
encodes commands for the CU

We study the MIT beta system, word = four bytes

- memory. byte ad.

- Three types of beta instructions

ALU instructions



how many registers?

ADD (RA, RB → RC) . SUB. MUL ...



ADDC (RA, C → RC) ...

Memory loading / storing

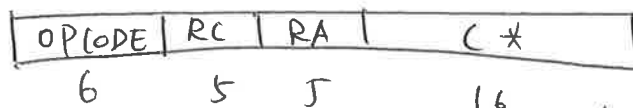


$REG[RC] \leftrightarrow MEM[REG[RA] + SXT(C)]$

LD, ST

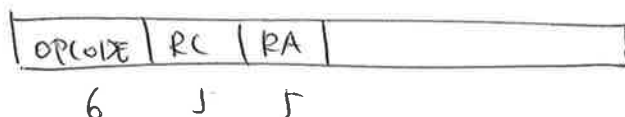
PC changes

$LDR \quad REG[RC] \leftrightarrow MEM[PC + 4 + SXT(C)]$



$PC \rightarrow PC + 4 + SXT(C)$, if $REG[RA] \neq 0$
 $PC \rightarrow PC + 4$

BEQ . BNE



JMP

$PC \rightarrow PC + REG[RA]$

$PC \rightarrow PC + 4$

