

Principles of 2D Printing

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Plan for Today

- 2D Printing
- Halftoning
- Digital Halftoning

History of Printing

- Woodblock printing (AD 220)
- Movable type printing (AD 1040)
 - Developed in Asia (ceramic, wood, metal)
 - Metal movable type in Europe (Johannes Gutenberg, 1439)
- Type-founding and typesetting



Lithography (1796)

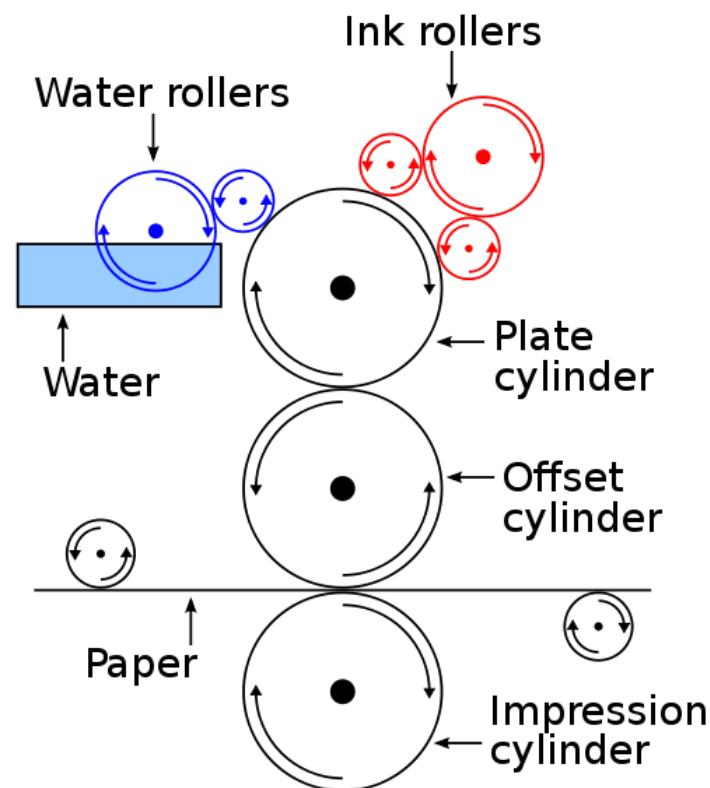
- Plate is a medium to transfer ink to a blank paper sheet
- Plate is divided into
 - **hydrophilic** regions accept water but repel ink
 - **hydrophobic** regions repel water and accept ink



Offset Printing (1870s - now)

- Inked image is transferred (offset) from a plate to rubber blanket, then to the printing surface
- High-quality, high-speed, low-cost for high-volume runs
- Requires fabricating plates

solopress



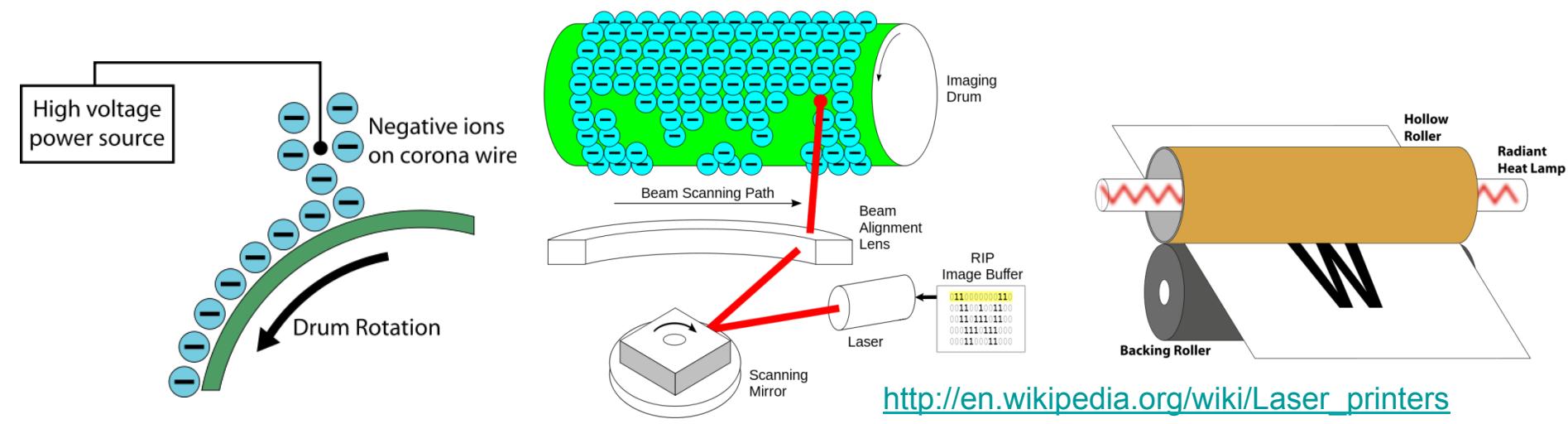
Digital Printing

- Printing from a digital image directly to a physical medium
- No printing plates
 - Faster, less expensive for a single copy or short runs
- Most popular methods
 - Inkjet printers
 - Laser printers



Laser Printing

- **Charging** - negative charge is applied to a photoreceptor (drum)
- **Exposing** - laser beam neutralizes the charge
- **Developing** - toner particles are attracted to areas touched by the laser
- **Transferring** - photoreceptor is pressed against the paper
- **Fusing** - heat and pressure bond the plastic powder to the paper
- **Cleaning** - excess toner is removed from the photoreceptor

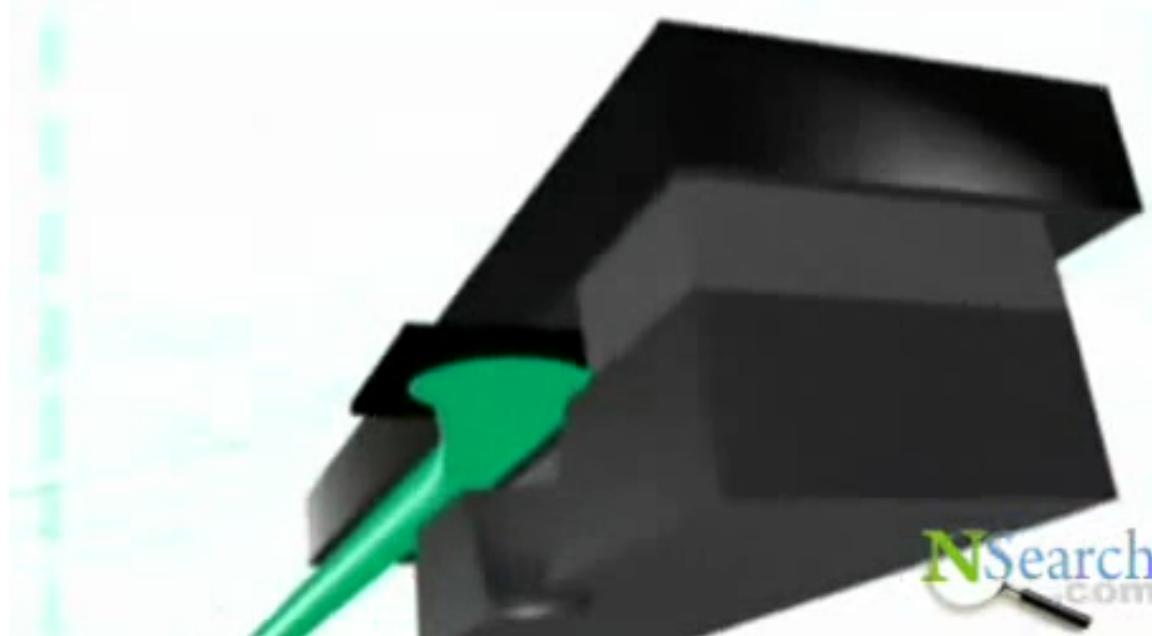


Laser Printing

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Inkjet Printing - Thermal (Canon, HP, Lexmark)

- Use a heating element in an ink-filled chamber behind each nozzle
- A pulse of current is passed through the heating element
- Vaporized ink in the chamber leads to a pressure change
- A droplet is jettted

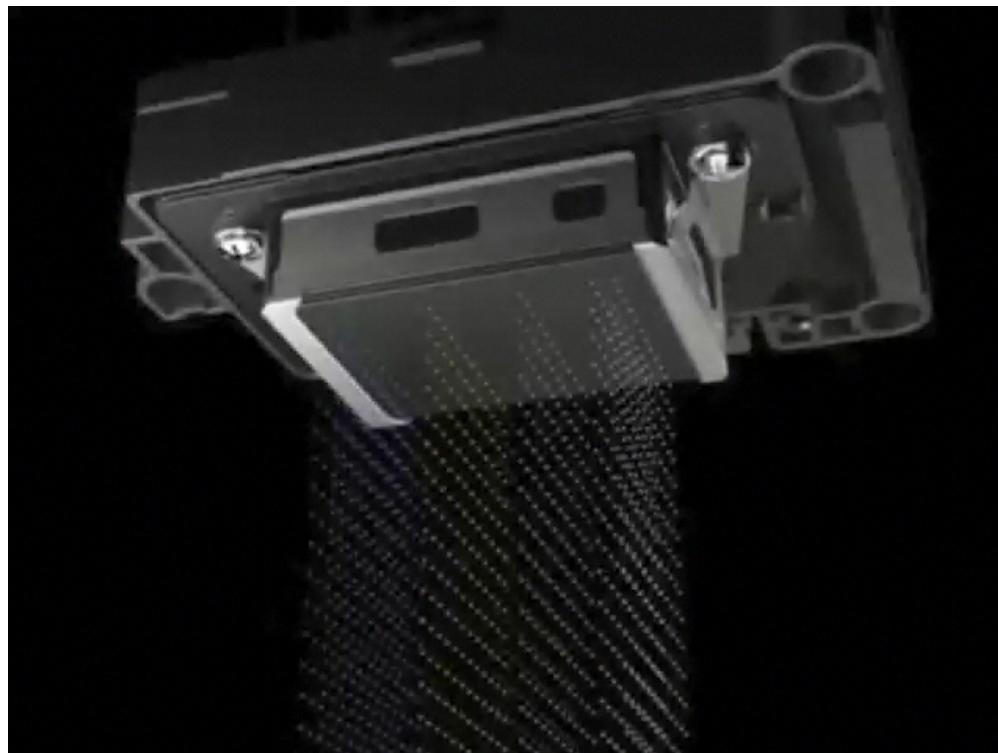


General principle:
Pressure to jet the ink



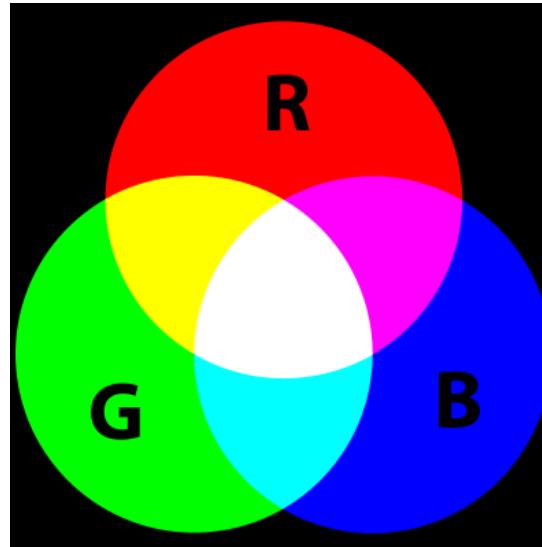
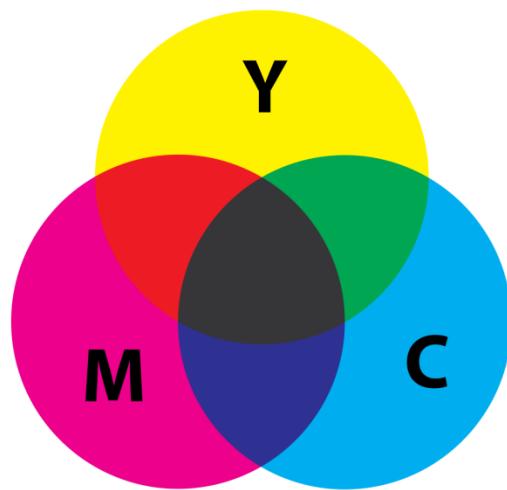
Inkjet Printing - Piezoelectric (Epson, Brother)

- Use piezoelectric material in an ink-filled chamber behind each nozzle
- Piezoelectric material changes shape when voltage is applied
- Generated pressure pulse ejects a droplet



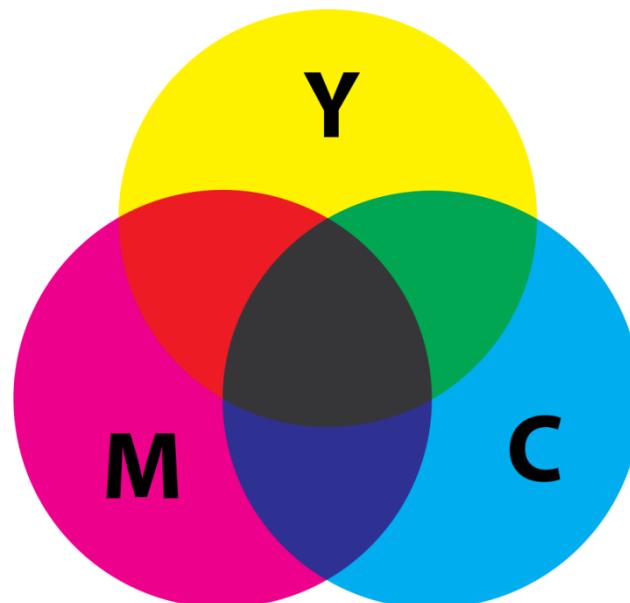
Subtractive vs. Additive Color

- Subtractive color
 - We start with white light (e.g., white paper)
 - Colored inks act as filters that remove light
- Additive color
 - We start with no light (e.g., darkness)
 - Light sources of different wavelengths are added



Color Printing Uses Subtractive Color Model

- CMYK color model
 - Cyan (C) - absorbs red, transparent to green and blue
 - Magenta (M) - absorbs green, transparent to red and blue
 - Yellow (Y) - absorbs blue, transparent to red and green
 - Black ink (K - key) - reduces unwanted color tints, improves sharpness, reduces cost



Cyan: 0,255, 255

Yellow: ?, ?, ?

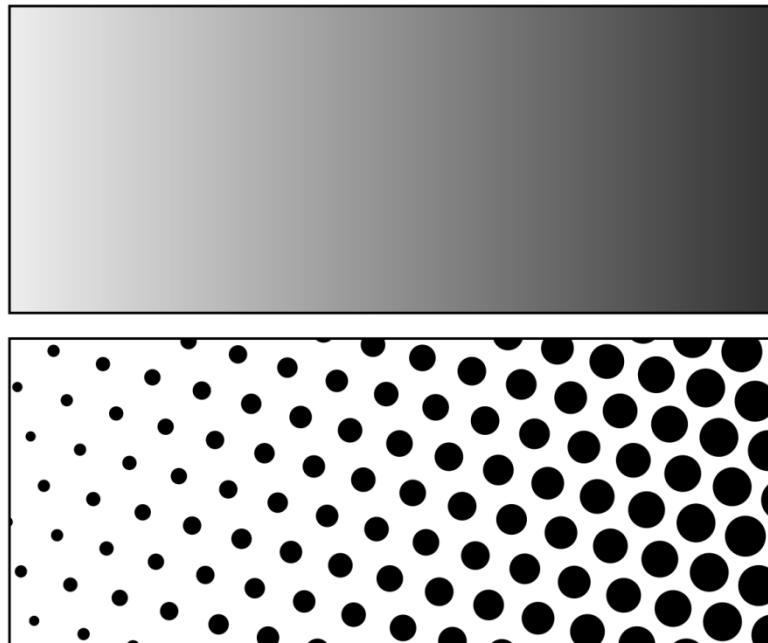
Cyan mix Yellow:
?, ?, ?

Plan for Today

- 2D Printing
- Halftoning
- Digital Halftoning

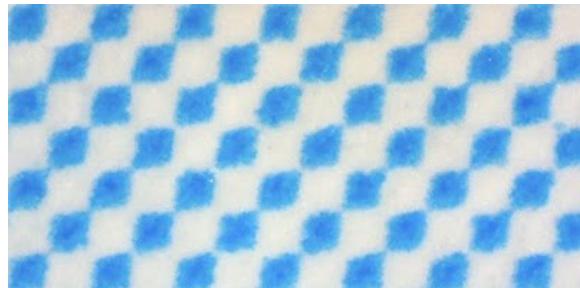
Halftoning

- A method to simulate a continuous tone through the use of dots that vary in **size, shape, spacing**
- E.g., continuous tone image contains infinite range of greys, the halftone process reduces it to an image with only one color of ink

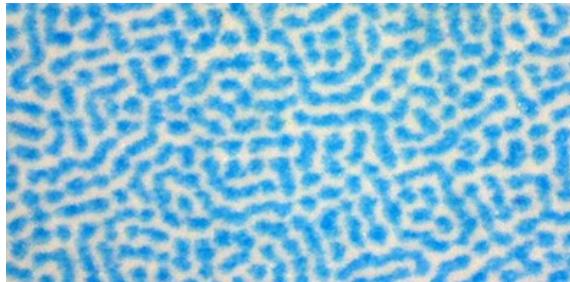


Halftoning

- Traditional screening
 - Amplitude Modulation (AM) - regular grid of dots, dots vary in size
 - Frequency Modulation (FM) - variable dot density
- Dot shapes
 - Round, elliptical, square



AM screen (elliptical)



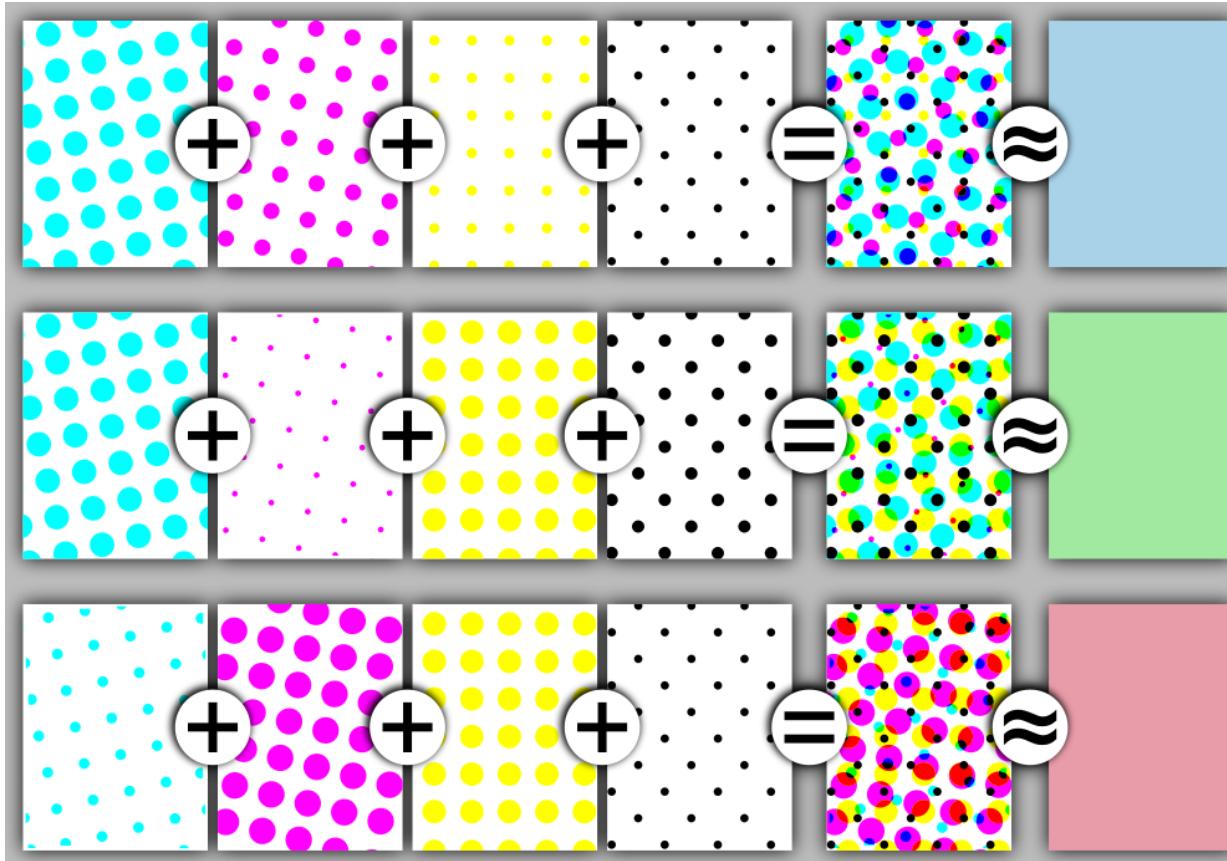
FM screen



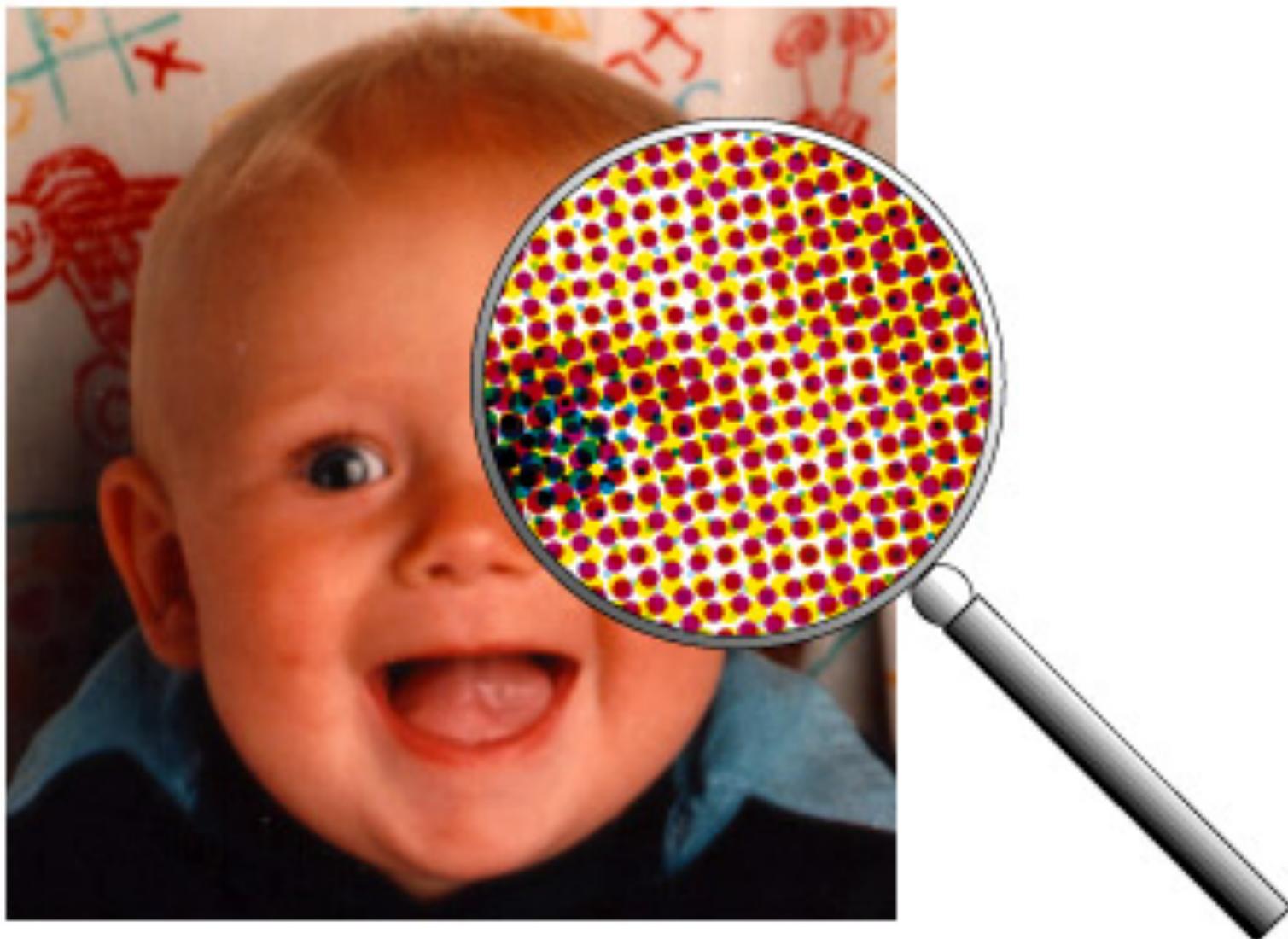
The first printed photo using a halftone, December 2, 1873.

Color Halftoning

- Multiple screens (CMYK)
- Screens are rotated to reduce moiré pattern



Color Halftoning

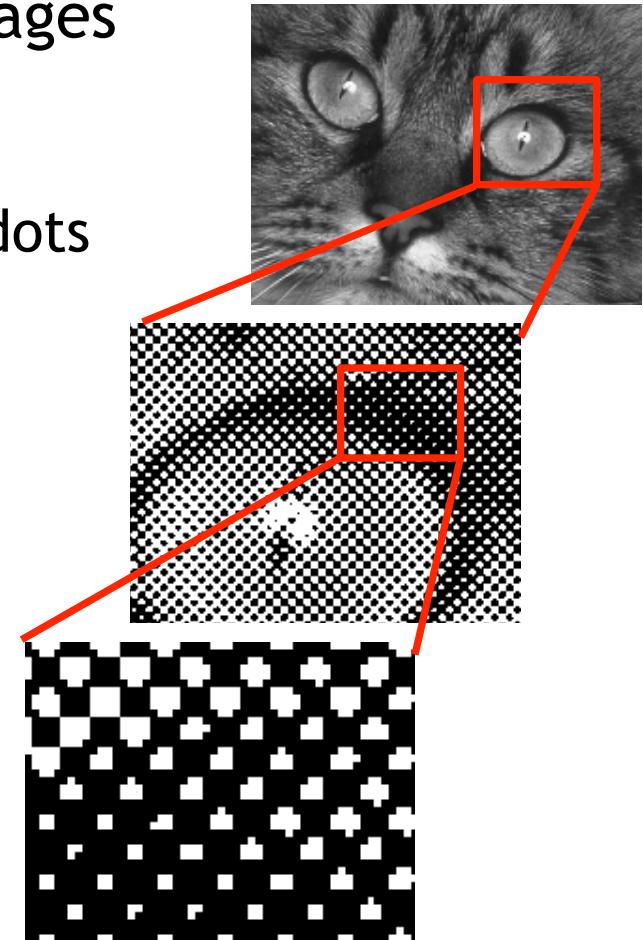


Plan for Today

- 2D Printing
- Halftoning
- Digital Halftoning
 - Point processes
 - Neighborhood processes
 - Global optimization

Digital Halftoning (since 1970s)

- To convert grayscale to bi-level images
- Used by all printers
 - Full color image -> CMY+Black ink dots
 - Grayscale image -> Black ink dots
- Digital world
 - Fixed pixel grid
 - $[0, 255] \rightarrow \{ \text{BLACK}, \text{WHITE} \}$

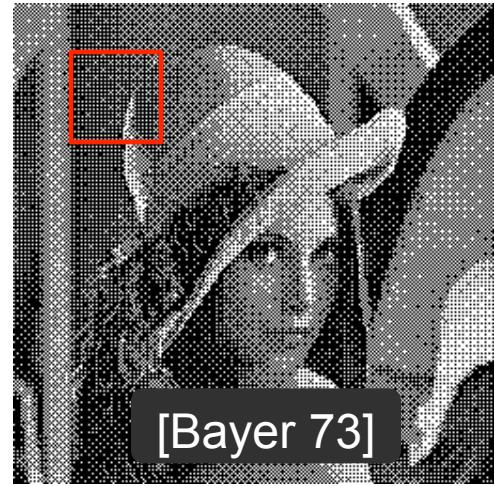


Classification

- General classification of halftoning [Model-based digital halftoning. Pappas et al. 03]
 - Class I: Point processes
 - Class II: Neighborhood processes
 - Class III: Global optimization
- Methods from a same class generally share
 - Quality
 - Speed

Class I: Point Processes

- One pixel at a time
 - Compare to threshold mask
- Speed
 - Ignore pixel interactions
 - Class I → **very fast**
- Quality
 - Regular patterns
 - Noise
 - Class I → **poor overall quality**



Thresholding

Original image



Simple threshold



$$v(x, y) = \text{trunc}(\hat{v}(x, y) + n)$$

Errors are low spatial frequencies.

Thresholding with Random Modulation (Roberts 1961)

- First add noise $\hat{v}(x, y) = \text{trunc}(K \times v(x, y) + \text{noise}(x, y))$
- Then quantize $0 \leq \text{noise} < 1$
- Pre-compute pseudo-random function and store in table

Thresholding



Thresholding + Noise

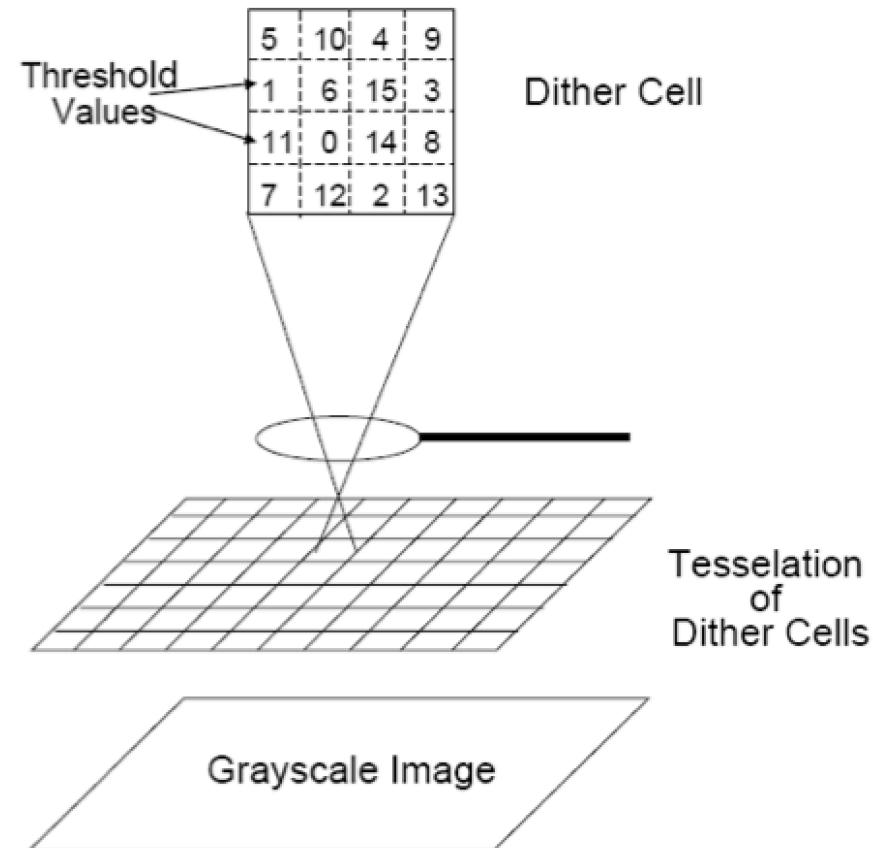


Dithering

- Each pixel produces a *quantization* error
- The quality of the result may be improved by adjusting the threshold locally, so that adjacent pixels in small areas are quantized with different thresholds.
- This reduces the **average** local quantization error.
Matrices of these threshold are called **dither** matrices

Dithering

- Every pixel in a region is thresholded using a different threshold value.



Thresholding + Noise

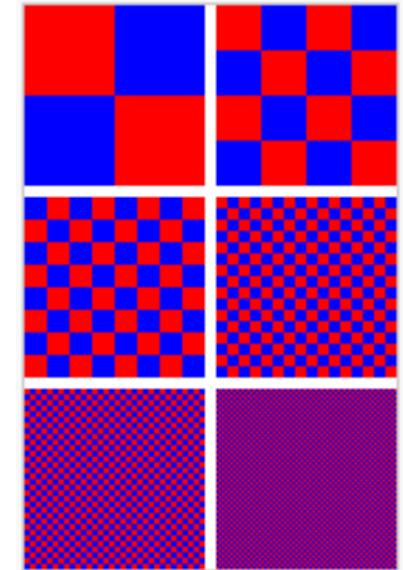


Dithering



Dithering

- Human visual system integrates information
- The human perception system do not have equal response to all spatial frequencies.
- As the spatial frequencies become higher and higher, our ability to perceive the pattern will be lower and lower.



Dithering

- Break the image into small blocks
- Define a *threshold matrix*
 - Use a different threshold for each pixel of the block
 - Compare each pixel to its own threshold
- The thresholds can be **clustered**
- The thresholds can be **dispersed**

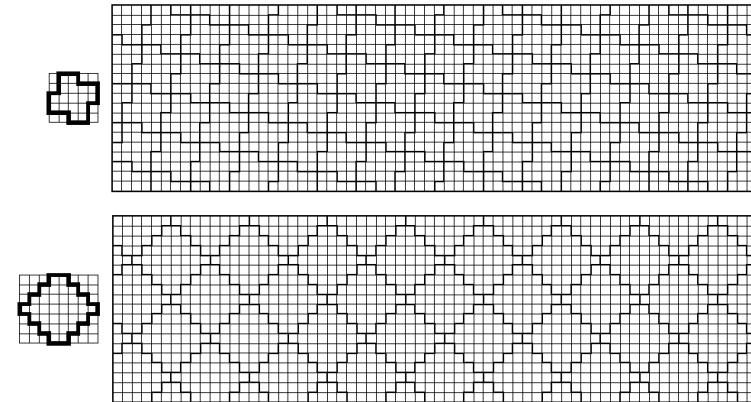
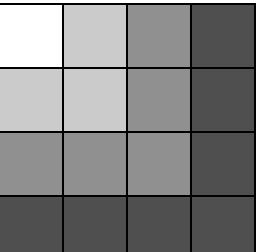
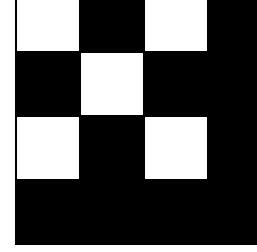


Image block	Threshold matrix	Result
 $\begin{bmatrix} 1 & 0.75 & 0.5 & 0.25 \\ 0.75 & 0.75 & 0.5 & 0.25 \\ 0.5 & 0.5 & 0.5 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \end{bmatrix}$	$\frac{1}{16} \begin{bmatrix} 2 & 16 & 3 & 13 \\ 14 & 6 & 11 & 7 \\ 4 & 10 & 1 & 15 \\ 12 & 8 & 9 & 5 \end{bmatrix}$	 $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Dithering Algorithm

For all X

For all Y

```
v = getImageValue(x, y)  
i = x mod 3  
j = y mod 3  
if v >= M[i, j] then  
    setPixel(x, y, WHITE)  
else  
    setPixel(x, y, BLACK)
```

The dithering matrix M (3x3)

3	7	5
6	1	2
9	4	8

Dithering Example

3	7	5
6	1	2
9	4	8

8	7	5	1	6	2	4	5	3	2
6	8	3	7	2	2	5	4	3	2
9	4	8	3	7	4	5	2	6	4
3	8	9	7	7	2	2	1	3	2
6	9	2	9	7	4	3	2	2	4
9	4	8	8	8	4	4	8	4	4

3	7	5
6	1	2
9	4	8

Dithering matrix

2	1	3
3	2	2
4	8	4

Image

?

Binary image

Dithering Example

3	7	5
6	1	2
9	4	8

Dithering matrix

8	7	5	1	6	2	4	5	3	2
6	8	3	7	2	2	5	4	3	2
9	4	8	3	7	4	5	2	6	4
3	8	9	7	7	2	2	1	3	2
6	9	2	9	7	4	3	2	2	4
9	4	8	8	8	4	4	8	4	4

3	7	5
6	1	2
9	4	8

Image

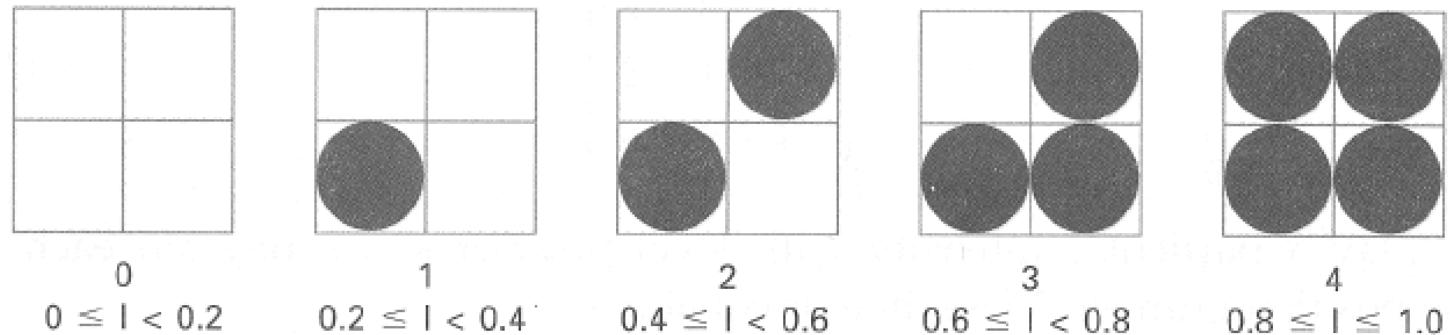
2	1	3
3	2	2
4	8	4

Binary image

0	0	0
0	1	1
0	1	0

Clustered-dot Dithering

- Values in a threshold matrix are set to produce a single cell
 - User cluster of pixels to represent intensity
- The cell size depends on the input value
 - Trade spatial resolution for intensity resolution



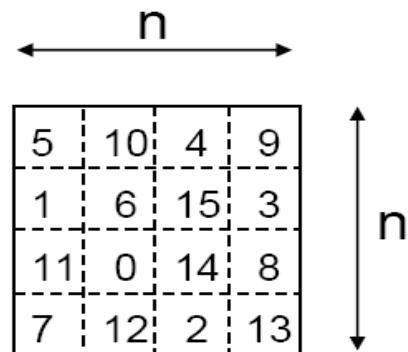
Clustered-dot Dithering

- How many intensities (gray scale values) in a $n \times n$ cluster?

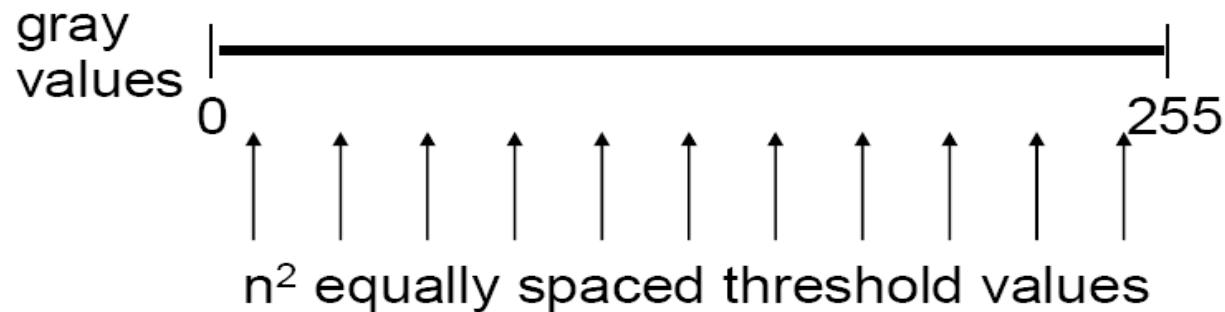
Clustered-dot Dithering

- How many intensities (gray scale values) in a $n \times n$ cluster?

Dither Cell



$n^2 + 1$ gray levels

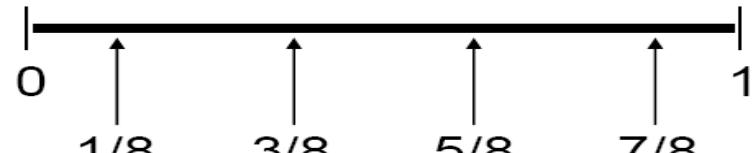


Clustered-dot Dithering

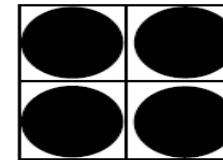
- Example:

dither cell

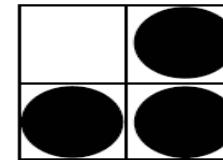
1/8	5/8
7/8	3/8



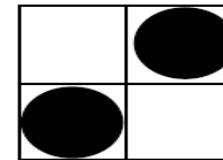
gray = 0...0.125



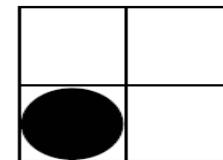
gray = 0.125...0.375



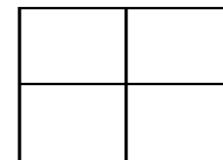
gray = 0.375...0.625



gray = 0.625...0.875

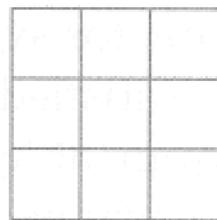


gray = 0.875...1.0

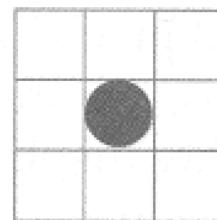


Clustered-dot Dithering

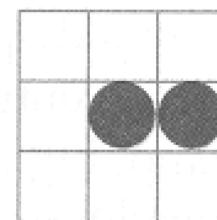
- Example:



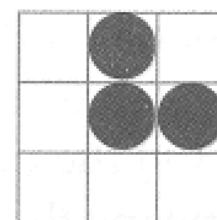
0
 $0 \leq I < 0.1$



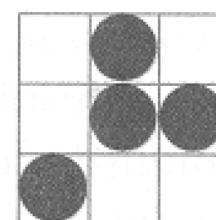
1
 $0.1 \leq I < 0.2$



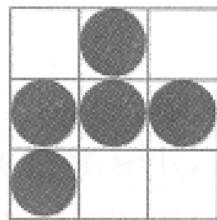
2
 $0.2 \leq I < 0.3$



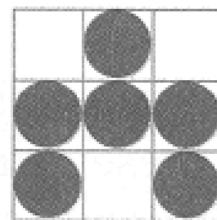
3
 $0.3 \leq I < 0.4$



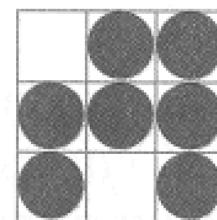
4
 $0.4 \leq I < 0.5$



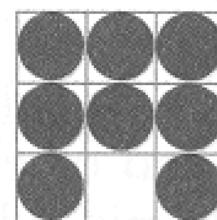
5
 $0.5 \leq I < 0.6$



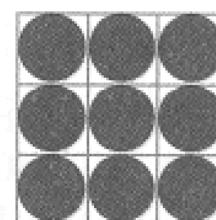
6
 $0.6 \leq I < 0.7$



7
 $0.7 \leq I < 0.8$



8
 $0.8 \leq I < 0.9$



9
 $0.9 \leq I \leq 1.0$

Black and white dots here are arbitrary,
e.g., it can represent the intensity level or the order of turning on/off pixels

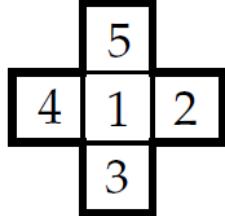
Clustered-dot Dithering

- If the consecutive thresholds are located in spatial proximity, it is called clustered-dot Dithering
- AM or FM halftoning?

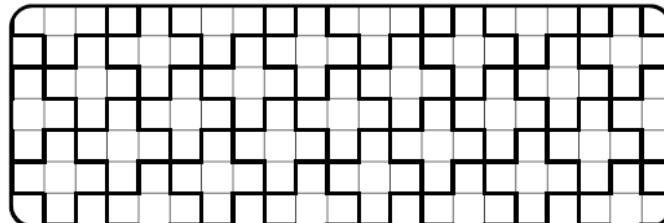
62	57	48	36	37	49	58	63
56	47	35	21	22	38	50	59
46	34	20	10	11	23	39	51
33	19	9	3	0	4	12	24
32	18	8	2	1	5	13	25
45	31	17	7	6	14	26	40
55	44	30	16	15	27	41	52
61	54	43	29	28	42	53	60

Clustered-dot Dithering

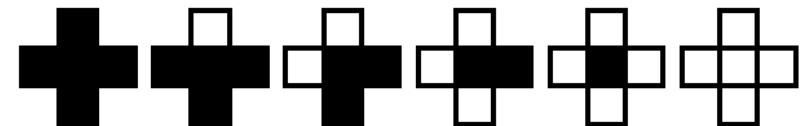
- Values in a threshold matrix are set to produce a single cell
 - User cluster of pixels to represent intensity
- The cell size depends on the input value
 - Trade spatial resolution for intensity resolution
- Properties
 - Dot shape, dot center of gravity motion, dot fusion, border to area ratio



Matrix with
5 values

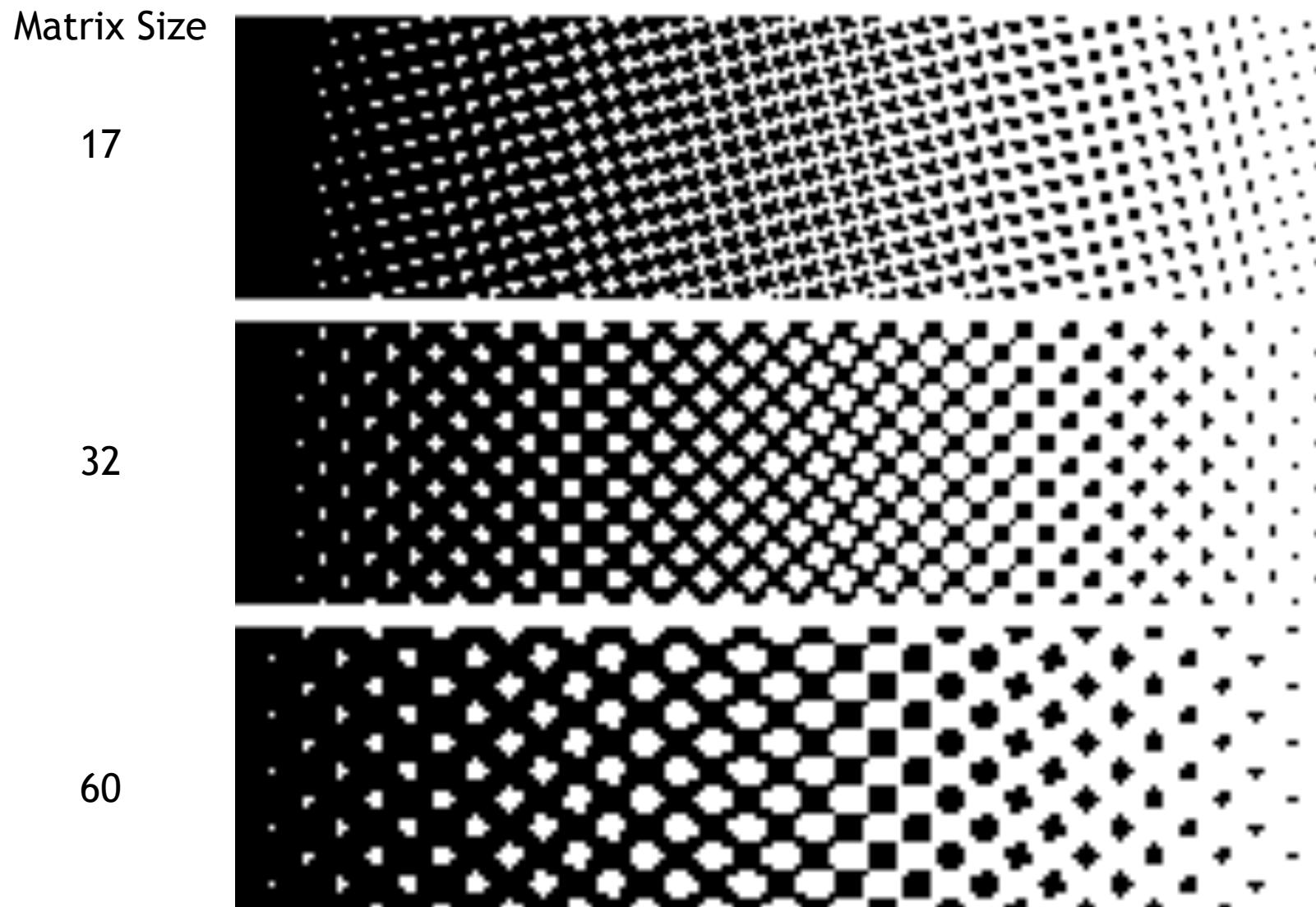


The corresponding
tiling

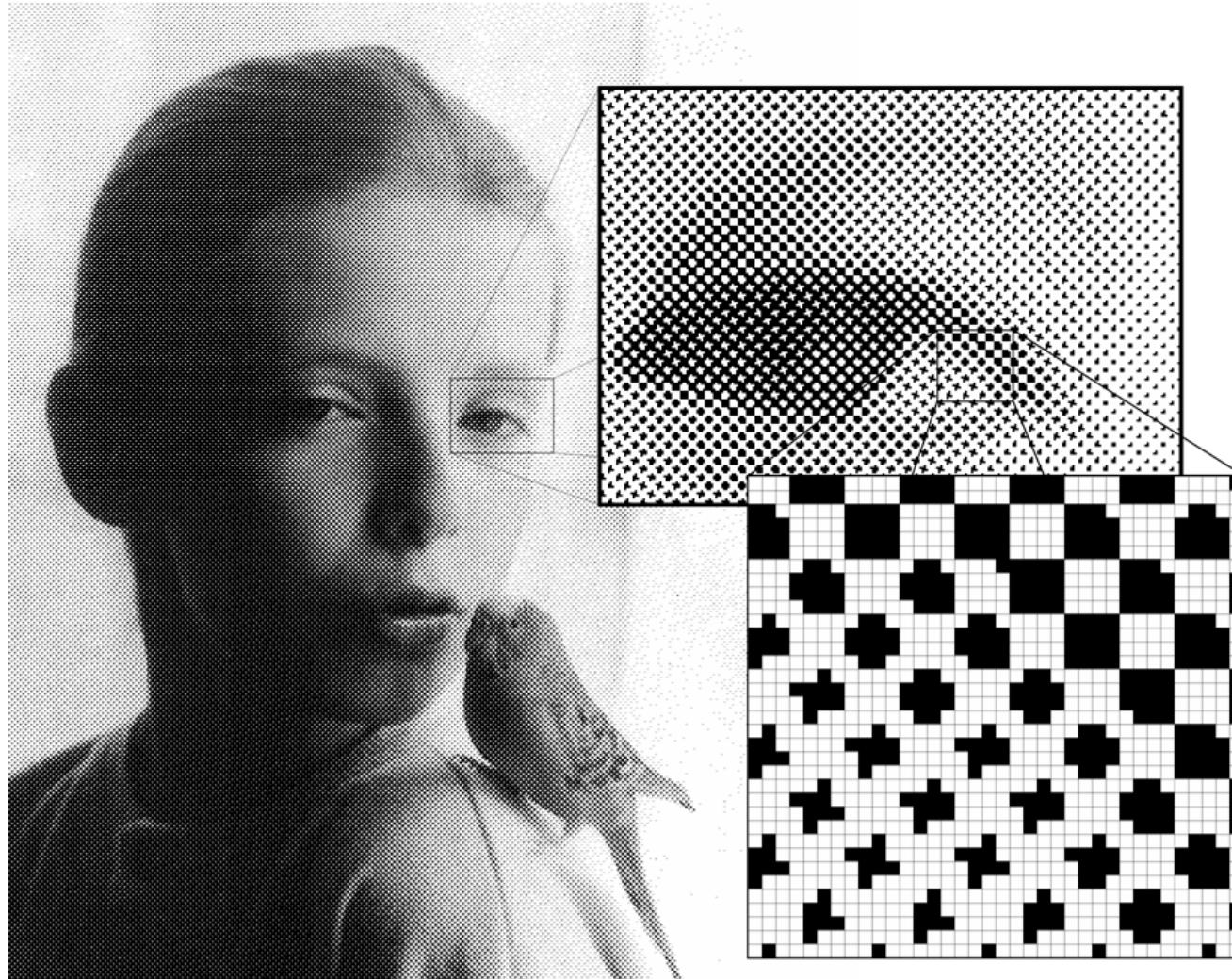


Micro dot selection for all
six possible tones

Clustered-dot Dithering Ramps



Clustered-dot Dithering Example



Clustered-dot Dithering - Supercells

- ~100 levels required for printing -> 10×10 matrix
- Supercell concatenates several threshold matrices
- Each threshold value is scaled/reassigned

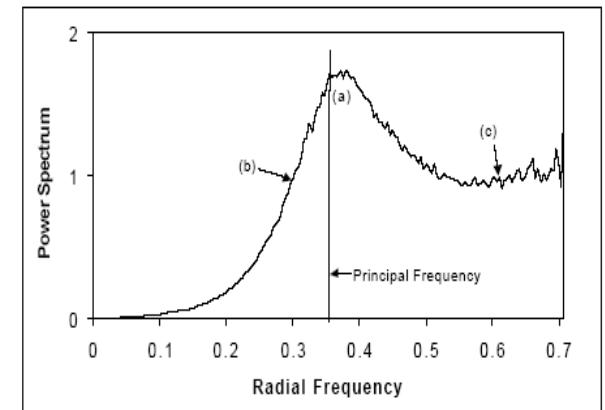
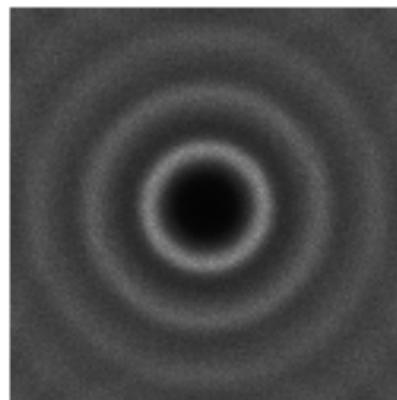
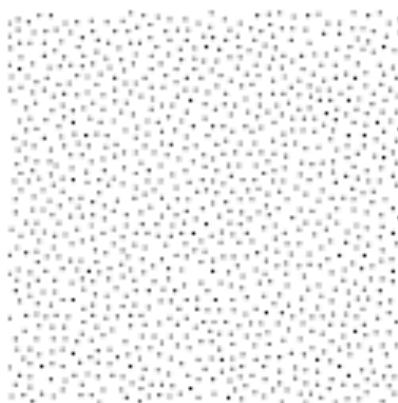
base matrix M	concatenation of 9 copies of M	M_s , after scaling threshold values
$\begin{matrix} 4 & 8 & 1 \\ 7 & 9 & 5 \\ 3 & 6 & 2 \end{matrix}$	$\begin{matrix} 4 & 8 & 1 & 4 & 8 & 1 & 4 & 8 & 1 \\ 7 & 9 & 5 & 7 & 9 & 5 & 7 & 9 & 5 \\ 3 & 6 & 2 & 3 & 6 & 2 & 3 & 6 & 2 \\ 4 & 8 & 1 & 4 & 8 & 1 & 4 & 8 & 1 \\ 7 & 9 & 5 & 7 & 9 & 5 & 7 & 9 & 5 \\ 3 & 6 & 2 & 3 & 6 & 2 & 3 & 6 & 2 \\ 4 & 8 & 1 & 4 & 8 & 1 & 4 & 8 & 1 \\ 7 & 9 & 5 & 7 & 9 & 5 & 7 & 9 & 5 \\ 3 & 6 & 2 & 3 & 6 & 2 & 3 & 6 & 2 \end{matrix}$	$\begin{matrix} 28 & 64 & 1 & 35 & 71 & 8 & 30 & 66 & 3 \\ 55 & 73 & 37 & 62 & 80 & 44 & 57 & 75 & 39 \\ 19 & 46 & 10 & 26 & 53 & 17 & 21 & 48 & 12 \\ 31 & 67 & 4 & 29 & 65 & 2 & 33 & 69 & 6 \\ 58 & 76 & 40 & 56 & 74 & 38 & 60 & 78 & 42 \\ 22 & 49 & 13 & 20 & 47 & 11 & 24 & 51 & 15 \\ 34 & 70 & 7 & 32 & 68 & 5 & 36 & 72 & 9 \\ 61 & 79 & 43 & 59 & 77 & 41 & 63 & 81 & 45 \\ 25 & 52 & 16 & 23 & 50 & 14 & 27 & 54 & 18 \end{matrix}$

Dispersed-dot Dithering

- Frequency Modulation (FM) halftoning
- The quality defined by the pattern of micro dots **for each** possible input constant value
- Each pattern should be as evenly dispersed as possible
 - **blue noise distribution**
- The distribution of threshold values inside the matrix is crucial to obtain a good distribution of micro dots

Blue Noise

- A pleasant distribution of points
 - I Points are away from each other with a nearly regular distance
 - II No alignment
 - In frequency domain: a preferred frequency, no preferred direction
- Blue noise (in frequency domain)
 - Radially symmetric
 - Null values for low frequencies, strong annulus with the frequency corresponding to the preferred distance period, white noise for high frequencies

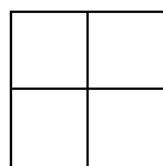


Bayer Ordered Dither Patterns

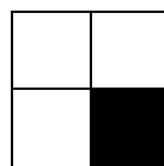
- The threshold matrix can be represented by an “index matrix”
 - The index matrix determines the order in which dots are “turned on” (change from white to black) as the images becomes darker

$$I_2(i, j) = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$$

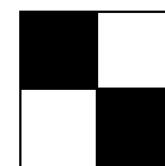
- Pixels are turned on (white to dark) in the following order:



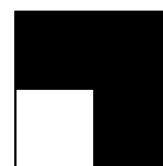
0



1



2



3



4

Bayer Ordered Dither Patterns

- Bayer's optimum index matrix (1973) can be defined recursively

$$I_2(i, j) = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$$

$$I_{2n} = \begin{bmatrix} 4 * I_n + 1 & 4 * I_n + 2 \\ 4 * I_n + 3 & 4 * I_n \end{bmatrix}$$

- Examples

1	2
3	0

5	9	6	10
13	1	14	2
7	11	4	8
15	3	12	0

2×2

4×4

21	37	25	41	22	38	26	42
53	5	57	9	54	6	58	10
29	45	17	33	30	46	18	34
61	13	49	1	62	14	50	2
23	39	27	43	20	36	24	40
55	7	59	11	52	4	56	8
31	47	19	35	28	44	16	32
63	15	51	3	60	12	48	0

8×8

Bayer Ordered Dither Patterns

B_4

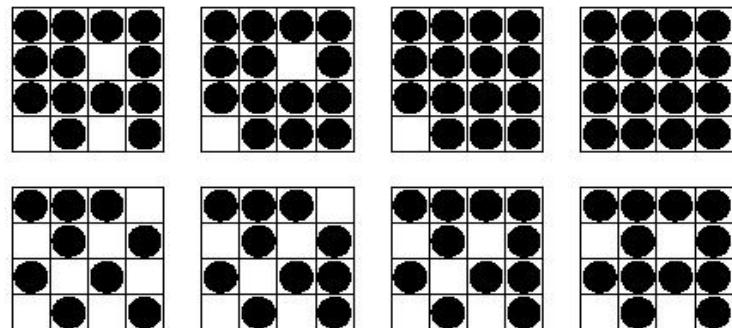
0	8	2	10
12	4	14	6
3	11	1	9
15	7	13	5

B_3

3	6	2
7	0	5
4	8	1

B_2

0	2
3	1



Exercise

Bayer Ordered Dither Patterns

B_4

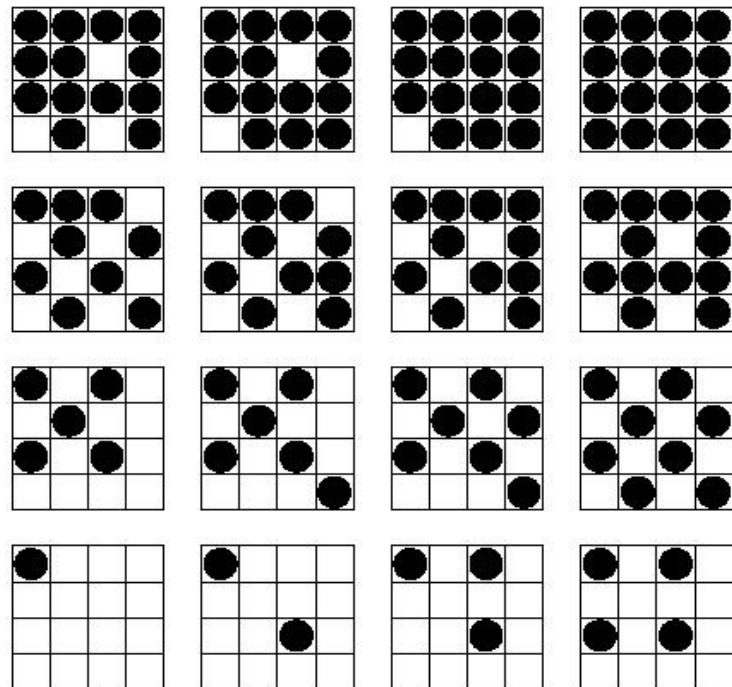
0	8	2	10
12	4	14	6
3	11	1	9
15	7	13	5

B_3

3	6	2
7	0	5
4	8	1

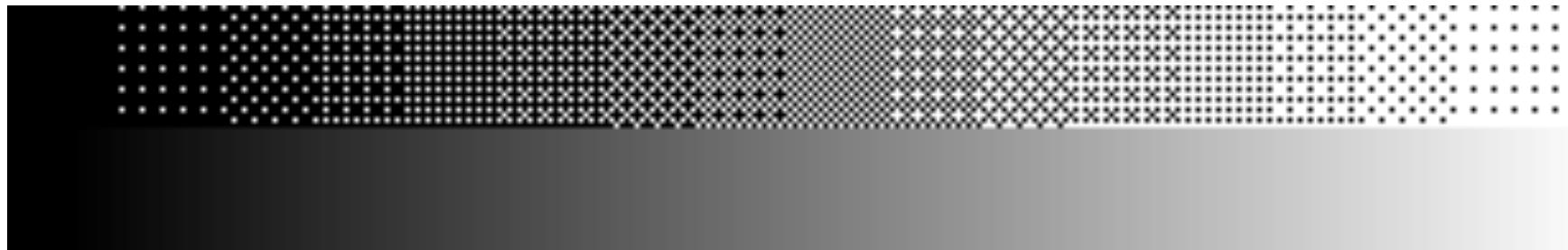
B_2

0	2
3	1



Bayer Ordered Dither Patterns

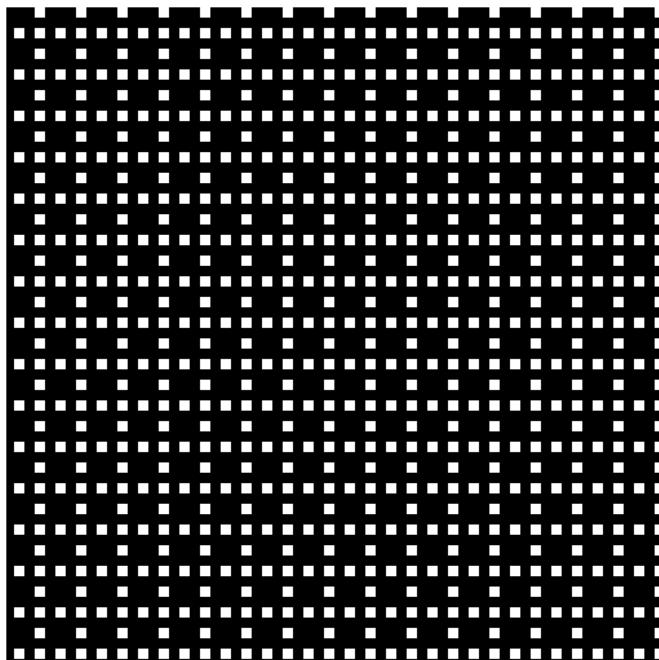
- Characteristic 15 patterns of the 4x4 ordered dithering matrix
 - Thresholds should be distributed as uniformly as possible compared with clustered dithering



Using multiple 4x4 dithering matrix to represent the bottom row
(continuous color)

Bayer Ordered Dither Patterns

- Optimal in terms of point to point distance, but produce strong alignments
 - These alignments can be seen in Fourier domain as strong values for some frequencies



Distribution for a Bayer
matrix of size 4x4



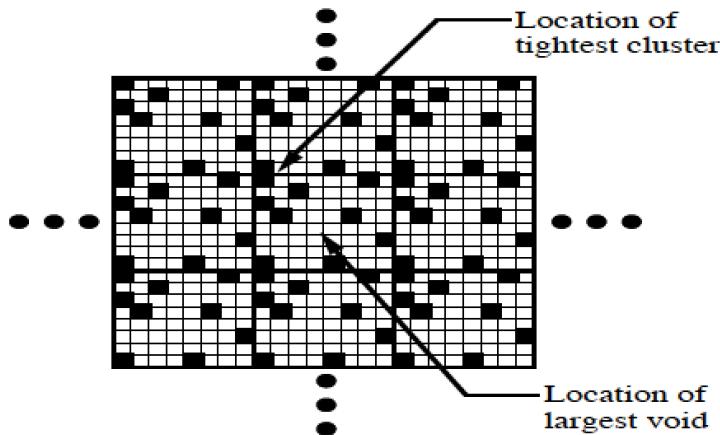
Fourier Amplitude
Spectrum

Void-and-Cluster & Blue Noise Mask Dithering

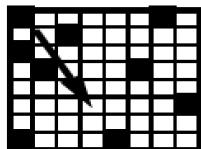
- [Ulichney, 1993]
- Breaks Bayer alignments (periodic pattern)
- Produces dot patterns that come close to blue noise
- Goal:
 - Determine threshold values of a rectangular matrix that produces blue noise pattern for each tone level
- Based entirely in the spatial domain

Void-and-Cluster & Blue Noise Mask Dithering

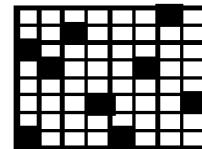
- Swapping pixels



c. swap hole closest to center of largest void with dot
closest to center of tightest cluster.



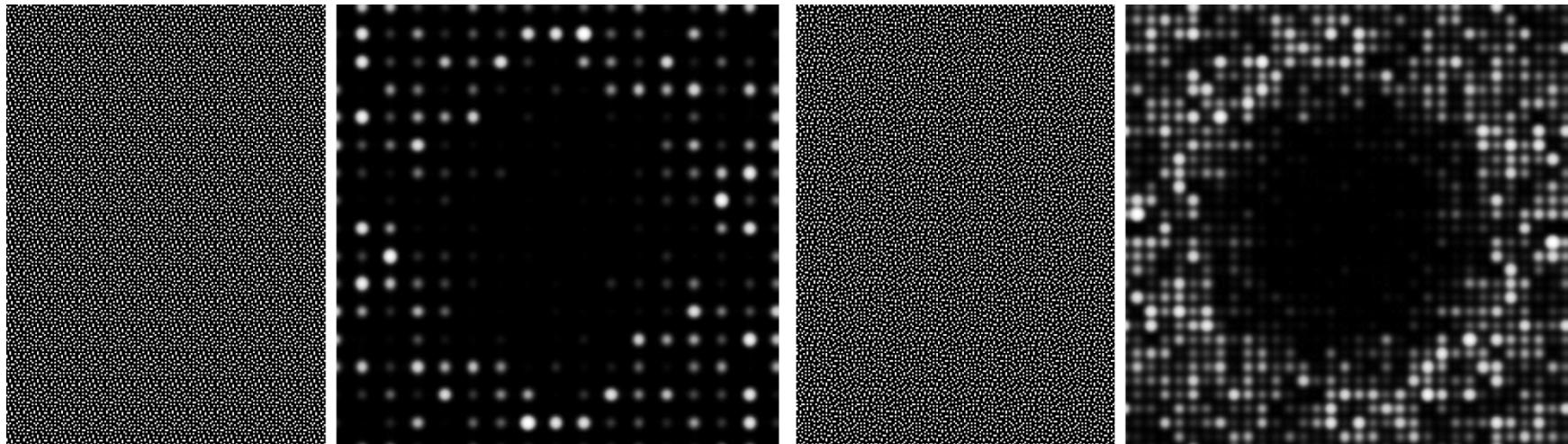
Before Move



After Move

Void-and-Cluster & Blue Noise Mask Dithering

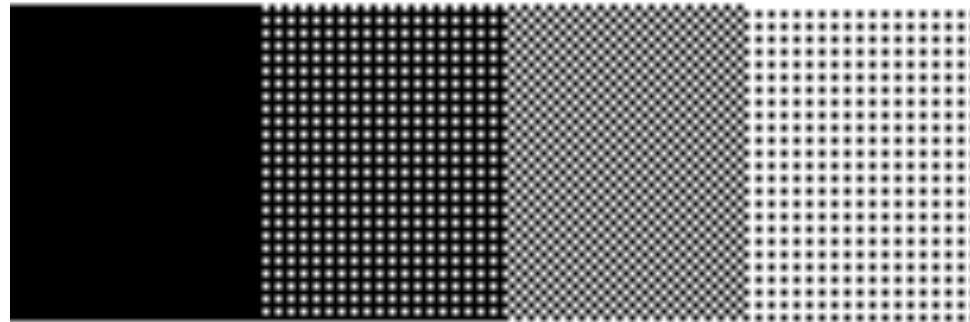
- Breaks Bayer alignments (periodic pattern)
- Produces dot patterns that come close to blue noise
- Goal:
 - Determine threshold values of a rectangular matrix that produces blue noise pattern for each tone level



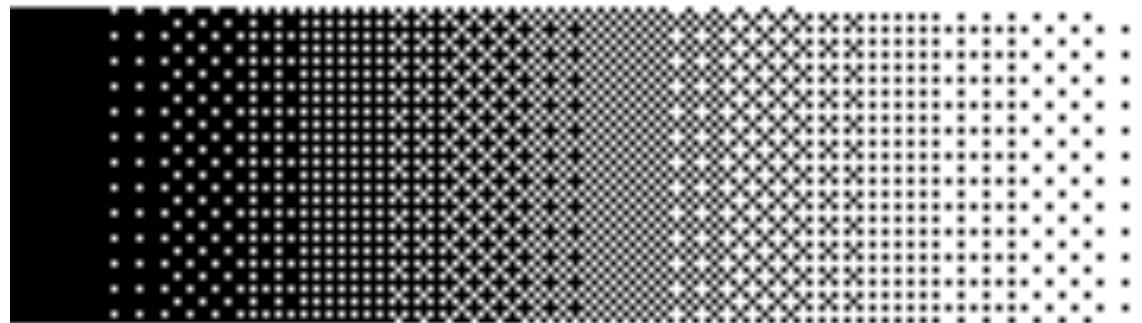
Void-and-cluster distribution and Fourier Amplitude Spectrum
Matrix size 16x16 and 32x32

Example

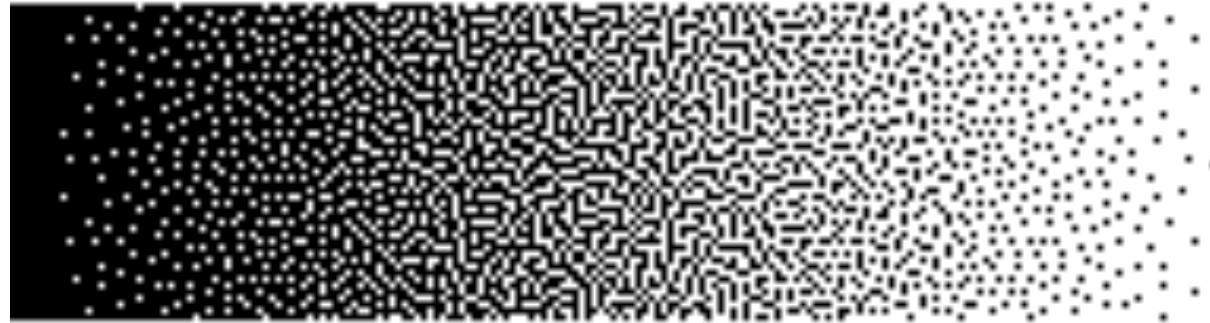
Bayer 2



Bayer 4



void-and-cluster 32



Clustered-dot Dithering - Color

- RGB converted to CMYK, each channel printed separately

- Dot-on-dot

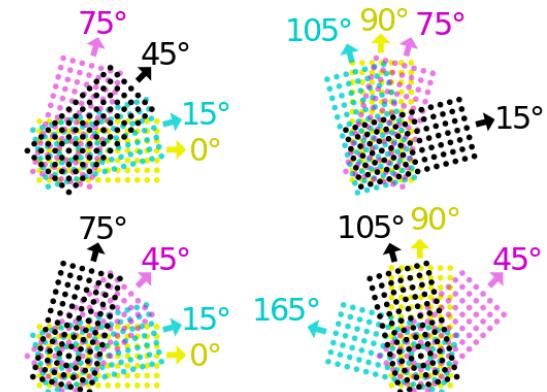
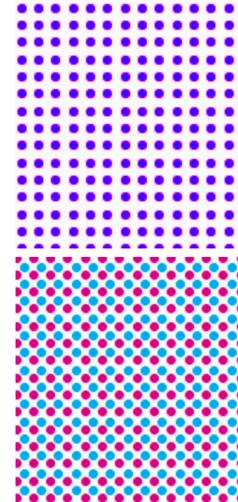
- Clusters centered at the same spatial position
 - Side-by-side drops produce different spectra than overlapping drops
 - Requires color calibration

- Dot-off-dot

- 4 color matrices are shifted in xy to minimize overlap

- Rotated dot

- CMYK layers have relative orientations



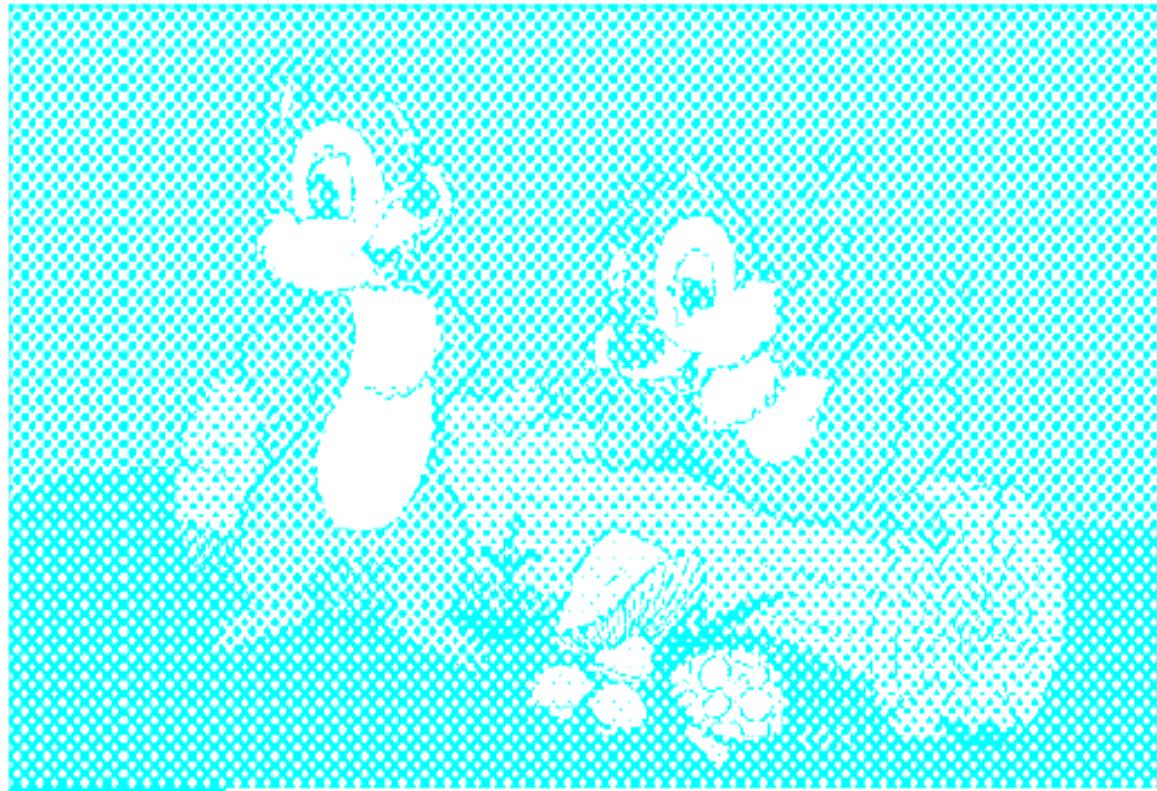
Rotated Dot Color Halftoning



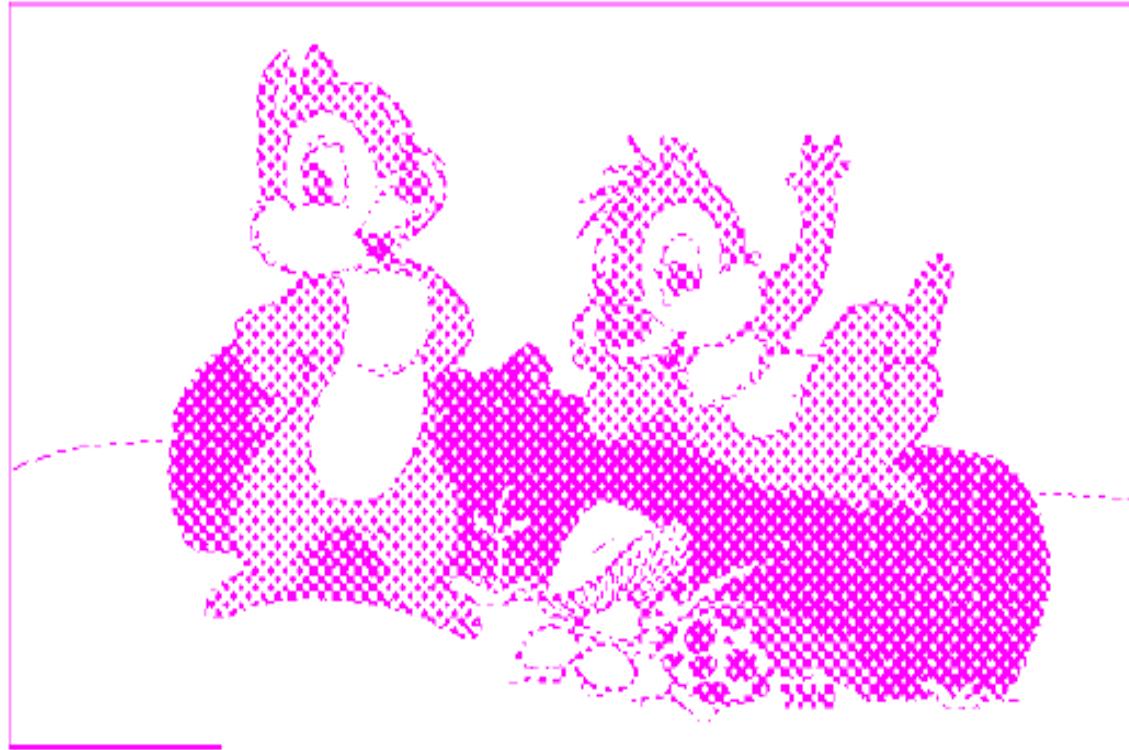
Rotated Dot Color Halftoning



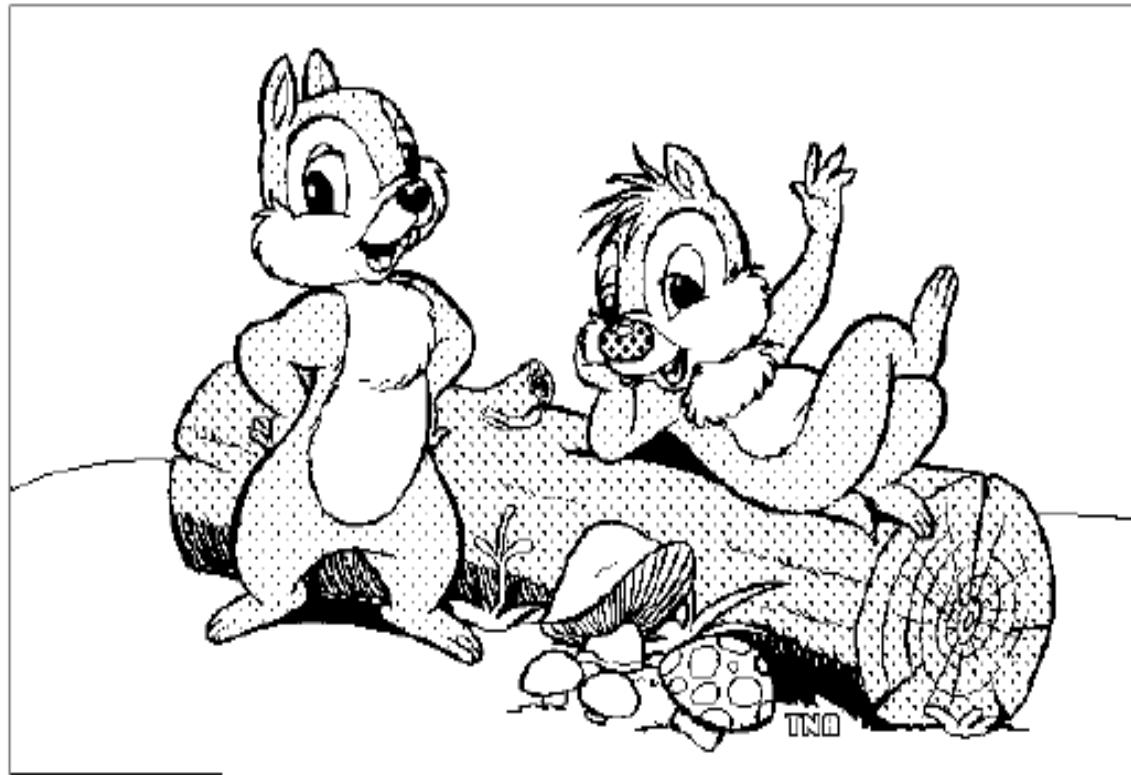
Rotated Dot Color Halftoning



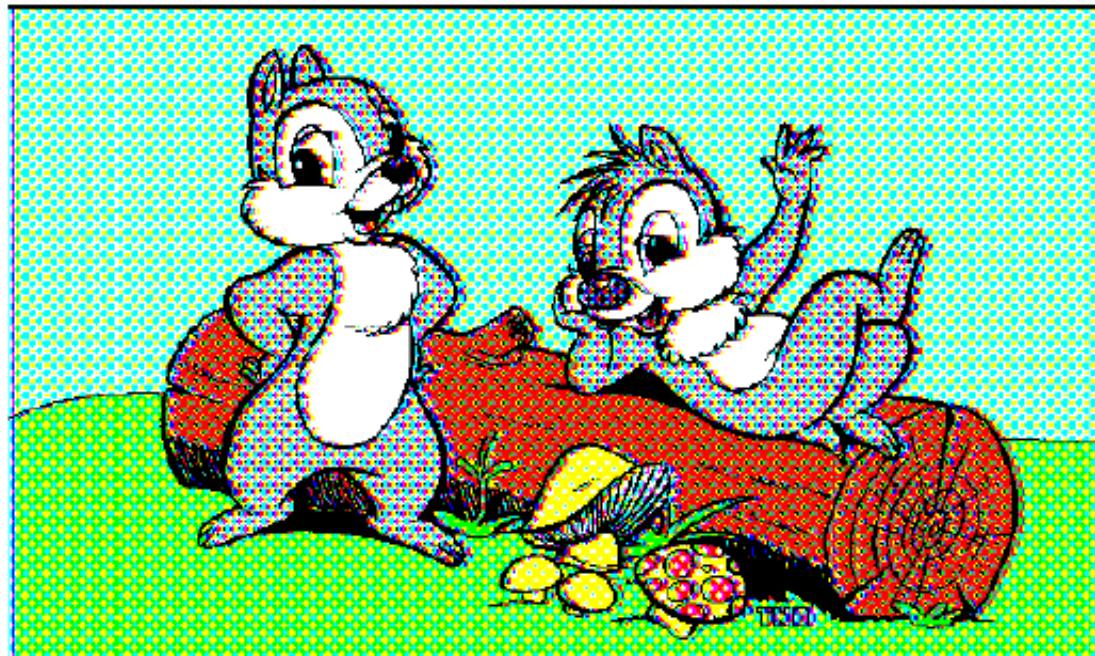
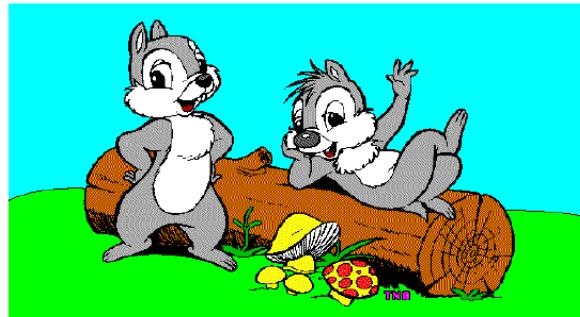
Rotated Dot Color Halftoning



Rotated Dot Color Halftoning



Rotated Dot Color Halftoning



Class II: Neighborhood Processes

- Error diffusion
 - [Floyd-Steinberg 1976, Jarvis et al. 1976, Eschbach & Knox 1991, Shiau & Fan 1994, Marcu & Abe 1996, Ostromoukhov 2001, Zhou & Fang 2003, Hwang et al. 2004, Kwak et al. 2006]
- Each pixel depends on multiple pixels
 - Extra computations
 - Class II → **moderately fast**
- Weak uniformity over the image
 - Loss of detail
 - Tradeoff between good tones and sharp edges
 - Class II → **average overall quality**

Floyd-Steinberg Error Diffusion

- With this method, the average quantization error is reduced by propagating the error from each pixel to some of its neighbors in the scan order



Floyd-Steinberg Error Diffusion

- With this method, the average quantization error is reduced by propagating the error from each pixel to some of its neighbors in the scan order

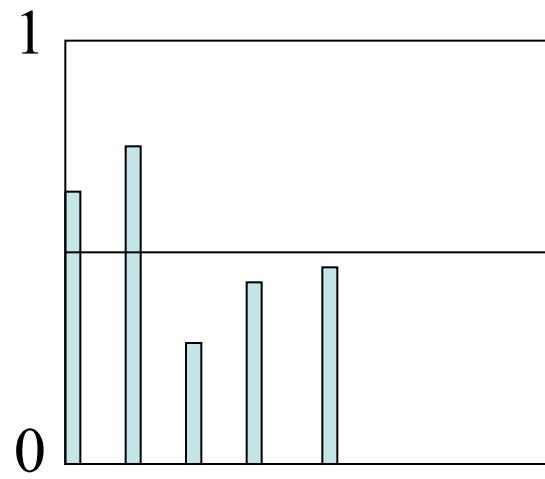


original image

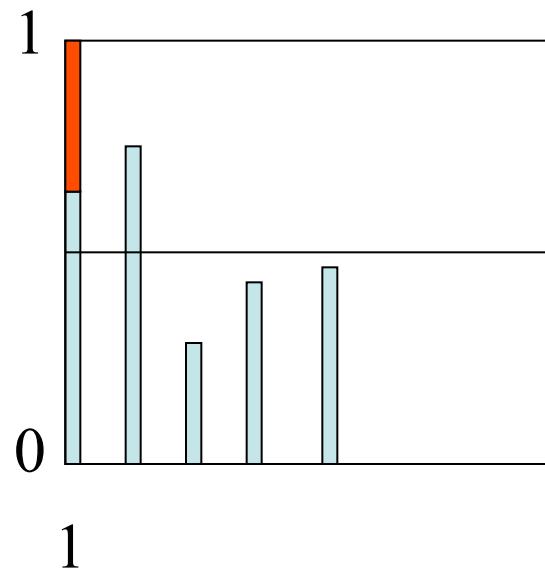


original image after the application of Floyd-Steinberg dithering

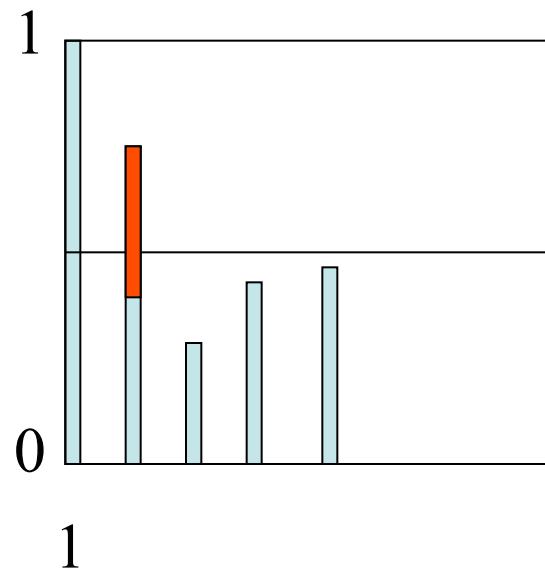
1D Error Diffusion



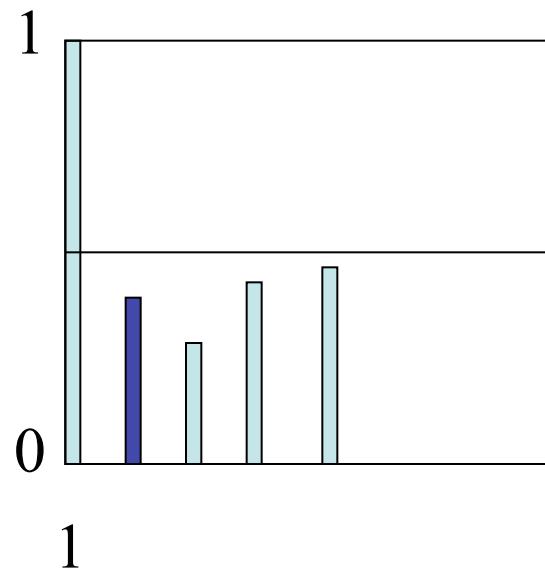
1D Error Diffusion



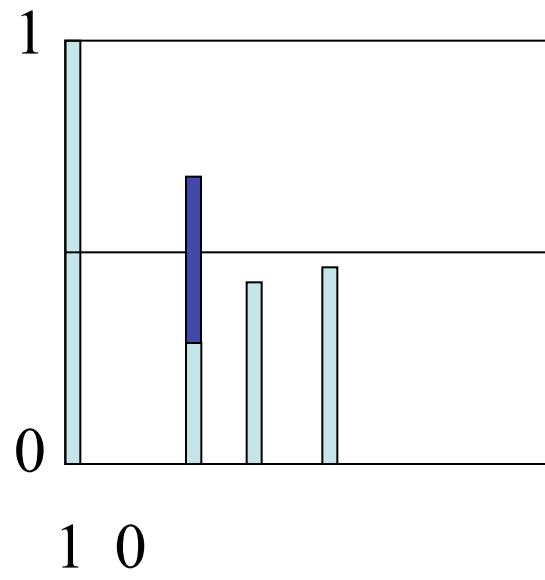
1D Error Diffusion



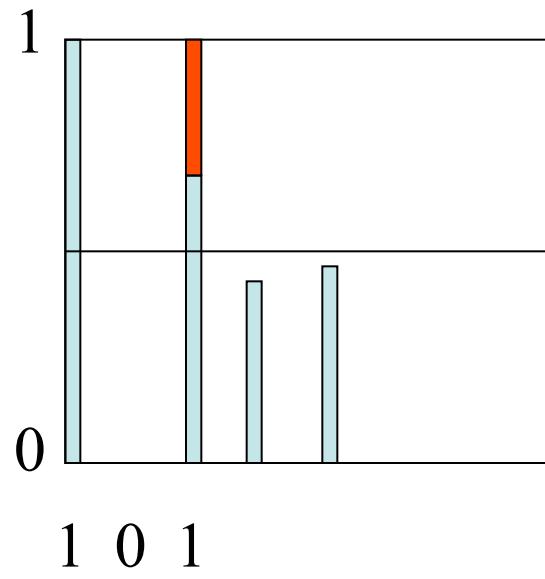
1D Error Diffusion



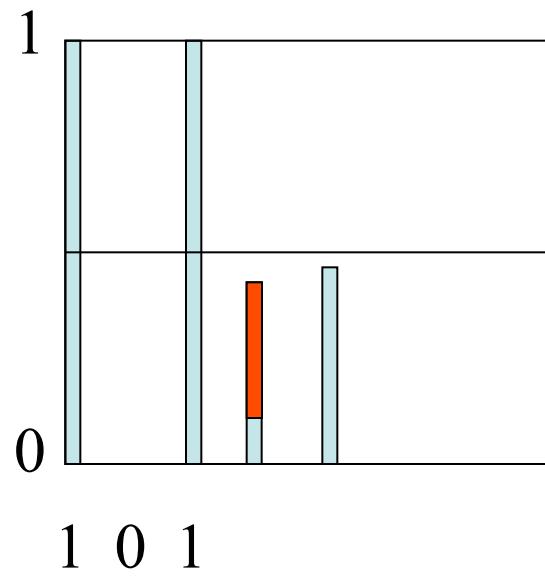
1D Error Diffusion



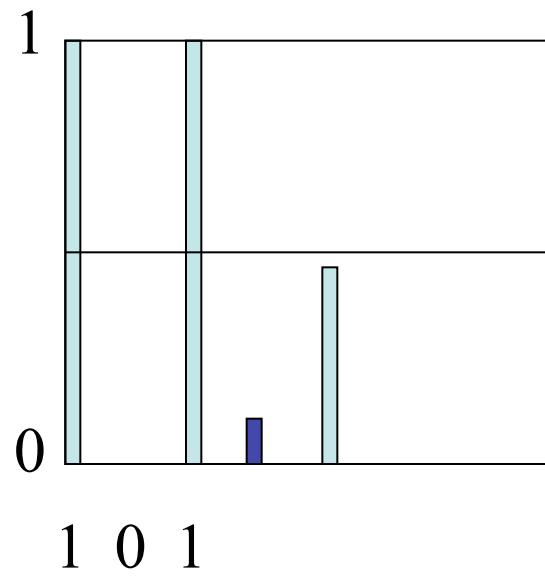
1D Error Diffusion



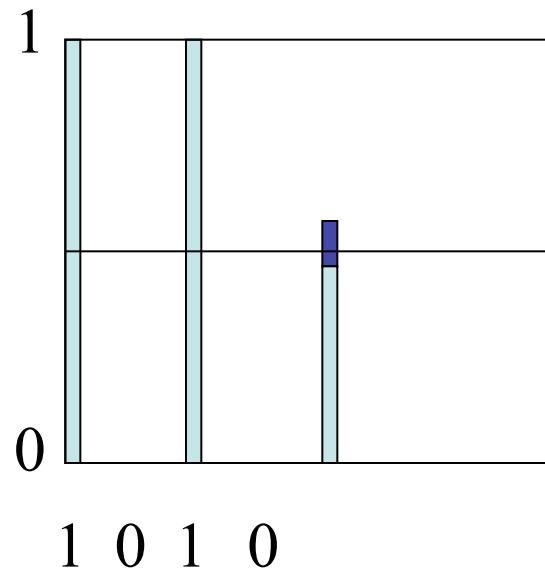
1D Error Diffusion



1D Error Diffusion

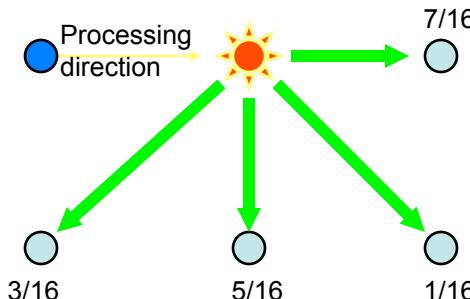
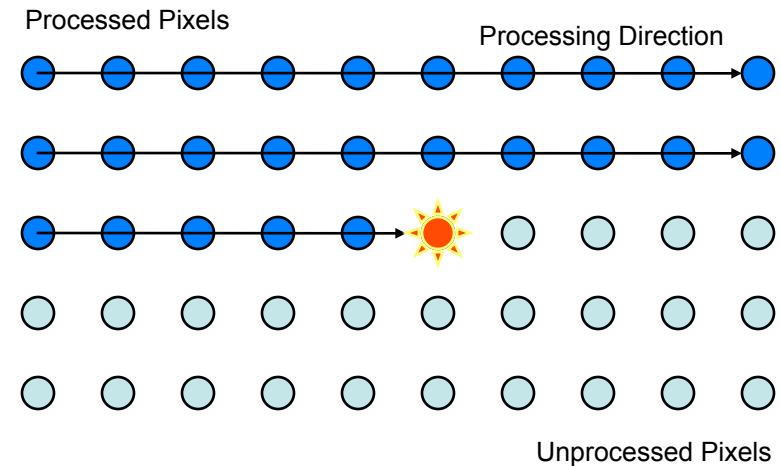


1D Error Diffusion



Floyd-Steinberg Error-Diffusion Algorithm (1975)

- Process pixels in order of scanlines
- Compare $\text{input}(x,y)$ with $\text{threshold}(x,y)=.5$
- Distribute quantization error to unprocessed pixels



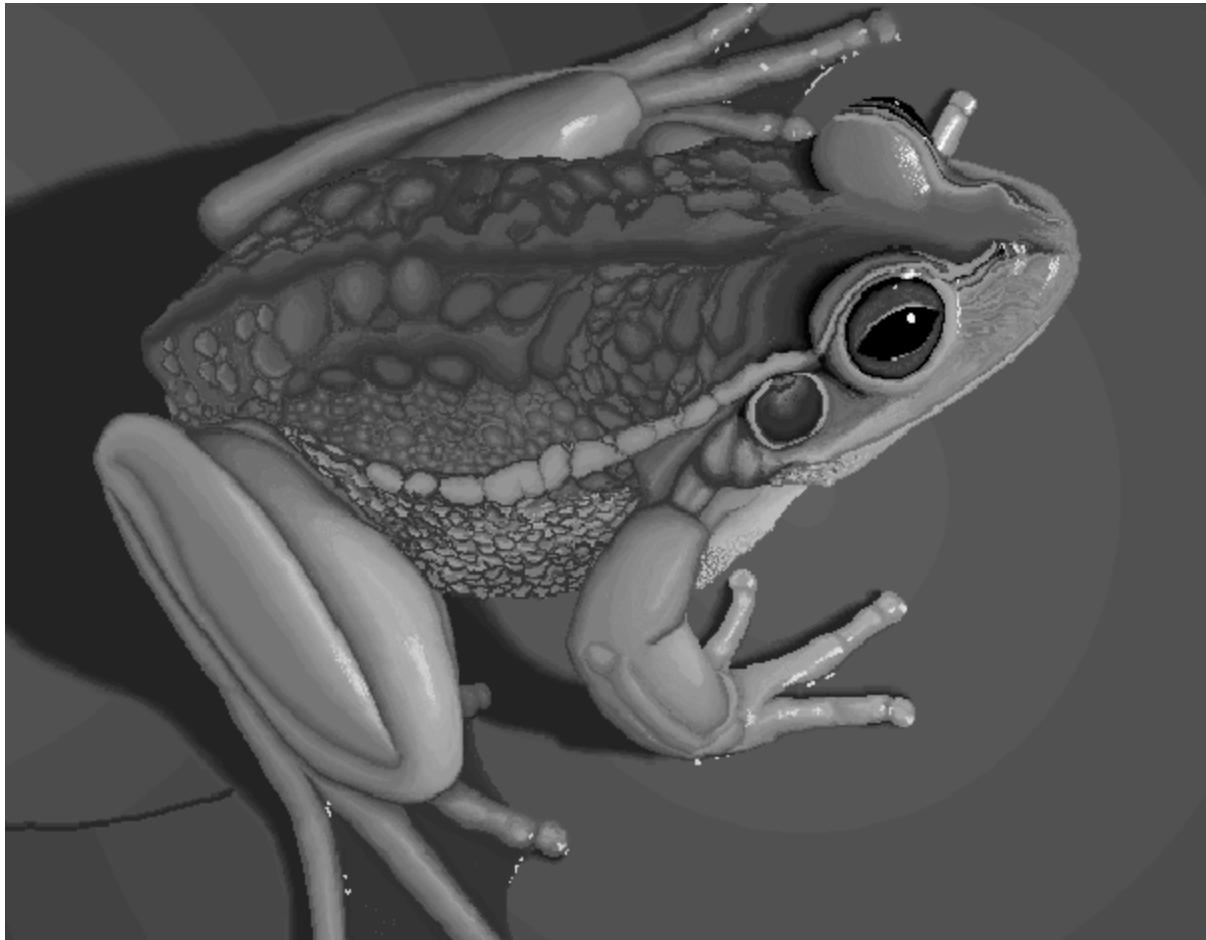
Note that the error propagation weights must sum to one

Error Diffusion Algorithm

```
Set AccErr[] to zero;  
For each pixel in the image scanning from left to right:  
    value= pixelValue(x,y) + AccErr[x,y];  
    if (value > WHITE/2)  
        setPixel(x,y, WHITE);  
        Error = value - WHITE;  
    else  
        setPixel(x,y, BLACK);  
        Error = value - BLACK;  
  
    AccErr[x+1, y]      += 7/16 * Error;  
    AccErr[x-1, y+1]    += 3/16 * Error;  
    AccErr[x , y+1]    += 5/16 * Error;  
    AccErr[x+1, y+1]   += 1/16 * Error;
```

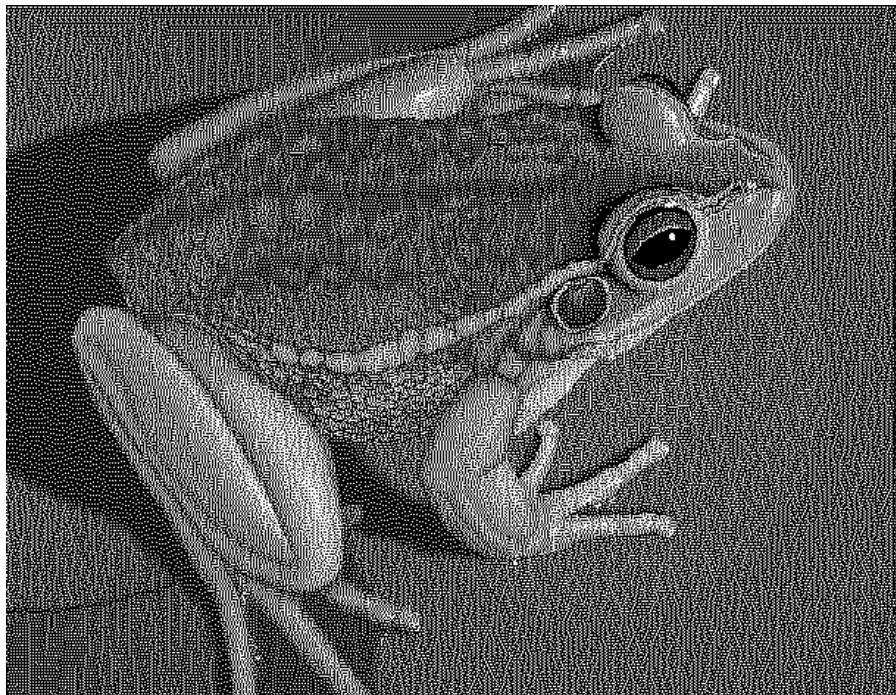
Examples

Original Picture

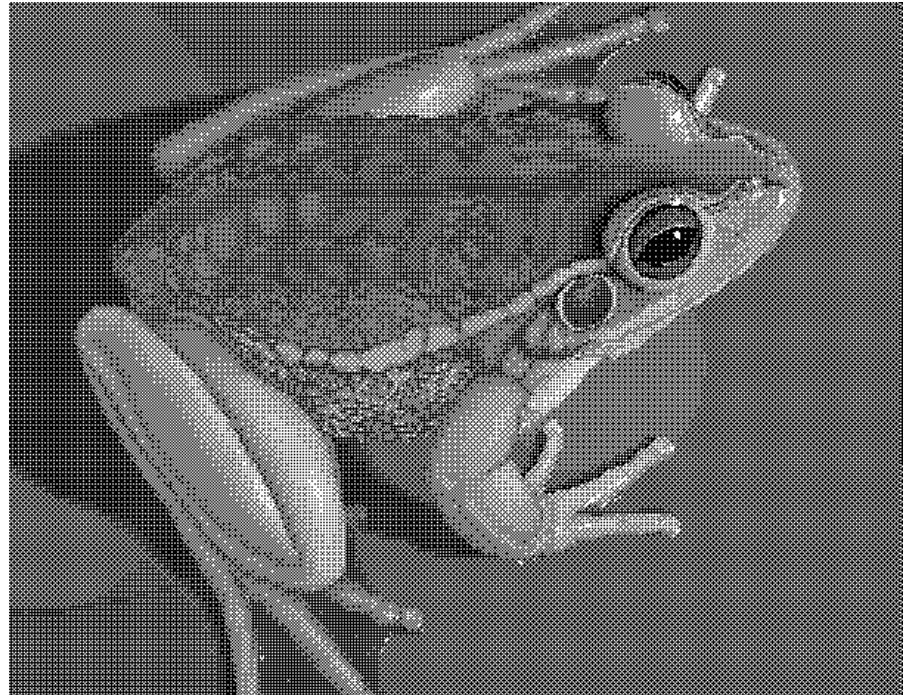


Examples

Error Diffusion

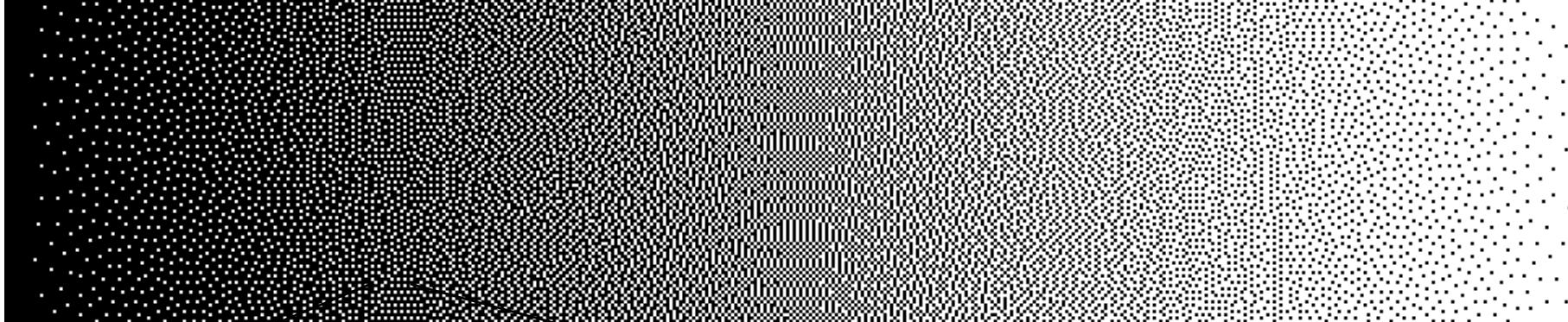


Dithering

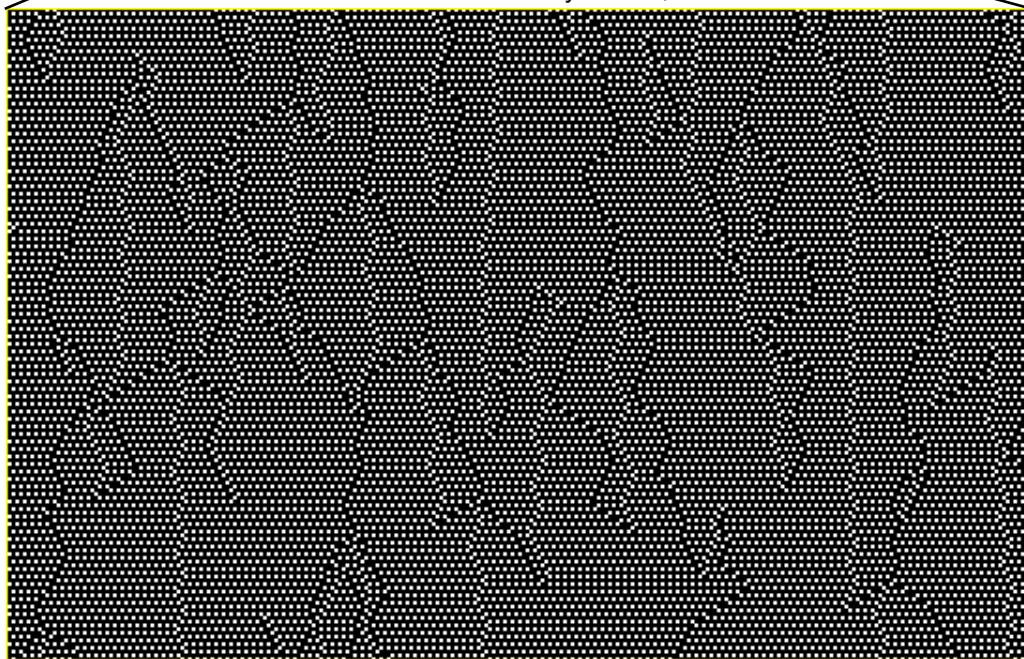


Floyd-Steinberg Error-Diffusion Algorithm

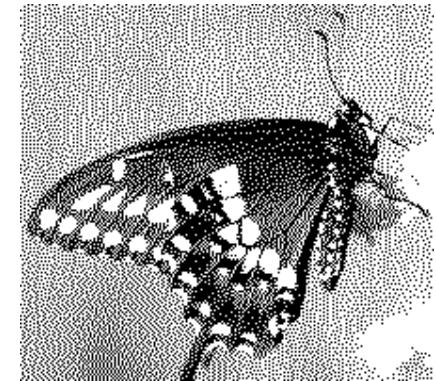
Gray Ramp



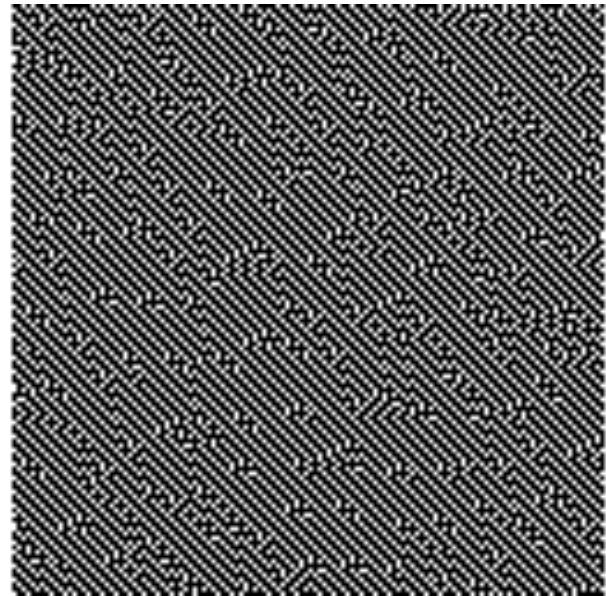
Intolerable Artifacts at Certain Intensity Levels, Out of Control



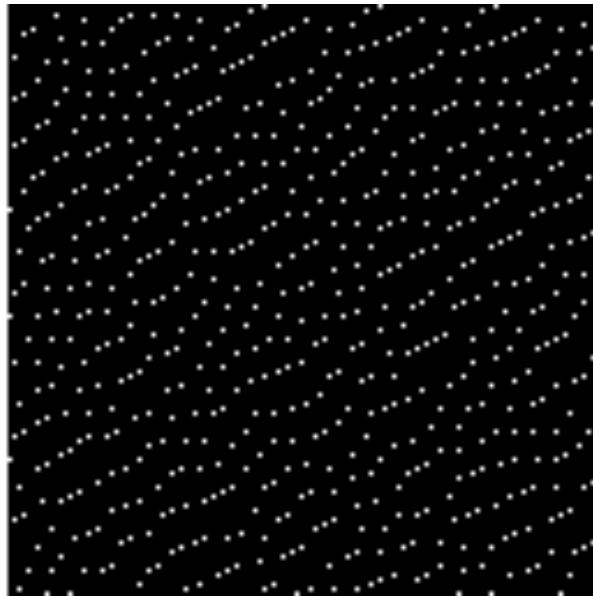
Sample Image



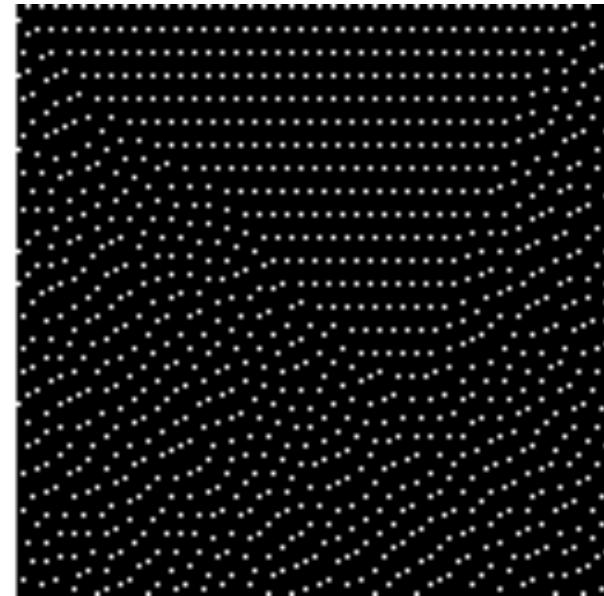
Floyd-Steinberg Error-Diffusion Artifacts



maze-like structures



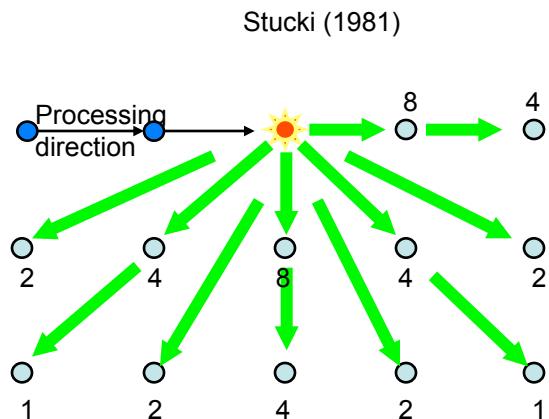
worm-like alignments



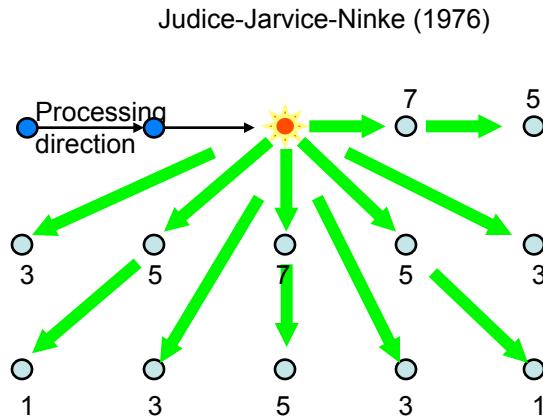
edges instabilities

Sophistication of Error-Diffusion Algorithms

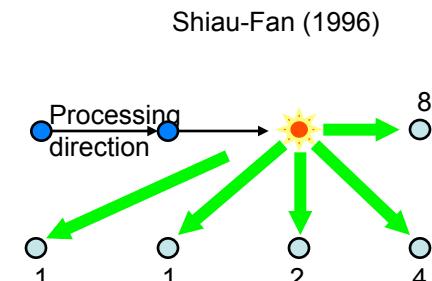
- Different error distribution coefficients
- Processing paths: serpentine, space-filling curves
- Threshold modulation: control of edge enhancement



/42



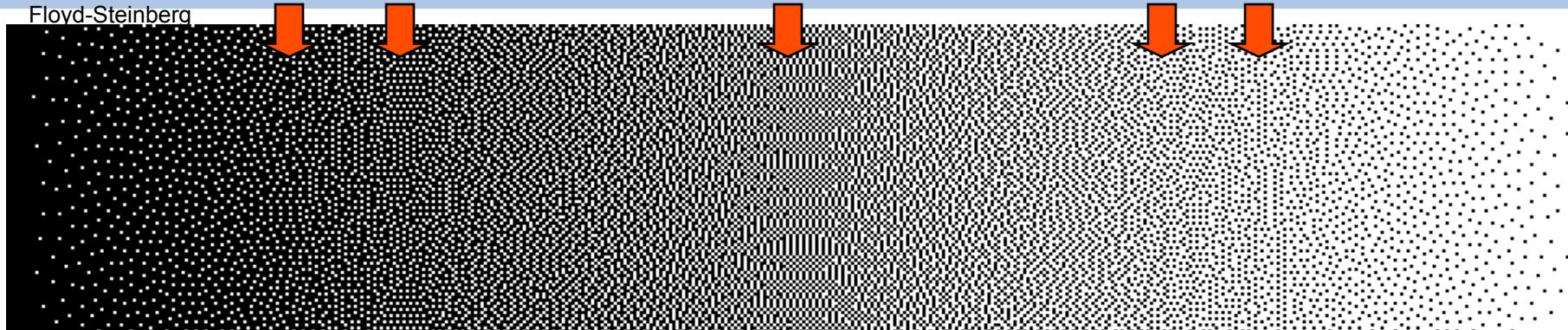
/48



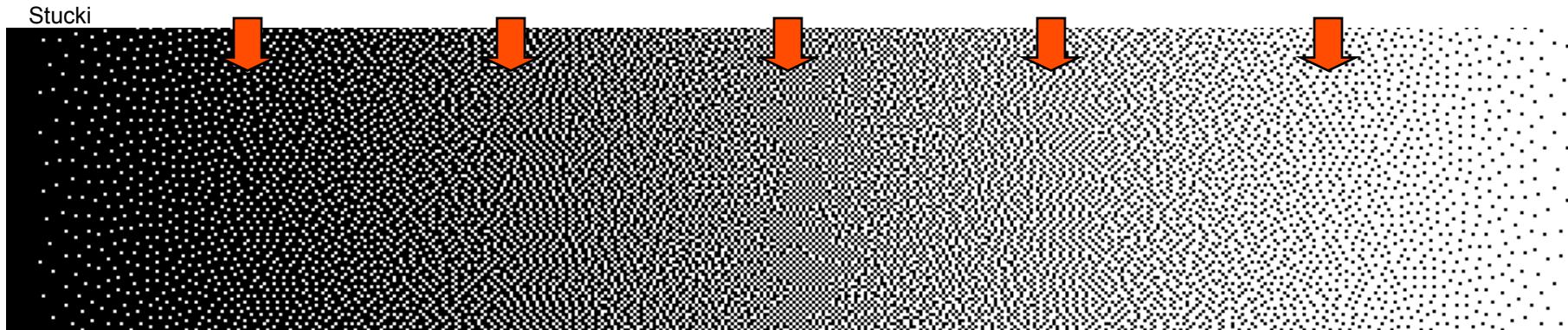
/16

Different Error-Diffusion Algorithms

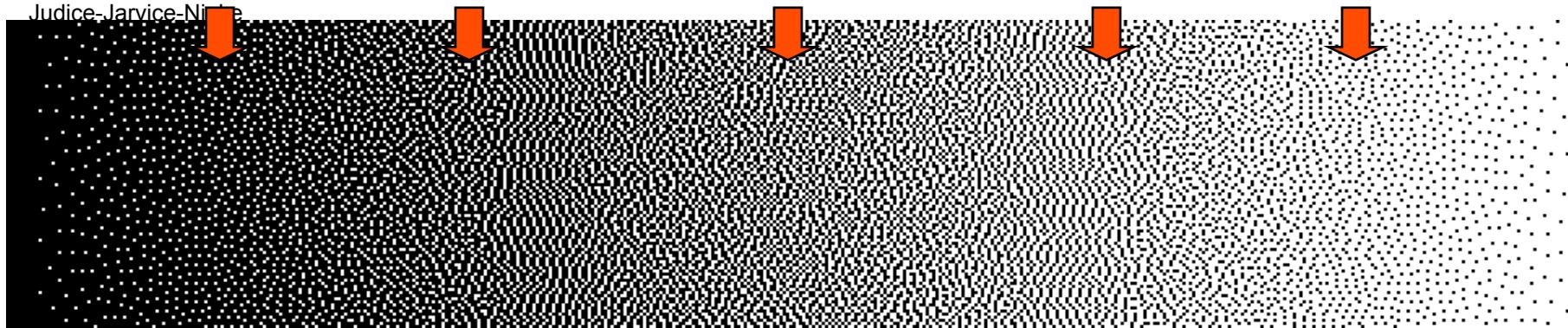
Floyd-Steinberg



Stucki

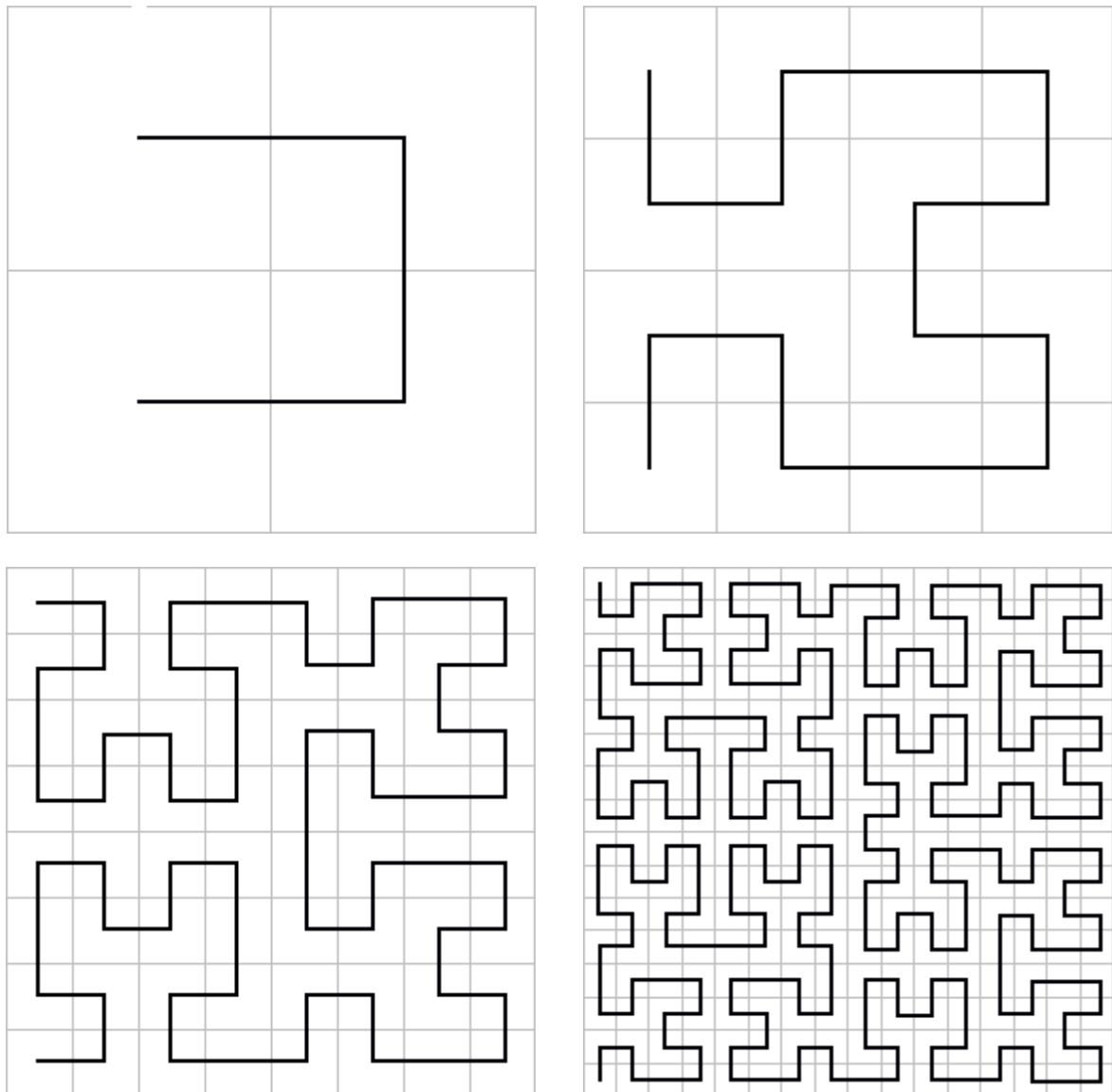


Judice-Jarvice-Ninke



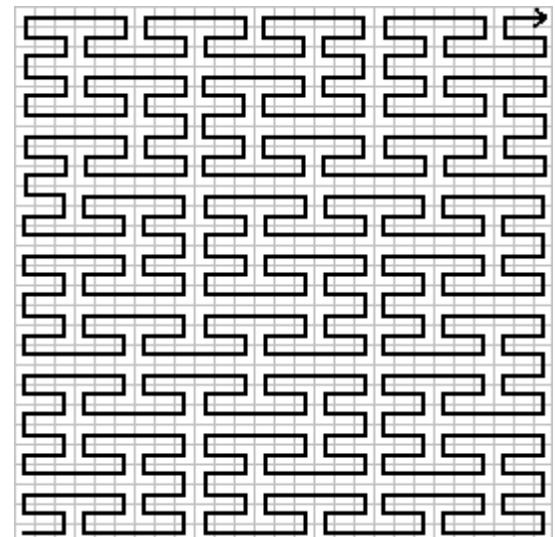
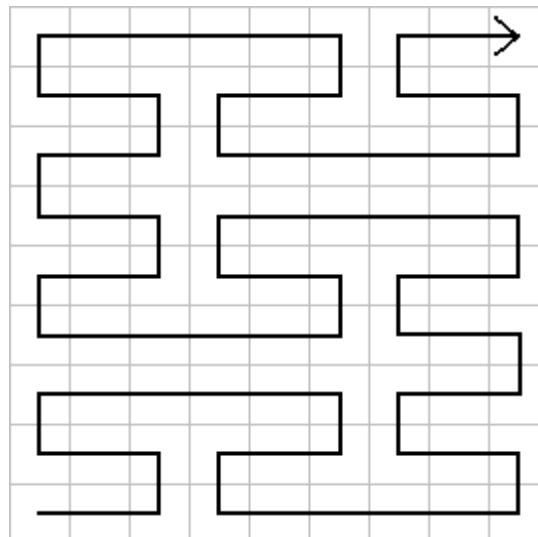
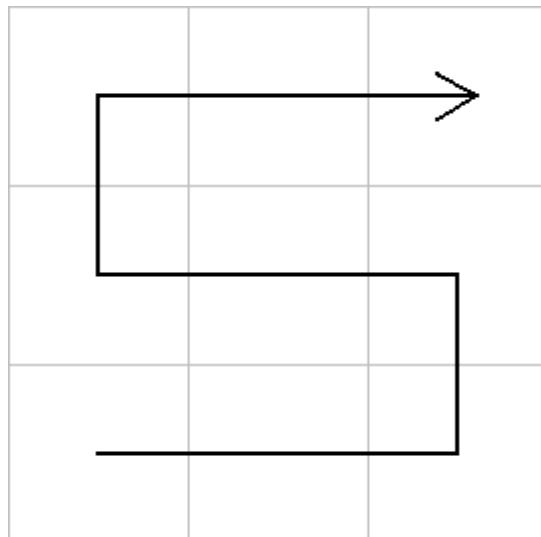
Space Filling Curves

Hilbert curve
(1-4)



Space Filling Curves

- Peano curve



Space Filling Curves



break regularity in raster order,
but introduce textured noise



Left to right



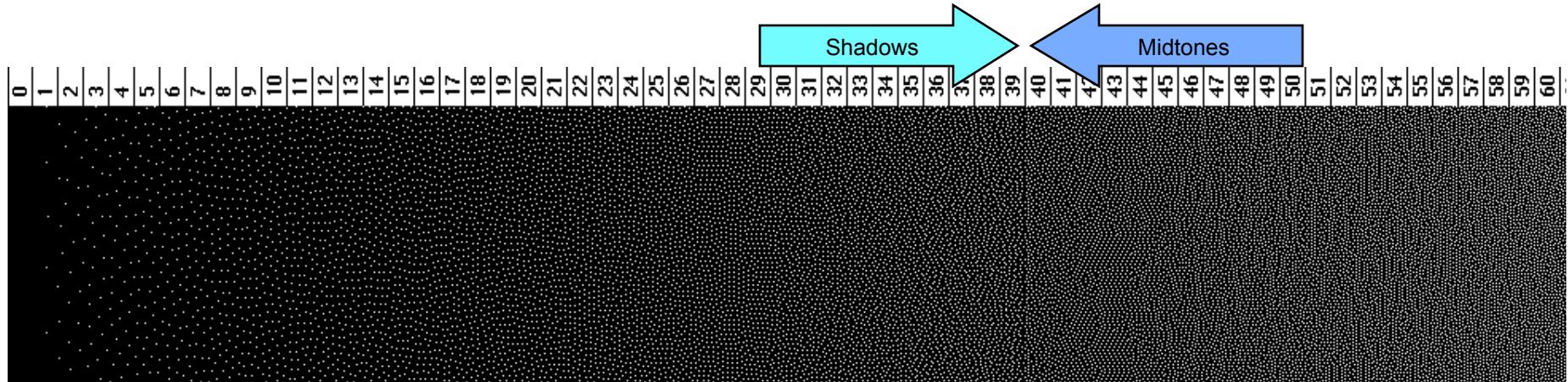
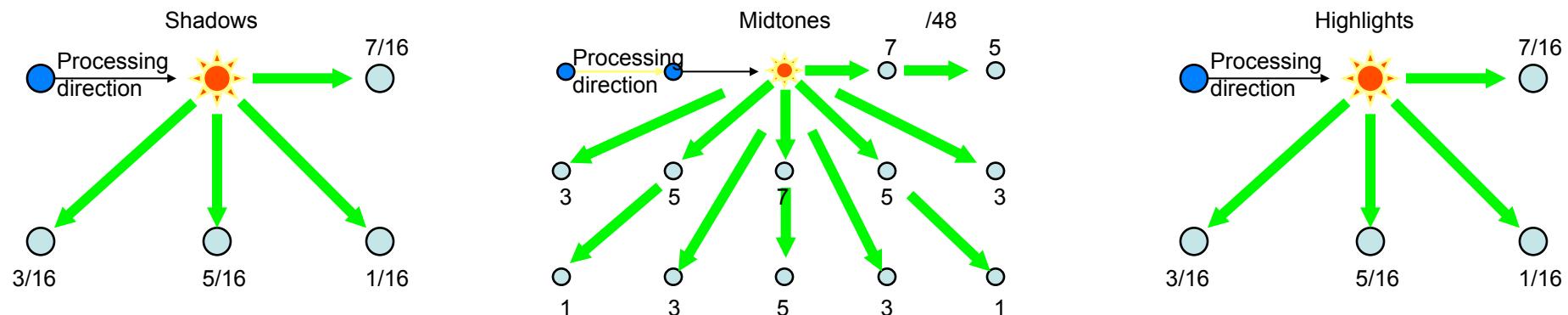
Hilbert Curve



Peano Curve

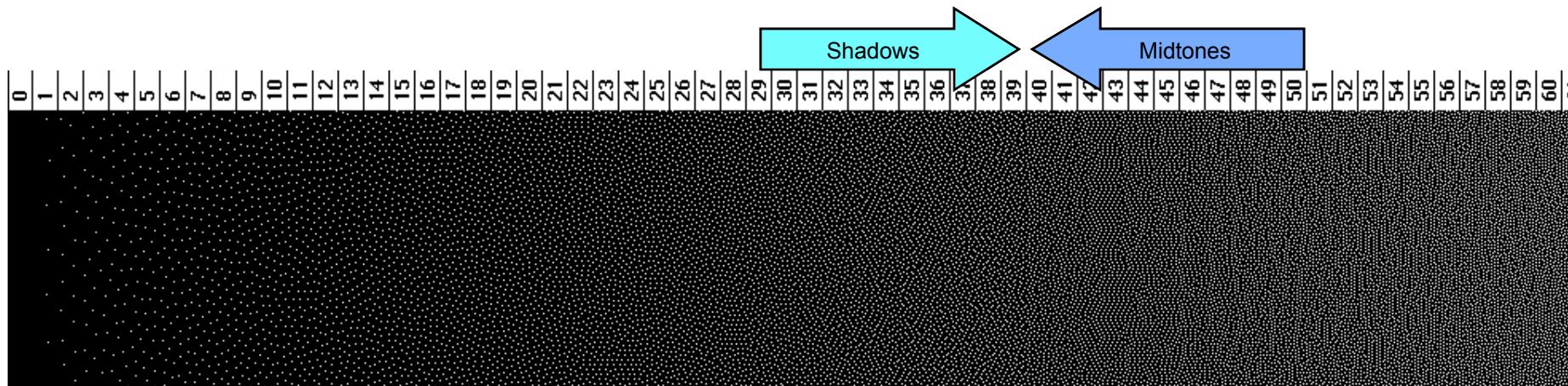
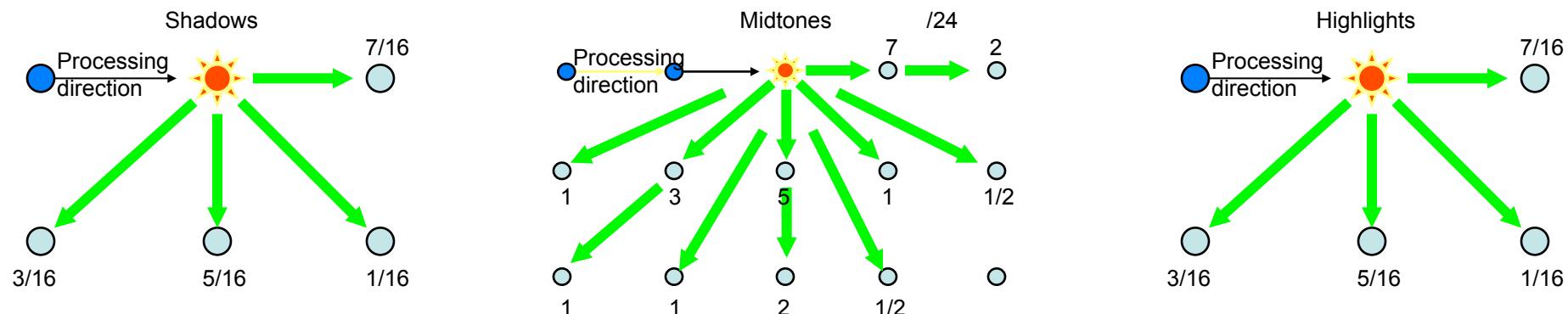
Tone-Dependent Weights

Eschbach



Tone-Dependent Weights

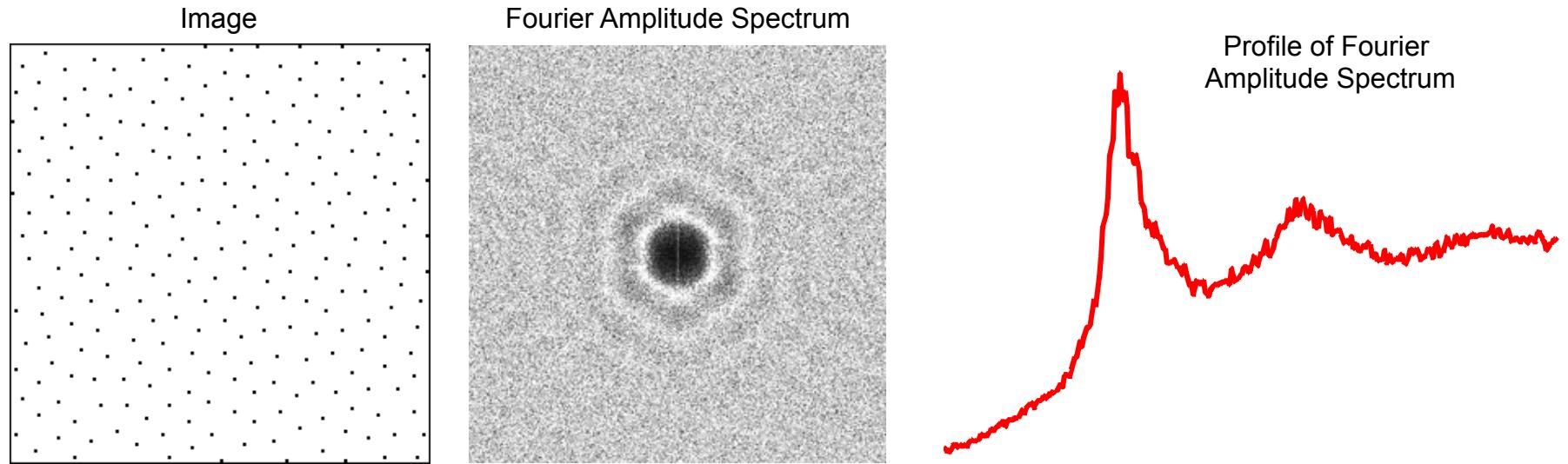
Eschbach



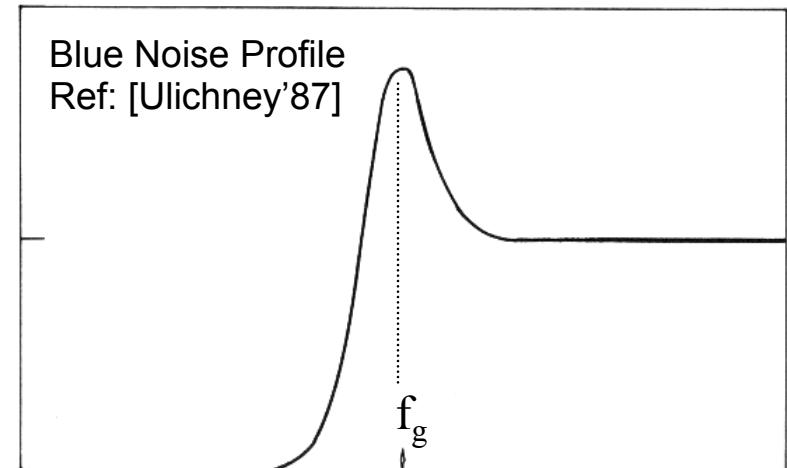
Class II: Neighborhood Processes

- Conclusion
 - Visual Artifacts can be improved Using Input-Dependent Weights
 - Correlation Between Matrices May Reduce Banding Effect
 - Weak Points of the Method
 - No Objective Criterion on Input-Dependent Weights
 - No Mechanism Proposed To Insure Correlation Between Weights

Objective Criterion of Visual Quality of Halftoning

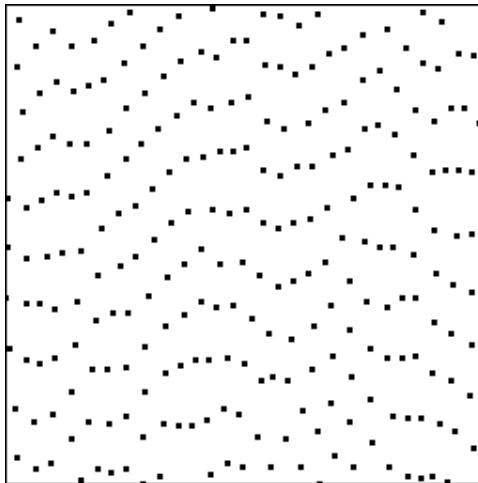


- Radial symmetry
- Characteristic blue noise profile

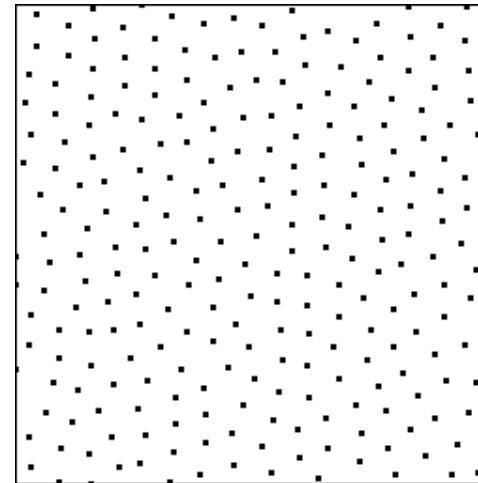


Objective Criterion of Visual Quality of Halftoning

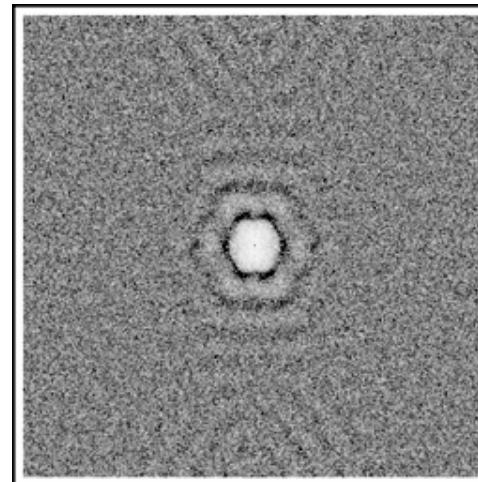
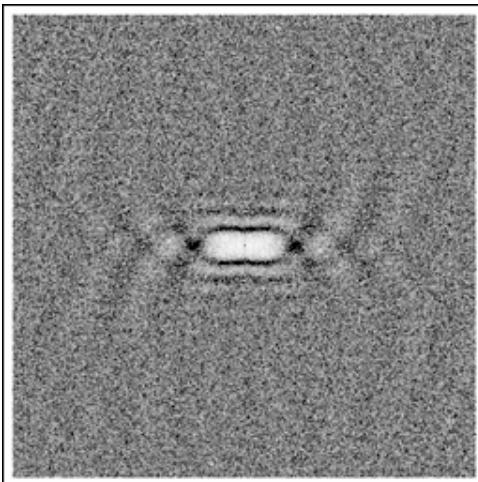
Bad Error-Diffusion
(Std. Floyd-Steinberg, scanlines)



Good Error-Diffusion



Images



Fourier
Amplitude Spectra

Tone-Dependent Weights

Ostromoukhov 2001

- Simplified set of distribution coefficients $d^i = \{d_{10}, d_{-11}, d_{01}\}$ for each intensity level i
- For each level, find $d^i = \{d_{10}, d_{-11}, d_{01}\}$ that approaches “Blue Noise”, by minimizing the difference between given Fourier Amplitude Spectrum and ideal “Blue Noise Profile”

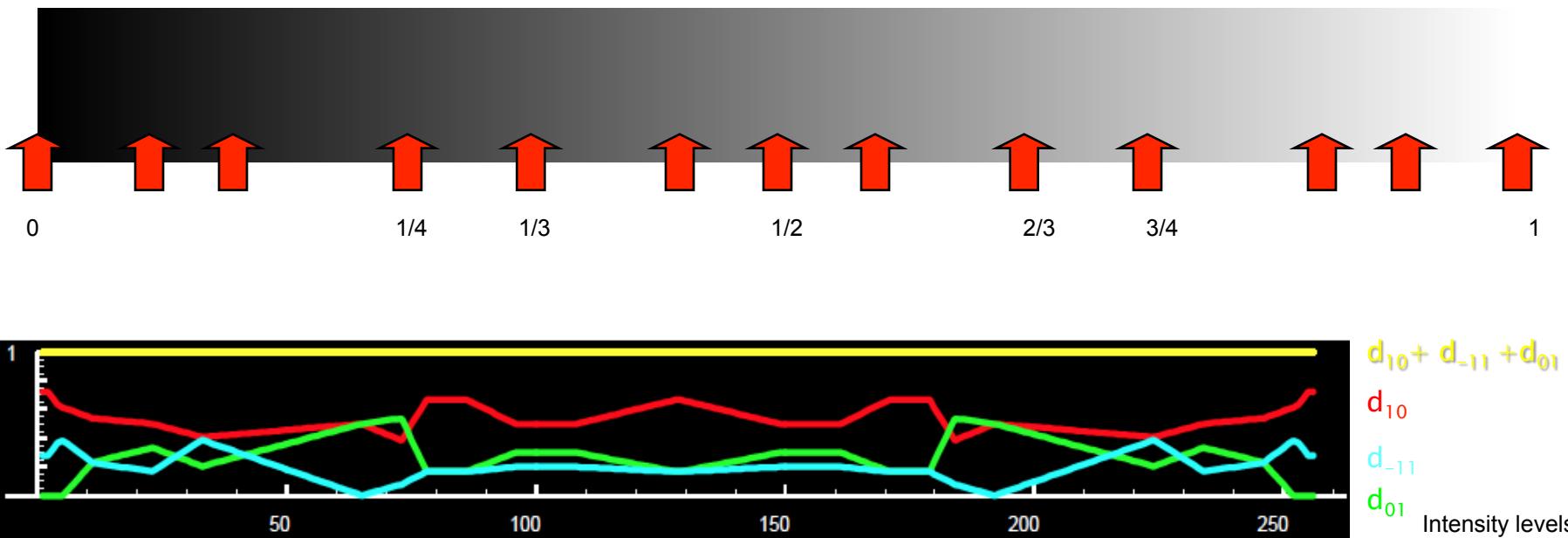


instead of presetting the values, compute the best values

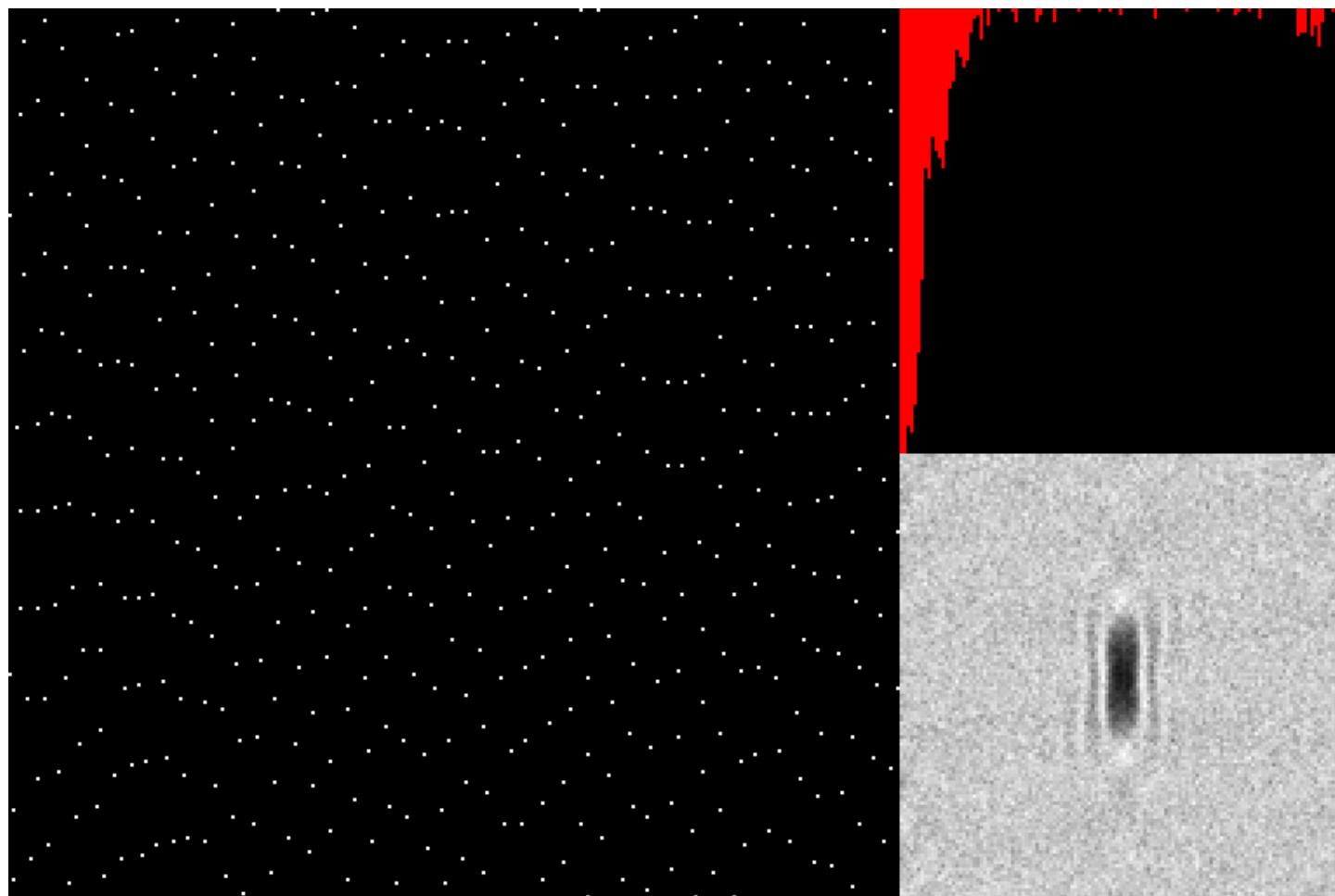
Tone-Dependent Weights

Ostromoukhov 2001

- Define distribution coefficient for a set of key levels
- Interpolate between coefficients



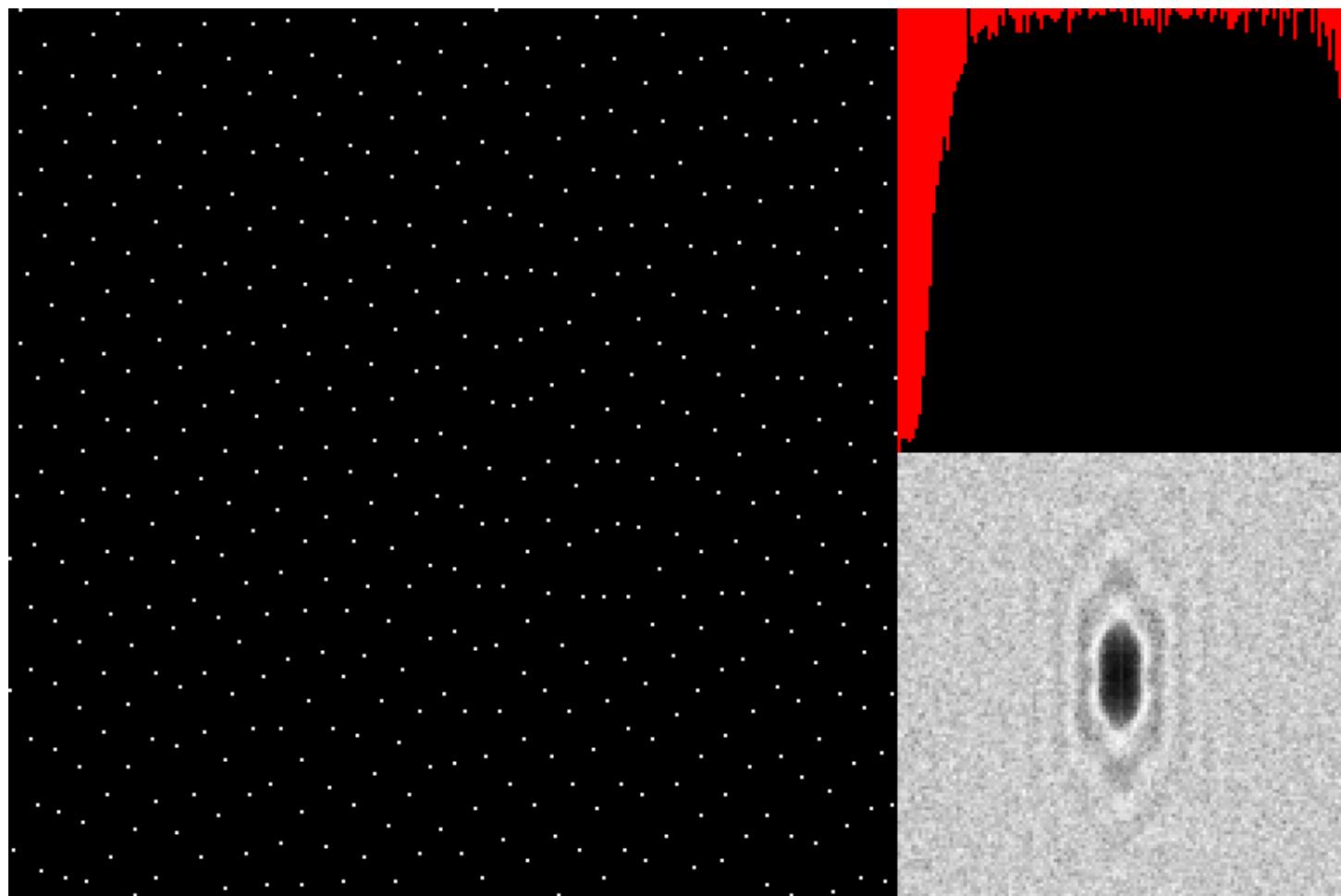
Computing Distribution Coefficients, Iteration # 1



$$\begin{aligned} d_{10} &= 0.333333 \\ d_{-11} &= 0.333333 \\ d_{01} &= 0.333333 \end{aligned}$$

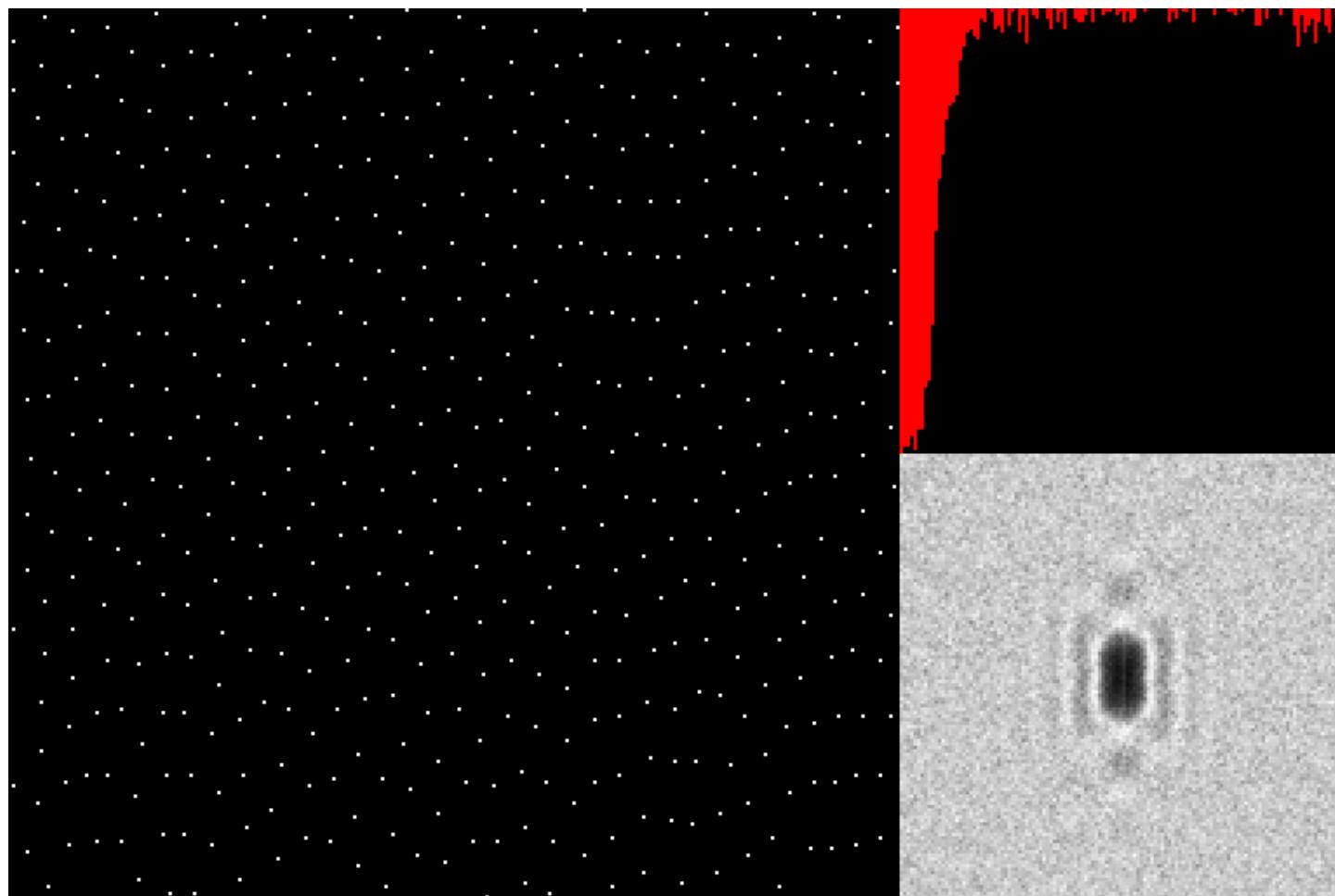
Here it is a particular intensity level (dark)

Computing Distribution Coefficients, Iteration # 3



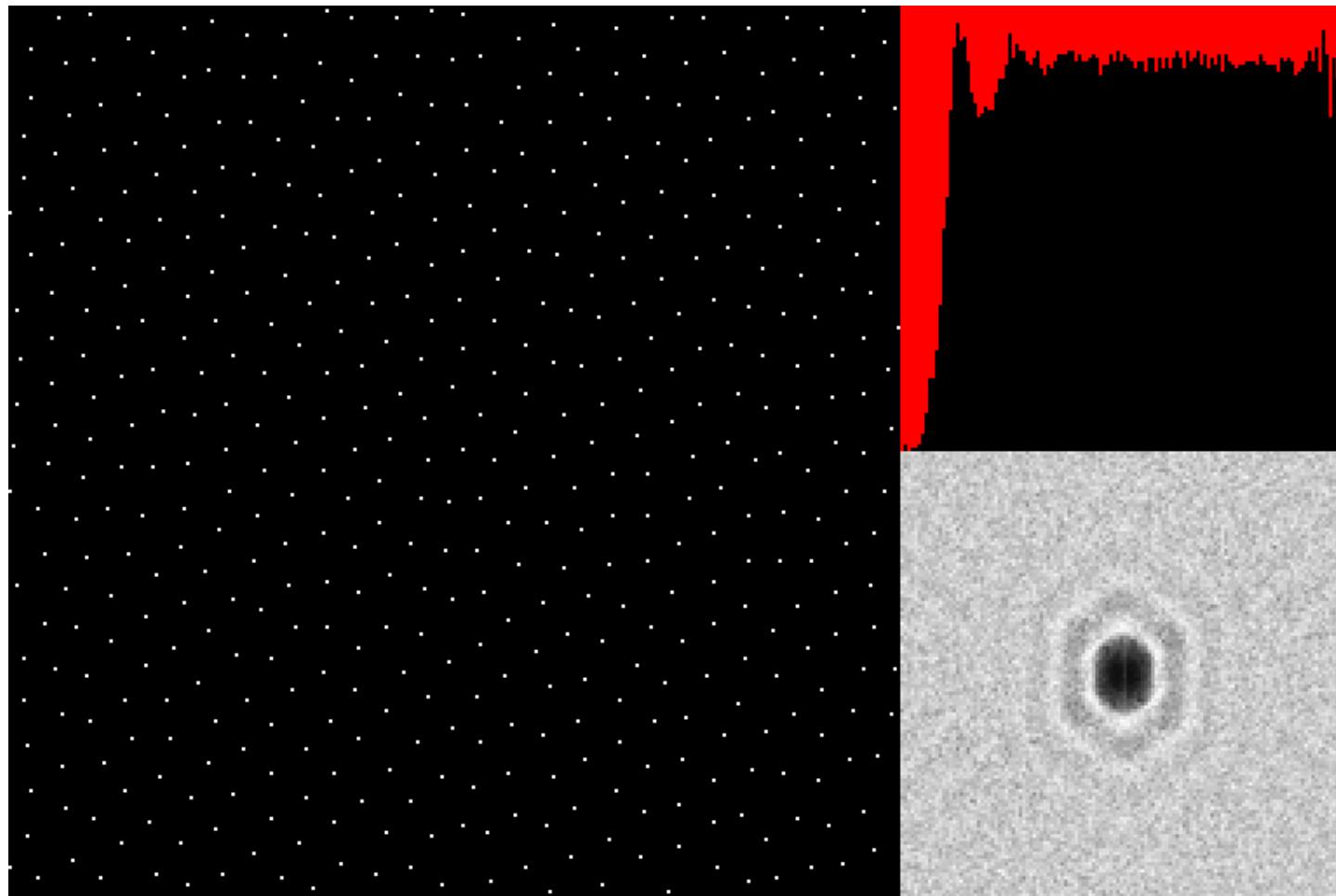
$$\begin{aligned} d_{10} &= 0.445736 \\ d_{-11} &= 0.306202 \\ d_{01} &= 0.248062 \end{aligned}$$

Computing Distribution Coefficients, Iteration # 7



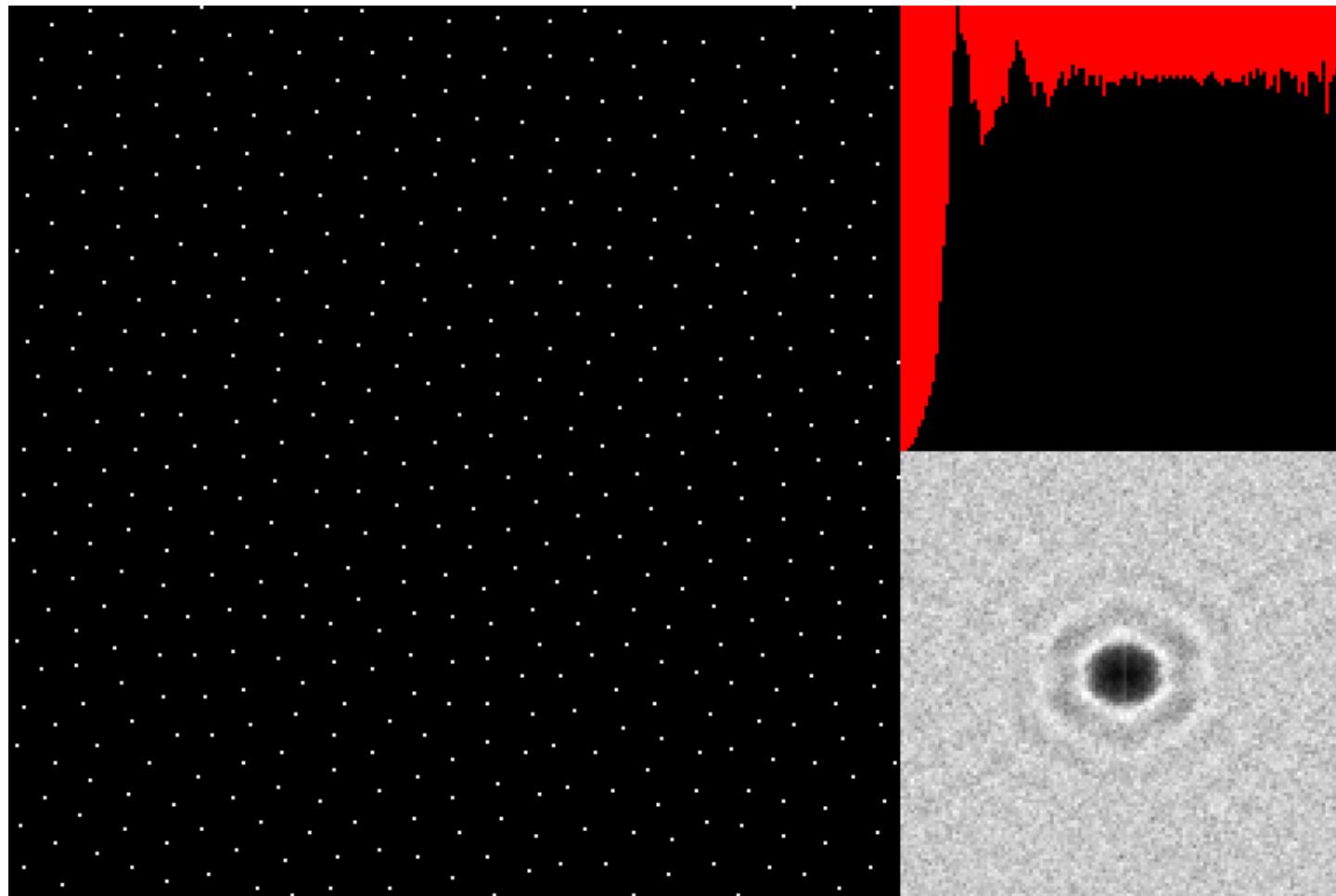
$$\begin{aligned} d_{10} &= 0.527778 \\ d_{-11} &= 0.444445 \\ d_{01} &= 0.0277776 \end{aligned}$$

Computing Distribution Coefficients, Iteration # 12



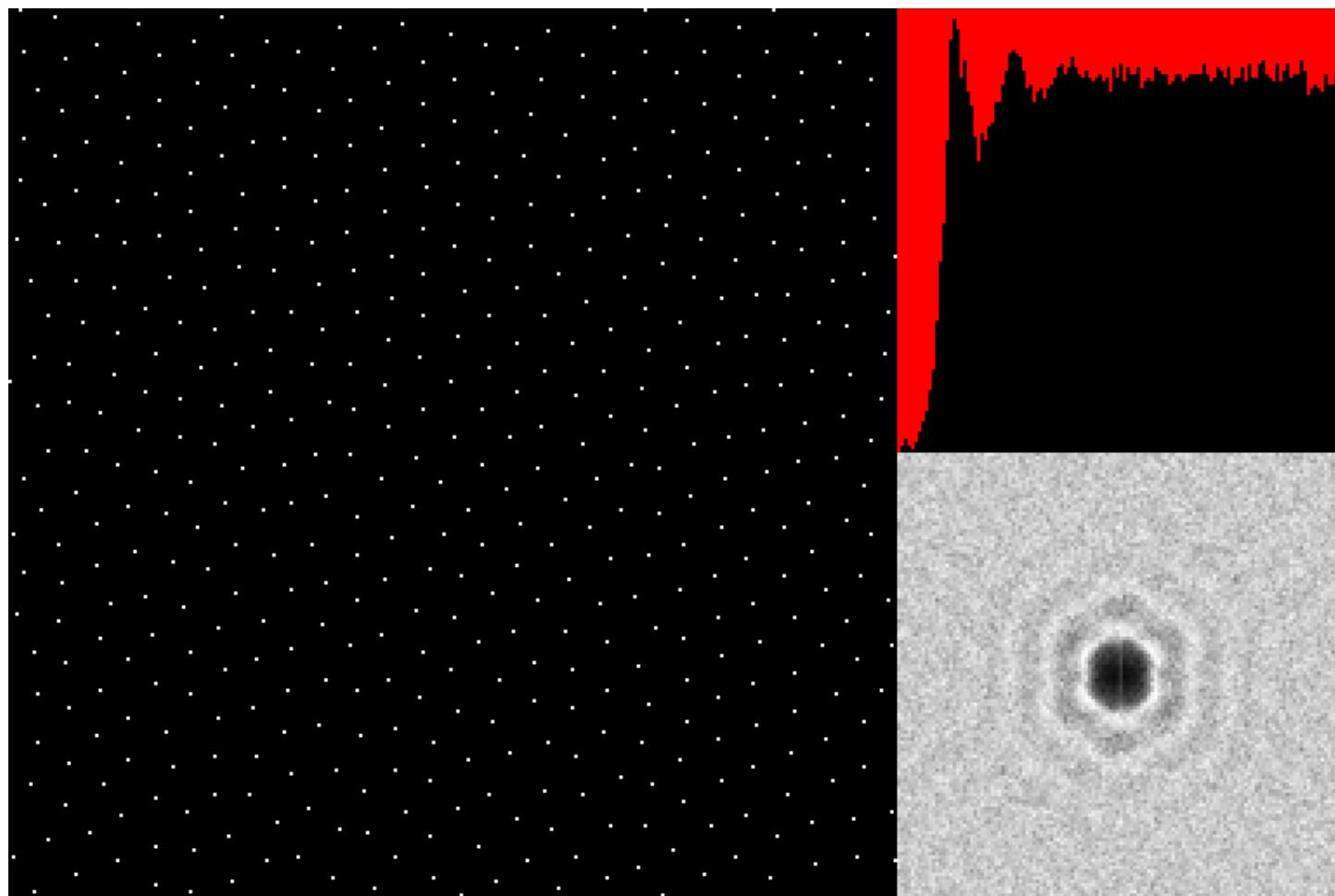
$$\begin{aligned} d_{10} &= 0.637344 \\ d_{-11} &= 0.186967 \\ d_{01} &= 0.175689 \end{aligned}$$

Computing Distribution Coefficients, Iteration # 15



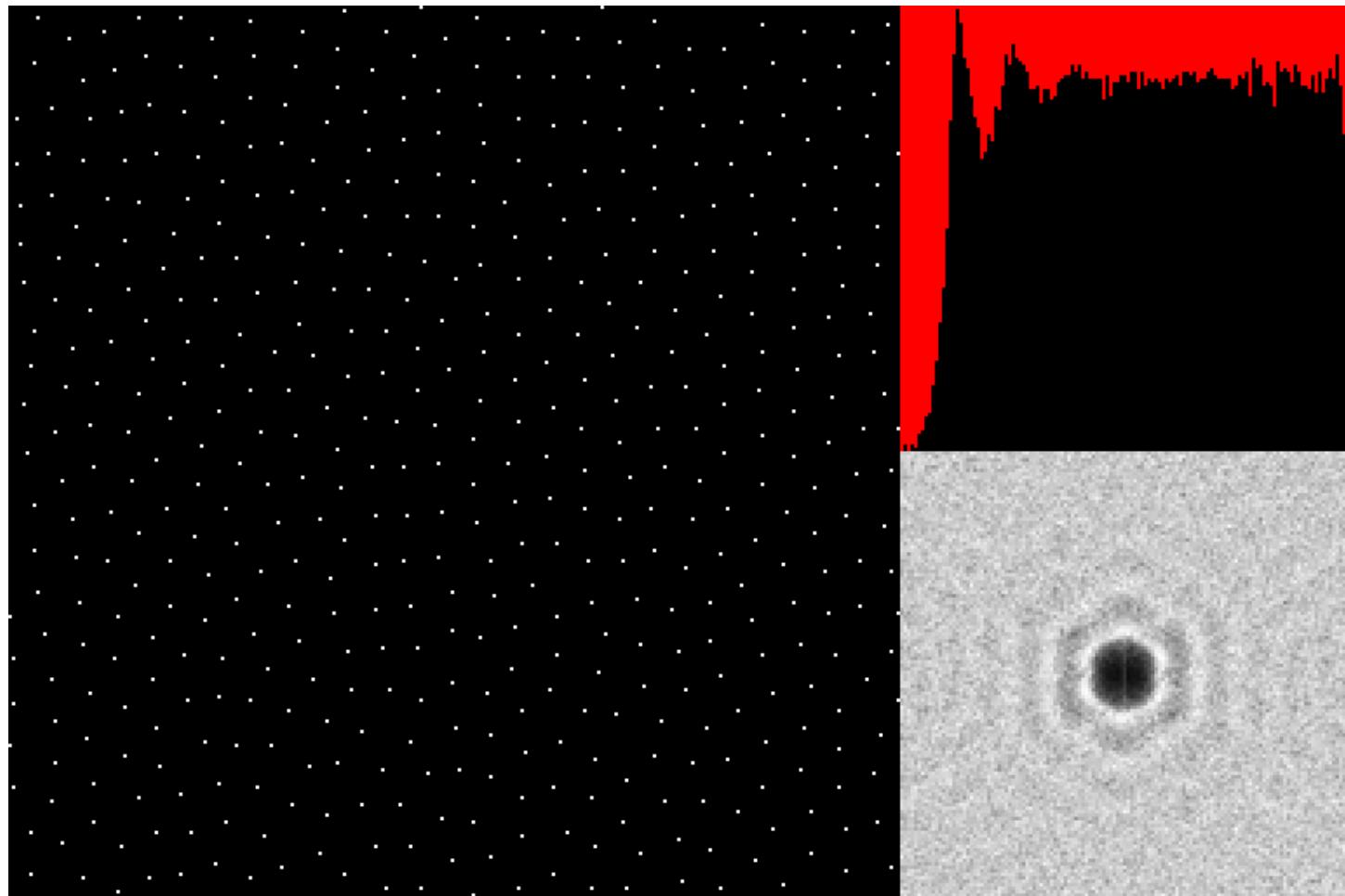
$$\begin{aligned} d_{10} &= 0.630038 \\ d_{-11} &= 0.218744 \\ d_{01} &= 0.151219 \end{aligned}$$

Computing Distribution Coefficients, Iteration # 20



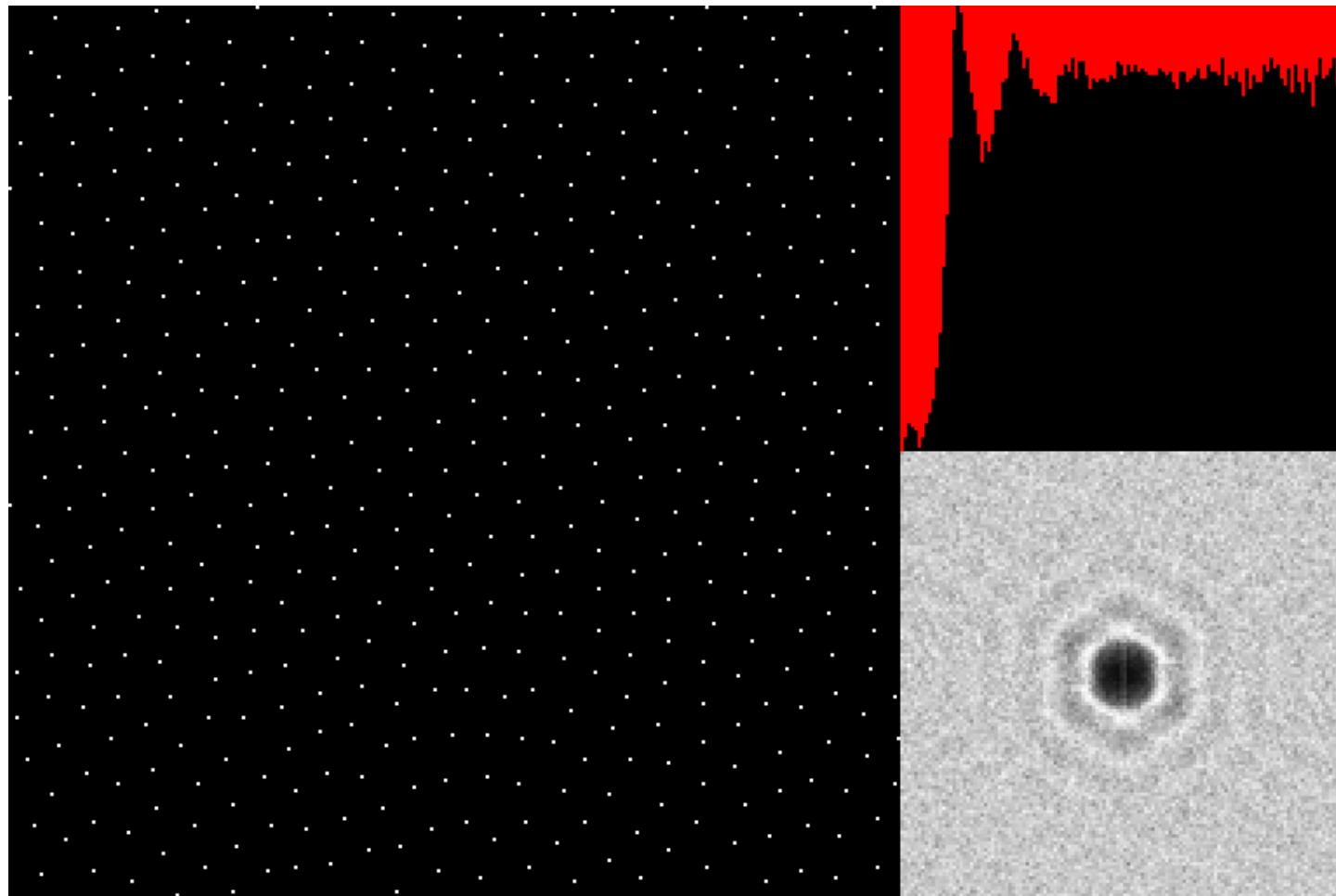
$$\begin{aligned} d_{10} &= 0.673123 \\ d_{-11} &= 0.114484 \\ d_{01} &= 0.212393 \end{aligned}$$

Computing Distribution Coefficients, Iteration # 33



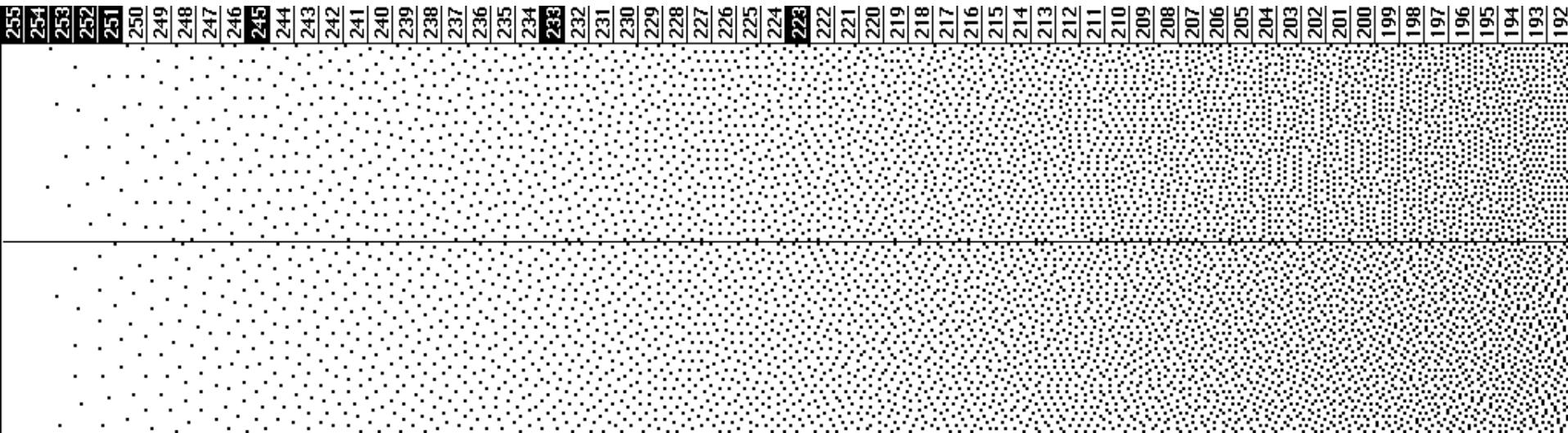
$$\begin{aligned} d_{10} &= 0.664381 \\ d_{-11} &= 0.185621 \\ d_{01} &= 0.149998 \end{aligned}$$

Computing Distribution Coefficients, Iteration # 41



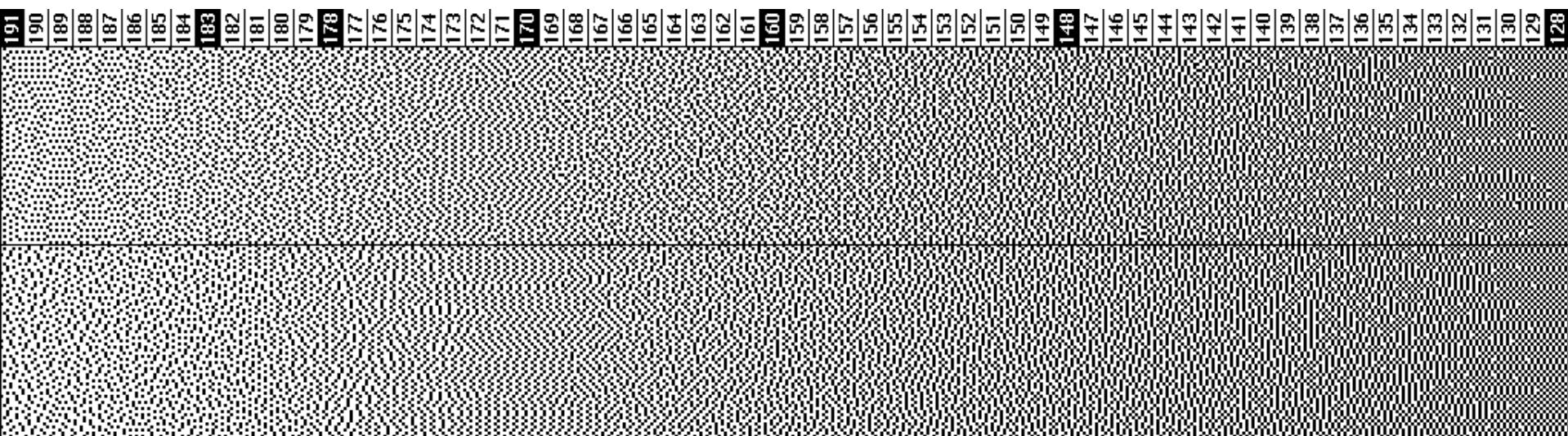
$$\begin{aligned} d_{10} &= 0.655903 \\ d_{-11} &= 0.182528 \\ d_{01} &= 0.16157 \end{aligned}$$

Results



Top: Std. Floyd-Steinberg E-D
Middle: Ostromoukhov 2001

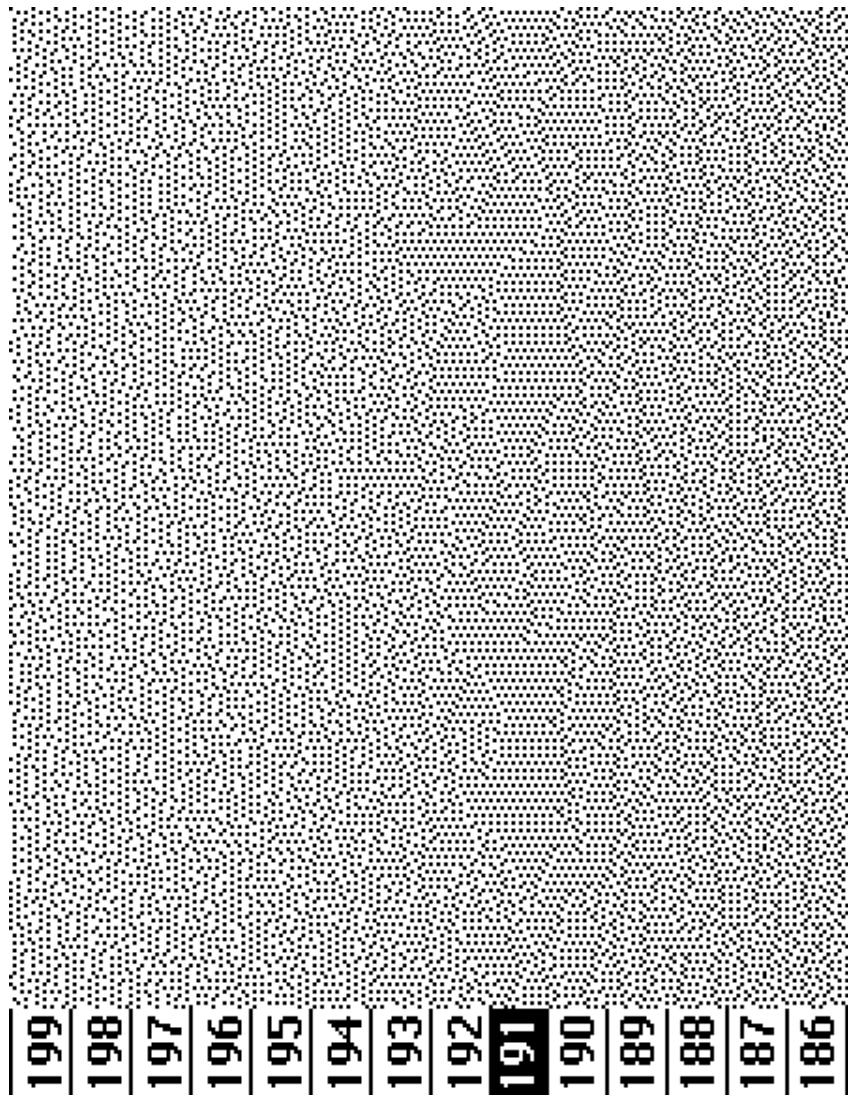
Results



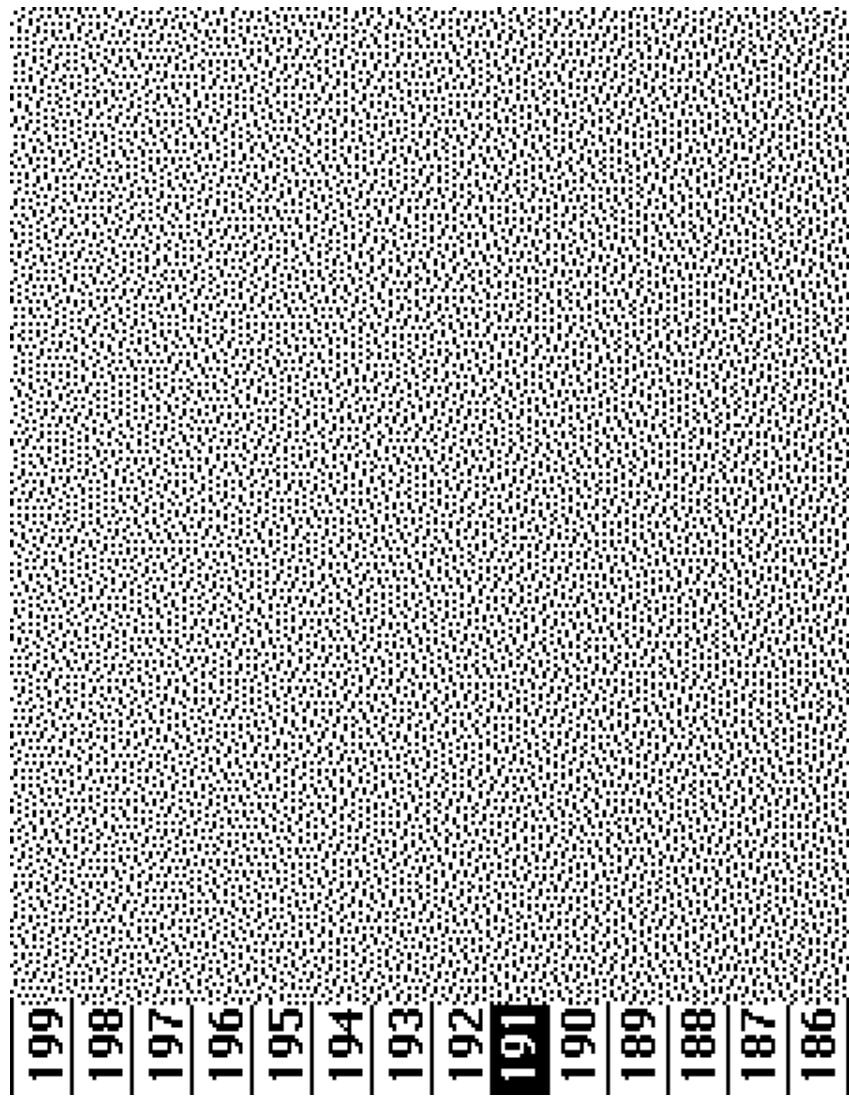
Top: Std. Floyd-Steinberg E-D
Middle: Ostromoukhov 2001

Results

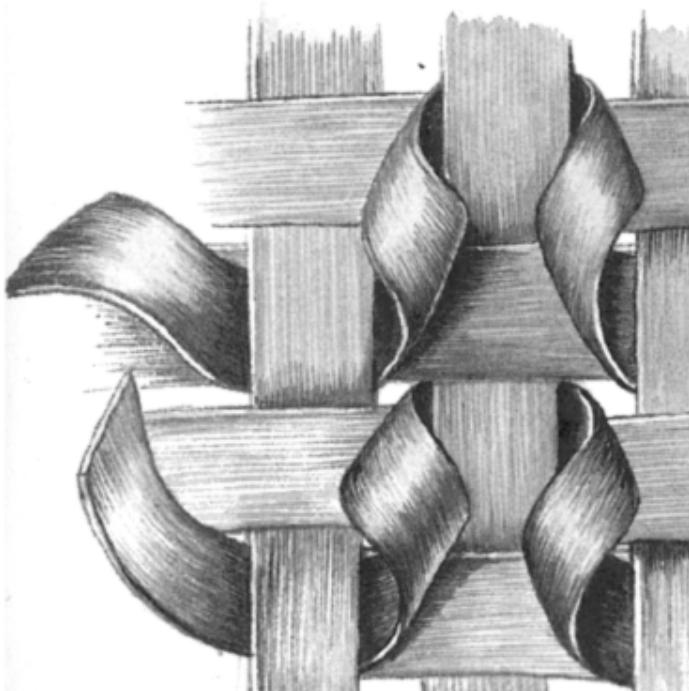
Std. Floyd-Steinberg E-D



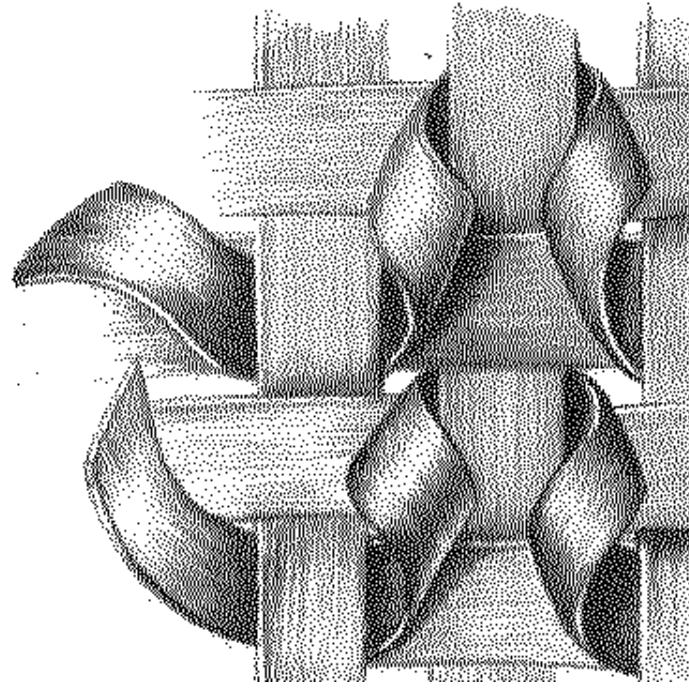
Ostromoukhov 2001



Results



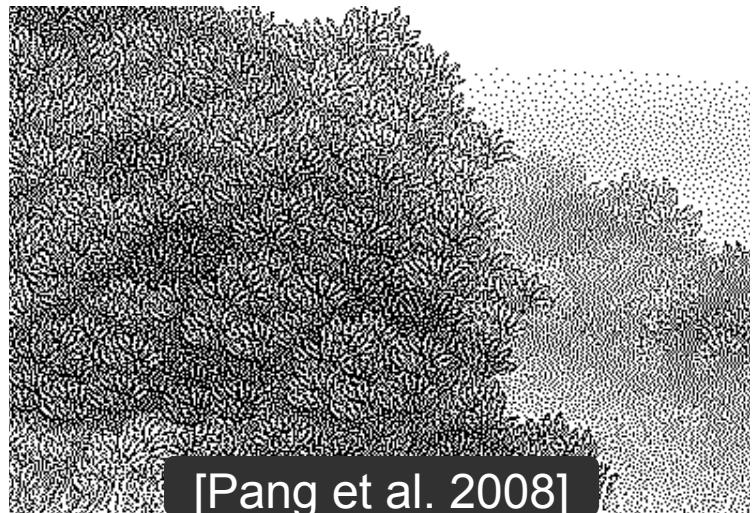
Original



Error Diffusion
[Ostromoukhov 2001]

Class III: Global Optimization

- Optimize whole image
 - Global objective function
 - Keep only good changes
 - Class III \rightarrow prohibitively slow
- Results
 - No visible artifacts
 - Accurate tones & structures
 - Class III \rightarrow the reference for quality

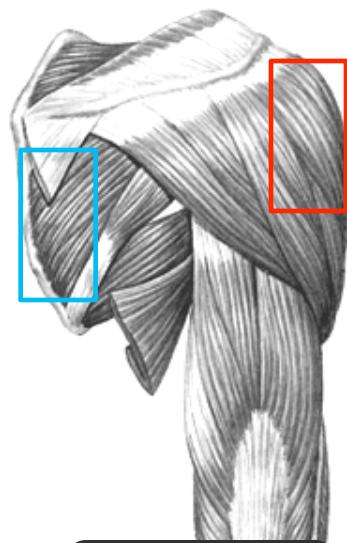


[Pang et al. 2008]

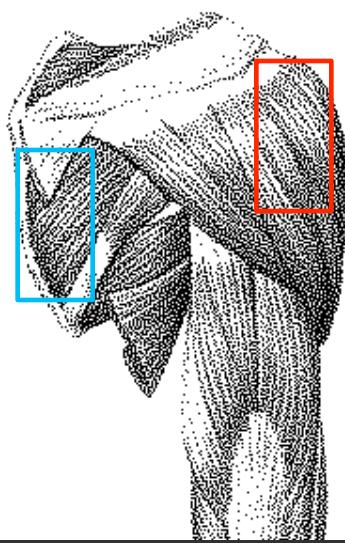


[Analoui & Allebach 1992]

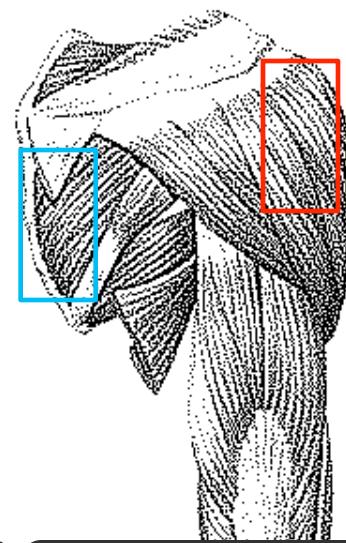
Results



Original



General-purpose ED



Structure-aware
[Pang et al. 2008]

Results



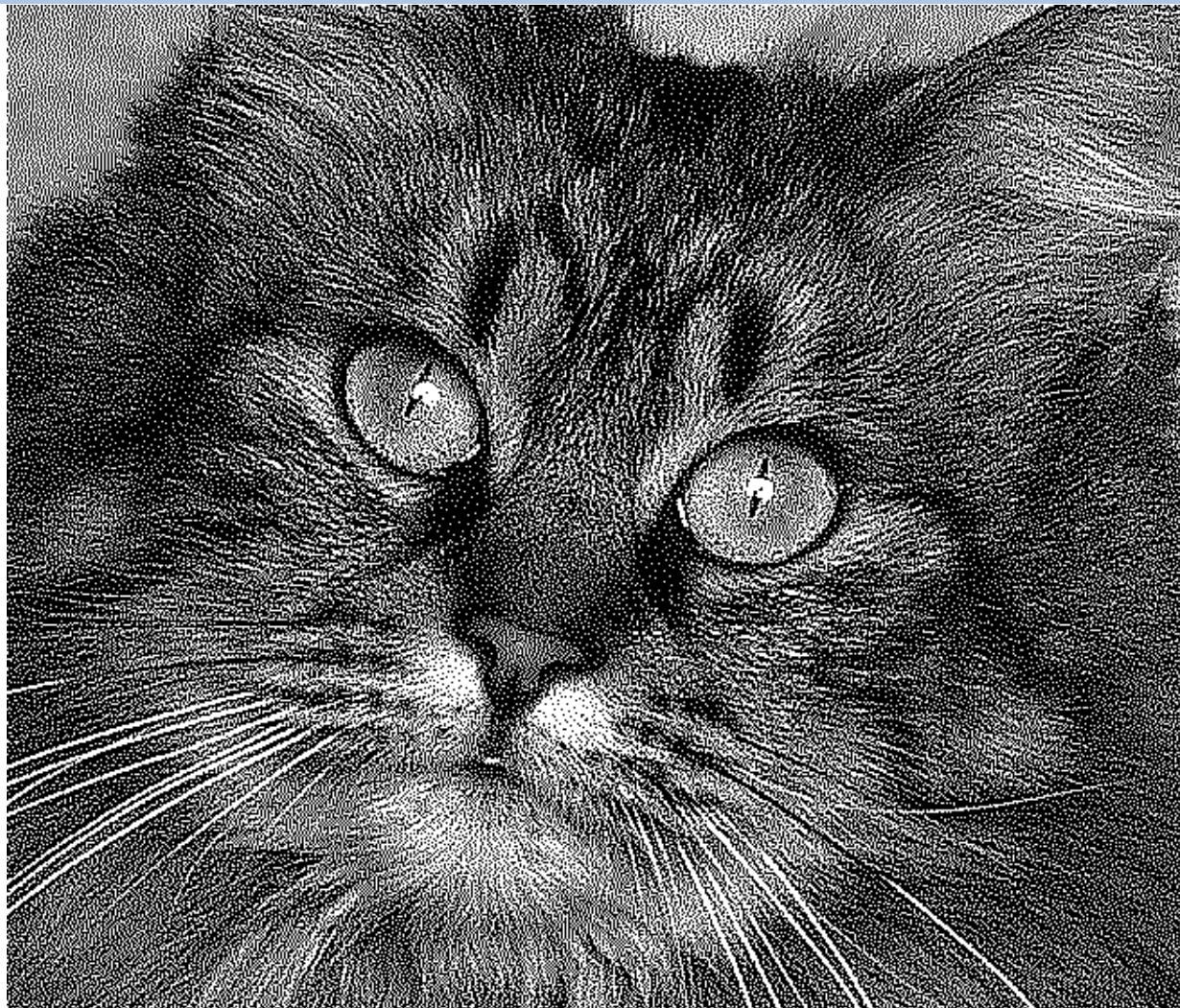
Original

Results



General-purpose ED

Results



Structure-aware
[Pang et al. 2008]

Results

Structure-Aware Halftoning

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Daniel Cohen-Or², Pheng-Ann Heng¹

¹The Chinese University of Hong Kong

²Tel Aviv University

That's All for Today!

More info:

- A Guide to Halftone Technologies

http://www.isisimaging.com/Isis_Library_files/Halftone_technology_guide.pdf