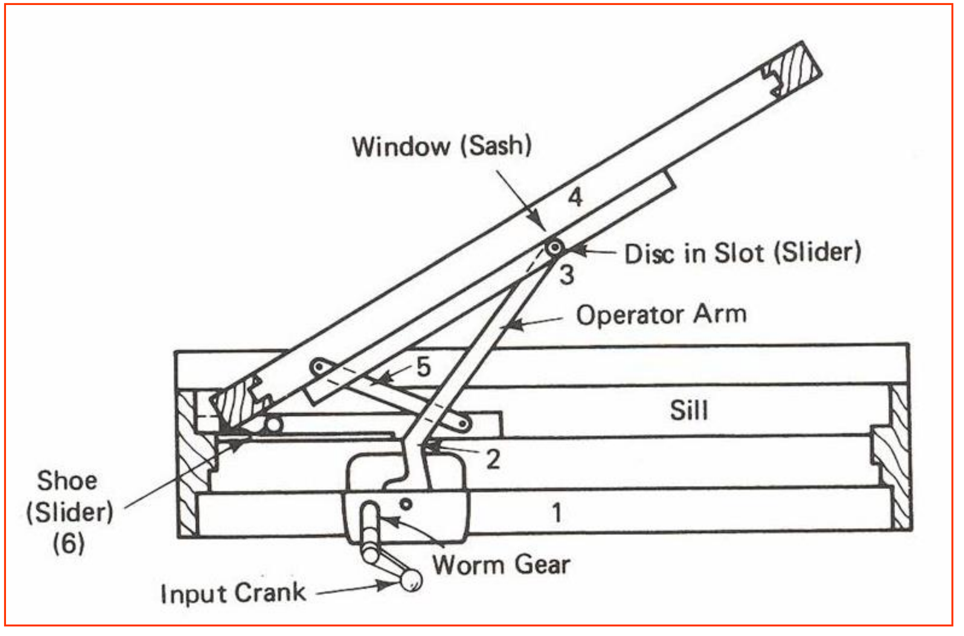
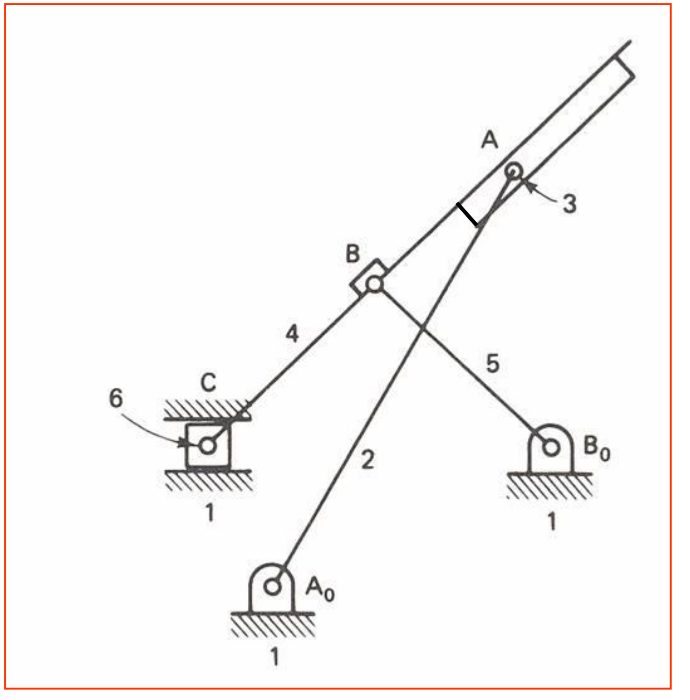
Exercise 1

- Determine the degrees of freedom for the window mechanism shown below



Mechanism to open and close a window



Associated Kinematic diagram

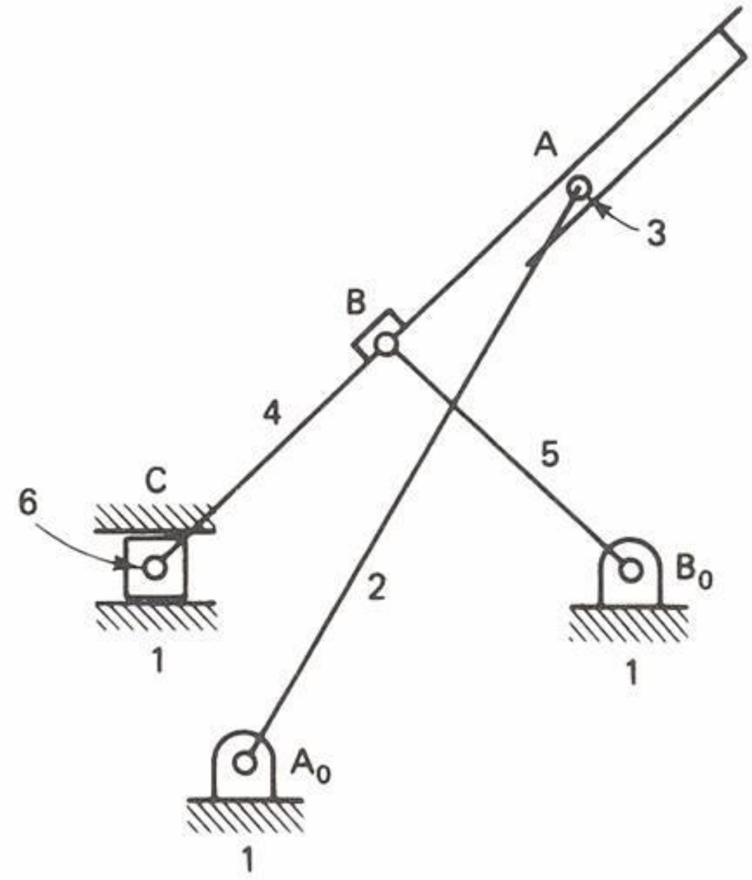
Exercise 1 Solution

$$n = 6, j = 7$$

$$F = 3n - 2j - 3$$

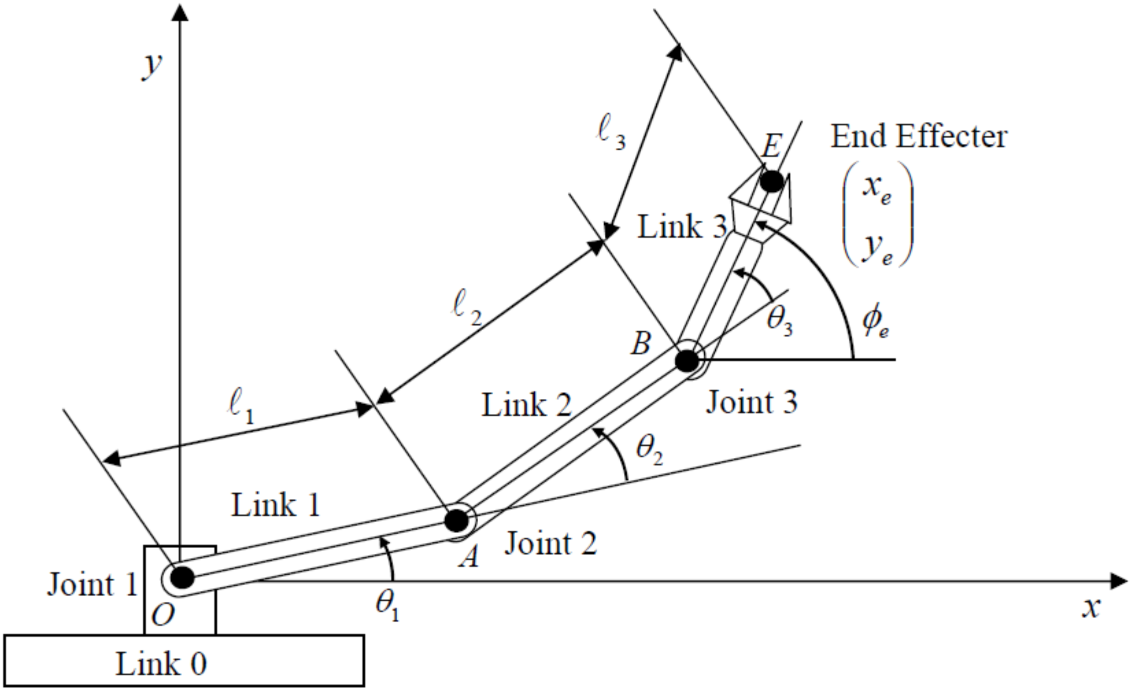
$$= 3(6) - 2(7) - 3$$

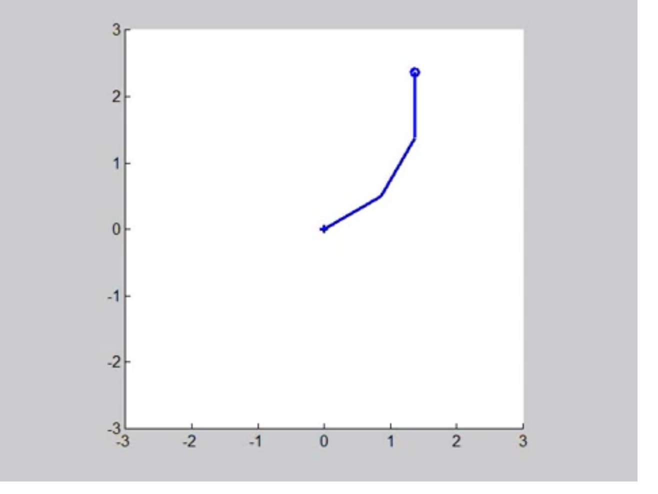
$$= 1$$



Exercise 2

- The planar robot arm is an open loop kinematic chain that consists of four links and three joints. Determine its degree-of-freedom

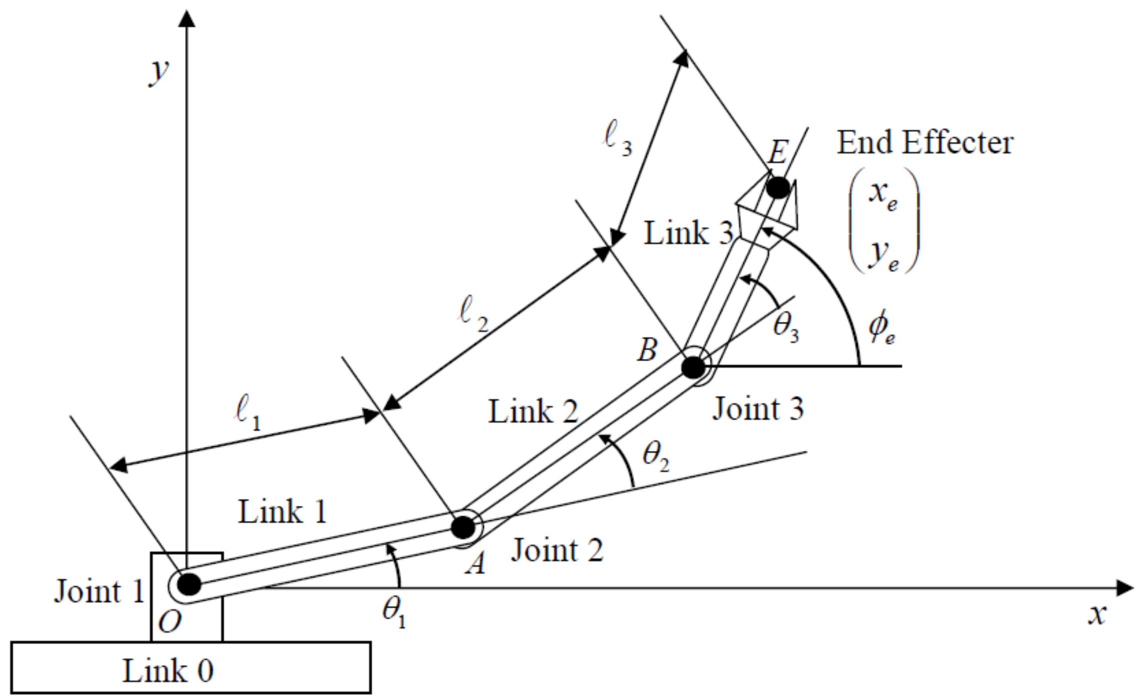




Exercise 2- Solution

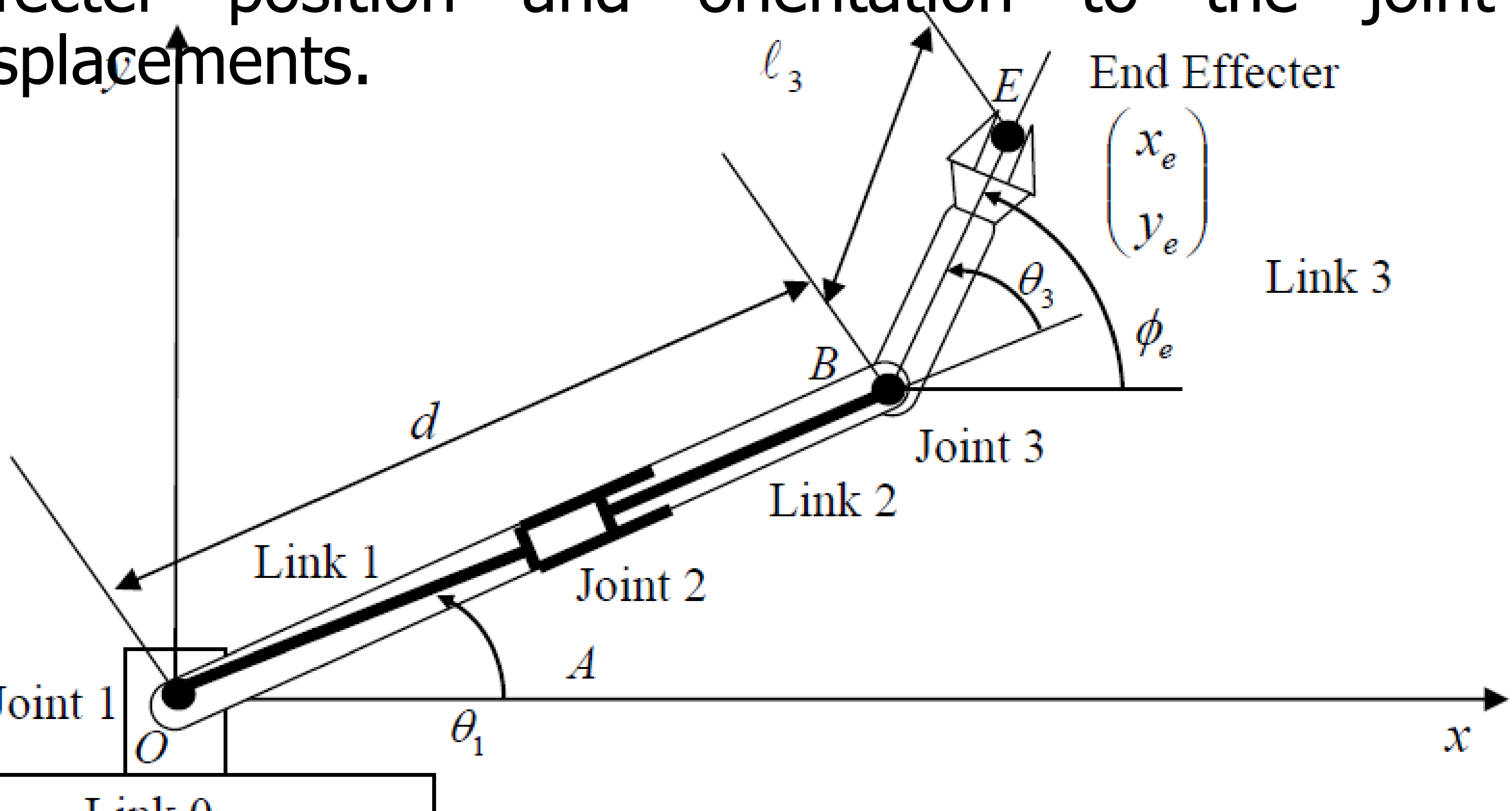
$$n = 4,\ j = 3$$

$$F = 3n - 2j - 3$$
$$= 3(4) - 2(3) - 3$$
$$= 3$$



Exercise 3

- Shown below in Figure is a planar robot arm with two revolute joints and one prismatic joint. Using the geometric parameters and joint displacements, obtain the kinematic equations relating the end-effecter position and orientation to the joint displacements.



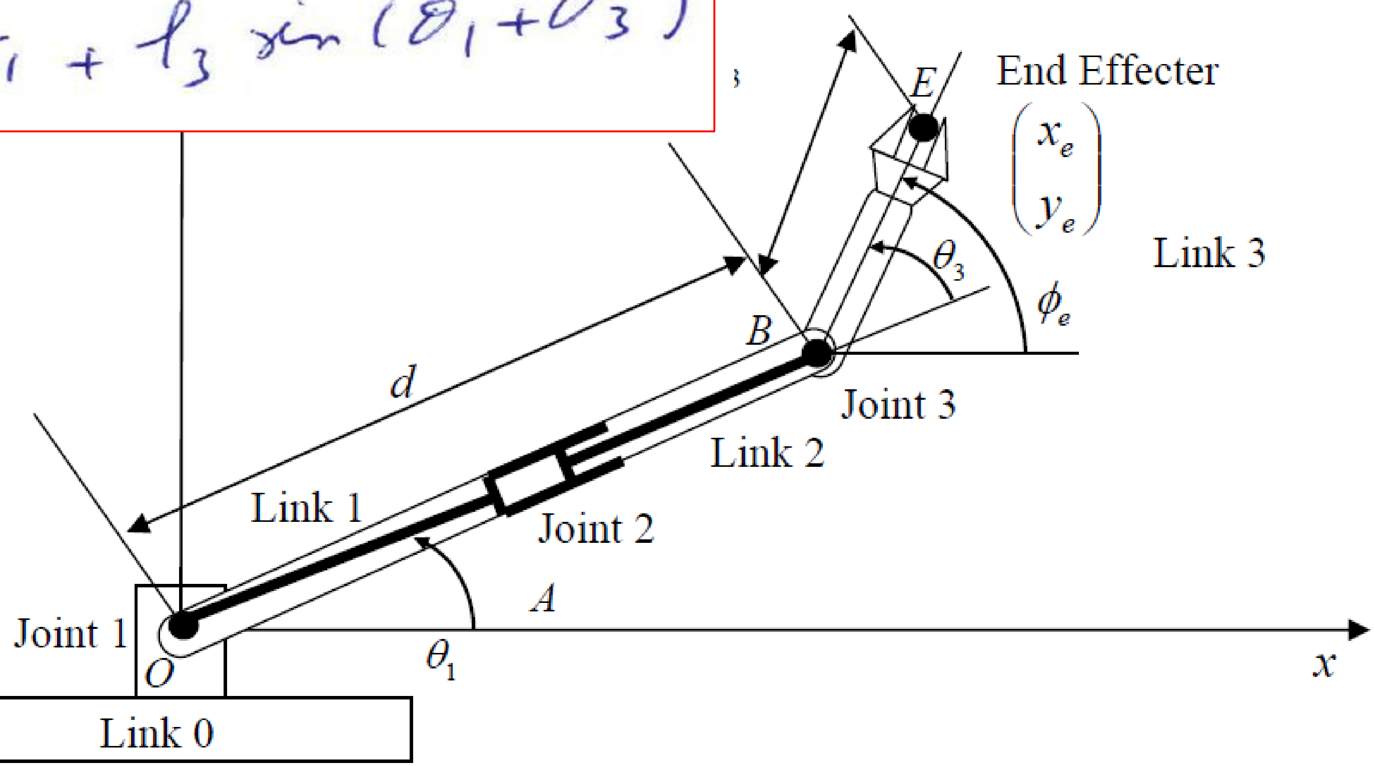
Exercise 3-Solution

$$\boxed{\theta_e = \theta_1 + \theta_3}$$

$$l_1 < d < l_1 + l_2$$

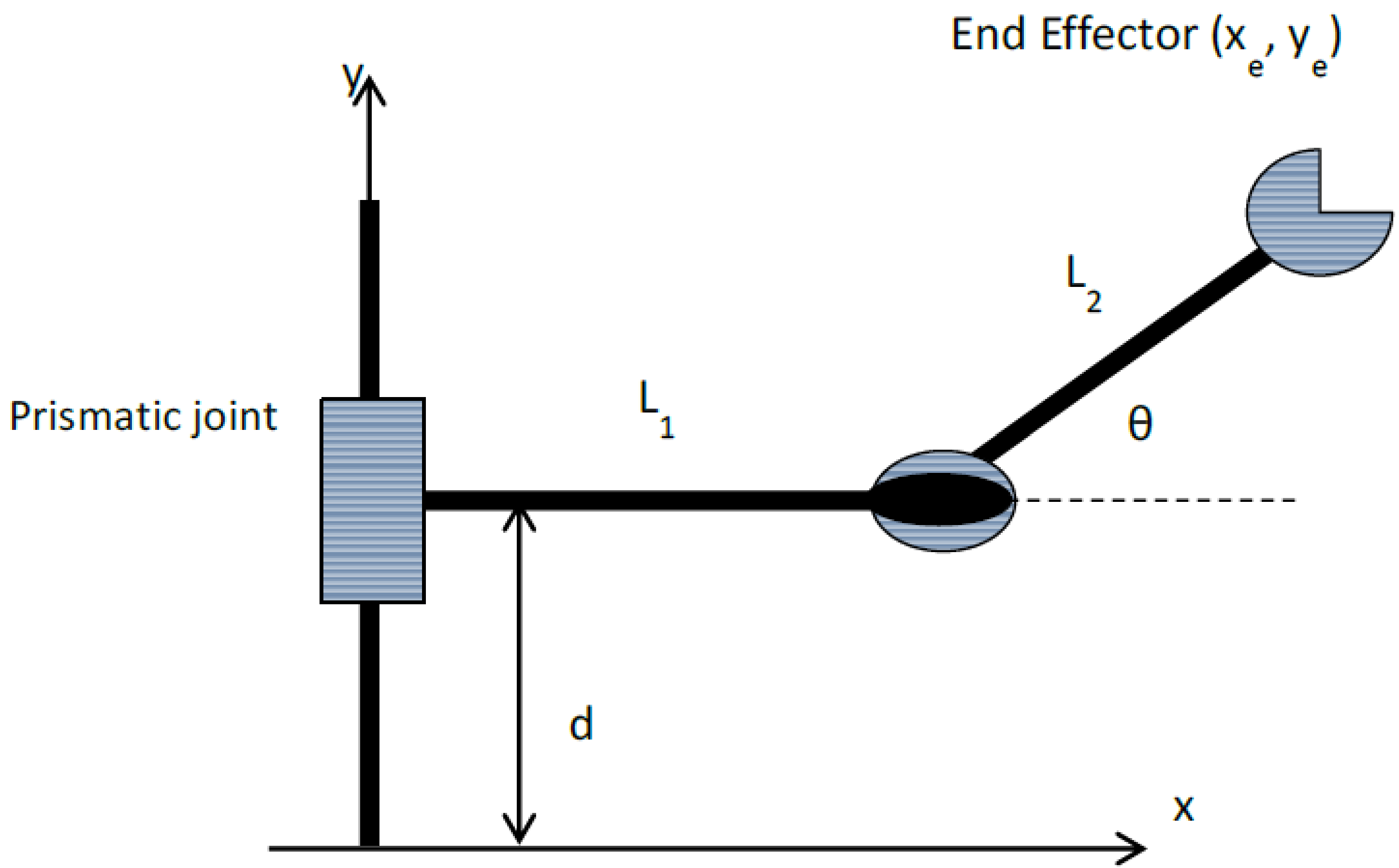
$$x_e = d \cos\theta_1 + l_3 \cos(\theta_1 + \theta_3)$$

$$y_e = d \sin\theta_1 + l_3 \sin(\theta_1 + \theta_3)$$



Exercise 4

- Given the desired end effector position, calculate the parameters d and θ for the robot arm shown



Exercise 4-Solution

Ans:

Forward kinematics:

$$\begin{bmatrix} x_e \\ y_e \end{bmatrix} = \begin{bmatrix} L_1 + L_2 \cos\theta \\ d + L_2 \sin\theta \end{bmatrix}$$

Hence,

$$x_e = L_1 + L_2 \cos\theta \rightarrow$$

$$\theta = \cos^{-1}\left(\frac{x_e - L_1}{L_2}\right) \text{ for } y_e > d \quad \text{and} \quad \theta = -\cos^{-1}\left(\frac{x_e - L_1}{L_2}\right) \text{ for } y_e < d$$

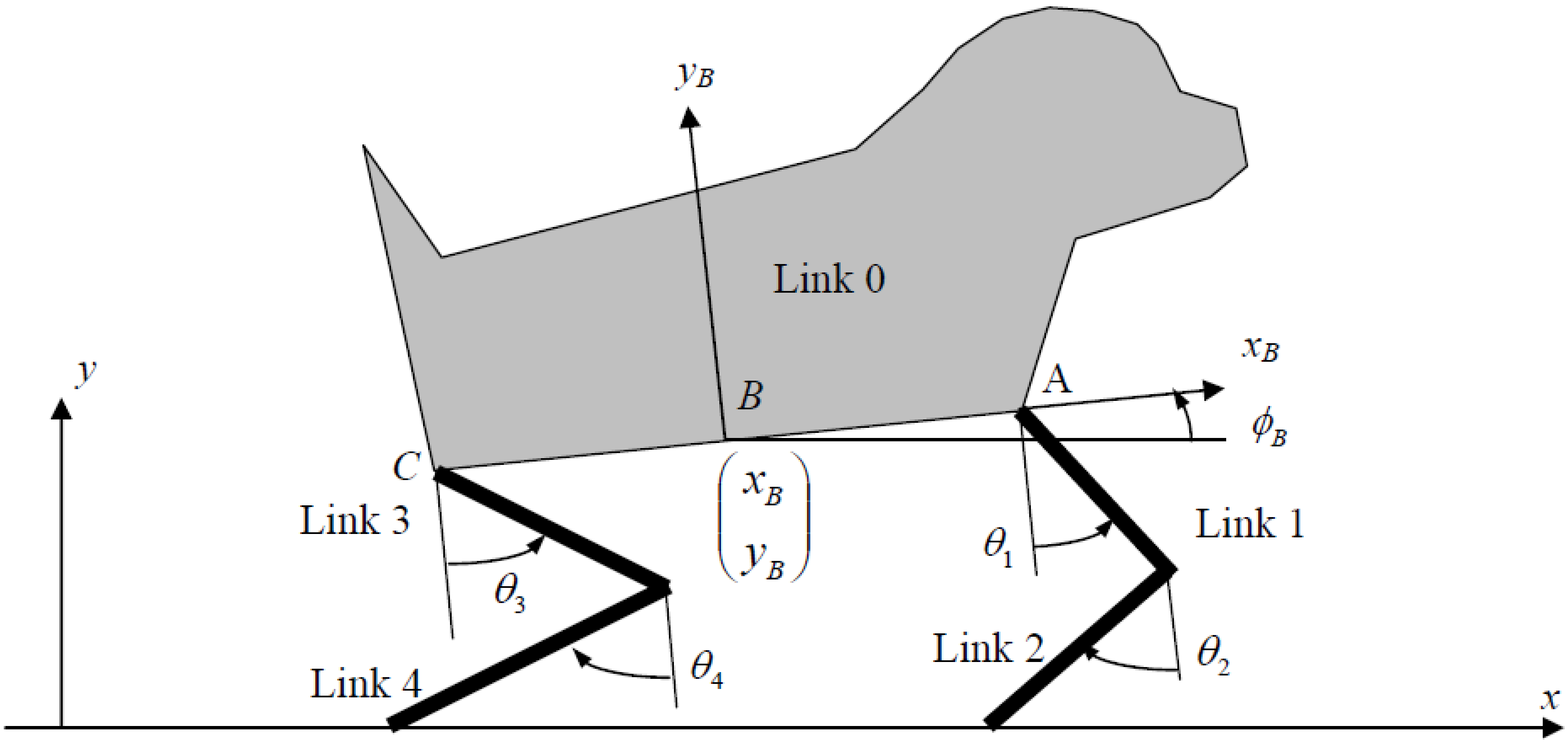
Since d is also an actuation and no limit is set in this situation, we can set $y_e > d$.

Hence, $\theta = \cos^{-1}\left(\frac{x_e - L_1}{L_2}\right)$.

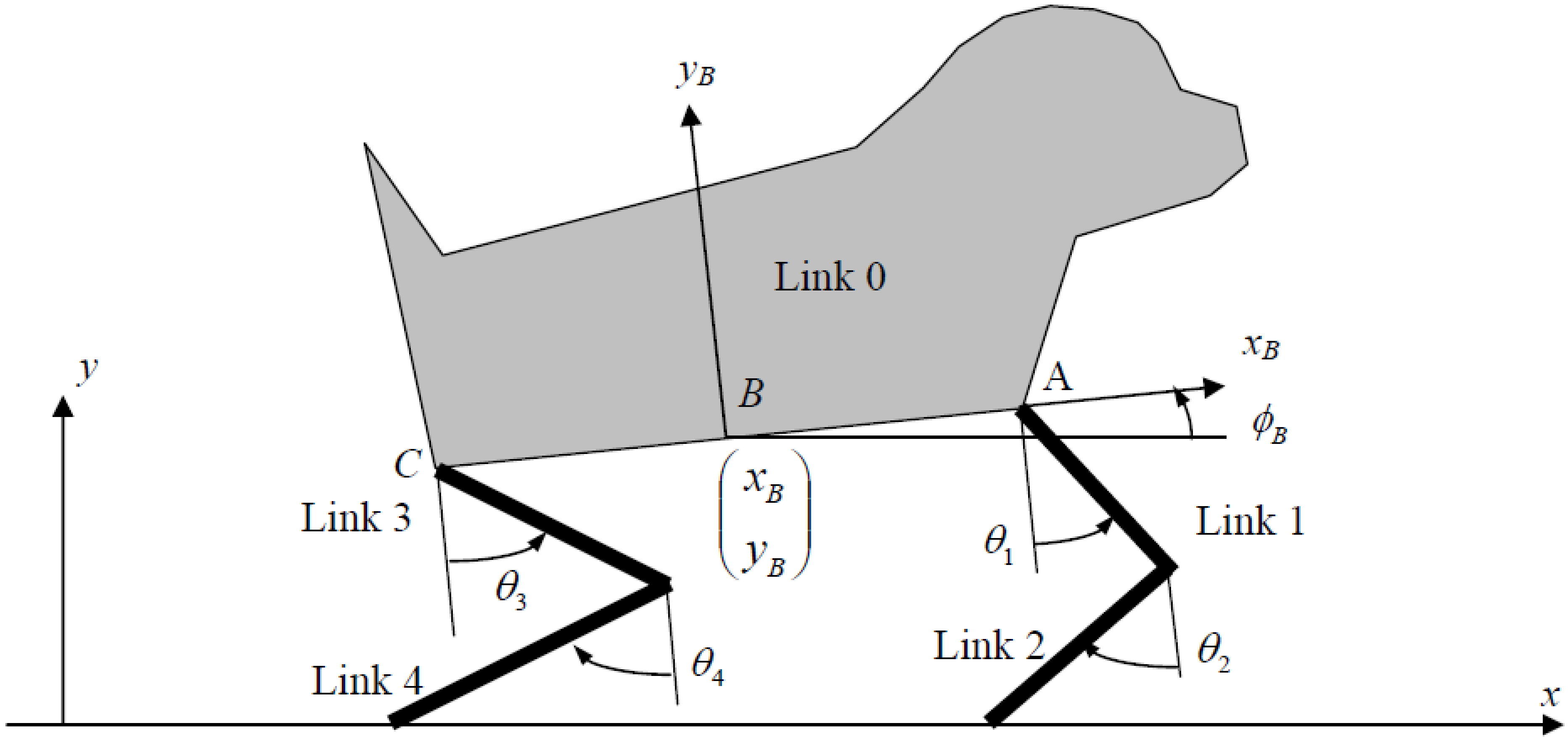
And also, $y_e = d + L_2 \sin\theta \rightarrow d = y_e - L_2 \sin\theta$

Exercise 5

- Obtain the joint angles of the dog's legs, given the body position and orientation (x_B, y_B, θ_B, x_{Link4}, x_{Link2})



Exercise 5-Solution



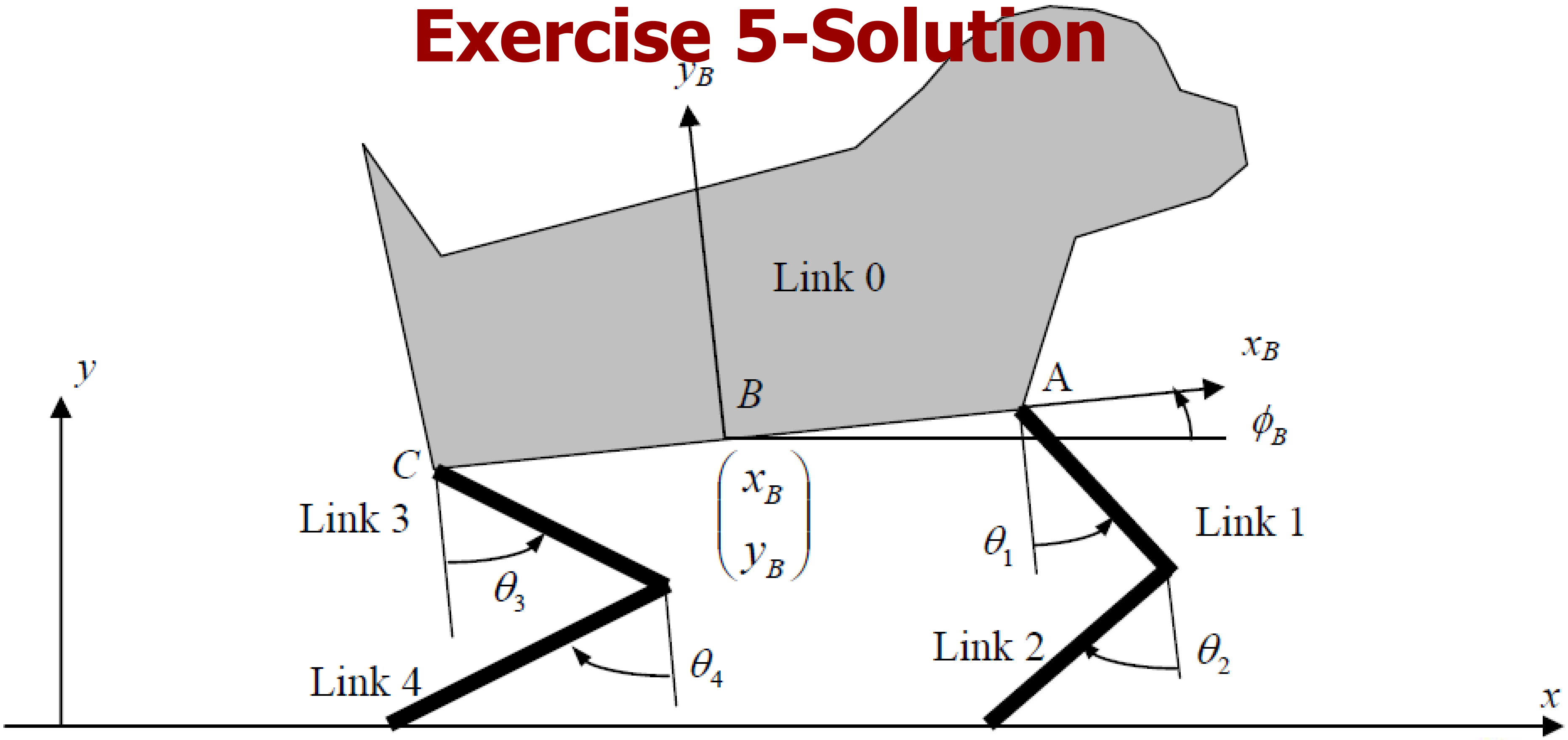
The inverse kinematics problem:

Step 1 Given x_B, y_B, ϕ_B, find x_A, y_A and x_C, y_C

Step 2 Given x_A, y_A, find θ_1, θ_2

Step 3 Given x_C, y_C, find θ_3, θ_4

Exercise 5-Solution



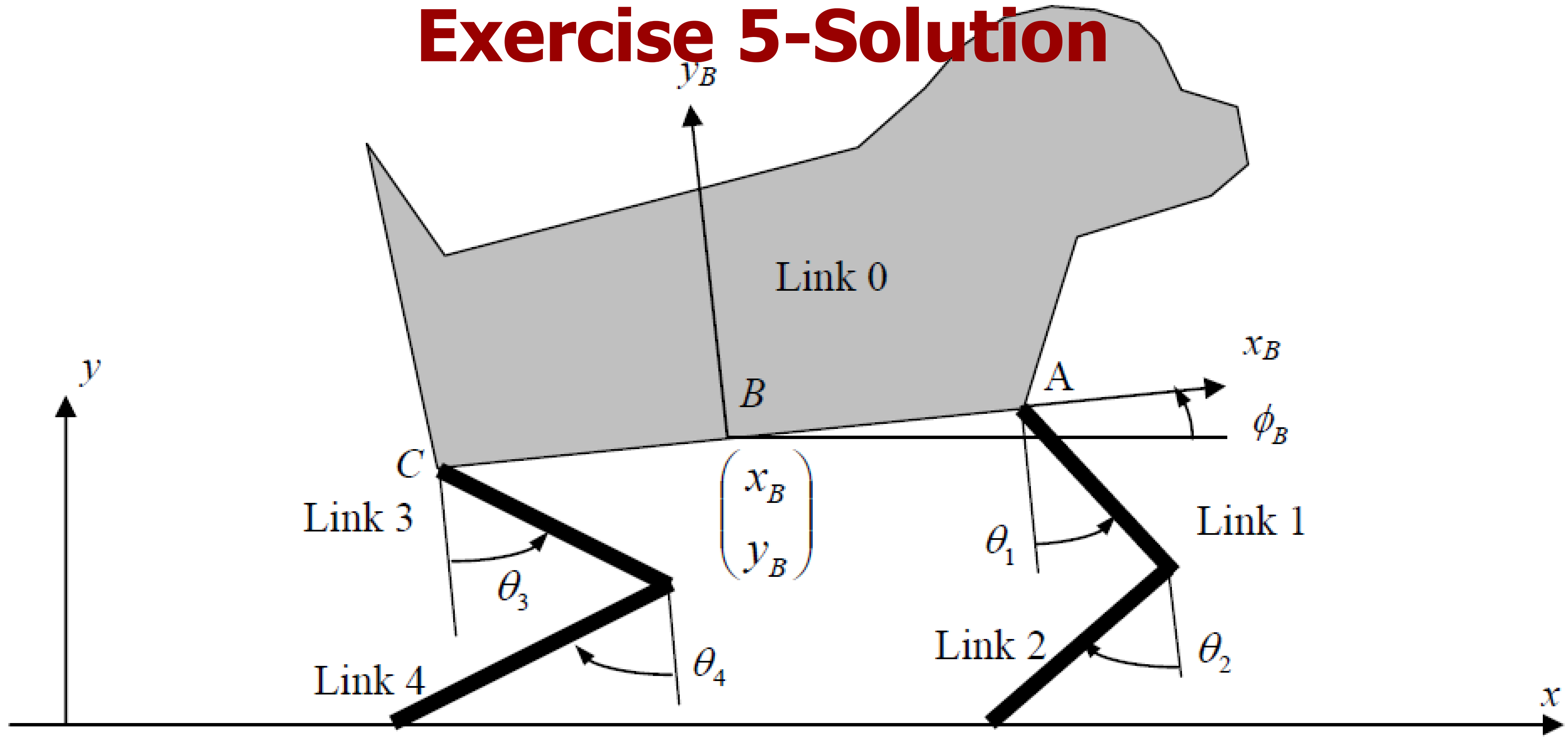
$$x_A = x_B + BA \cos\phi_B$$

$$y_A = y_B + BA \sin\phi_B$$

$$x_c = x_B - BC \cos\phi_B$$

$$y_c = y_B - BC \sin\phi_B$$

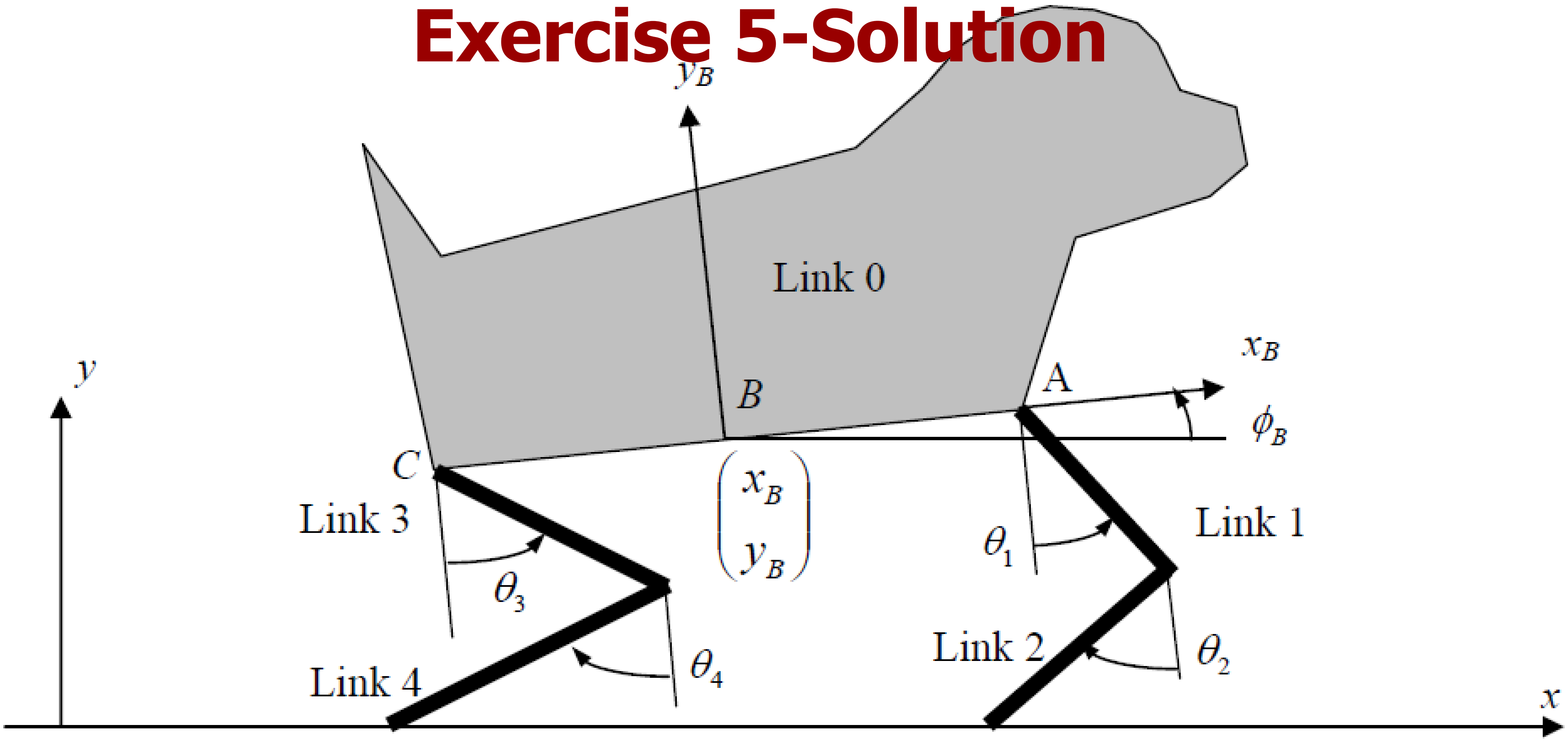
Exercise 5-Solution



$$x_A = \left(-l_1 \sin\theta_1 + l_2 \sin\theta_2\right)\cos\phi_B + x_{link\,2}$$

$$y_A = \left(l_1 \cos\theta_1 + l_2 \cos\theta_2\right)\sin\phi_B$$

Exercise 5-Solution



$$x_c = (-l_3 \sin\theta_3 + l_4 \sin\theta_4)\cos\phi_B + x_{link4}$$

$$y_c = (l_3 \cos\theta_3 + l_4 \cos\theta_4)\sin\phi_B$$