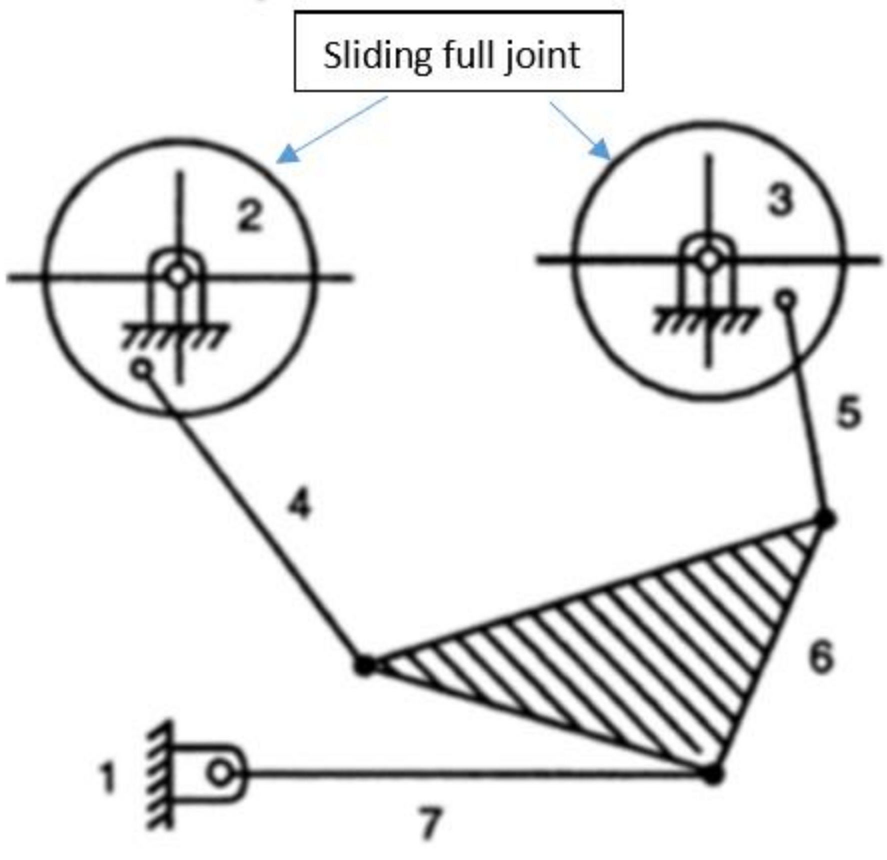
01.110: Computational Fabrication Summer 2017

Exercise 1

Determine the degrees of freedom for the system shown below

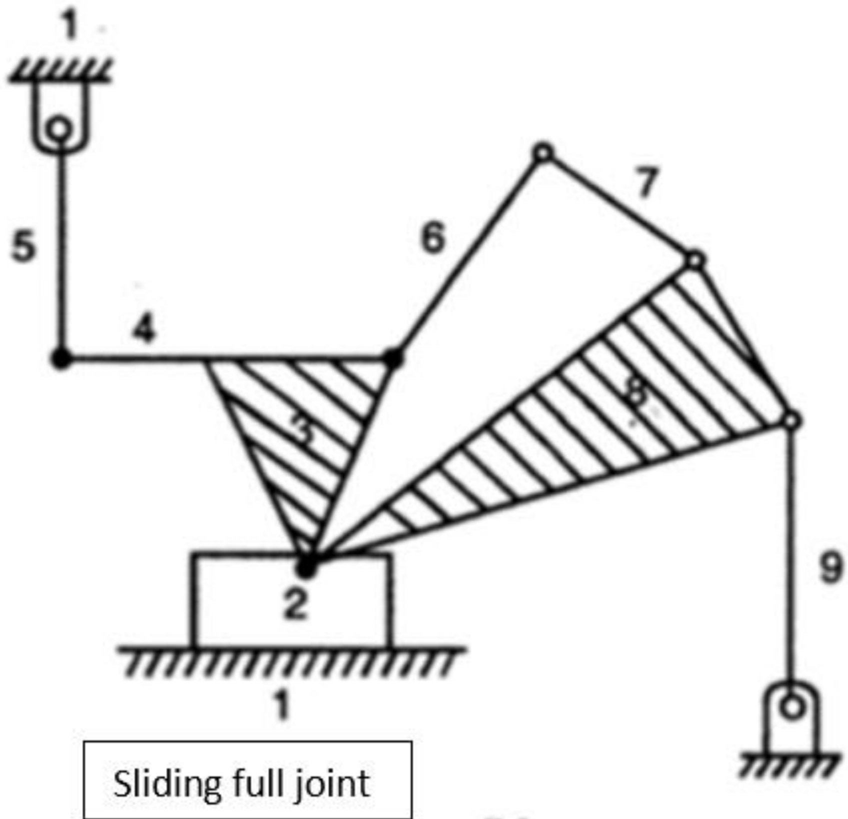


(a)

(a) Number of links, $n = 7$

Number of joints, $j = 8$

Degree of freedom, $F = 3n - 2j - 3 = 3(7) - 2(8) - 3 = 2$

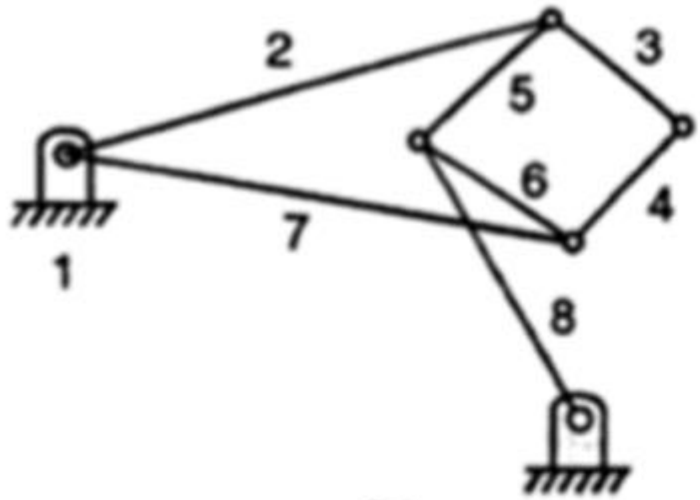


(b)

(b) Number of links, $n = 9$

Number of joints, $j = 11$

Degree of freedom, $F = 3n - 2j - 3 = 3(9) - 2(11) - 3 = 2$



(b)

(c) Number of links, $n = 8$
(the graph below)

Number of joints, $j = 10$

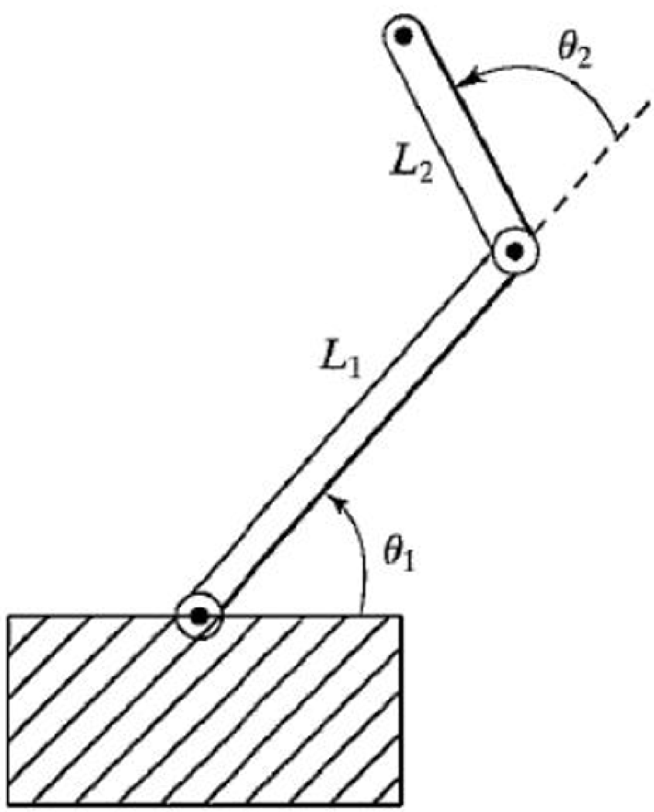
Degree of freedom, $F = 3n - 2j - 3 = 3(8) - 2(10) - 3 = 1$

Exercise 2

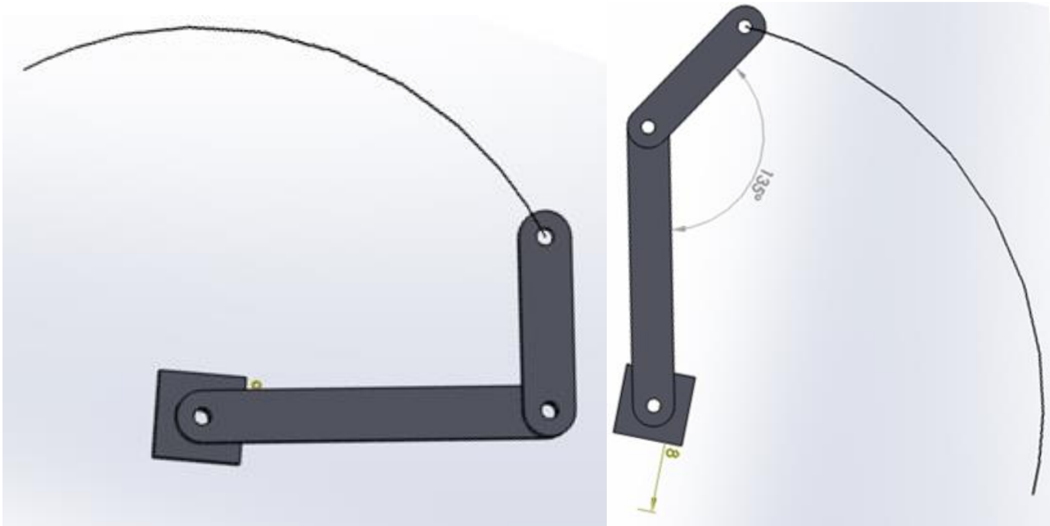
The following figure shows a two-link planar arm with rotary joints. For this arm, the second link is a half long of the first. The joint limits are as follows:

$$\begin{cases} 0 < \theta_1 < \pi \\ -\pi/2 < \theta_2 < \pi \end{cases}$$

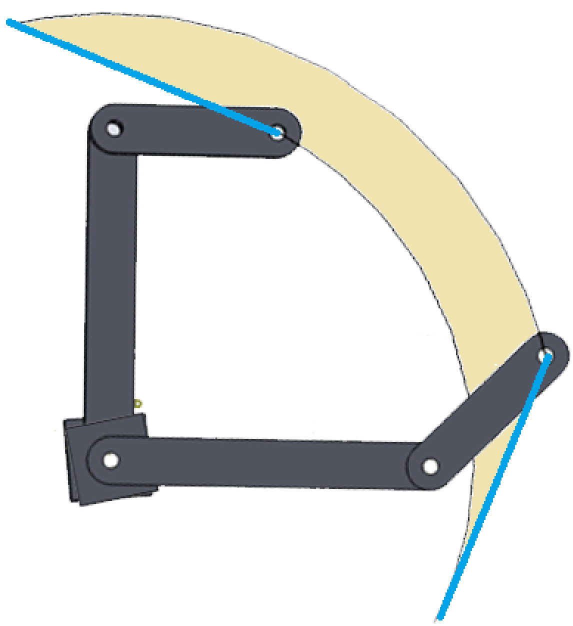
Sketch the reachable workspace (an area) of the tip of link 2. You may assume that the box will not affect the movement of the linkage [You may use Matlab or solidworks to answer this question].



Using Solidworks, I set the motion of the joint between L1 and the box to be oscillating **90 degrees** (theta1 is between 0 and PI), and check the trace path when theta2 is **-45 degrees**, or **90 degrees** (theta2 is between -PI/2 and PI).



Overlapping the two, we get:

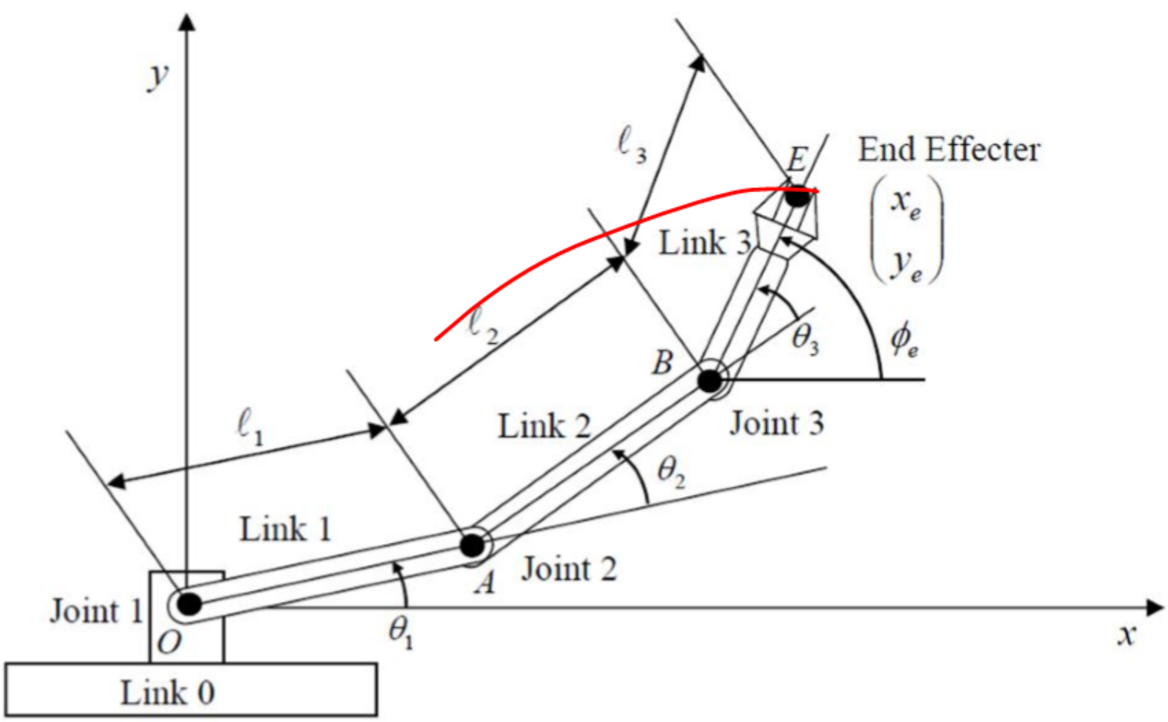


The yellow region is thus the reachable workspace.

Exercise 3

The RRR planar manipulator in Figure below is composed of three revolute joints, θ_1, θ_2 and θ_3 with full range of motion.

a. Find the algebraic relationship between the joint coordinates $(\theta_1, \theta_2, \theta_3)$ of the manipulator and the Cartesian coordinates (x_e, y_e, ϕ_e) of the end effector at E. Is this mapping 1 to 1? i.e, a choice of $(\theta_1, \theta_2, \theta_3)$ implies a unique (x_e, y_e, ϕ_e).
b. Sketch by hand the workspace and the dexterous workspace of the robot (case of $l_1 = l_2 = l_3$ and $l_1 \neq l_2 \neq l_3$)
c. Is the inverse kinematics map 1 to 1? Use a sketch argument to illustrate your answer.
d. Find the algebraic expression for the inverse kinematics and discuss the multiplicity of solutions.
e. Is there any benefit to this manipulator over RR (2 revolute joints) manipulator?



(a) $x_e = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3)$

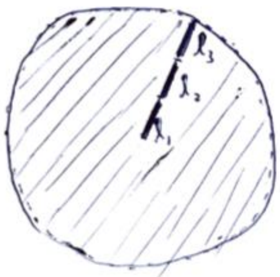
$y_e = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3)$

$\Phi_e = \theta_1 + \theta_2 + \theta_3$

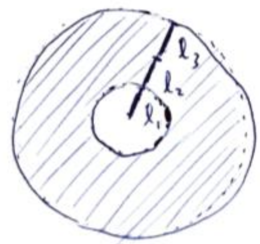
This mapping is one-to-one.

(b) when $l_1 = l_2 = l_3$:

Workspace :

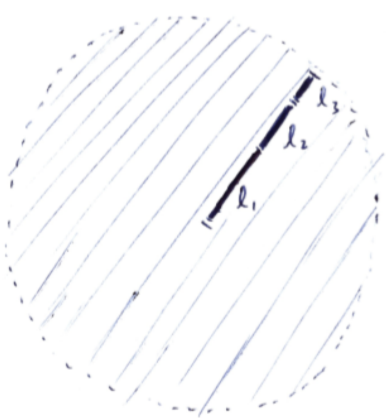


Dexterous workspace:

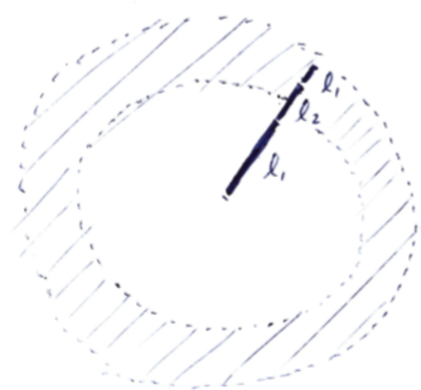


(b) when $l_1 \neq l_2 \neq l_3$

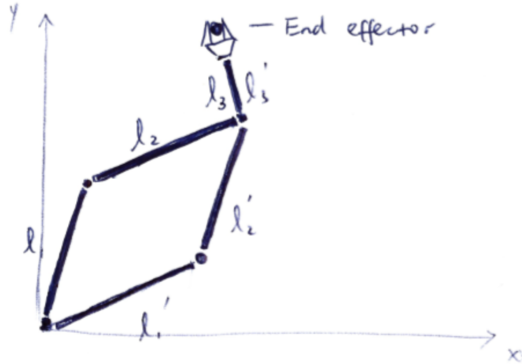
Workspace:



Dexterous workspace:



(c) The inverse kinematics is not one-to-one. For instance, the following two link & joint combinations will result in the same end effector position:



Let γ be the angle between link 1 (l_1) and the line connecting origin O and joint betw. link 2 (l_2) and link 3 (l_3)

(d) Let ~~&~~ α be the angle between link 1 (l_1) and x-axis, Let β be the angle between link 1 (l_1) and link 2 (l_2), and let (x_w, y_w) be the position of the joint between link 2 (l_2) and link 3 (l_3).

Then:

$$\begin{cases} \theta_1 = \tan^{-1}\dfrac{y_w}{x_w} - \cos^{-1}\dfrac{x_w^2 + y_w^2 + l_1^2 \cancel{4} - l_2^2}{2 l_1 \sqrt{x_w^2 + y_w^2}} \\ \theta_2 = \pi - \cos^{-1}\dfrac{l_1^2 + l_2^2 - x_w^2 - y_w^2}{2 l_1 l_2} \\ \theta_3 = \phi_e - \theta_1 - \theta_2 \end{cases}$$

Since there are multiple solutions for $\cos^{-1} x$ (for instance, both $45°$ and $-45°$ are solutions for $\cos^{-1}(\frac{\sqrt{2}}{2})$), there will be multiple solutions for inverse kinematics.

(e) The degree of freedom of RRR is $3(4) - 2(3) - 3 = 3$, whereas the degree of freedom of RR is $3(3) - 2(2) - 3 = 2$.

This shows that the RRR has benefits over RR in the way that there is greater degree of freedom in RRR, a greater region may be reached by RRR end effector.