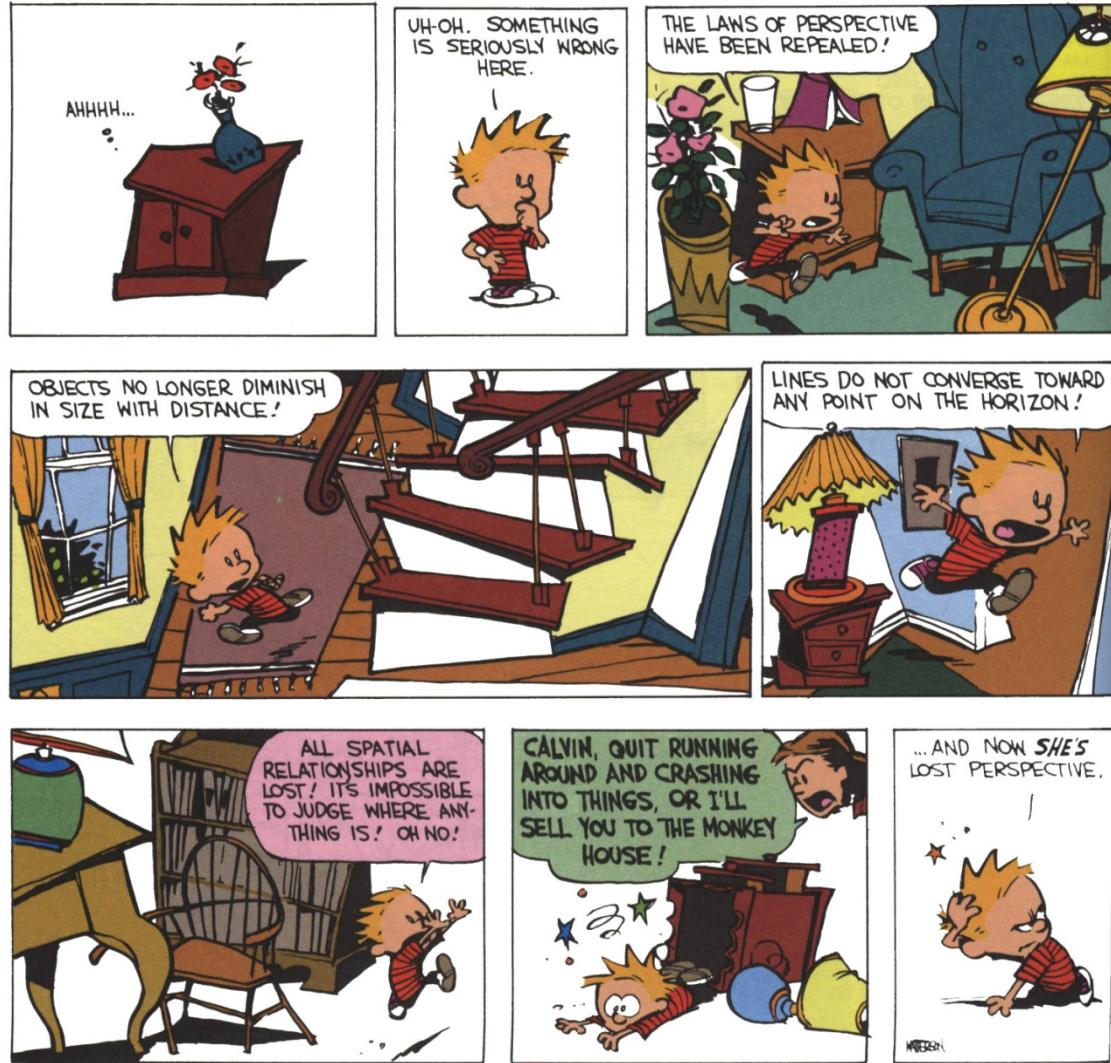


# Graphics Pipeline & Rasterization

calvin  
and  
hobbes

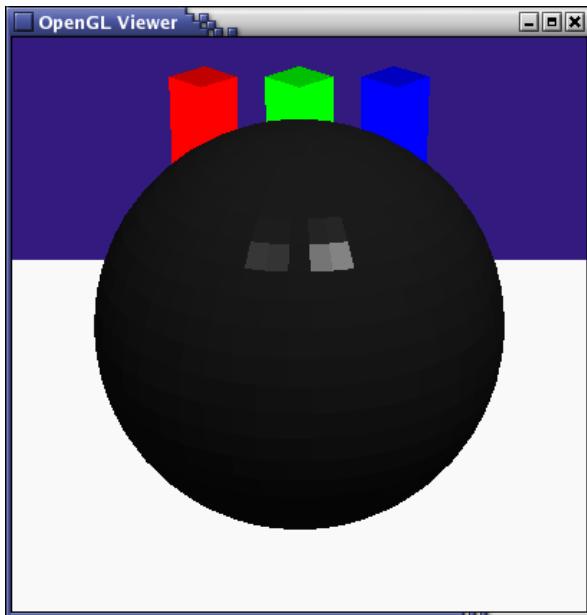
WATTERSON



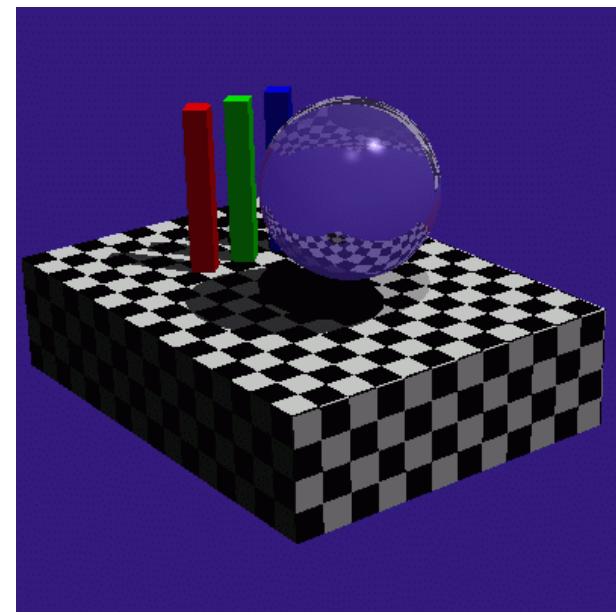
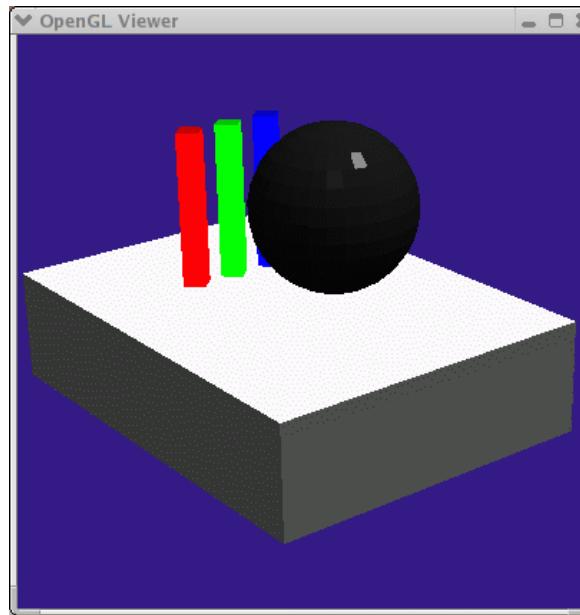
# How Do We Render Interactively?

---

- Use graphics hardware, via OpenGL or DirectX
  - OpenGL is multi-platform, DirectX is MS only



OpenGL rendering

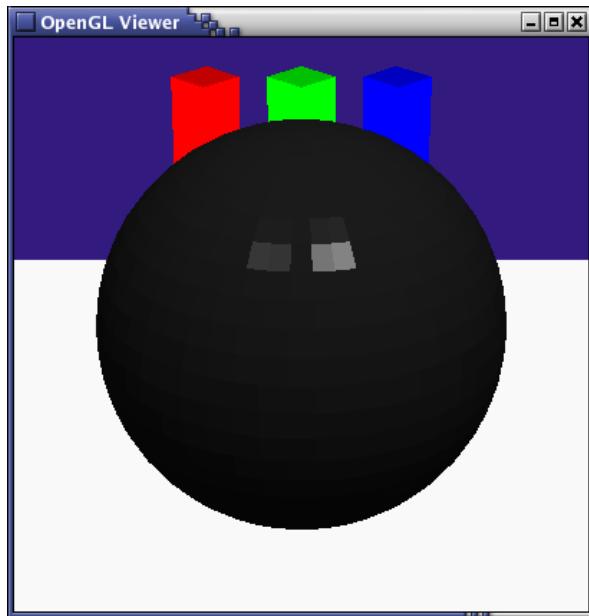


Our ray tracer

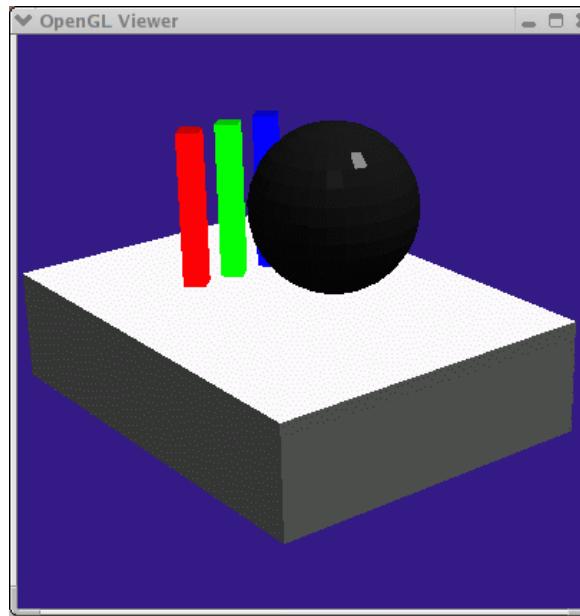
# How Do We Render Interactively?

---

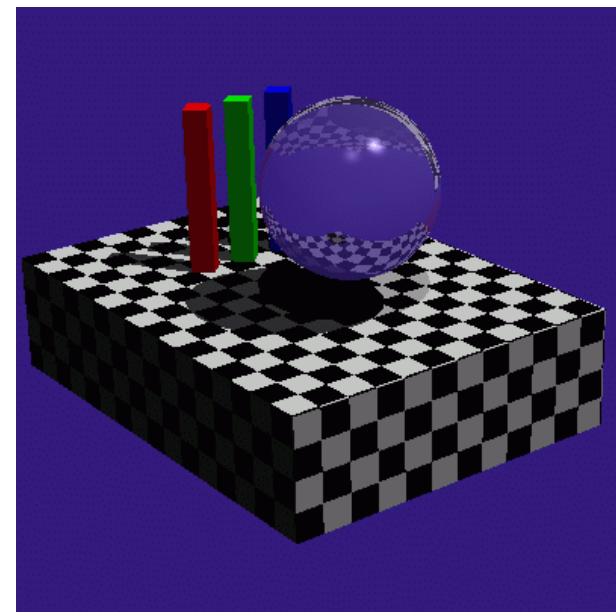
- Use graphics hardware, via OpenGL or DirectX
  - OpenGL is multi-platform, DirectX is MS only



OpenGL rendering



Our ray tracer



- Most global effects available in ray tracing will be sacrificed for speed, but some can be approximated

# Ray Casting vs. GPUs for Triangles

---

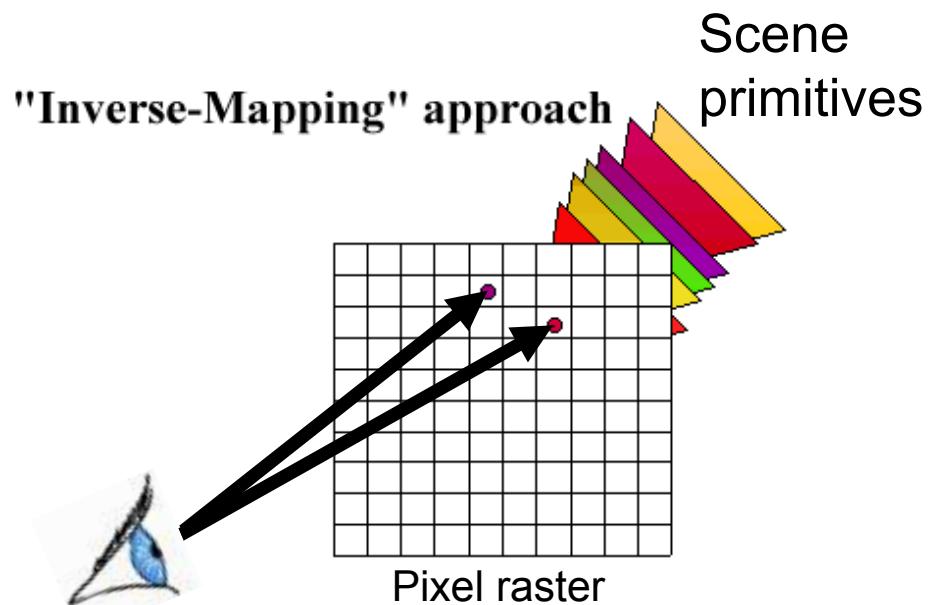
## Ray Casting

For each pixel (ray)

For each triangle

Does ray hit triangle?

Keep closest hit



# Ray Casting vs. GPUs for Triangles

## Ray Casting

For each pixel (ray)

For each triangle

Does ray hit triangle?

Keep closest hit

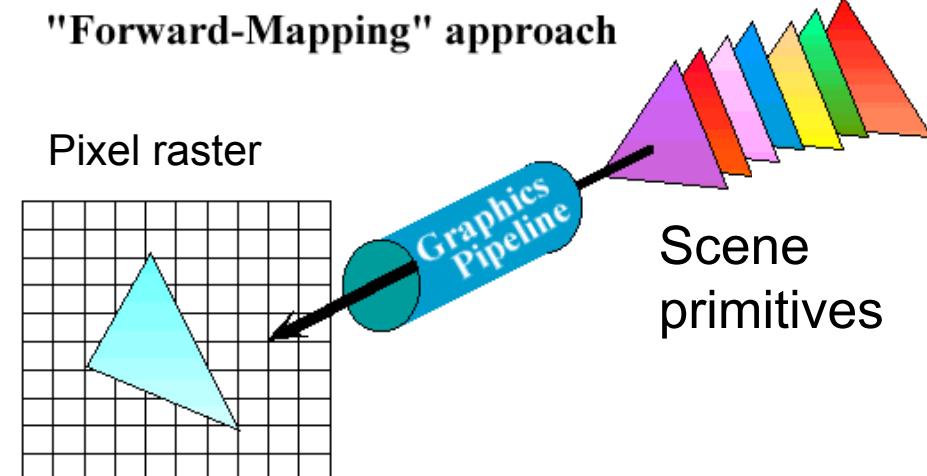
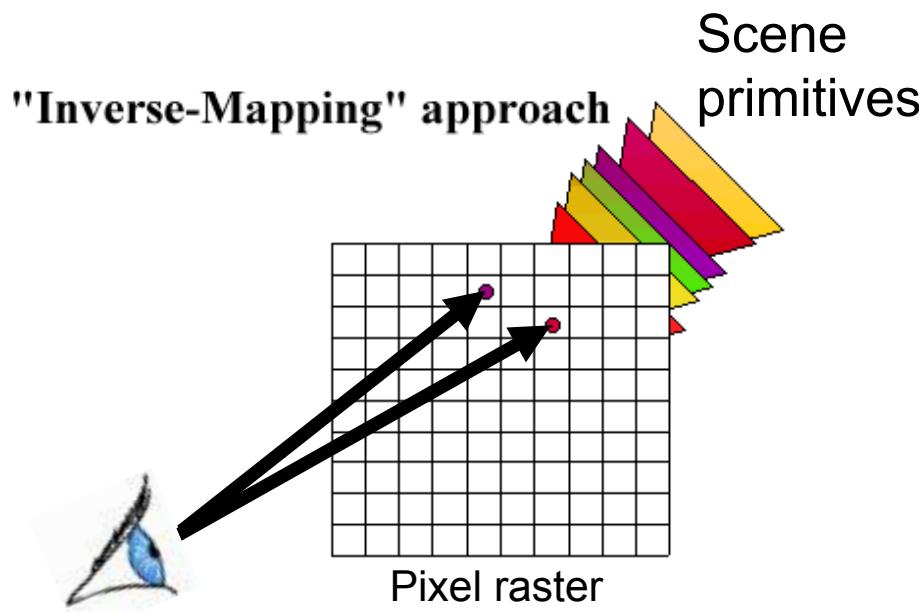
## GPU

For each triangle

For each pixel

Does triangle cover pixel?

Keep closest hit



# Ray Casting vs. GPUs for Triangles

---

## Ray Casting

For each pixel (ray)

For each triangle

Does ray hit triangle?

Keep closest hit

## GPU

For each triangle

For each pixel

Does triangle cover pixel?

Keep closest hit



It's just a different order of the loops!

# GPUs do Rasterization

- The process of taking a triangle and figuring out which pixels it covers is called rasterization

GPU

For each triangle

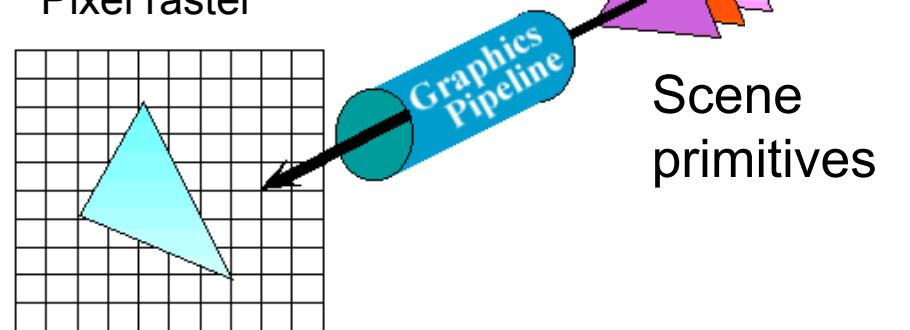
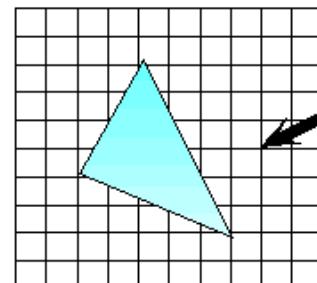
For each pixel

Does triangle cover pixel?

Keep closest hit

"Forward-Mapping" approach

Pixel raster



# GPUs do Rasterization

- The process of taking a triangle and figuring out which pixels it covers is called rasterization
- We've seen acceleration structures for ray tracing; rasterization is not stupid either
  - We're not actually going to test all pixels for each triangle

GPU

For each triangle

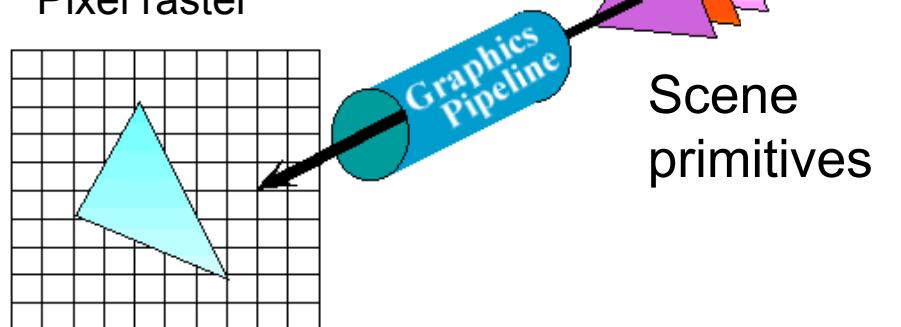
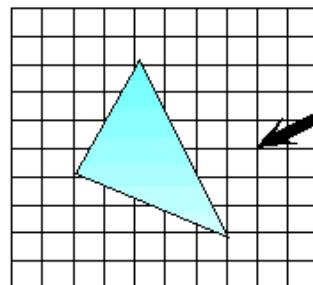
For each pixel

Does triangle cover pixel?

Keep closest hit

"Forward-Mapping" approach

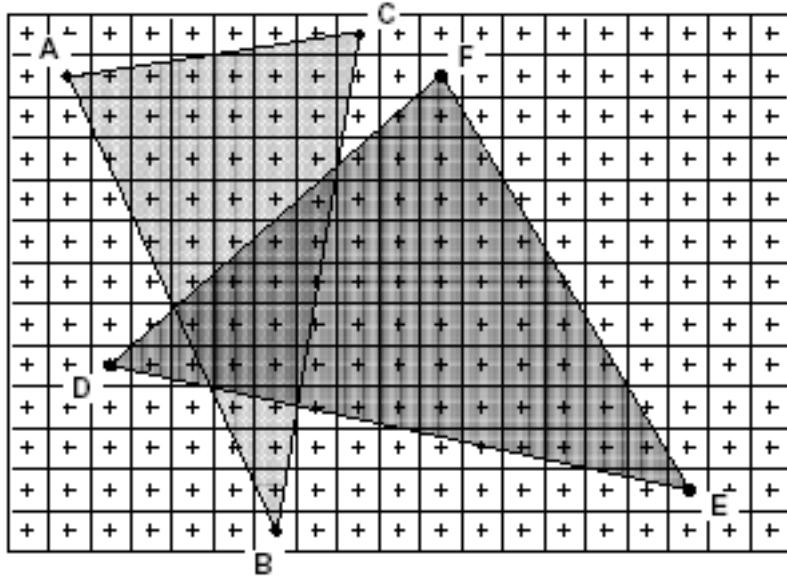
Pixel raster



# Rasterization (“Scan Conversion”)

- Given a triangle's vertices & extra info for shading, figure out which pixels to "turn on" to render the primitive
- Compute illumination values to "fill in" the pixels within the primitive
- At each pixel, keep track of the closest primitive (z-buffer)
  - Only overwrite if triangle being drawn is closer than the previous triangle in that pixel

```
glBegin(GL_TRIANGLES)
glNormal3f(...)
glVertex3f(...)
glVertex3f(...)
glVertex3f(...)
glEnd();
```



# What are the Main Differences?

---

## Ray Casting

For each pixel (ray)

For each triangle

Does ray hit triangle?

Keep closest hit

## GPU

For each triangle

For each pixel

Does triangle cover pixel?

Keep closest hit

Ray-centric

Triangle-centric

- What needs to be stored in memory in each case?

# What are the Main Differences?

---

## Ray Casting

For each pixel (ray)

For each triangle

Does ray hit triangle?

Keep closest hit

## GPU

For each triangle

For each pixel

Does triangle cover pixel?

Keep closest hit

Ray-centric

Triangle-centric

- In this basic form, ray tracing needs the entire scene description in memory at once
  - Then, can sample the image completely freely
- The rasterizer only needs one triangle at a time, plus the entire image and associated depth information for all pixels

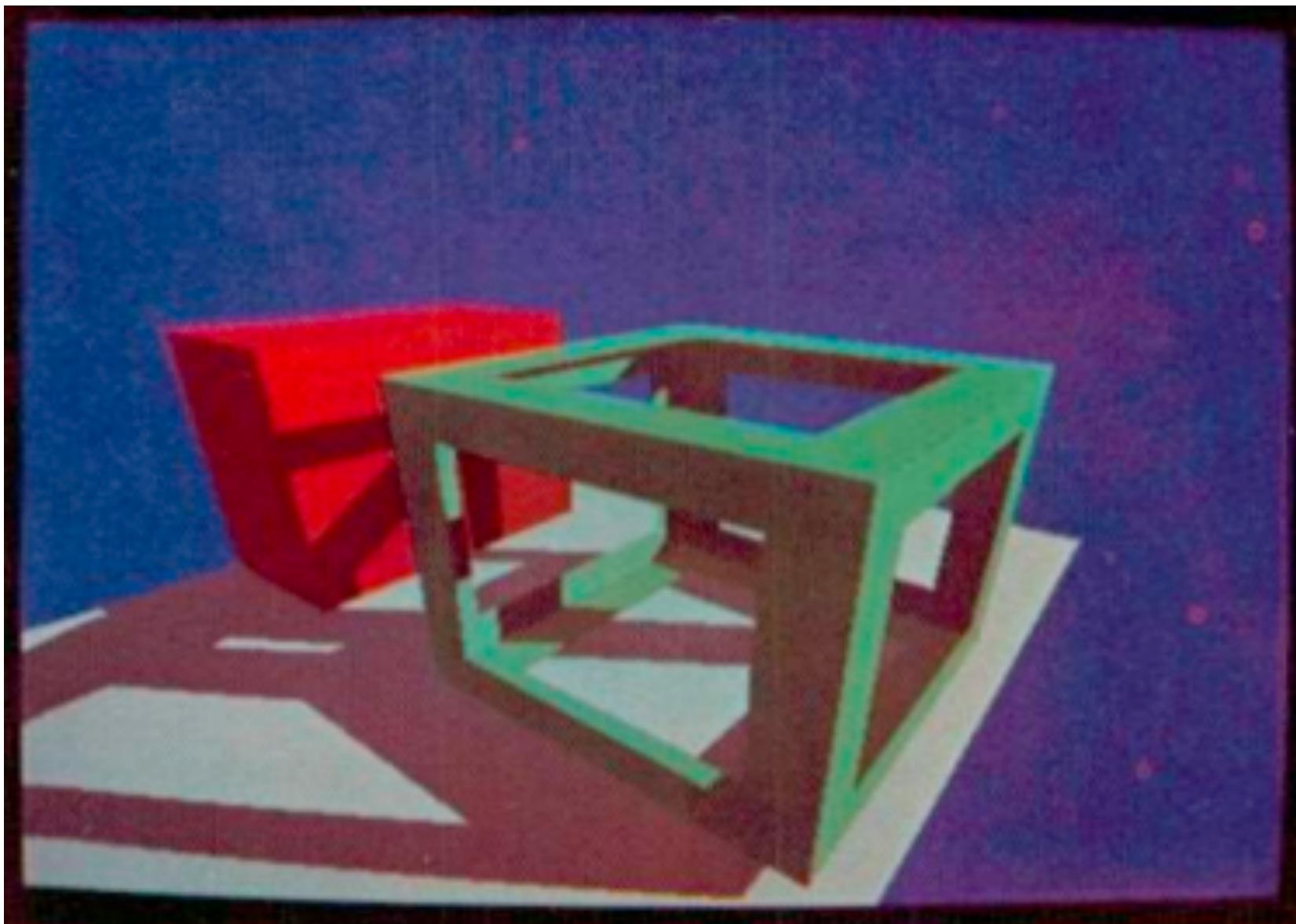
# Rasterization Advantages

---

- Modern scenes are more complicated than images
  - A 1920x1080 frame at 64-bit color and 32-bit depth per pixel is 24MB (not that much)
    - Of course, if we have more than one sample per pixel this gets larger, but e.g. 4x supersampling is still a relatively comfortable ~100MB
  - Our scenes are routinely larger than this
    - This wasn't always true

# Rasterization Advantages

[Weiler, Atherton 1977](#)



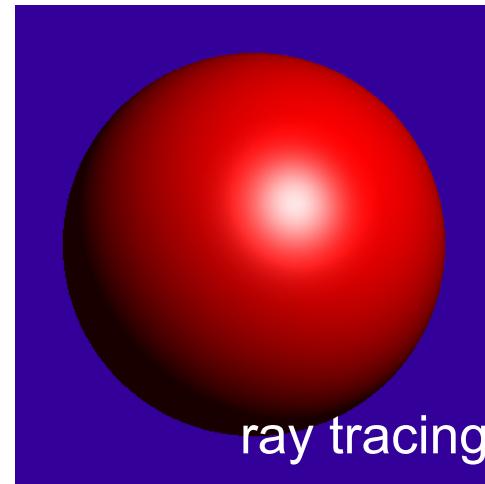
# Rasterization Advantages

---

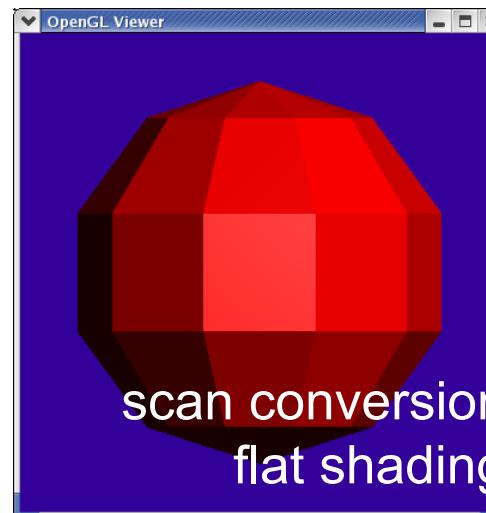
- Modern scenes are more complicated than images
  - A 1920x1080 frame (1080p) at 64-bit color and 32-bit depth per pixel is 24MB (not that much)
    - Of course, if we have more than one sample per pixel (later) this gets larger, but e.g. 4x supersampling is still a relatively comfortable ~100MB
  - Our scenes are routinely larger than this
    - This wasn't always true
- A rasterization-based renderer can stream over the triangles, no need to keep entire dataset around
  - Allows parallelism and optimization of memory systems

# Rasterization Limitations

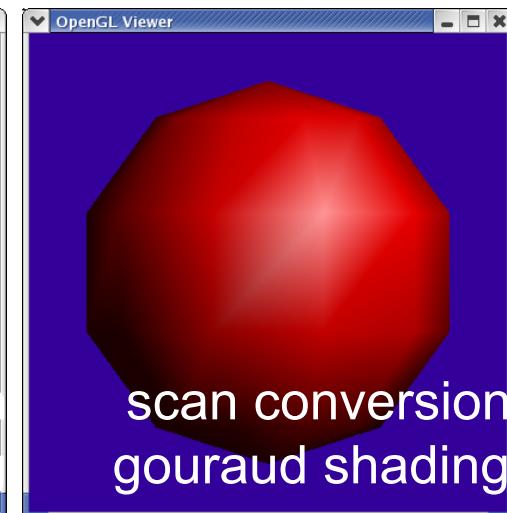
- Restricted to scan-convertible primitives
  - Pretty much: triangles
- Faceting, shading artifacts
  - This is largely going away with programmable per-pixel shading, though
- No unified handling of shadows, reflection, transparency
- Potential problem of overdraw (high depth complexity)
  - Each pixel touched many times



ray tracing



Interpolate vertex normal



Interpolate vertex color

# Ray Casting / Tracing

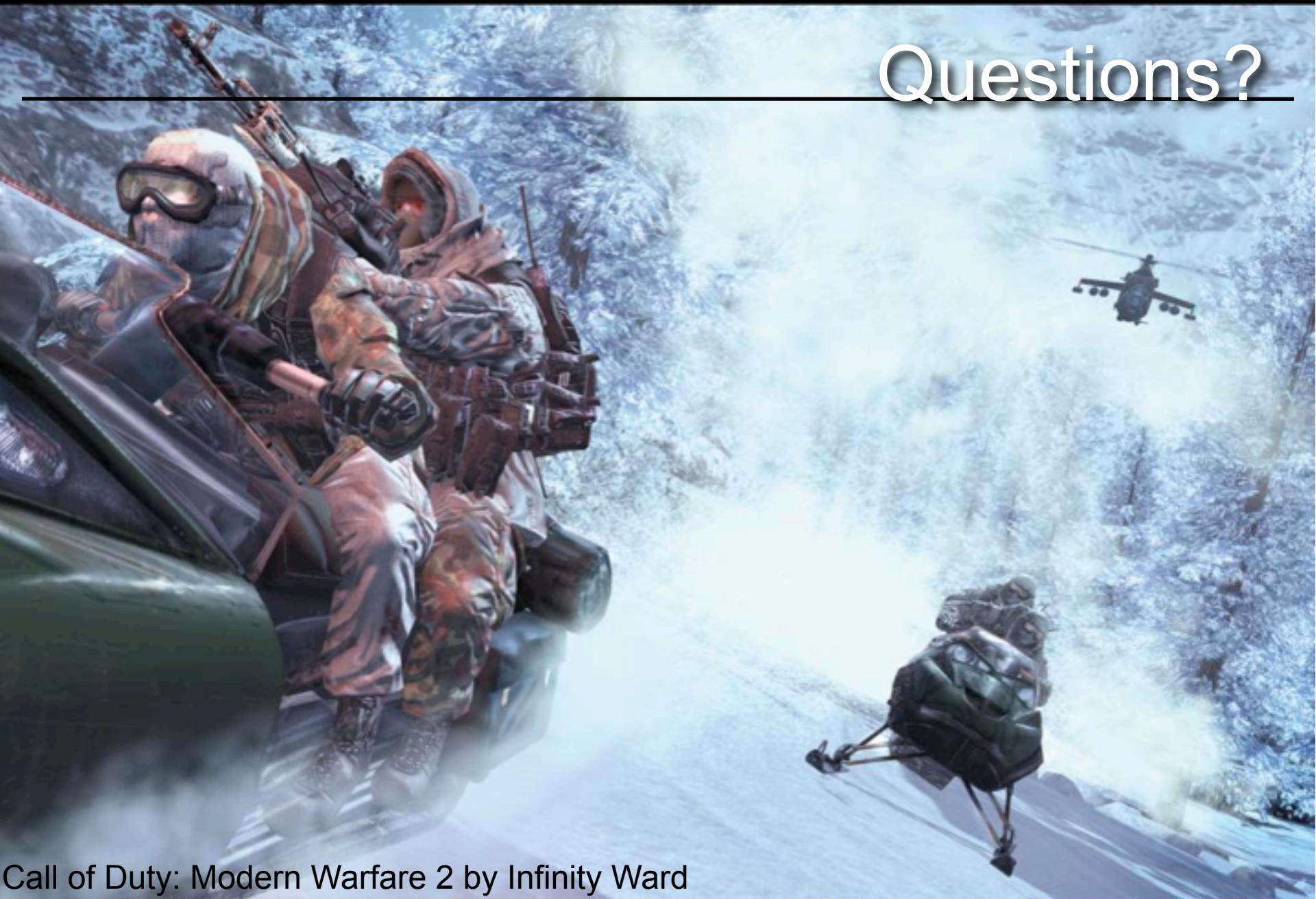
---



- Advantages
  - Generality: can render anything that can be intersected with a ray
  - Easily allows recursion (shadows, reflections, etc.)
  
- Disadvantages
  - Hard to implement in hardware (lacks computation coherence, must fit entire scene in memory, bad memory behavior)
    - Not such a big point any more given general purpose GPUs
  - Has traditionally been too slow for interactive applications
  - Both of the above are changing rather rapidly right now!

---

# Questions?



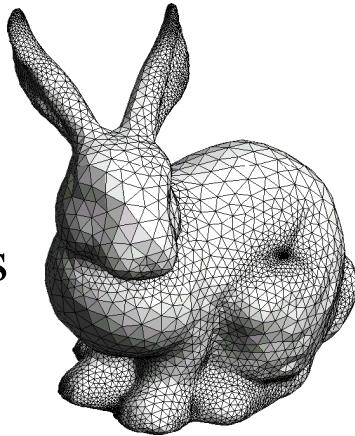
Call of Duty: Modern Warfare 2 by Infinity Ward

# Modern Graphics Pipeline

---

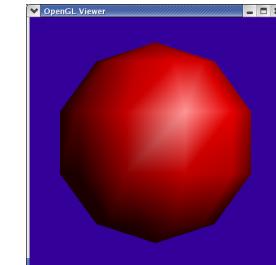
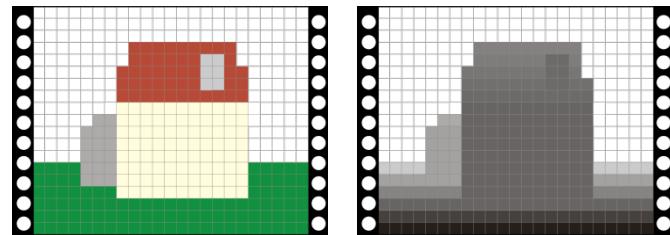
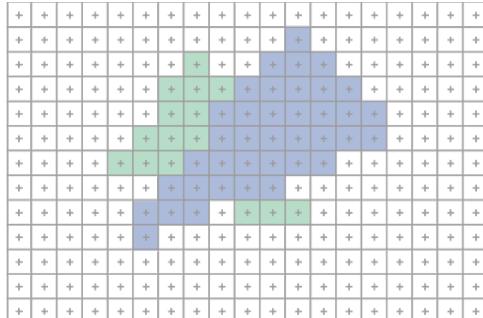
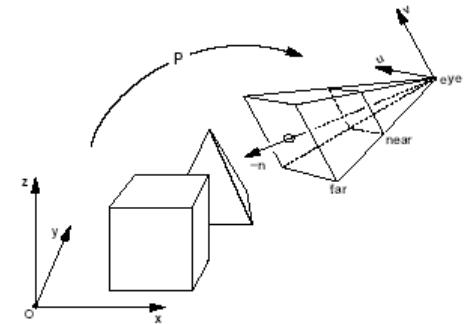
- Input
  - Geometric model
    - Triangle vertices, vertex normals, texture coordinates
  - Lighting/material model (shader)
    - Light source positions, colors, intensities, etc.
    - Texture maps, specular/diffuse coefficients, etc.
  - Viewpoint + projection plane
- Output
  - Color (+depth) per pixel

Colbert & Krivanek



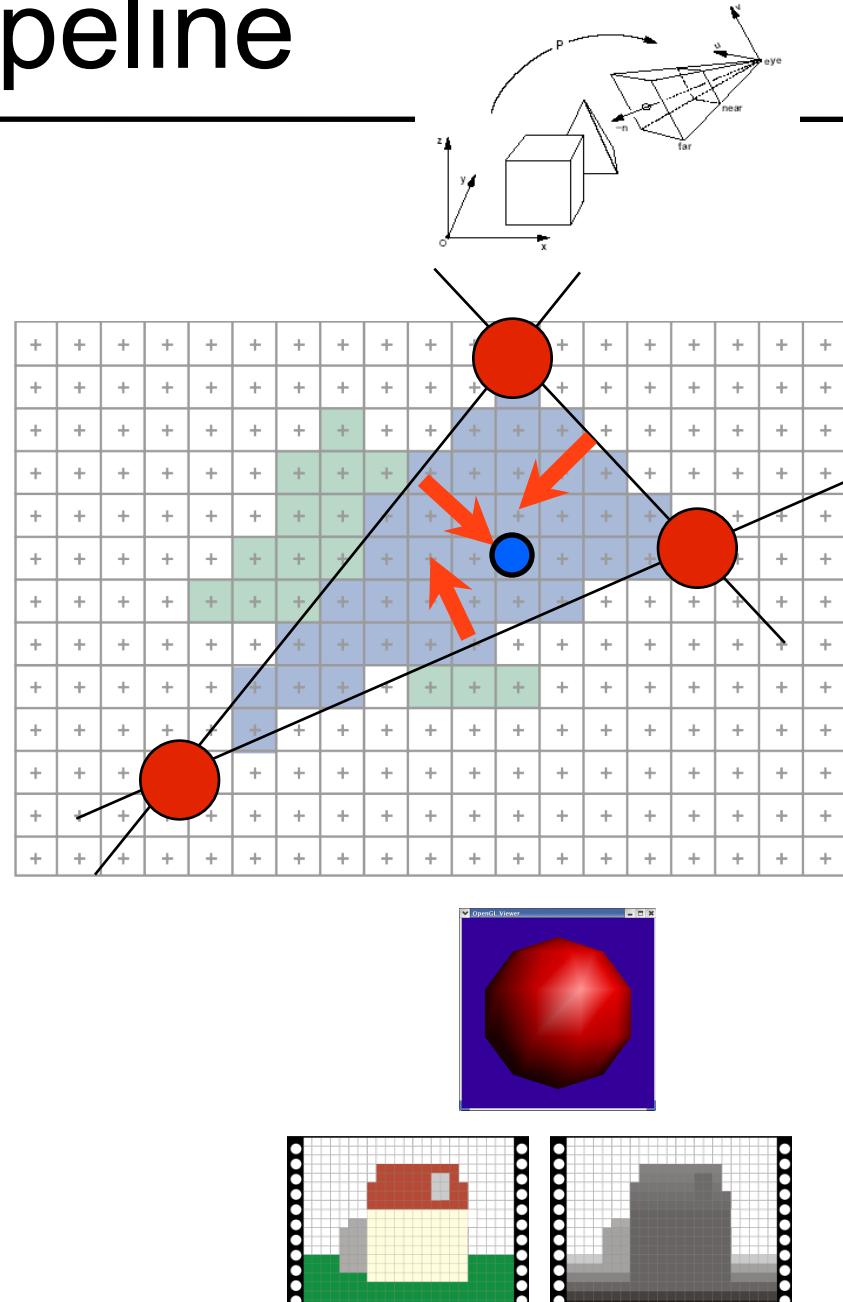
# Modern Graphics Pipeline

- Project vertices to 2D (image)
- Rasterize triangle: find which pixels should be lit
- Test visibility (Z-buffer), update frame buffer color
- Compute per-pixel color



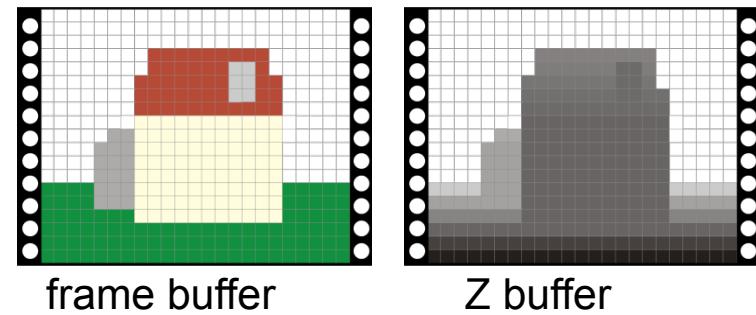
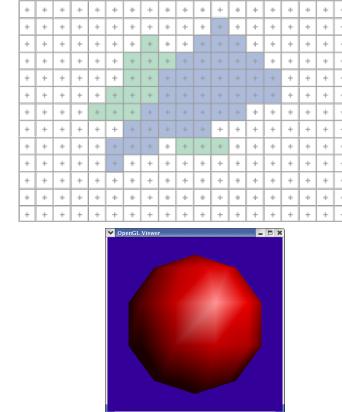
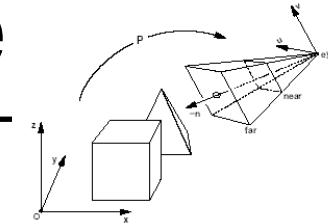
# Modern Graphics Pipeline

- Project vertices to 2D (image)
- Rasterize triangle: find which pixels should be lit
  - For each pixel, test 3 edge equations
    - if all pass, draw pixel
- Compute per-pixel color
- Test visibility (Z-buffer), update frame buffer color



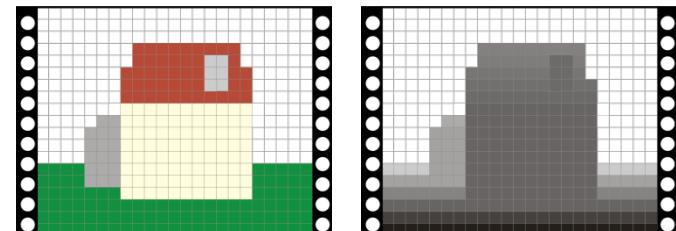
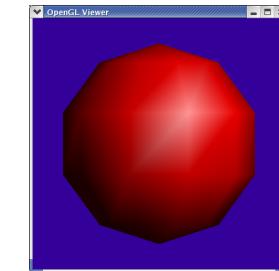
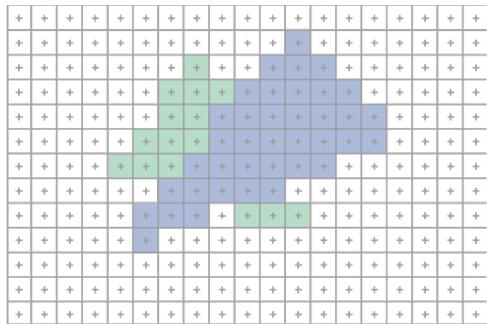
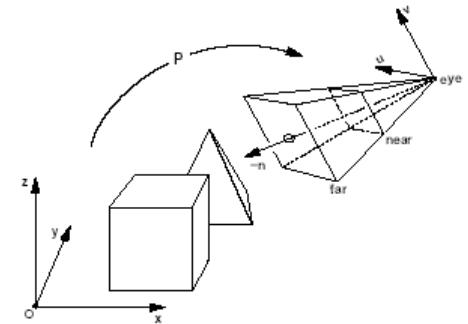
# Modern Graphics Pipeline

- Perform projection of vertices
- Rasterize triangle: find which pixels should be lit
- Compute per-pixel color
- Test visibility, update frame buffer color
  - Store minimum distance to camera for each pixel in “Z-buffer”
    - ~same as  $t_{min}$  in ray casting!
  - **if**  $newz < zbuffer[x,y]$   
 $zbuffer[x,y] = new\_z$   
 $framebuffer[x,y] = new\_color$



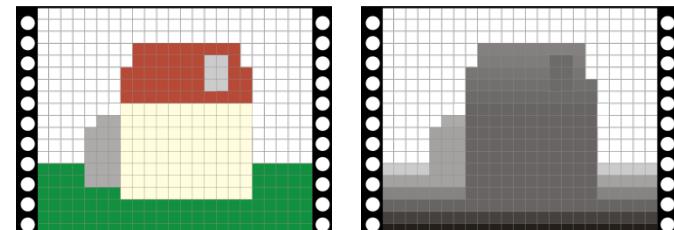
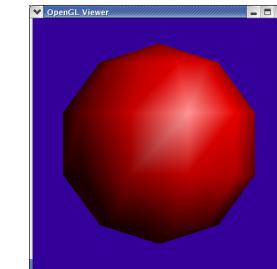
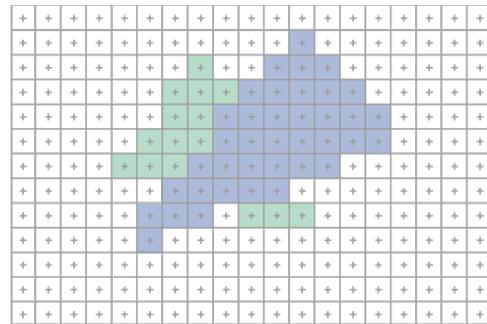
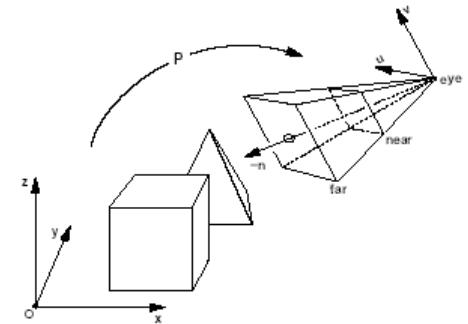
# Modern Graphics Pipeline

For each triangle  
transform into eye space  
(perform projection)  
setup 3 edge equations  
for each pixel  $x, y$   
if passes all edge equations  
compute  $z$   
if  $z < z\text{buffer}[x, y]$   
 $z\text{buffer}[x, y] = z$   
 $\text{framebuffer}[x, y] = \text{shade}()$



# Modern Graphics Pipeline

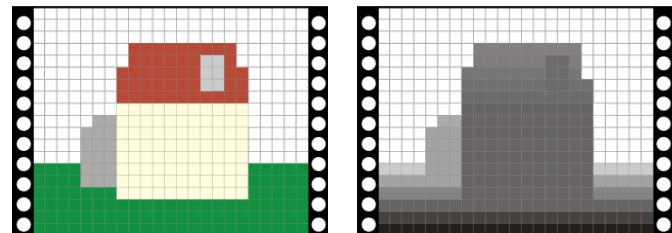
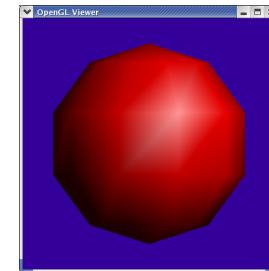
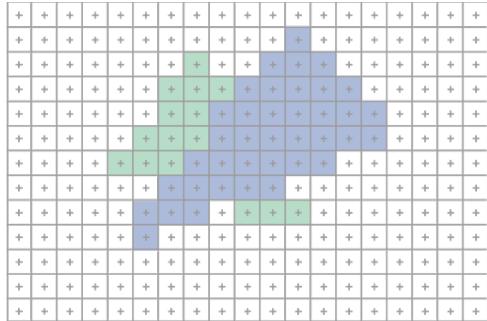
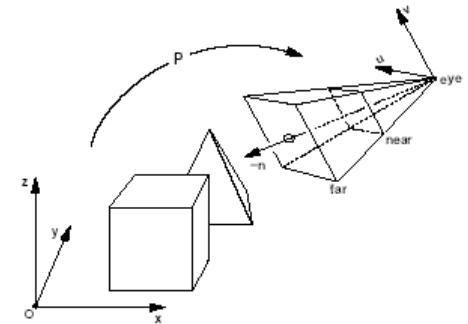
For each triangle  
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compute  $z$   
if  $z < z\text{buffer}[x, y]$   
 $z\text{buffer}[x, y] = z$   
 $\text{framebuffer}[x, y] = \text{shade}()$



## Questions?

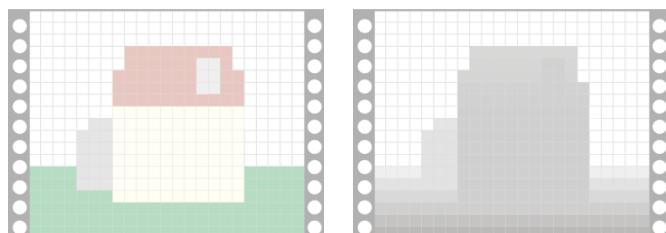
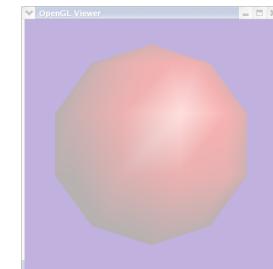
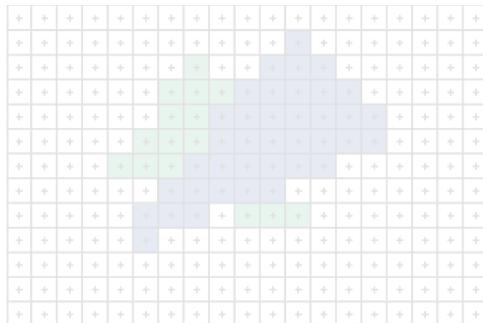
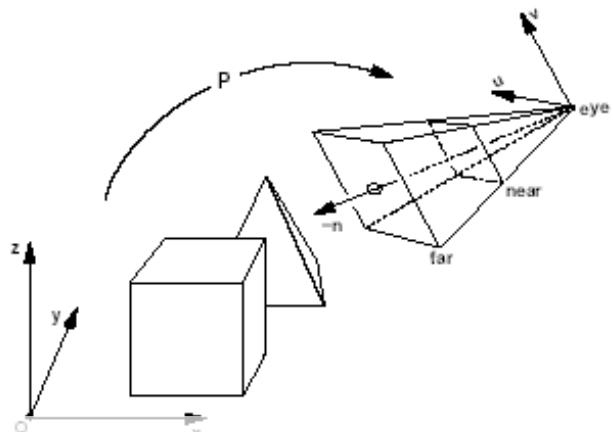
# Modern Graphics Pipeline

- Project vertices to 2D (image)
- Rasterize triangle: find which pixels should be lit
- Compute per-pixel color
- Test visibility (Z-buffer), update frame buffer



# Projection

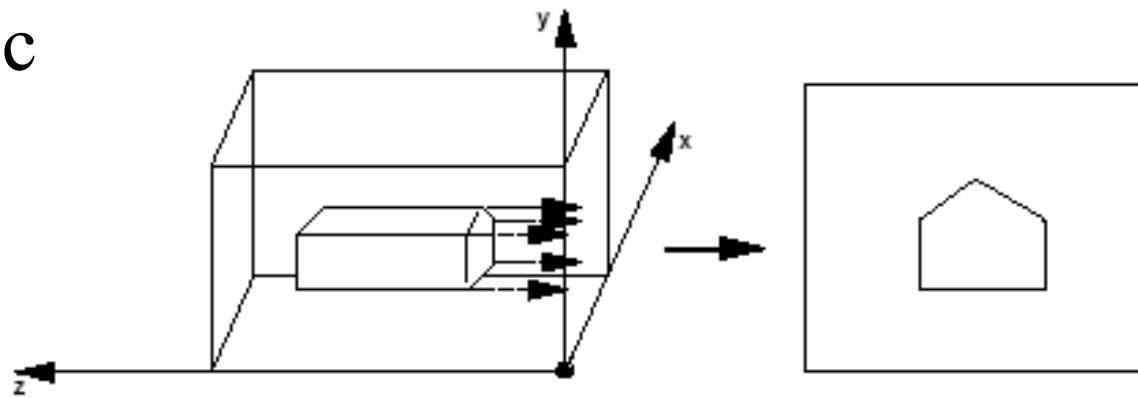
- Project vertices to 2D (image)
- Rasterize triangle: find which pixels should be lit
- Compute per-pixel color
- Test visibility (Z-buffer), update frame buffer



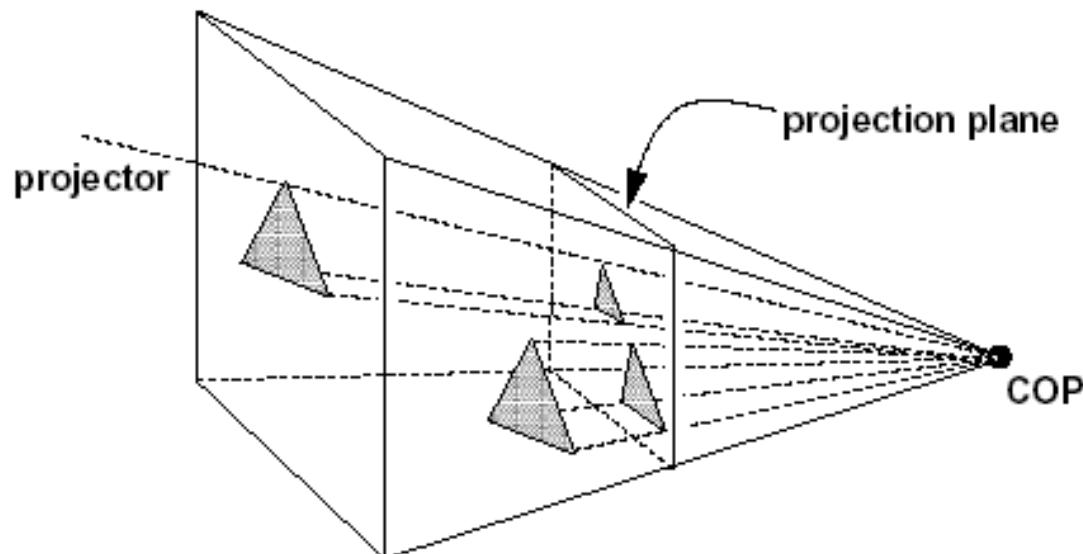
# Orthographic vs. Perspective

---

- Orthographic

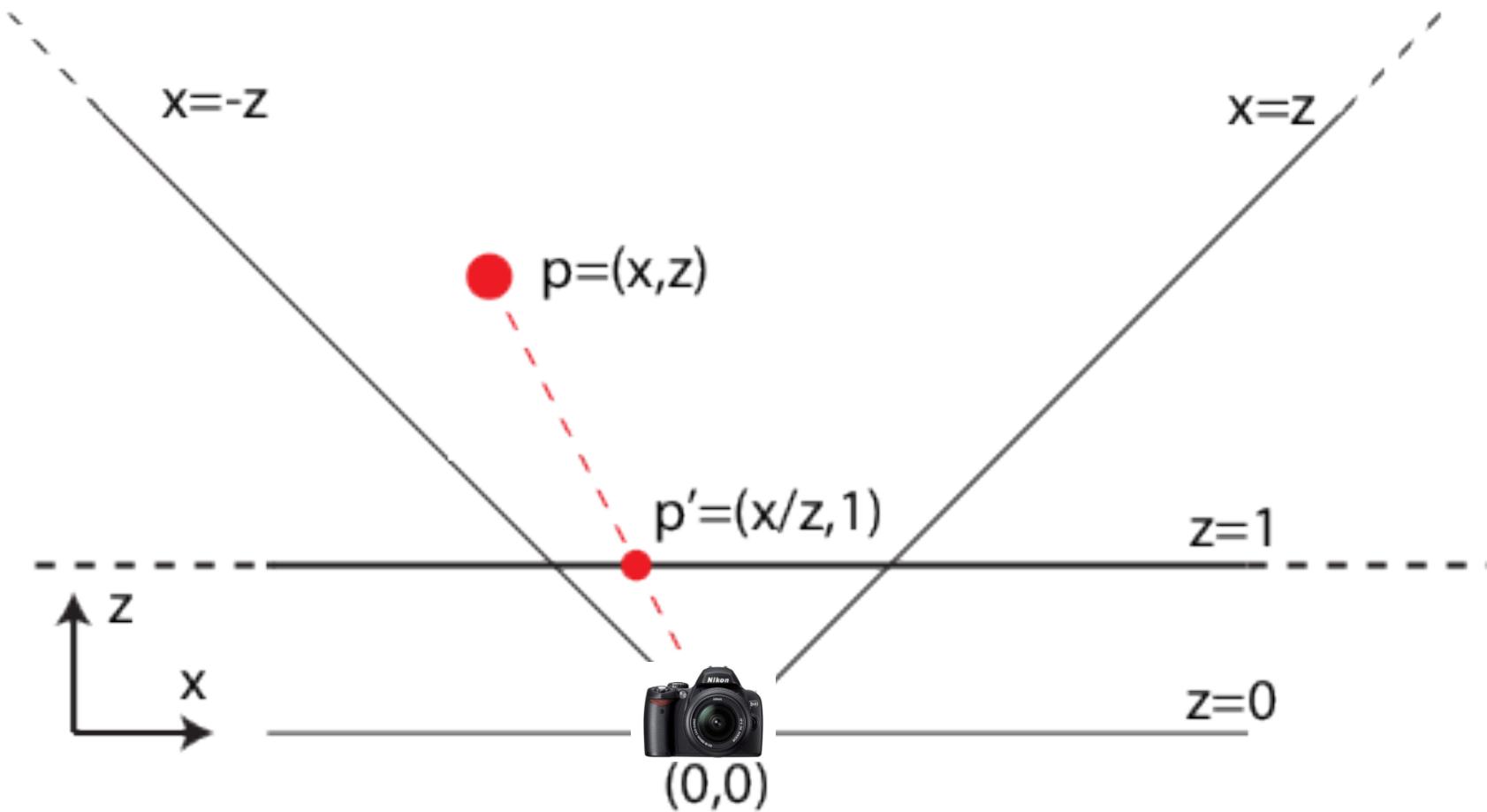


- Perspective



# Perspective in 2D

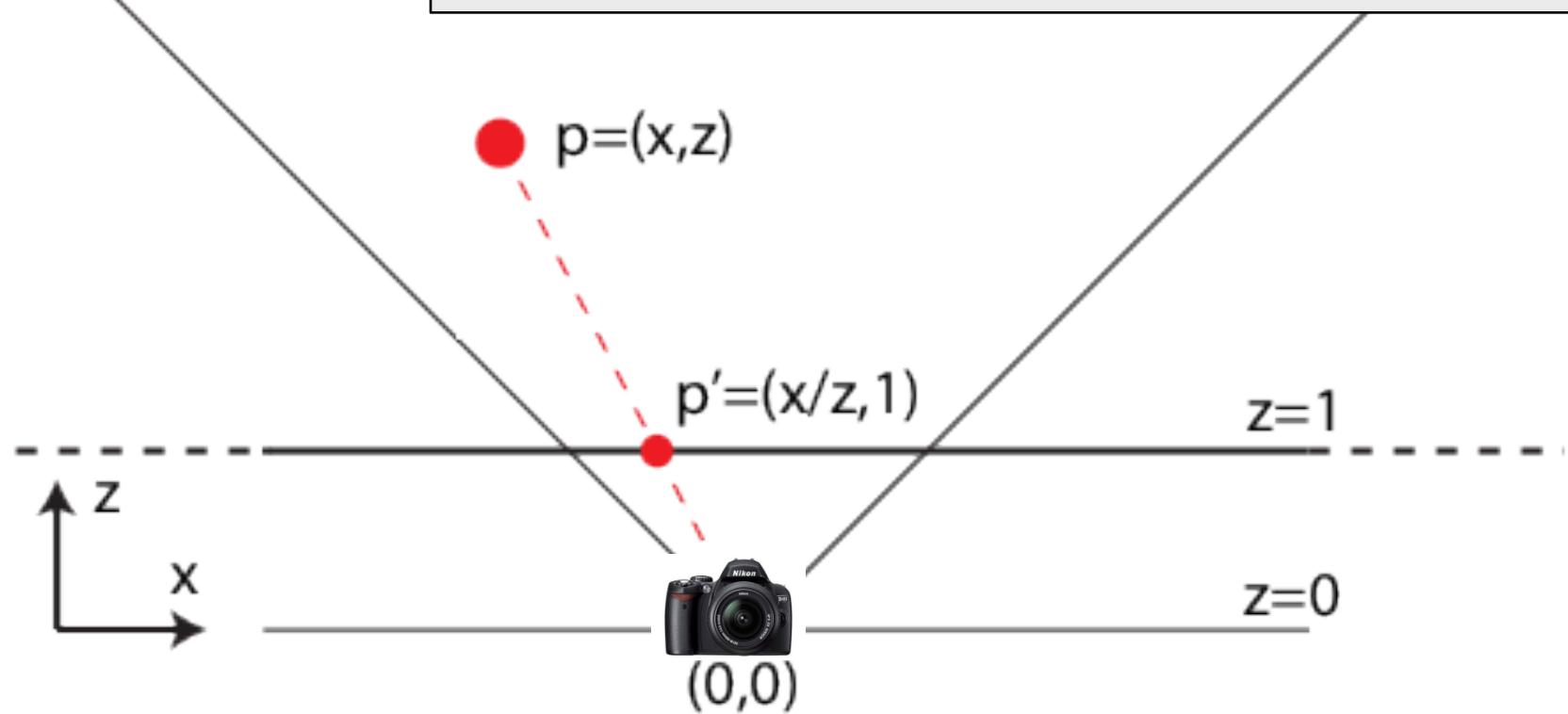
---



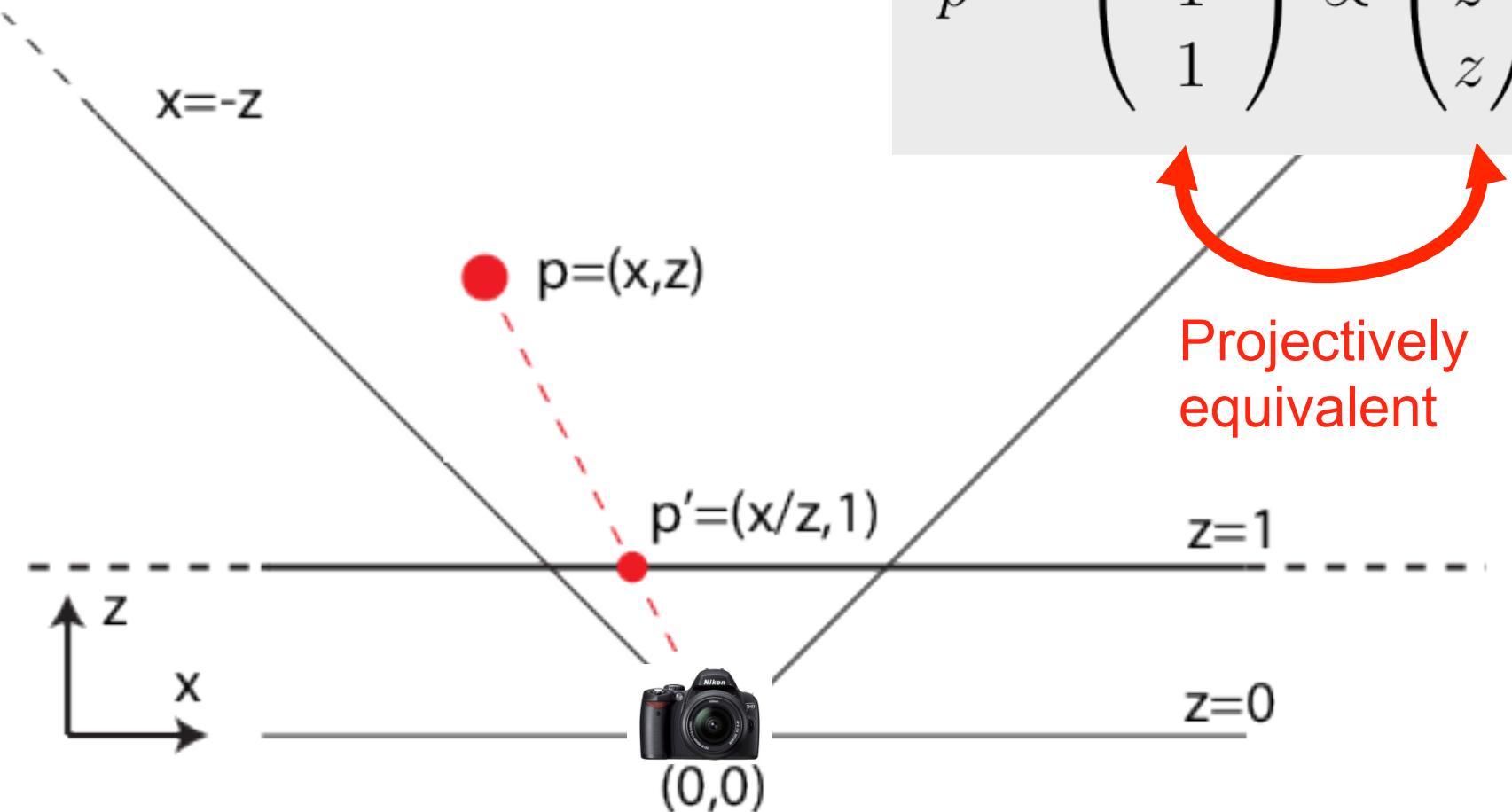
# Perspective in 2D

The projected point in homogeneous coordinates  
(we just added  $w=1$ ):

$$p' = \begin{pmatrix} x/z \\ 1 \\ 1 \end{pmatrix}$$



# Perspective in 2D



$$p' = \begin{pmatrix} x/z \\ 1 \\ 1 \end{pmatrix} \propto \begin{pmatrix} x \\ z \\ z \end{pmatrix}$$

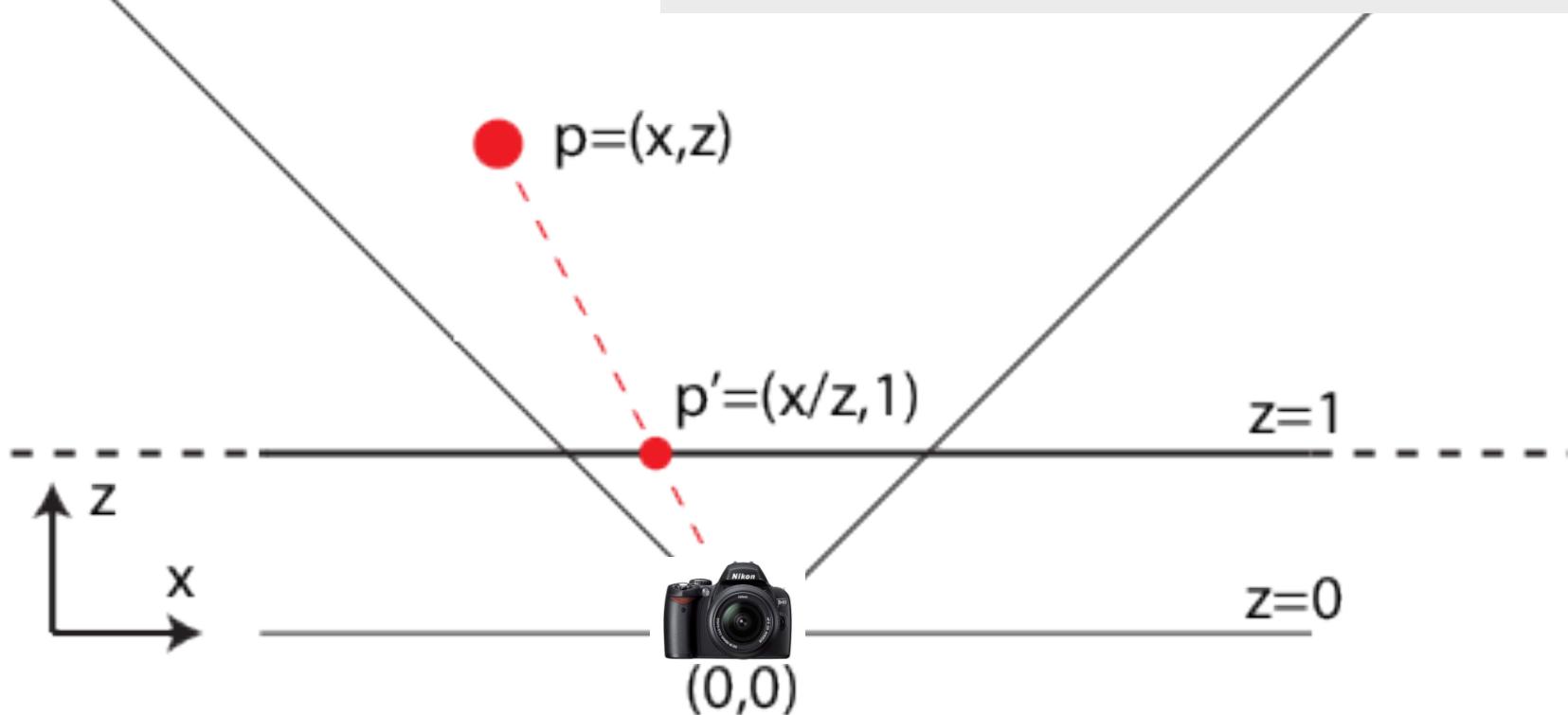
Projectively  
equivalent

# Perspective in 2D

We'll just copy z to w, and get the projected point after homogenization!

$$x = -z$$

$$p' \propto \begin{pmatrix} x \\ z \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ z \\ 1 \end{pmatrix}$$



# Extension to 3D

---

- Trivial: Just add another dimension  $y$  and treat it like  $x$
- Different fields of view and non-square image aspect ratios can be accomplished by simple scaling of the  $x$  and  $y$  axes.
- $z$  is the only special dimension

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

# Caveat

---

- These projections matrices work perfectly in the sense that you get the proper 2D projections of 3D points.
- However, since we are flattening the scene onto the  $z=1$  plane, we've lost all information about the distance to camera.
  - We need the distance for Z buffering, i.e., figuring out what is in front of what!

# Basic Idea: store $1/z$

---

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

# Basic Idea: store $1/z$

---

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} x \\ y \\ 1 \\ z \end{pmatrix}$$

- $z' = 1$  before homogenization
- $z' = 1/z$  after homogenization

# Basic Idea: store $1/z$

---

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} x \\ y \\ 1 \\ z \end{pmatrix}$$

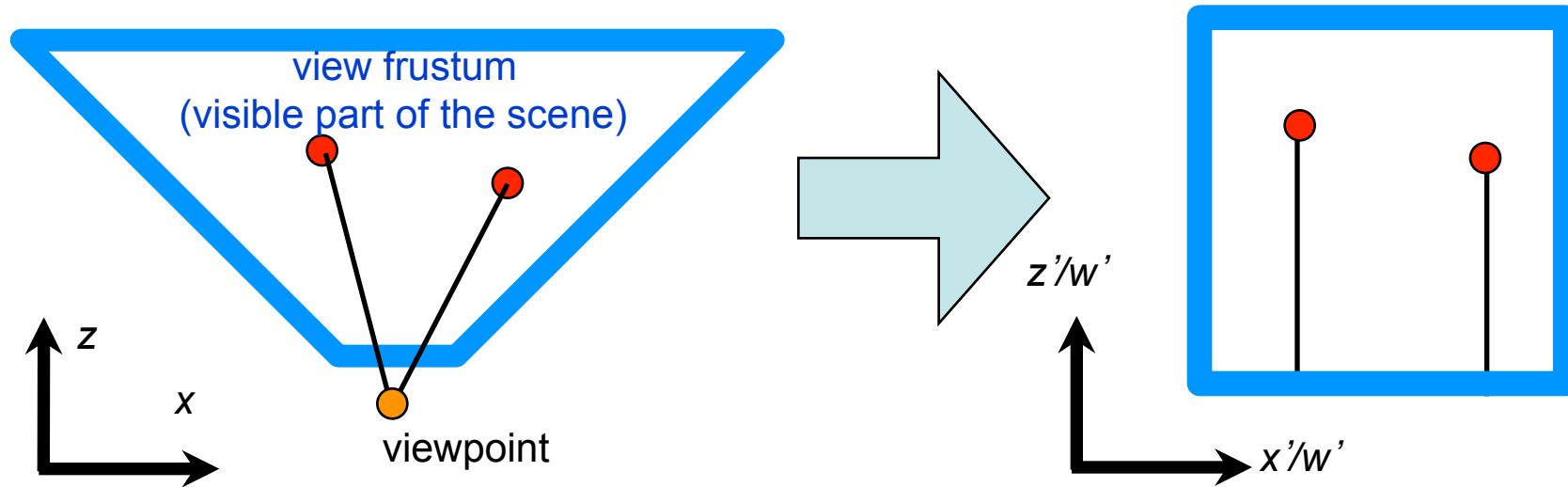
- $z' = 1$  before homogenization
- $z' = 1/z$  after homogenization

This could cause problem

# Full Idea: Remap the View Frustum

---

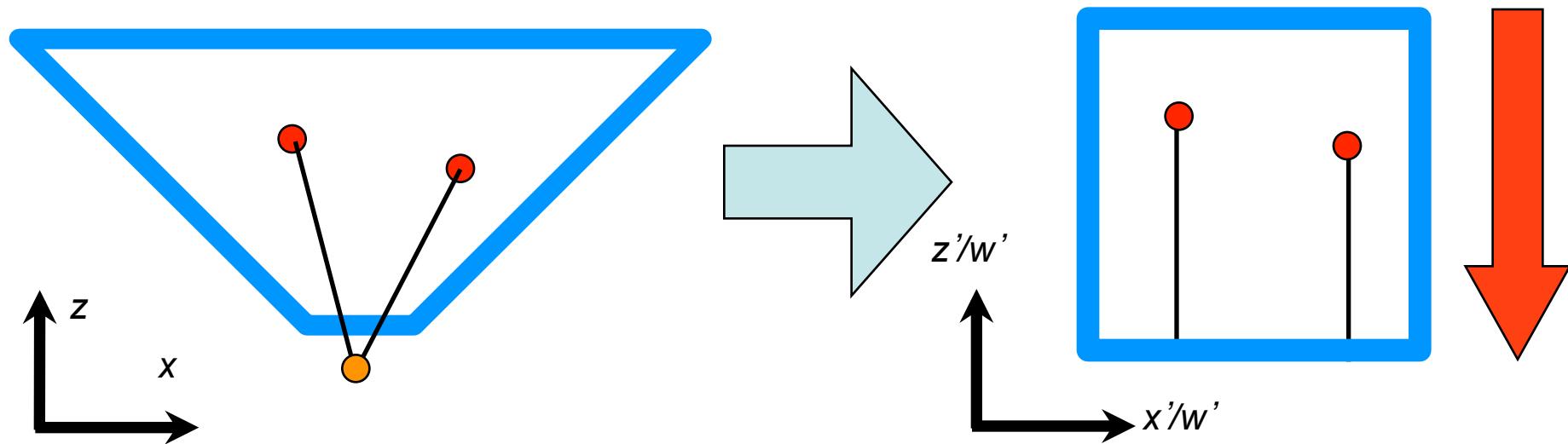
- We can transform the frustum by a modified projection in a way that makes it a square (cube in 3D) after homogenization (division by  $w'$ ).



# The View Frustum in 2D

---

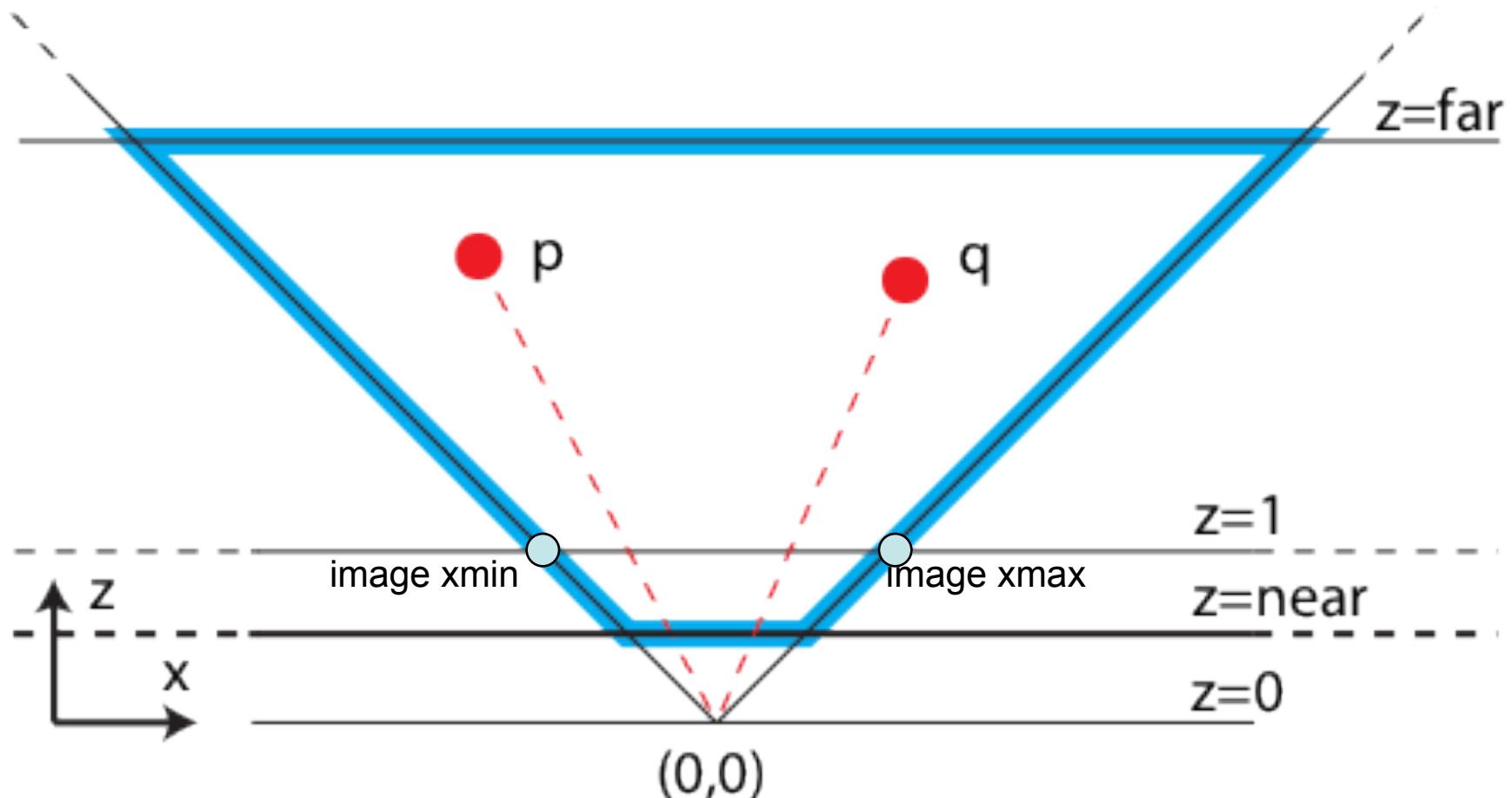
- We can transform the frustum by a modified projection in a way that makes it a square (cube in 3D) after homogenization (division by  $w'$ ).



The final image is obtained by merely dropping the  $z$  coordinate after projection  
(orthogonal projection)

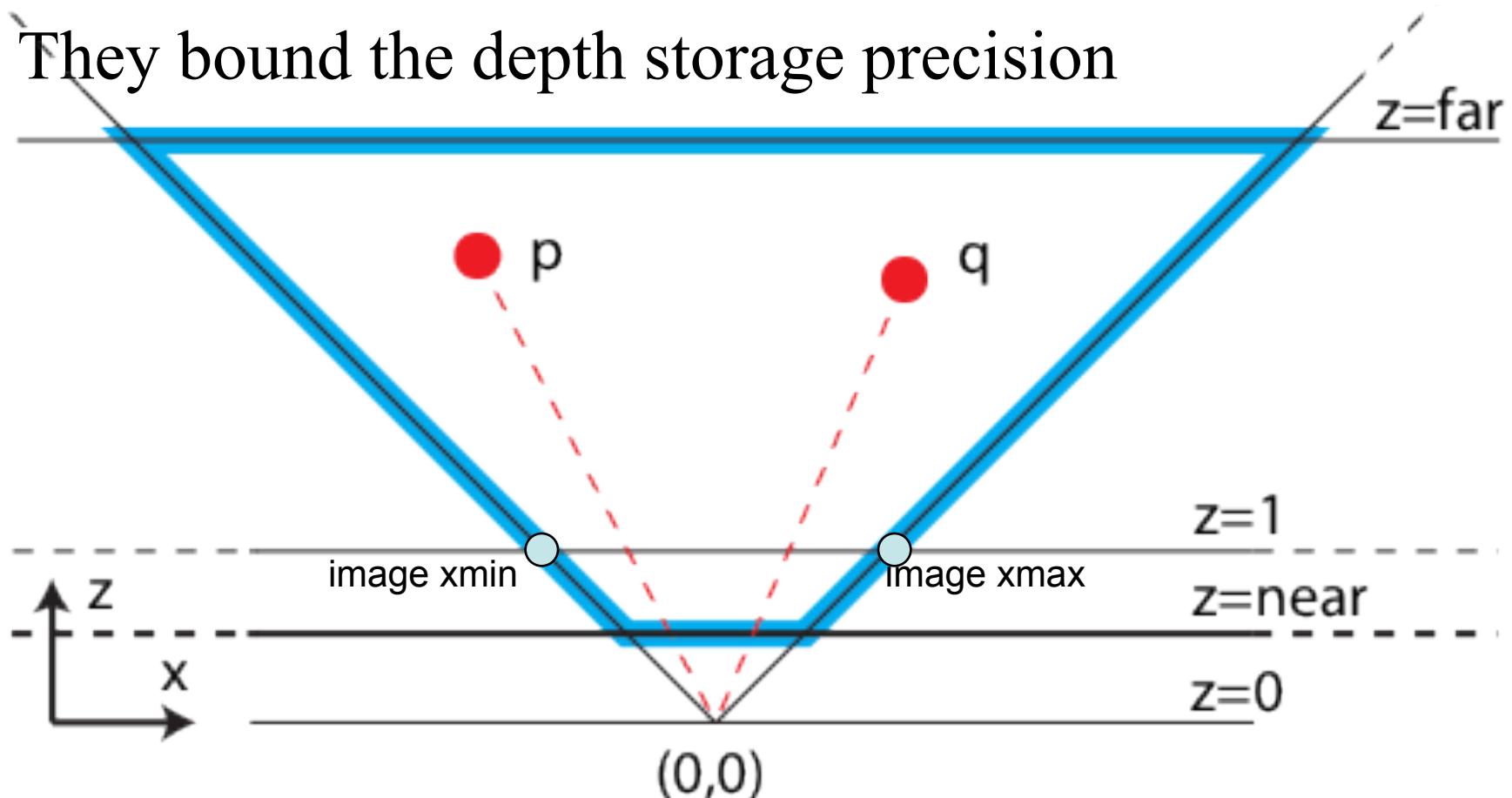
# The View Frustum in 2D

- (In 3D this is a truncated pyramid.)



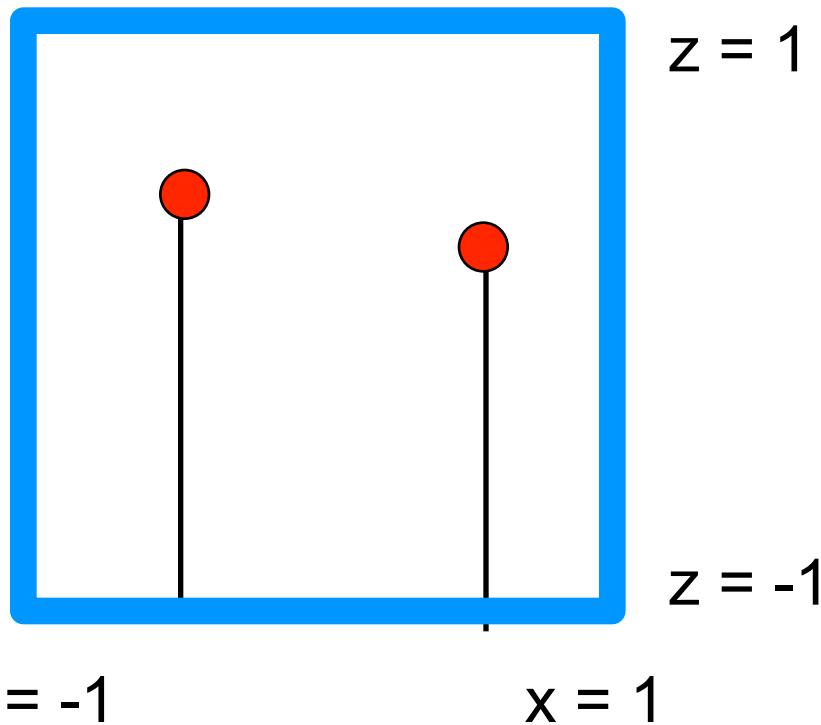
# The View Frustum in 2D

- Far and near are kind of arbitrary
- They bound the depth storage precision



# The Canonical View Volume

---



- Point of the exercise: This gives screen coordinates and depth values for Z-buffering with unified math
  - Caveat: OpenGL and DirectX define Z differently  $[0,1]$  vs.  $[-1,1]$

# OpenGL Form of the Projection

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{\text{far}+\text{near}}{\text{far}-\text{near}} & -\frac{2*\text{far}*\text{near}}{\text{far}-\text{near}} \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Homogeneous coordinates within  
canonical view volume

Input point in view  
coordinates

Check for general case derivation:  
[http://www.songho.ca/opengl/gl\\_projectionmatrix.html](http://www.songho.ca/opengl/gl_projectionmatrix.html)

# OpenGL Form of the Projection

---

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{\text{far}+\text{near}}{\text{far}-\text{near}} & -\frac{2*\text{far}*\text{near}}{\text{far}-\text{near}} \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

- $z' = (az+b)/z = a + b/z$ 
  - where  $a$  &  $b$  depend on near & far
- Similar enough to our basic idea:

- $z' = 1/z$

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

# Recap: Projection

---

- Perform rotation/translation/other transforms to put viewpoint at origin and view direction along z axis
  - This is the OpenGL “modelview” matrix
- Combine with projection matrix (perspective or orthographic)
  - Homogenization achieves foreshortening
  - This is the OpenGL “projection” matrix
- Corollary: The entire transform from object space to canonical view volume  $[-1,1]^3$  is a single matrix

# Recap: Projection

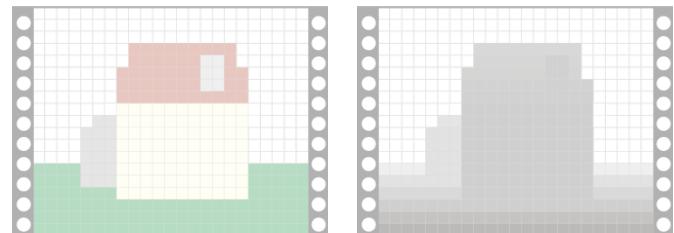
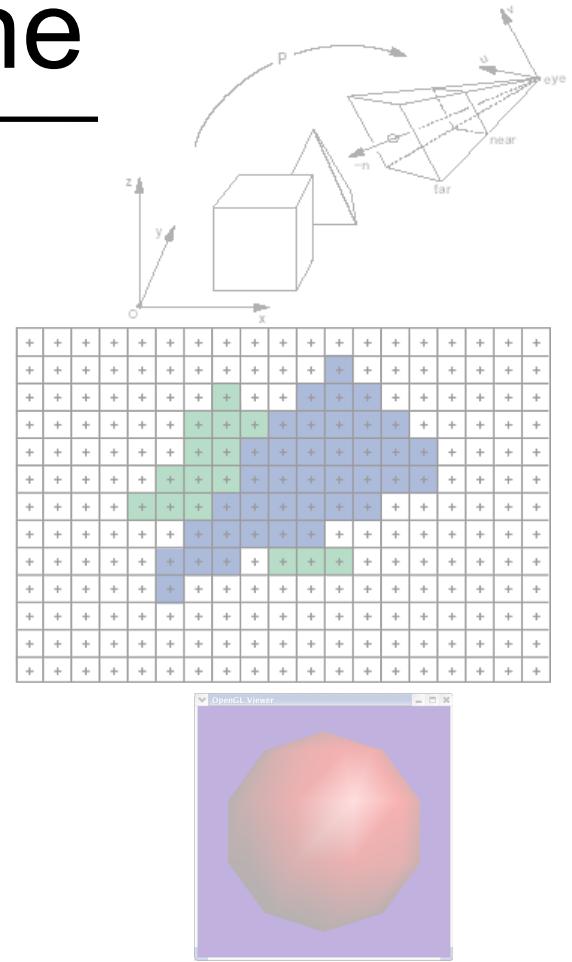
# Questions?

---

- Perform rotation/translation/other transforms to put viewpoint at origin and view direction along z axis
  - This is the OpenGL “modelview” matrix
- Combine with projection matrix (perspective or orthographic)
  - Homogenization achieves foreshortening
  - This is the OpenGL “projection” matrix
- Corollary: The entire transform from object space to canonical view volume  $[-1,1]^3$  is a single matrix

# Modern Graphics Pipeline

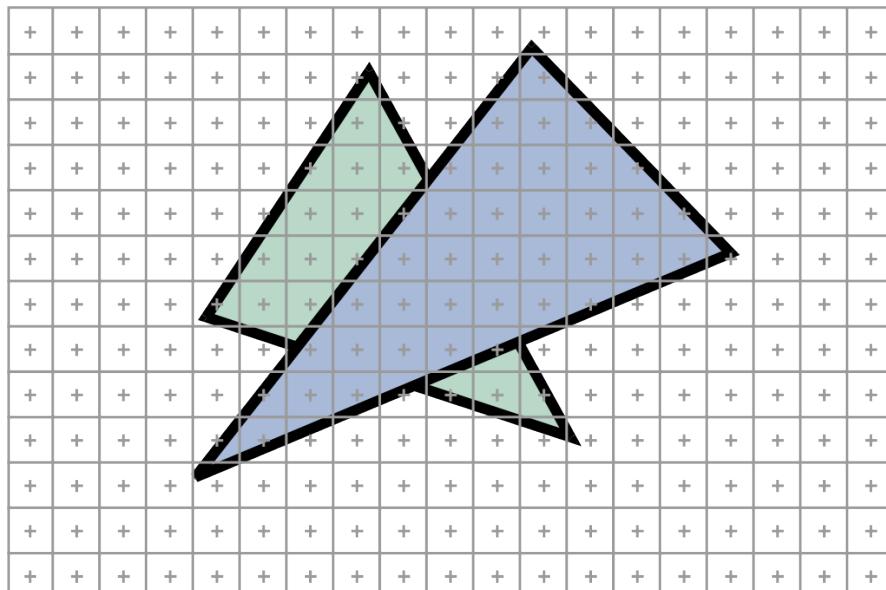
- Project vertices to 2D (image)
  - We now have screen coordinates
- Rasterize triangle: find which pixels should be lit
- Compute per-pixel color
- Test visibility (Z-buffer), update frame buffer



# 2D Scan Conversion

---

- Primitives are “continuous” geometric objects; screen is discrete (pixels)



# 2D Scan Conversion

---

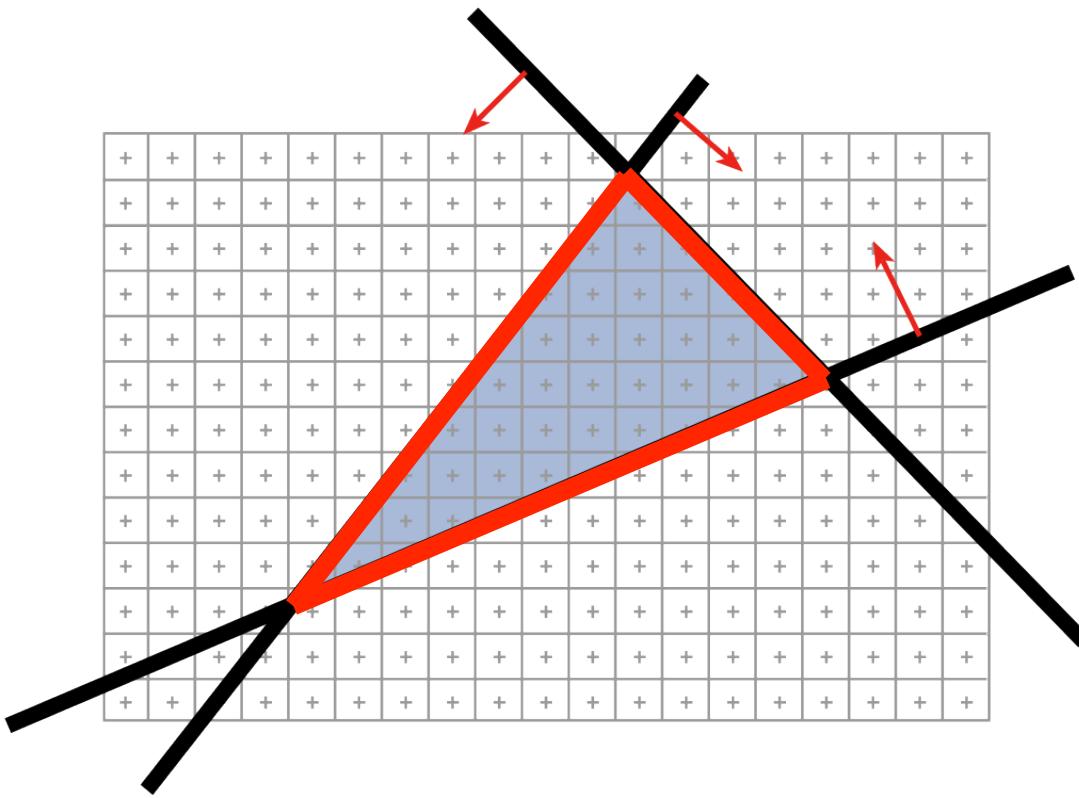
- Primitives are “continuous” geometric objects; screen is discrete (pixels)
- Rasterization computes a discrete approximation in terms of pixels (**how?**)

+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
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# Edge Functions

---

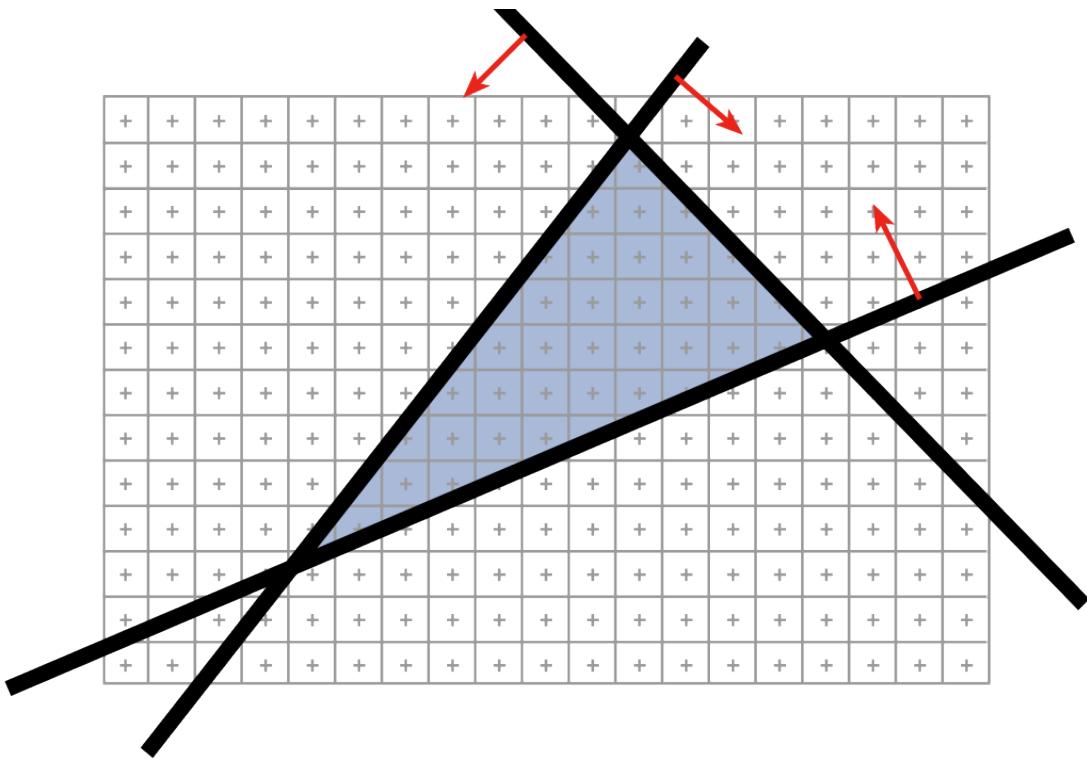
- The triangle's 3D edges project to line segments in the image (thanks to planar perspective)
  - Lines map to lines, not curves



# Edge Functions

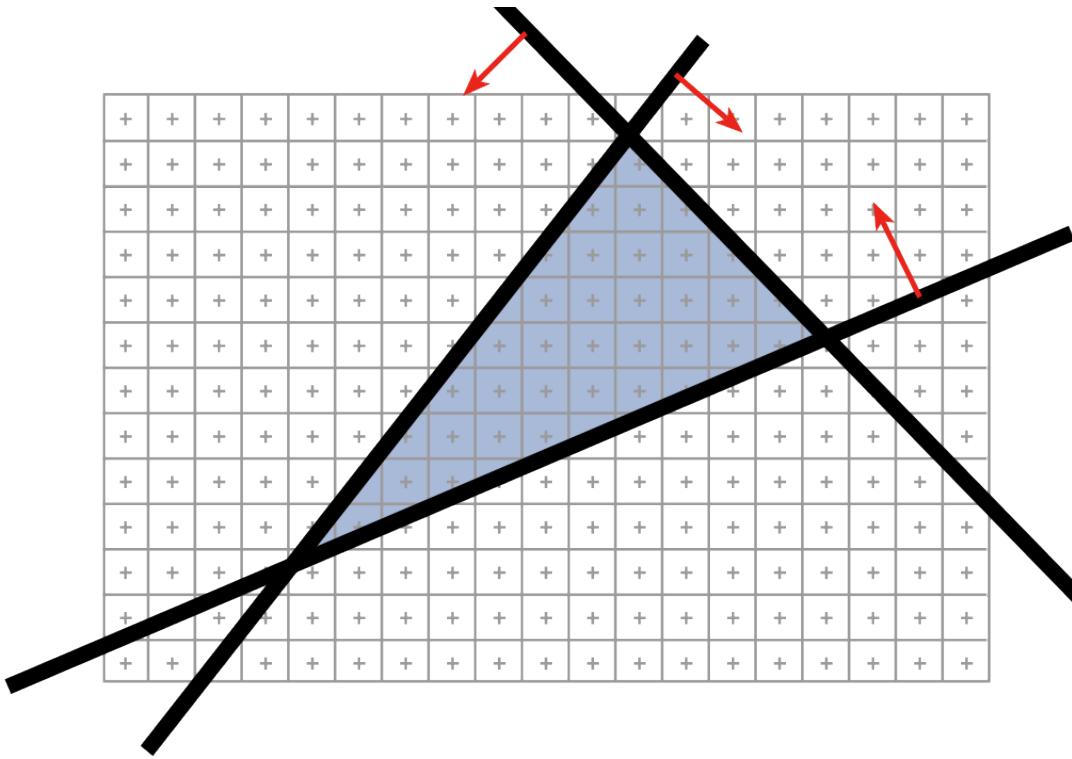
---

- The triangle's 3D edges project to line segments in the image (thanks to planar perspective)
- The interior of the triangle is the set of points that is inside all three halfspaces defined by these lines



# Edge Functions

- The triangle's 3D edges project to line segments in the image (thanks to planar perspective)
- The interior of the triangle is the set of points that is inside all three halfspaces defined by these lines



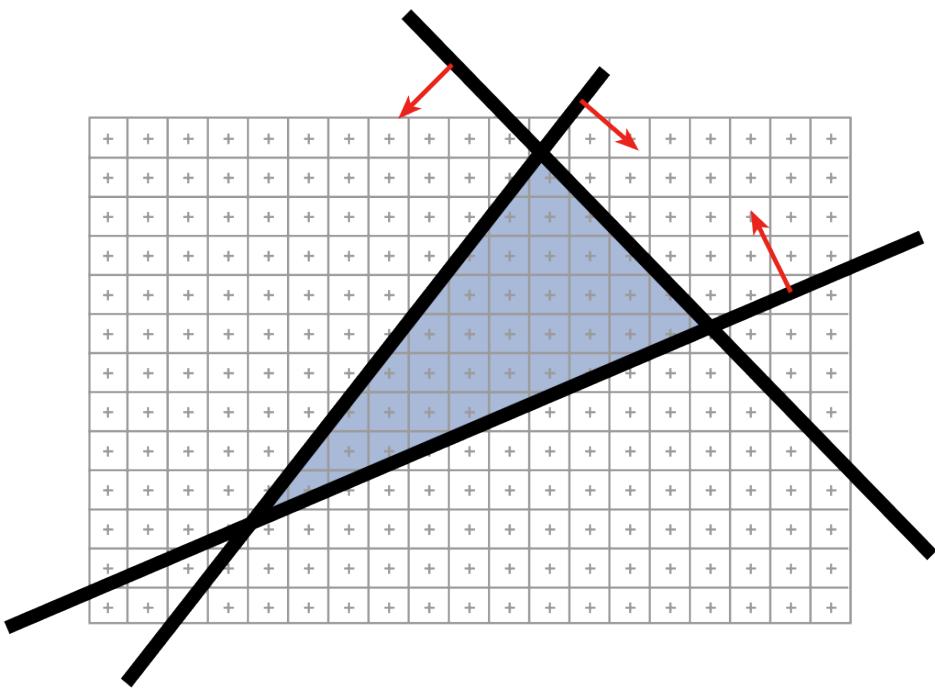
$$E_i(x, y) = a_i x + b_i y + c_i$$

$$(x, y) \text{ within triangle} \Leftrightarrow E_i(x, y) \geq 0, \quad \forall i = 1, 2, 3$$

# Brute Force Rasterizer

---

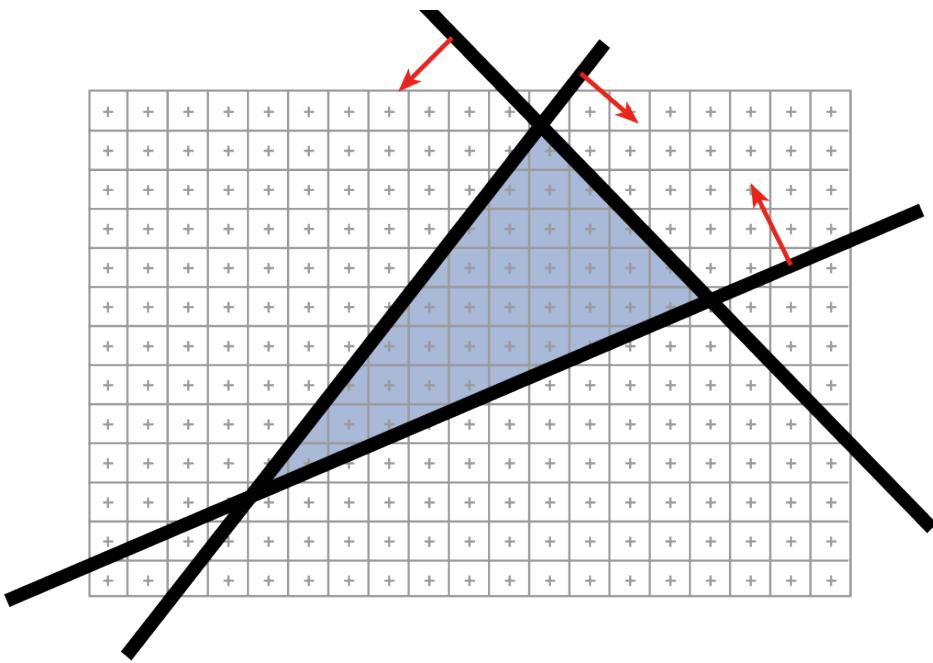
- Compute  $E_1, E_2, E_3$  coefficients from projected vertices
  - Called “triangle setup”, yields  $a_i, b_i, c_i$  for  $i=1,2,3$



# Brute Force Rasterizer

---

- Compute  $E_1, E_2, E_3$  coefficients from projected vertices
- For each pixel  $(x, y)$ 
  - Evaluate edge functions at pixel center
  - If all non-negative, pixel is in!

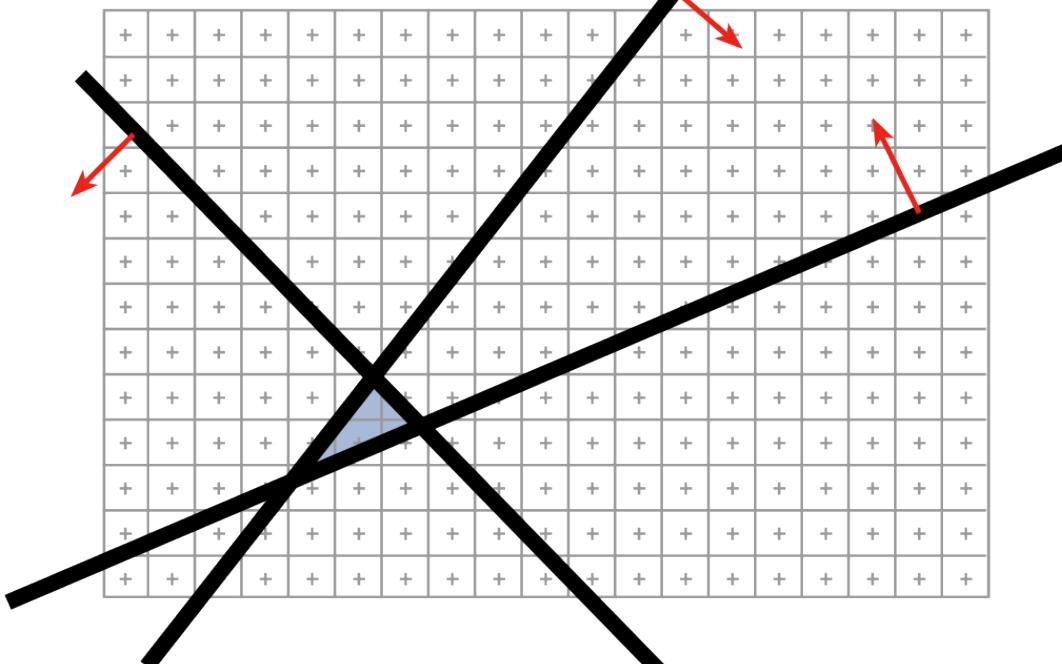


Problem?

# Brute Force Rasterizer

---

- Compute  $E_1, E_2, E_3$  coefficients from projected vertices
- For each pixel  $(x, y)$ 
  - Evaluate edge functions at pixel center
  - If all non-negative, pixel is in!

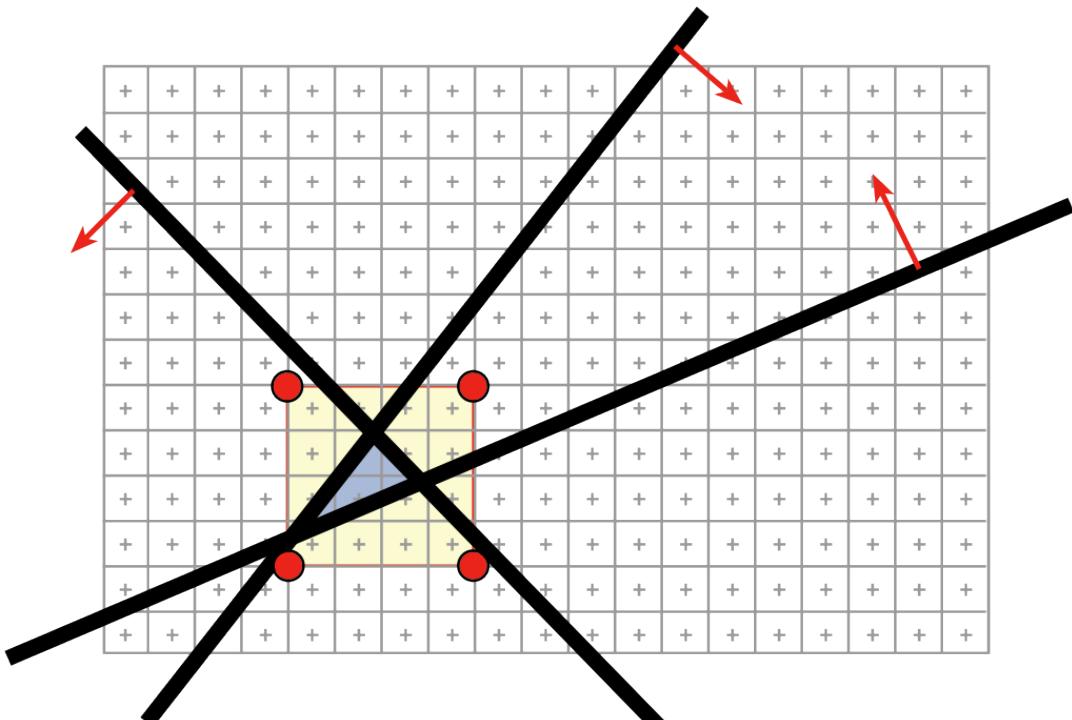


If the triangle is small, lots of useless computation if we really test all pixels

# Easy Optimization

---

- Improvement: Scan over only the pixels that overlap the screen bounding box of the triangle
- How do we get such a bounding box?
  - $X_{\min}, X_{\max}, Y_{\min}, Y_{\max}$  of the projected triangle vertices



# Rasterization Pseudocode

For every triangle

    Compute projection for vertices, compute the  $E_i$

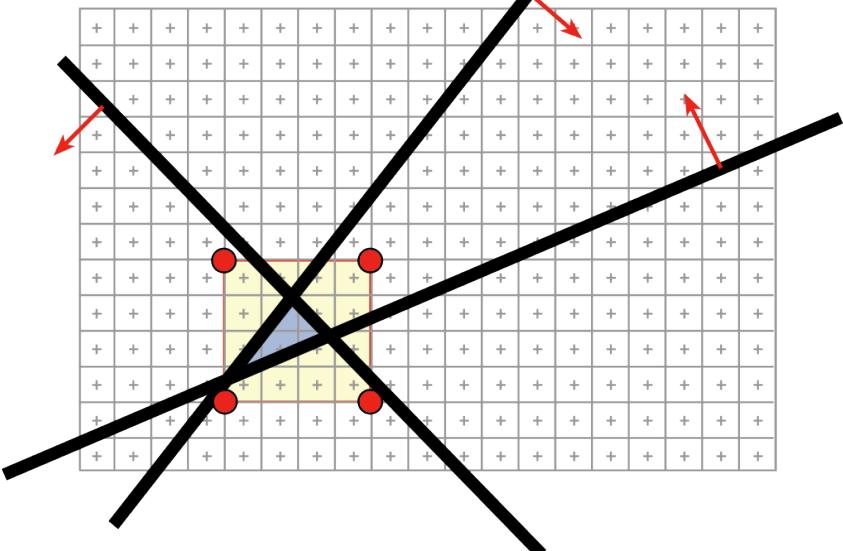
    Compute bbox, clip bbox to screen limits

    For all pixels in bbox

        Evaluate edge functions  $E_i$

        If all  $> 0$

$\text{Framebuffer}[x, y] = \text{triangleColor}$



Bounding box clipping is easy,  
just clamp the coordinates to the  
screen rectangle

# Rasterization Pseudocode

For every triangle

    Compute projection for vertices, compute the  $E_i$

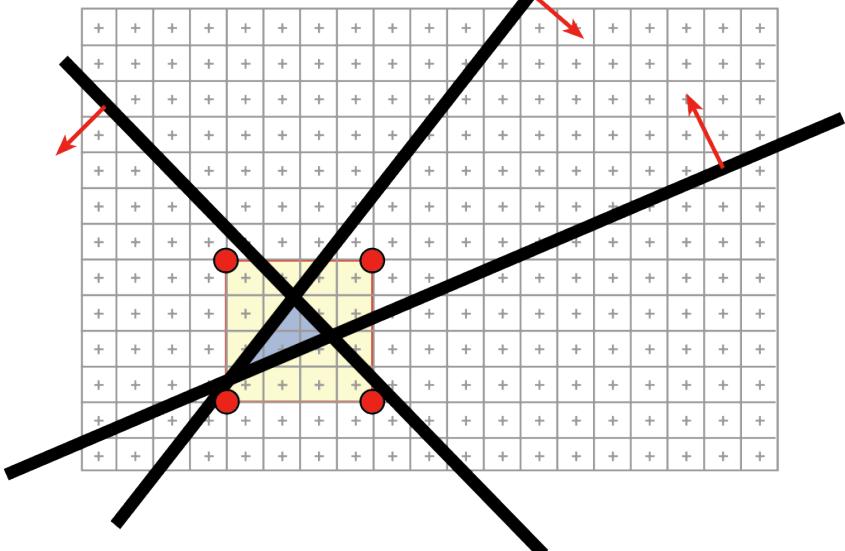
    Compute bbox, clip bbox to screen limits

    For all pixels in bbox

        Evaluate edge functions  $E_i$

        If all  $> 0$

$\text{Framebuffer}[x, y] = \text{triangleColor}$



Bounding box clipping is easy,  
just clamp the coordinates to the  
screen rectangle

## Questions?

# Can We Do Better?

---

For every triangle

Compute projection for vertices, compute the  $E_i$

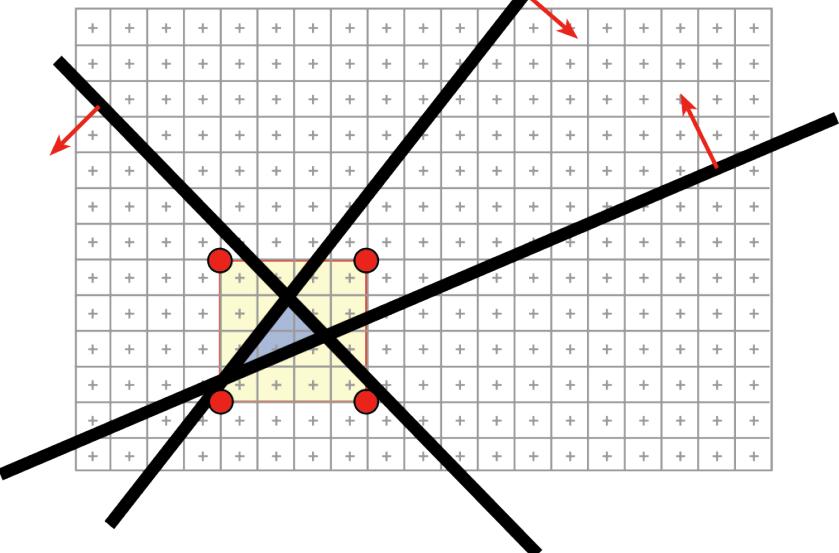
Compute bbox, clip bbox to screen limits

For all pixels in bbox

Evaluate edge functions  $a_{ix} + b_{iy} + c_i$

If all  $> 0$

Framebuffer[x, y] = triangleColor



# Can We Do Better?

---

For every triangle

Compute projection for vertices, compute the  $E_i$

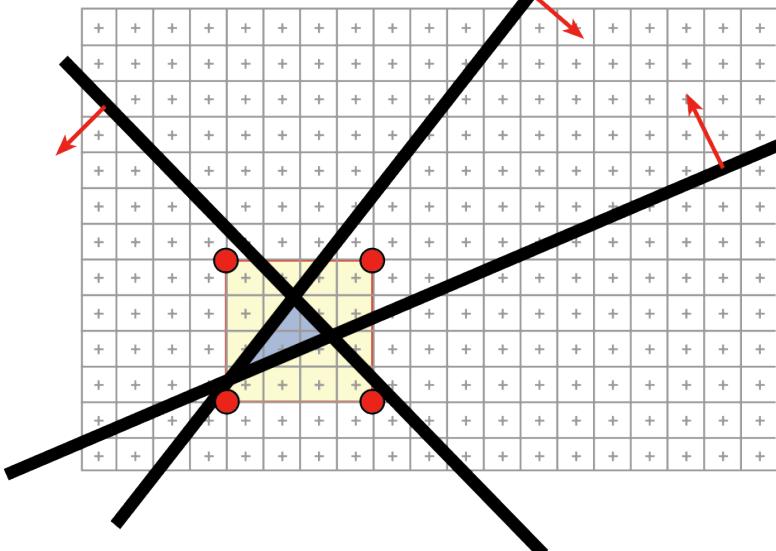
Compute bbox, clip bbox to screen limits

For all pixels in bbox

Evaluate edge functions  $a_{ix} + b_{iy} + c_i$

If all  $> 0$

$\text{Framebuffer}[x, y] = \text{triangleColor}$



These are linear functions of the pixel coordinates  $(x, y)$ , i.e., they only change by a constant amount when we step from  $x$  to  $x+1$  (resp.  $y$  to  $y+1$ )

# Incremental Edge Functions

---

For every triangle

    ComputeProjection

    Compute bbox, clip bbox to screen limits

    For all scanlines  $y$  in bbox

        Evaluate all  $E_i$ 's at  $(x_0, y)$ :  $E_i = a_i x_0 + b_i y + c_i$

        For all pixels  $x$  in bbox

            If all  $E_i > 0$

                Framebuffer[x, y] = triangleColor

                Increment line equations:  $E_i += a_i$

- We save ~two multiplications and two additions per pixel when the triangle is large

# Incremental Edge Functions

---

For every triangle

    ComputeProjection

    Compute bbox, clip bbox to screen limits

    For all scanlines  $y$  in bbox

        Evaluate all  $E_i$ 's at  $(x_0, y)$ :  $E_i = a_i x_0 + b_i y + c_i$

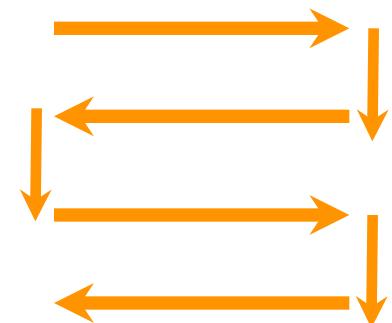
        For all pixels  $x$  in bbox

            If all  $E_i > 0$

                Framebuffer[x, y] = triangleColor

                Increment line equations:  $E_i += a_i$

- We save ~two multiplications and two additions per pixel when the triangle is large

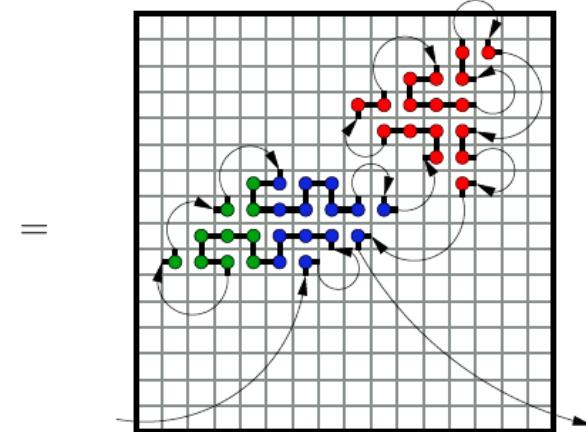
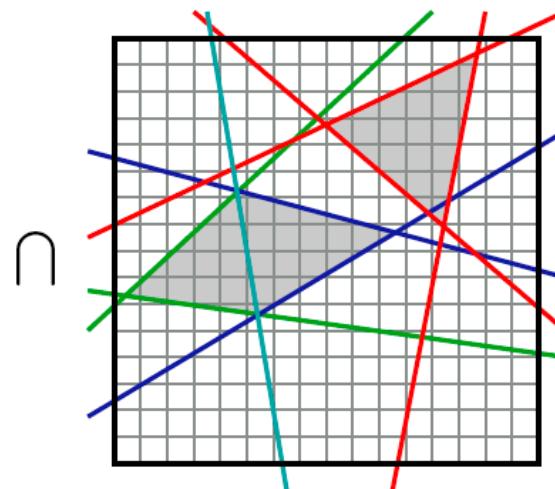
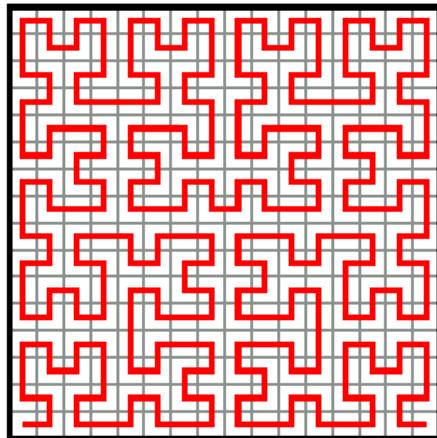


Can also zig-zag to avoid reinitialization per scanline, just initialize once at  $x_0, y_0$

# Questions?

---

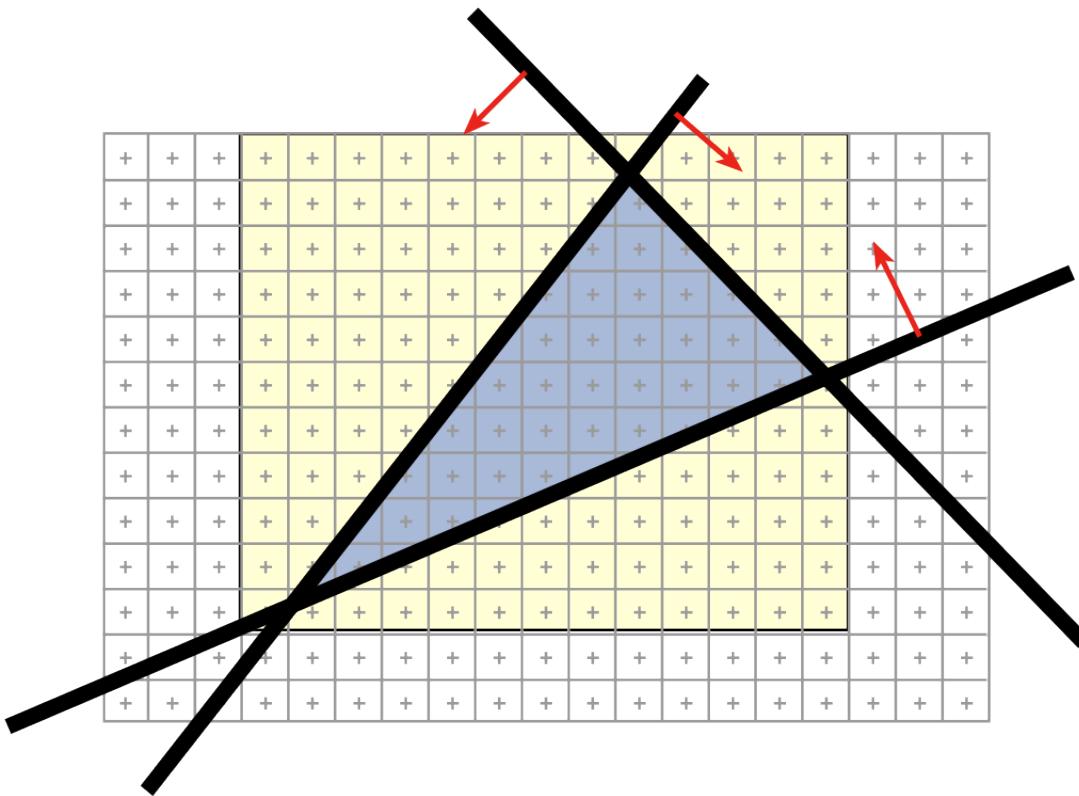
- For a really HC piece of rasterizer engineering, see the hierarchical [Hilbert curve rasterizer by McCool, Wales and Moule.](#)
  - (Hierarchical? We'll look at that next..)



# Can We Do Even Better?

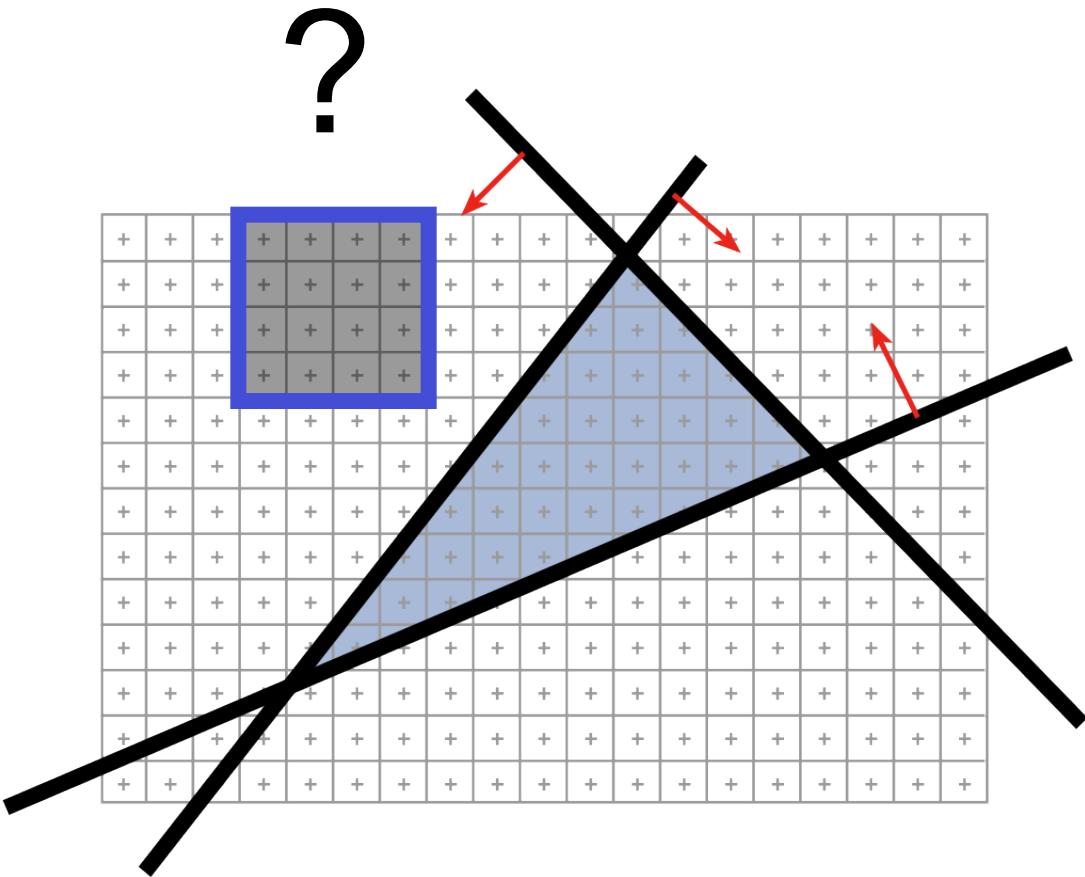
---

- We compute the line equation for many useless pixels
- What could we do?



# Indeed, We Can Be Smarter

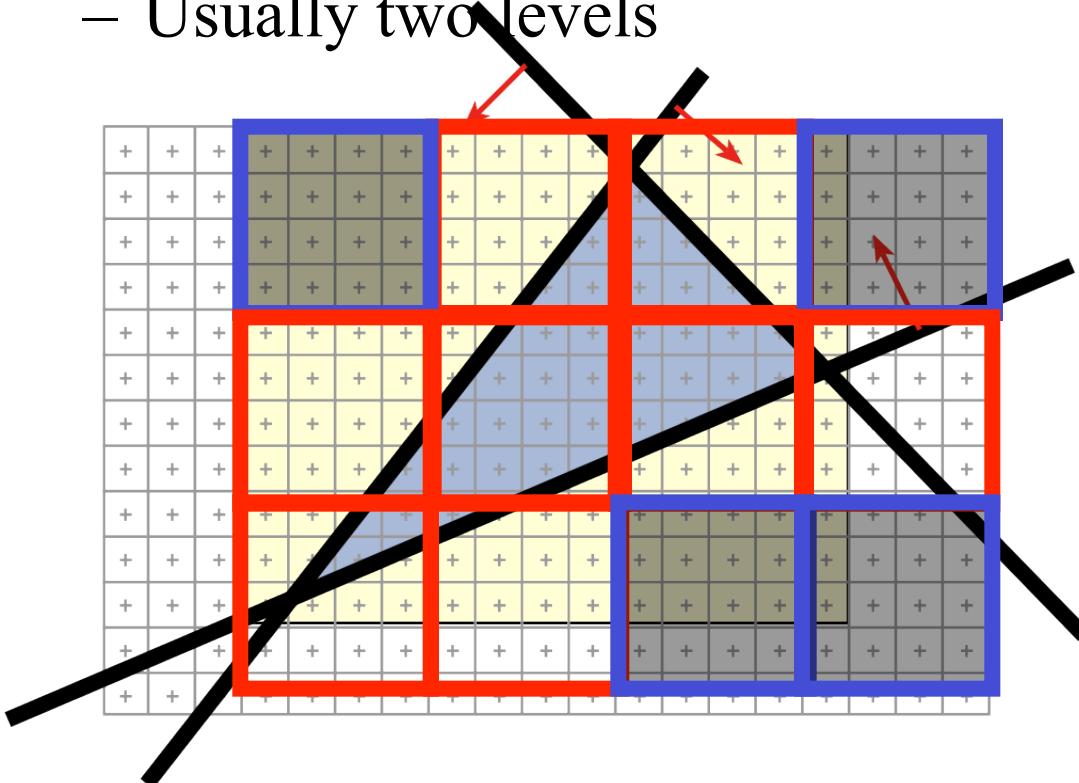
---



# Indeed, We Can Be Smarter

---

- Hierarchical rasterization!
  - Conservatively test blocks of pixels before going to per-pixel level (can skip large blocks at once)
  - Usually two levels

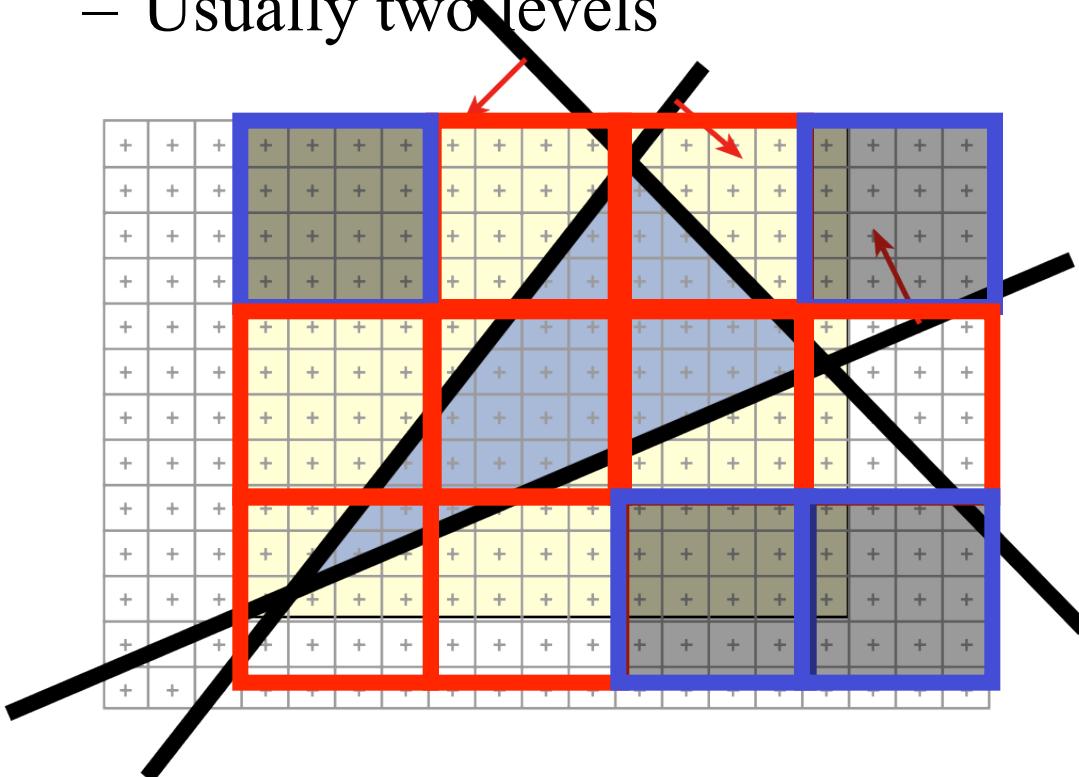


Conservative tests of axis-aligned blocks vs. edge functions are not very hard, thanks to linearity. See [Akenine-Möller and Aila, Journal of Graphics Tools 10\(3\), 2005.](#)

# Indeed, We Can Be Smarter

---

- Hierarchical rasterization!
  - Conservatively test blocks of pixels before going to per-pixel level (can skip large blocks at once)
  - Usually two levels



Can also test if an entire block is inside the triangle; then, can skip edge functions tests for all pixels for even further speedups.(Must still test Z, because they might still be occluded.)

# Further References

---

- Henry Fuchs, Jack Goldfeather, Jeff Hultquist, Susan Spach, John Austin, Frederick Brooks, Jr., John Eyles and John Poulton, “Fast Spheres, Shadows, Textures, Transparencies, and Image Enhancements in Pixel-Planes”, Proceedings of SIGGRAPH ‘85 (San Francisco, CA, July 22–26, 1985). In Computer Graphics, v19n3 (July 1985), ACM SIGGRAPH, New York, NY, 1985.
- Juan Pineda, “A Parallel Algorithm for Polygon Rasterization”, Proceedings of SIGGRAPH ‘88 (Atlanta, GA, August 1–5, 1988). In Computer Graphics, v22n4 (August 1988), ACM SIGGRAPH, New York, NY, 1988. Figure 7: Image from the spinning teapot performance test.
- Marc Olano Trey Greer, “Triangle Scan Conversion using 2D Homogeneous Coordinates”, Graphics Hardware 97  
<http://www.cs.unc.edu/~olano/papers/2dh-tri/2dh-tri.pdf>

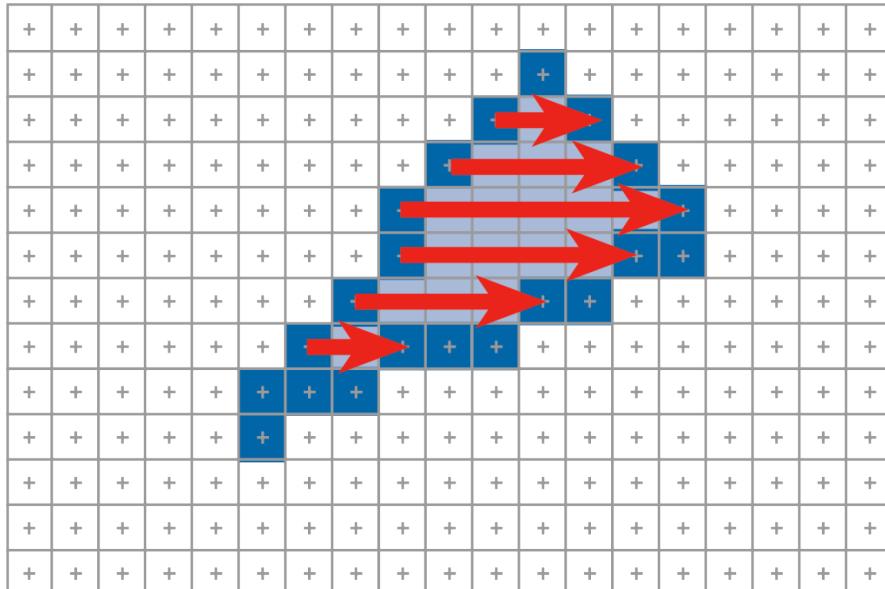
# Olschool Rasterization

- Compute the boundary pixels using line rasterization

# Oldschool Rasterization

---

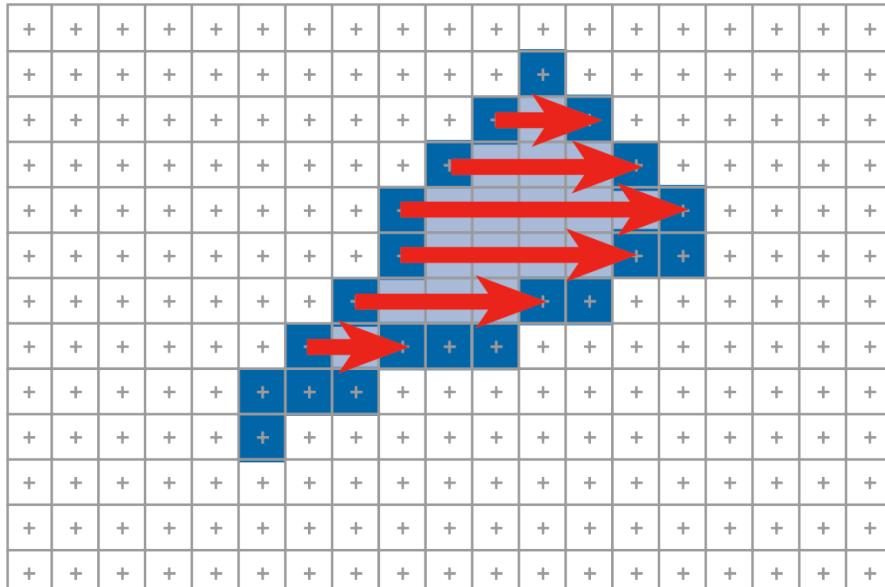
- Compute the boundary pixels using line rasterization
- Fill the spans



# Oldschool Rasterization

---

- Compute the boundary pixels using line rasterization
- Fill the spans

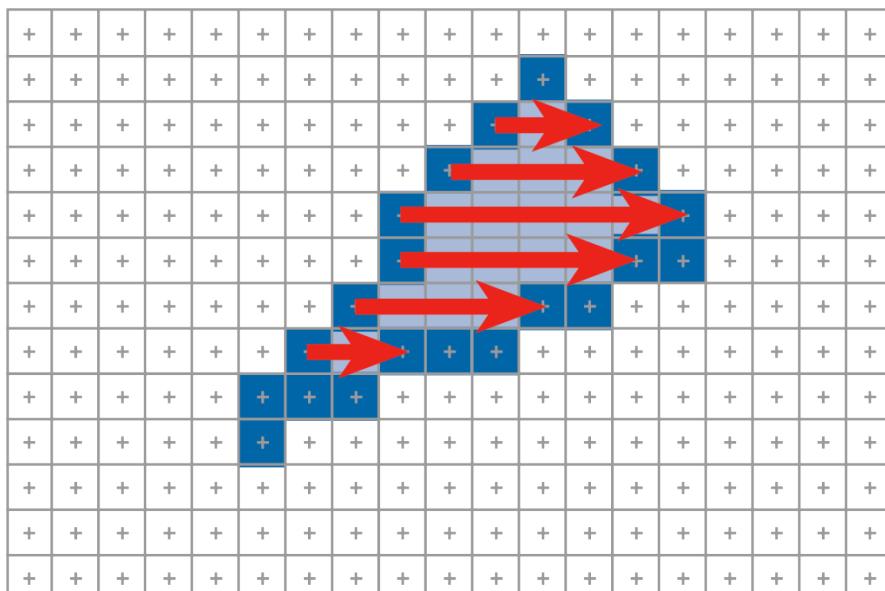


More annoying to implement than edge functions

Not faster unless triangles are huge

# Oldschool Rasterization Questions?

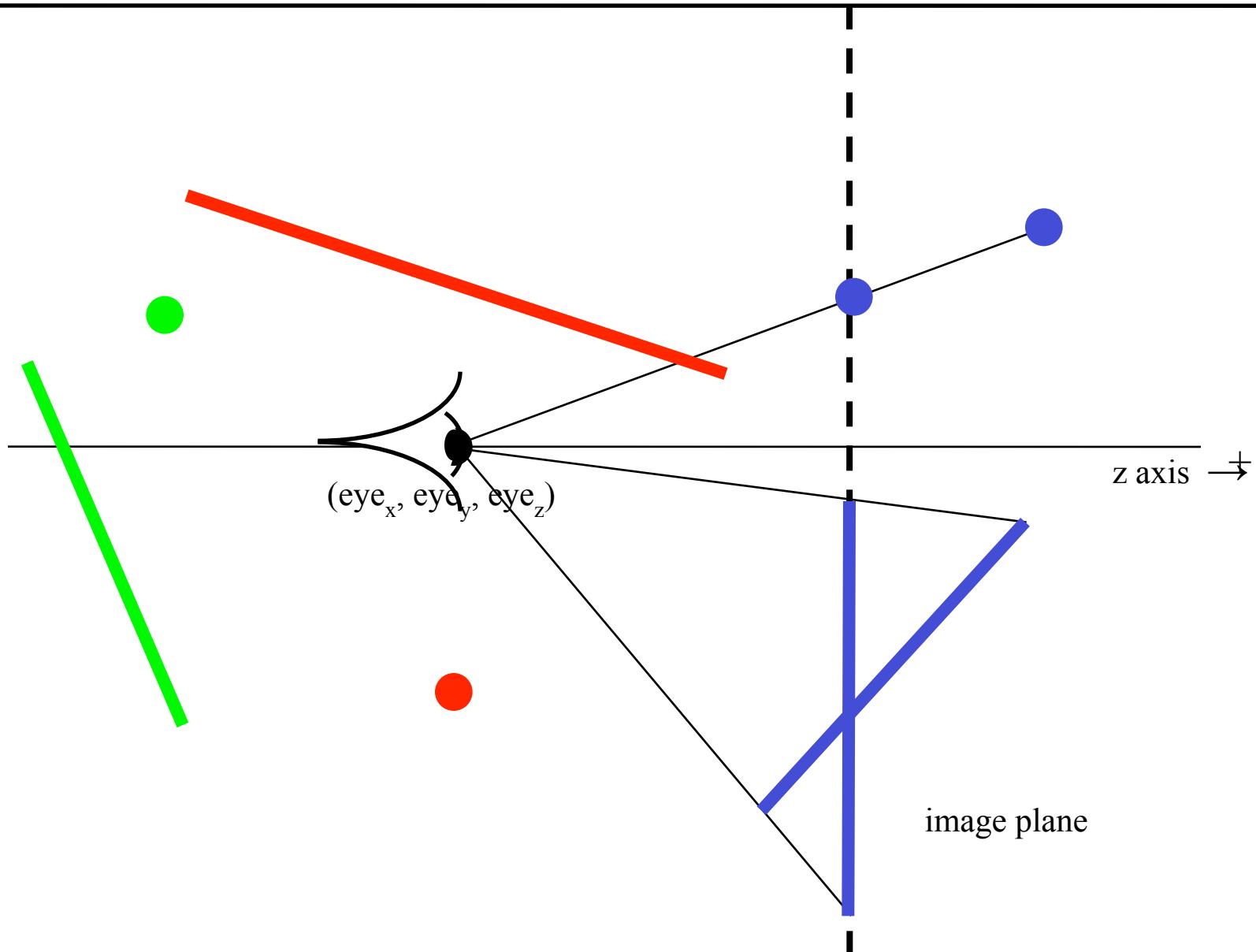
- Compute the boundary pixels using line rasterization
- Fill the spans



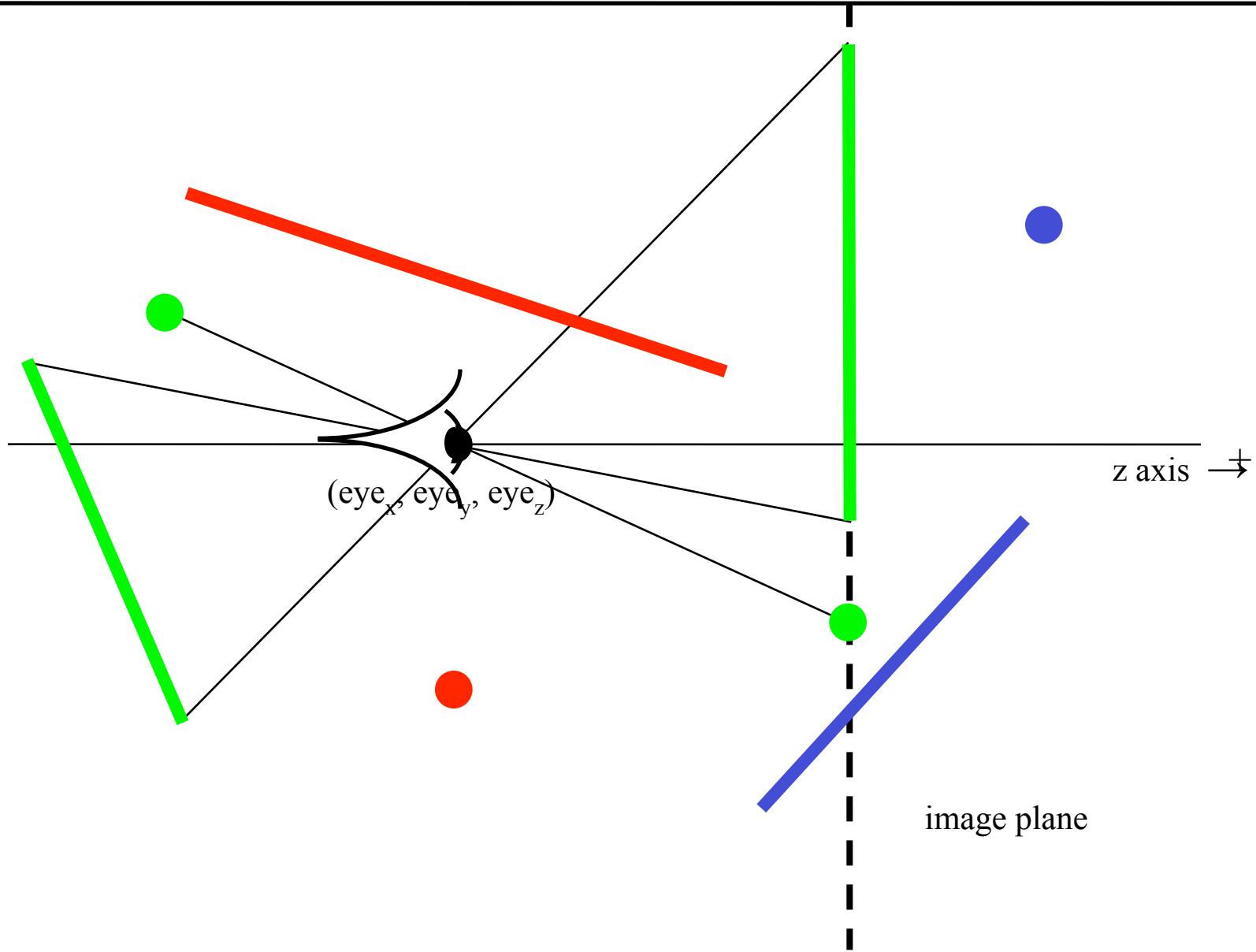
More annoying to implement than edge functions

Not faster unless triangles are huge

# What if the $p_z$ is $> eye_z$ ?

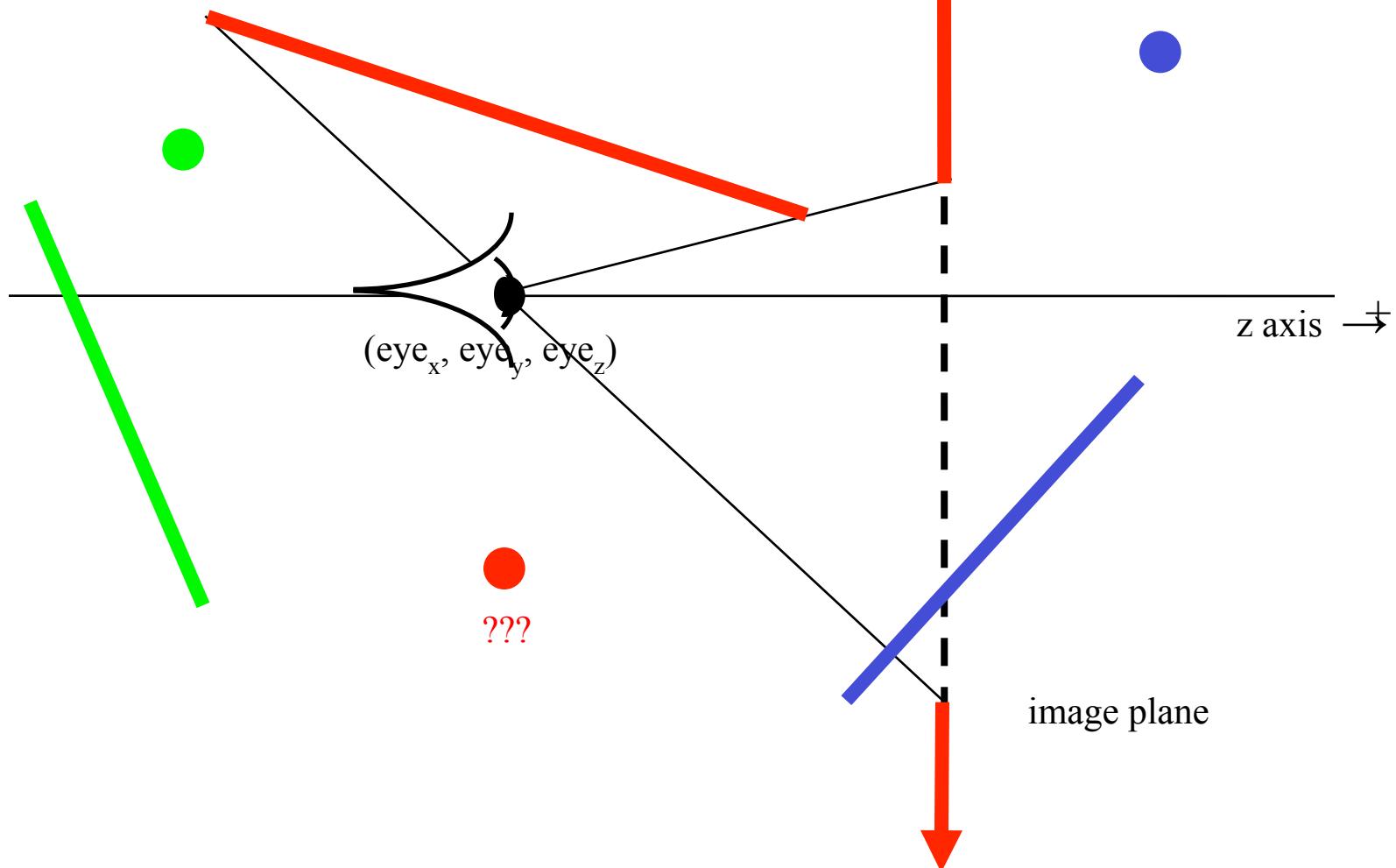


# What if the $p_z$ is < $eye_z$ ?

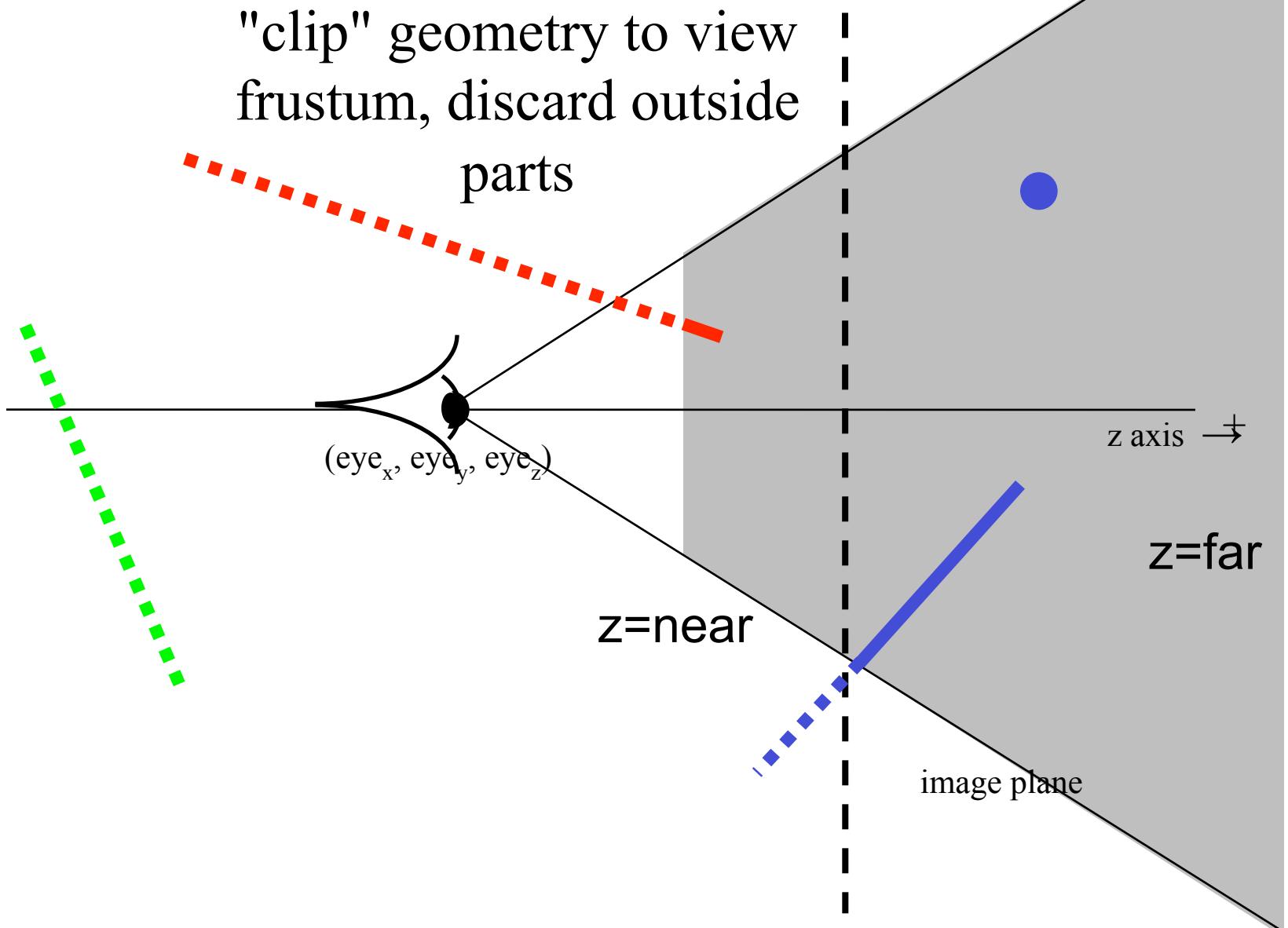


# What if the $p_z = \text{eye}_z$ ?

When  $w' = 0$ , point projects to infinity  
(homogenization is division by  $w'$ )



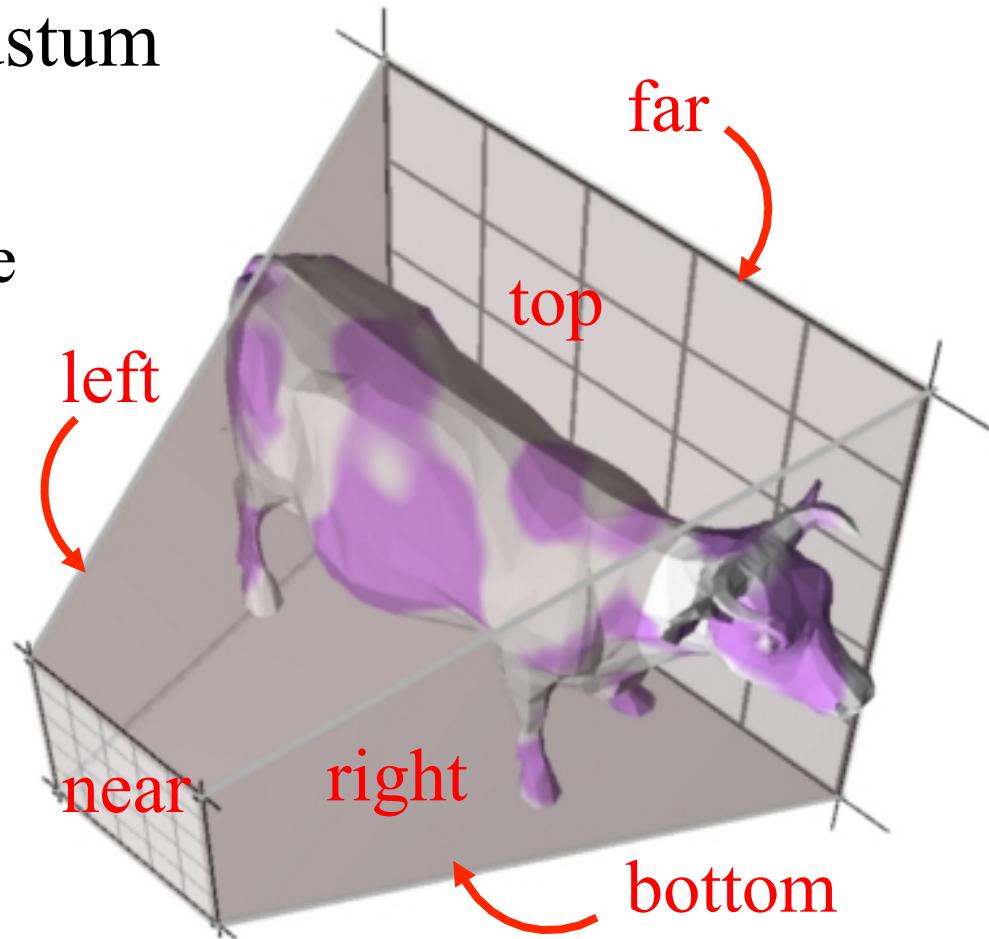
# A Solution: Clipping



# Clipping

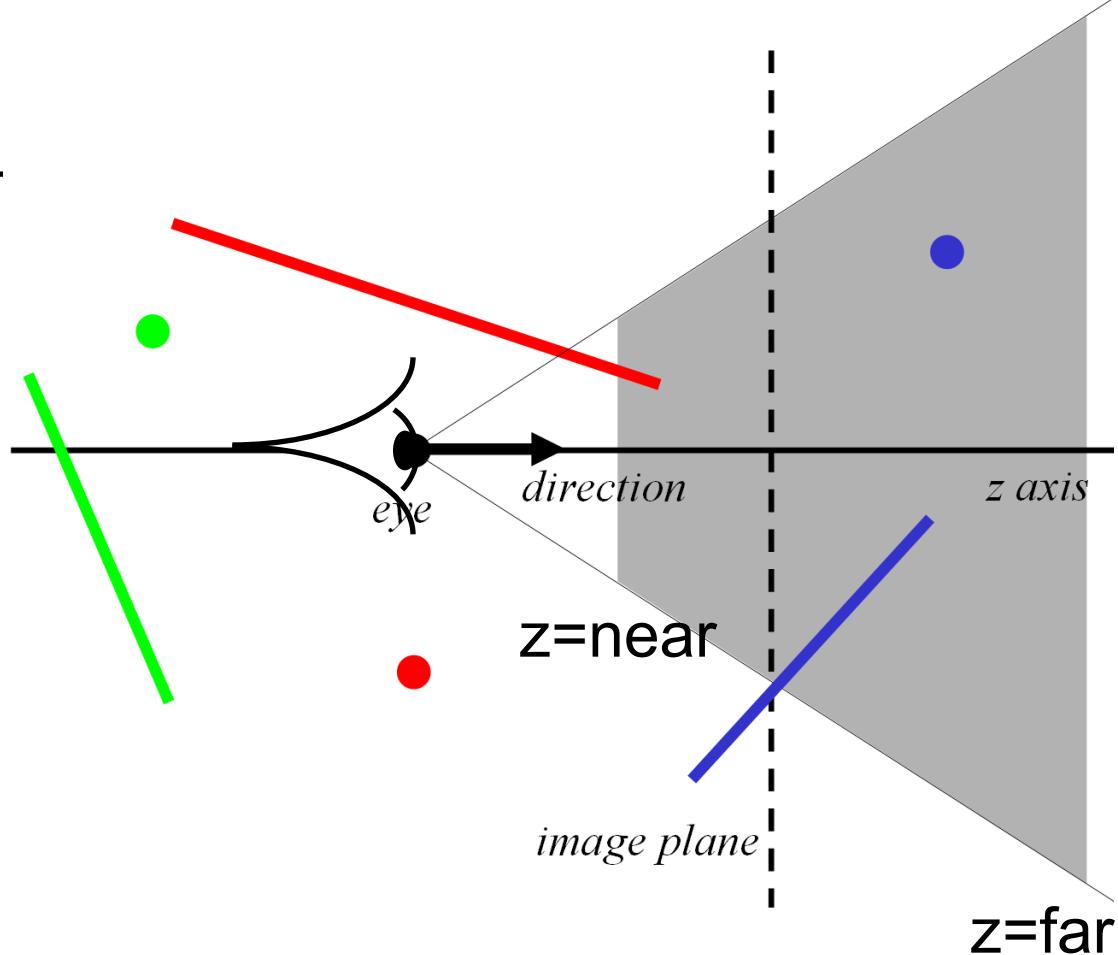
---

- Eliminate portions of objects outside the viewing frustum
- View Frustum
  - boundaries of the image plane projected in 3D
  - a near & far clipping plane
- User may define additional clipping planes



# Why Clip?

- Avoid degeneracies
  - Don't draw stuff behind the eye
  - Avoid division by 0 and overflow



# Related Idea

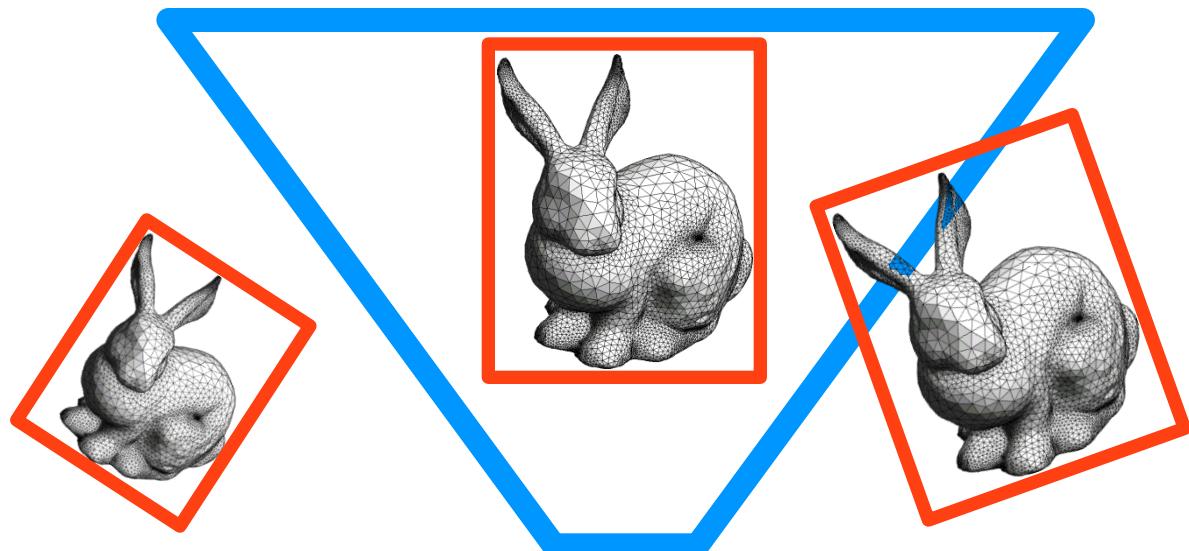
---

- “View Frustum Culling”
  - Use bounding volumes/hierarchies to test whether any part of an object is within the view frustum
    - Need “frustum vs. bounding volume” intersection test
    - Crucial to do hierarchically when scene has lots of objects!
    - Early rejection (different from clipping)

See e.g.

[Optimized view frustum culling algorithms for bounding boxes](#), Ulf

Assarsson and Tomas Möller, journal of graphics tools, 2000.



# Related Idea

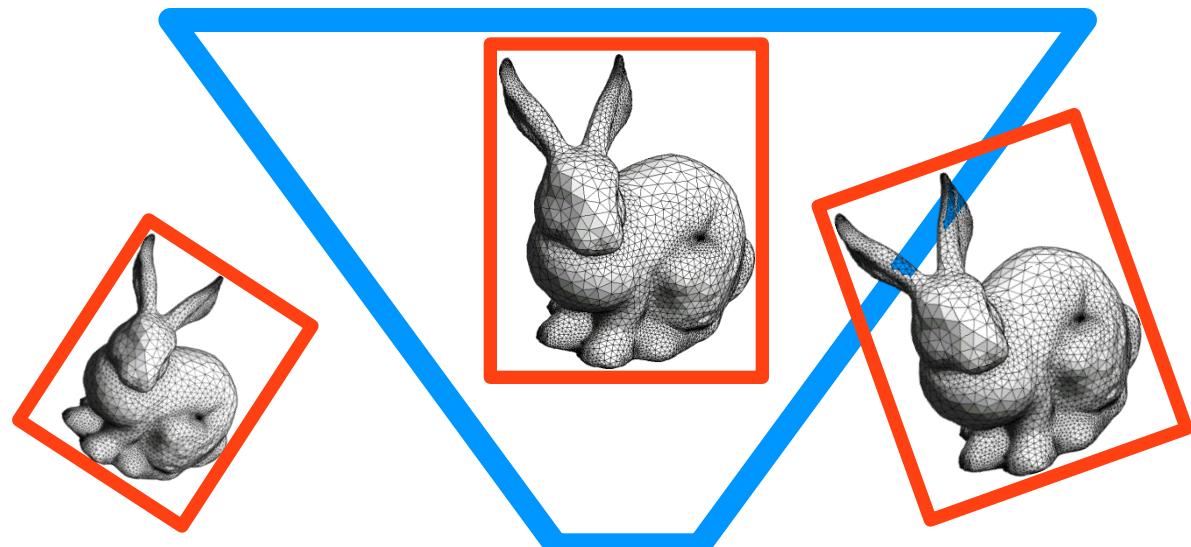
# Questions?

- “View Frustum Culling”
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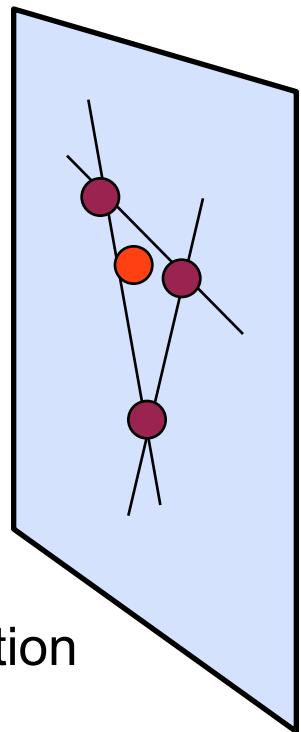
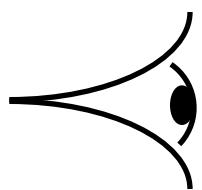
# Homogeneous Rasterization

---

- Idea: avoid projection (and division by zero) by performing rasterization in 3D
  - Or equivalently, use 2D homogenous coordinates ( $w' = z$  after the projection matrix, remember)
- Motivation: clipping is annoying
- Marc Olano, Trey Greer: Triangle scan conversion using 2D homogeneous coordinates, Proc. ACM SIGGRAPH/Eurographics Workshop on Graphics Hardware 1997

# Homogeneous Rasterization

---

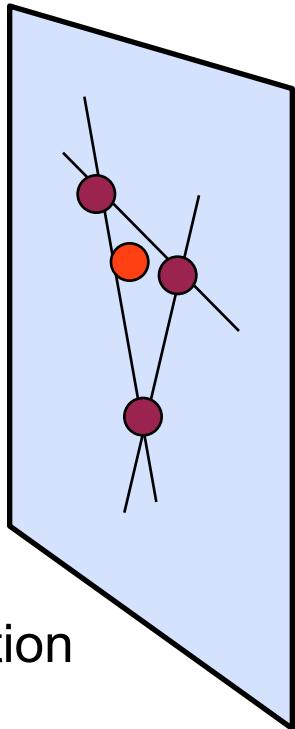
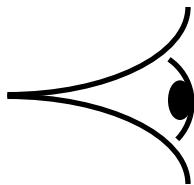


2D rasterization

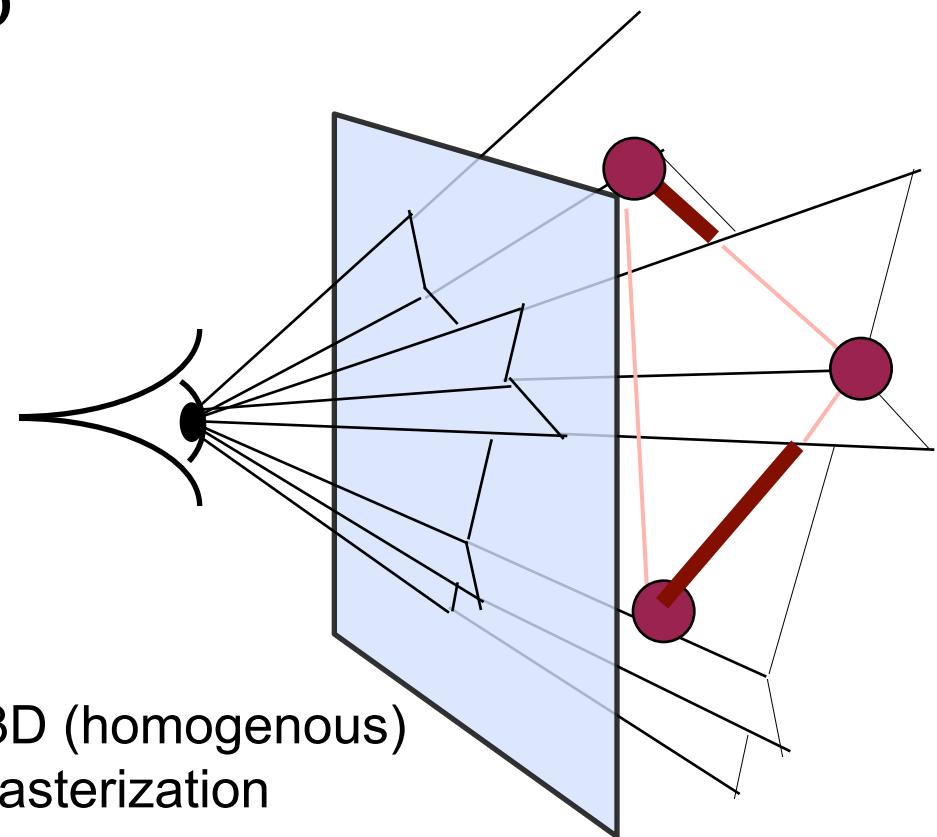
# Homogeneous Rasterization

---

- Replace 2D edge equation by 3D plane equation
  - Plane going through 3D edge and viewpoint
  - Still a halfspace, just 3D



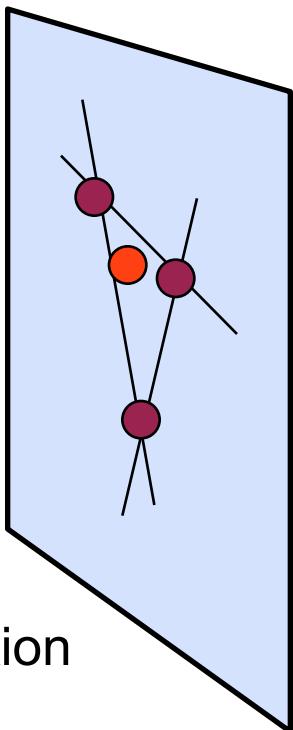
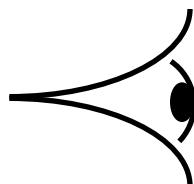
2D rasterization



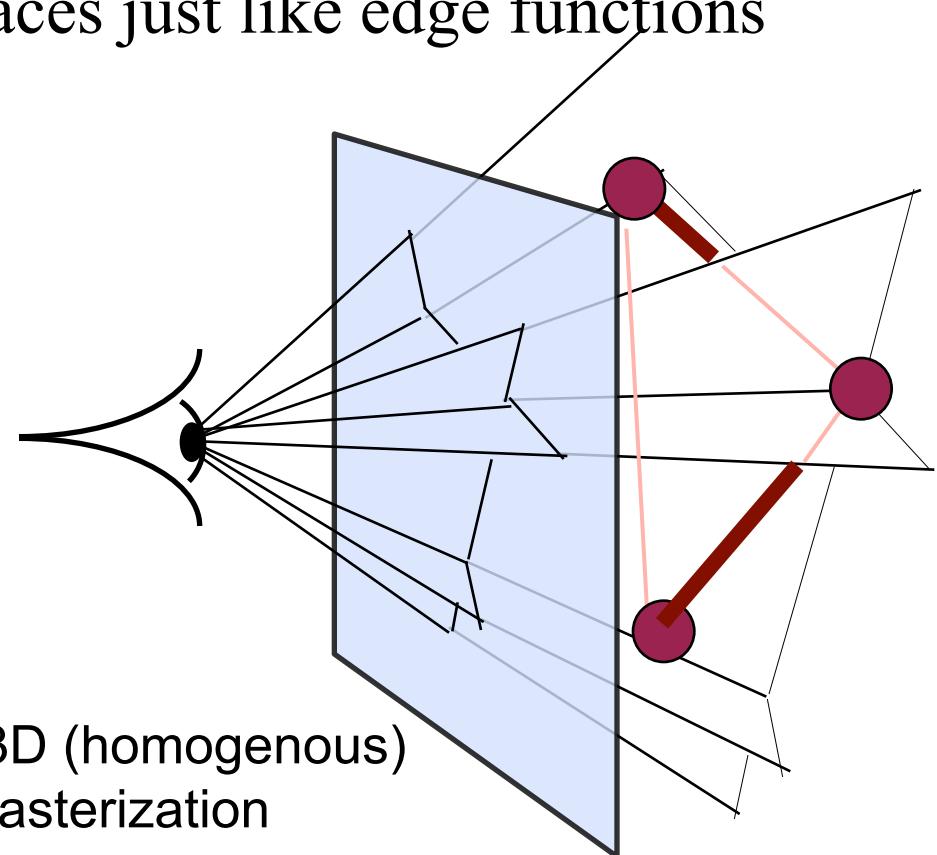
3D (homogenous)  
rasterization

# Homogeneous Rasterization

- Replace 2D edge equation by 3D plane equation
  - Treat pixels as 3D points  $(x, y, 1)$  on image plane, test for containment in 3 halfspaces just like edge functions



2D rasterization



3D (homogenous)  
rasterization

# Homogeneous Rasterization

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Given 3D triangle

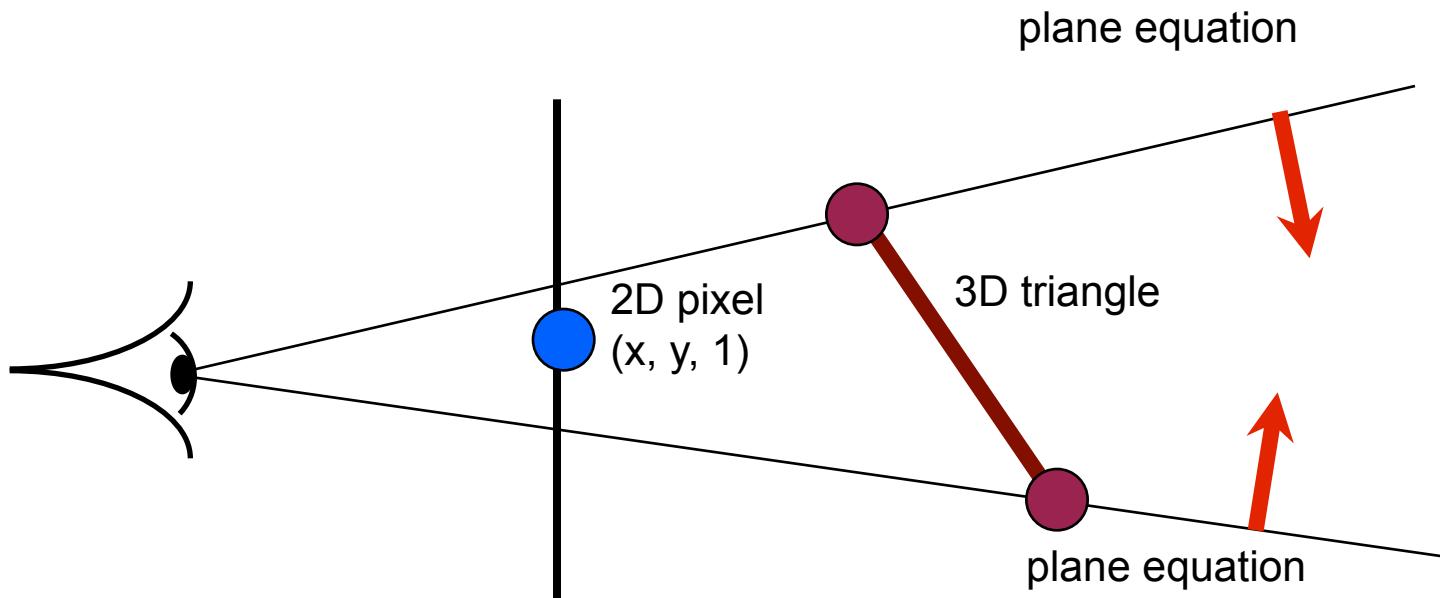
setup plane equations

(plane through viewpoint & triangle edge)

For each pixel  $x, y$

compute plane equations for  $(x, y, 1)$

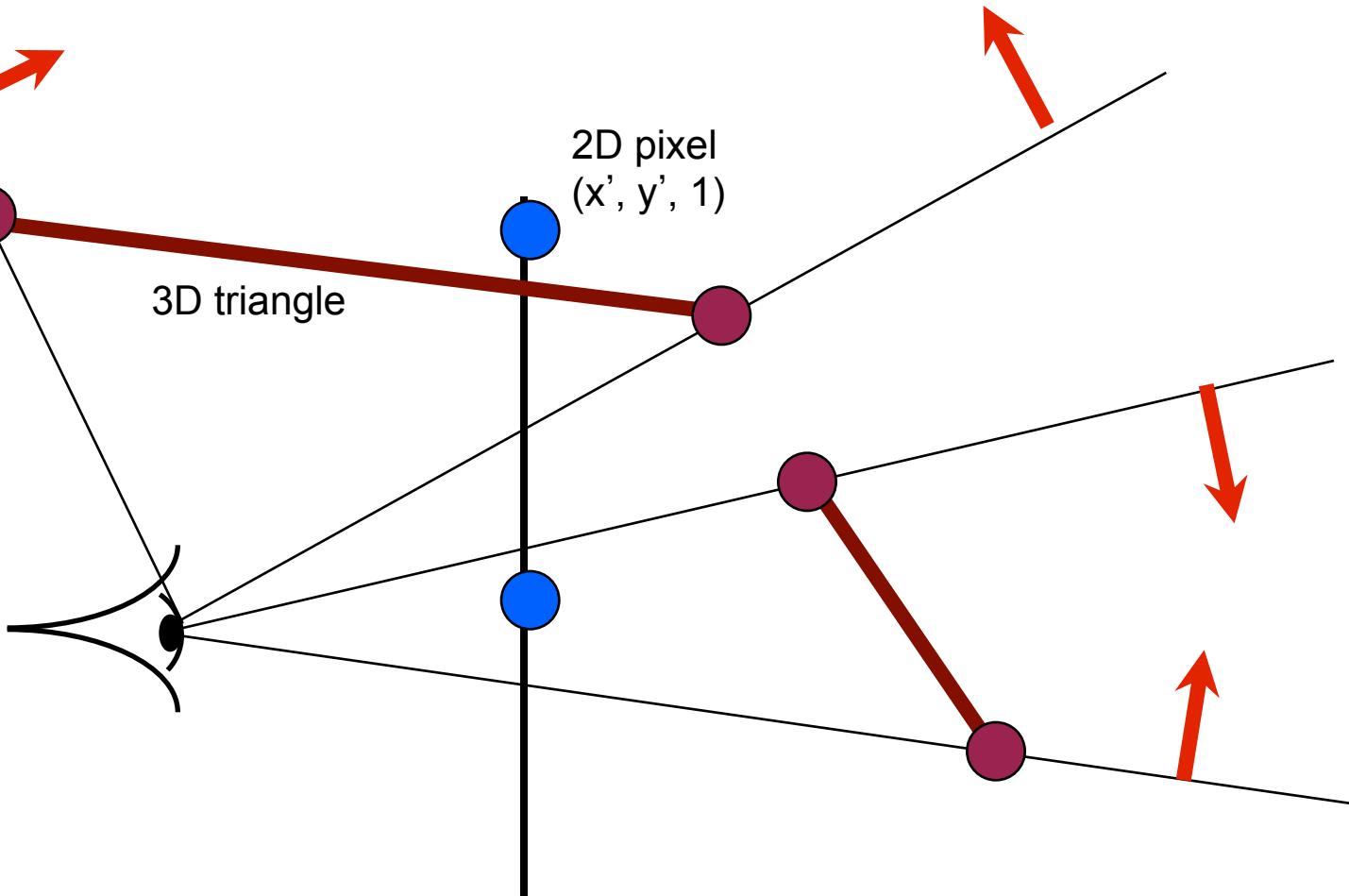
if all pass, draw pixel



# Homogeneous Rasterization

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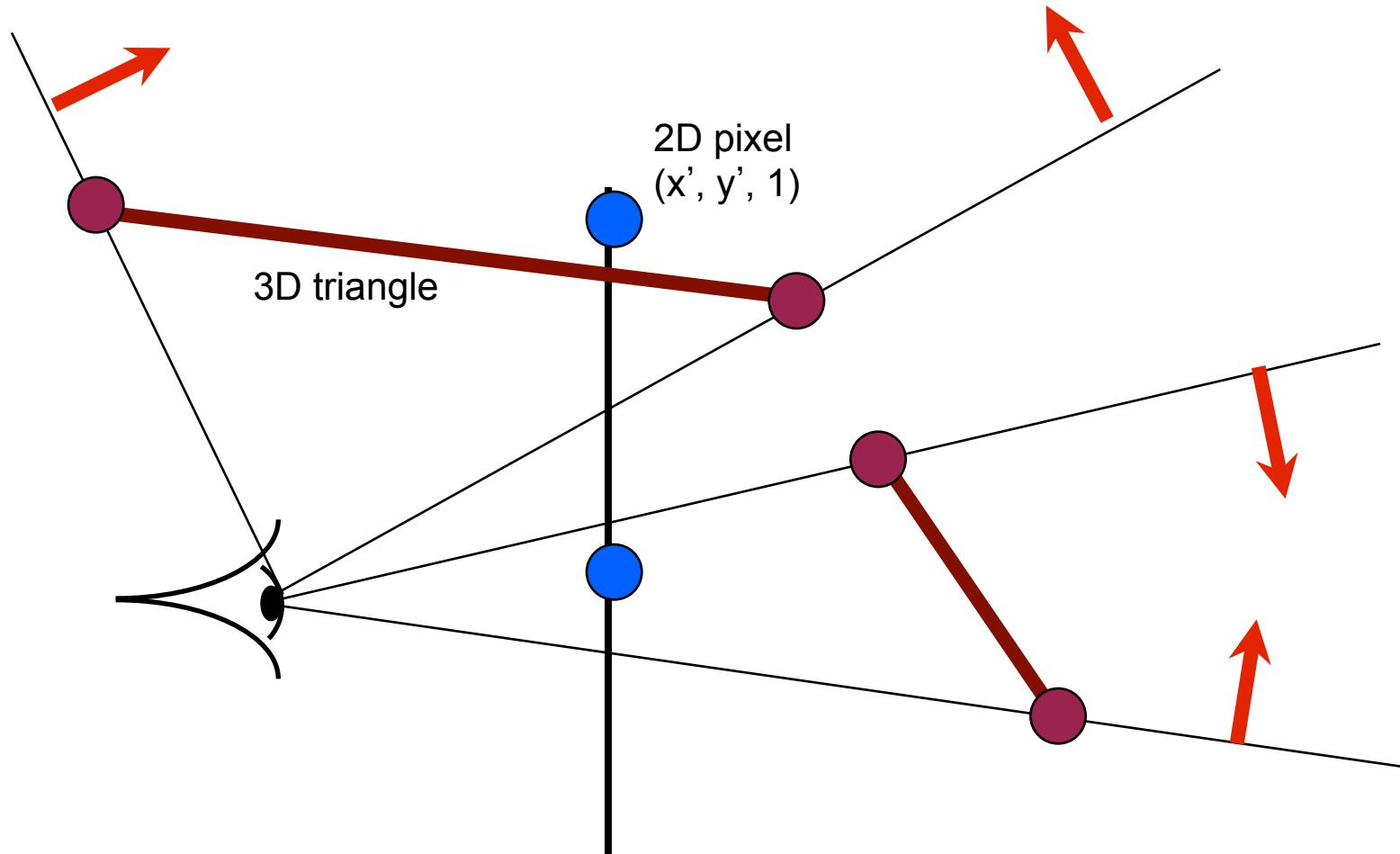
- Works for triangles behind eye
- Still linear, can evaluate incrementally/hierarchically like 2D



# Homogeneous Rasterization Recap

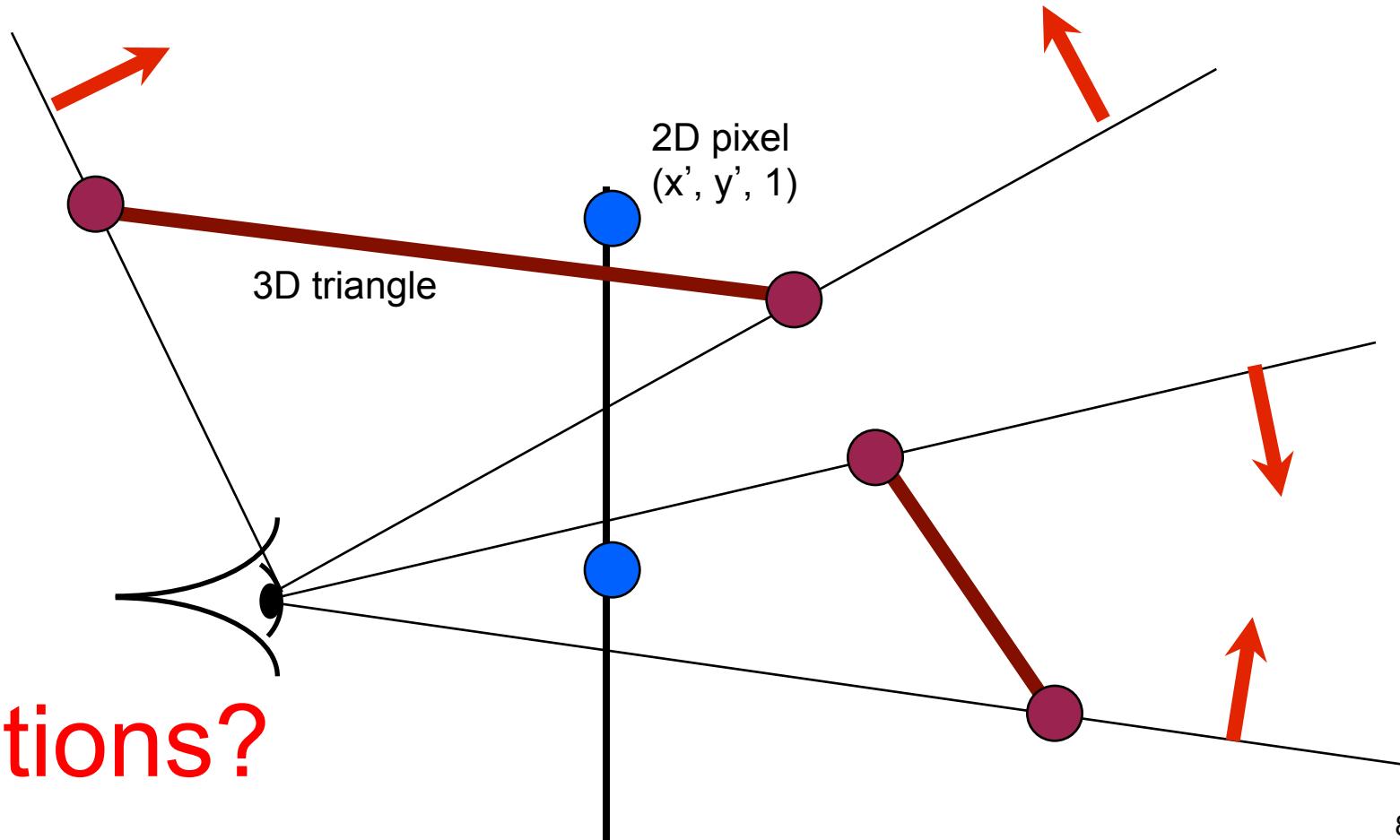
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- Rasterizes with plane tests instead of edge tests
- Removes the need for clipping!



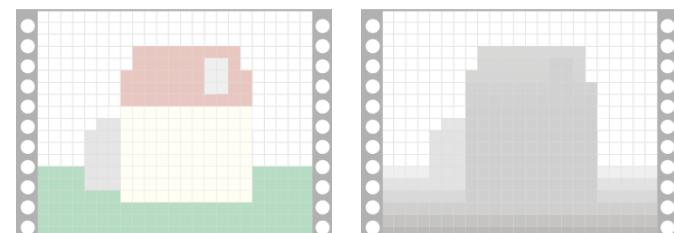
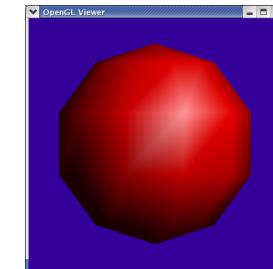
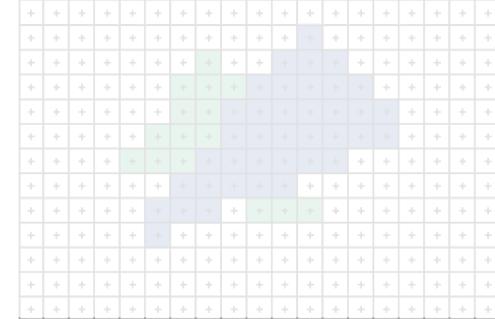
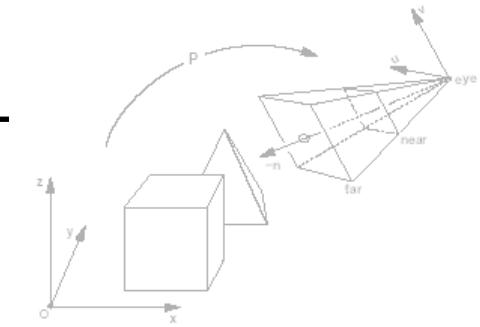
# Homogeneous Rasterization Recap

- Rasterizes with plane tests instead of edge tests
- Removes the need for clipping!



# Modern Graphics Pipeline

- Perform projection of vertices
- Rasterize triangle: find which pixels should be lit
- Compute per-pixel color
- Test visibility, update frame buffer



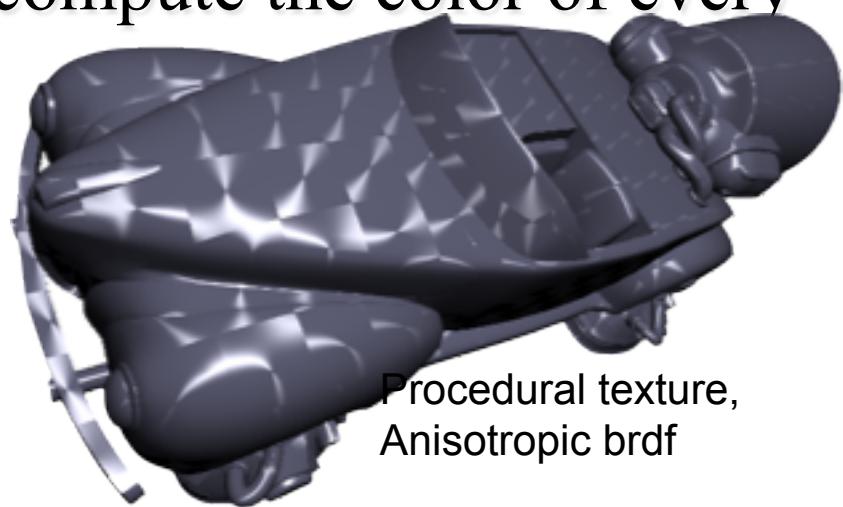
# Pixel Shaders

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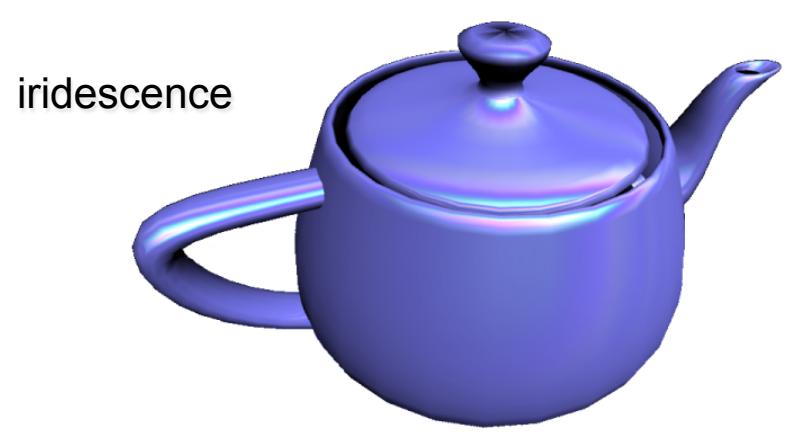
- Modern graphics hardware enables the execution of rather complex programs to compute the color of every single pixel
- More later



Translucence  
Backlighting



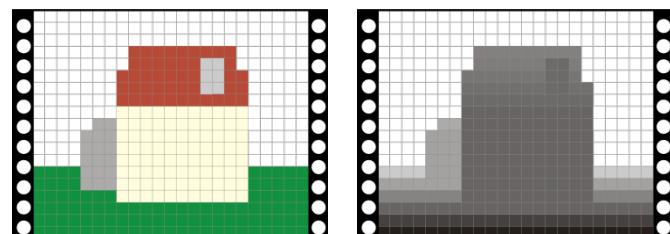
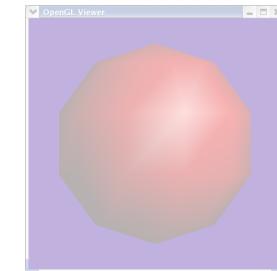
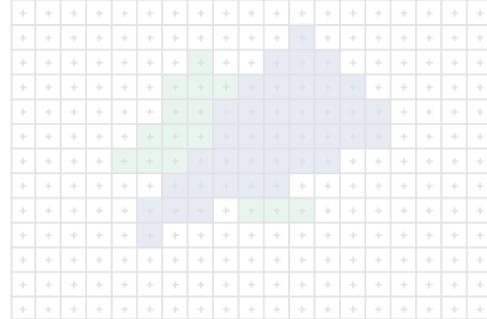
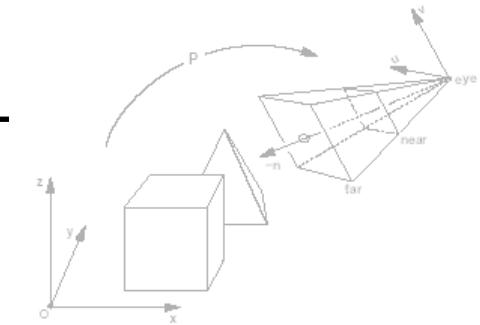
Procedural texture,  
Anisotropic brdf



iridescence

# Modern Graphics Pipeline

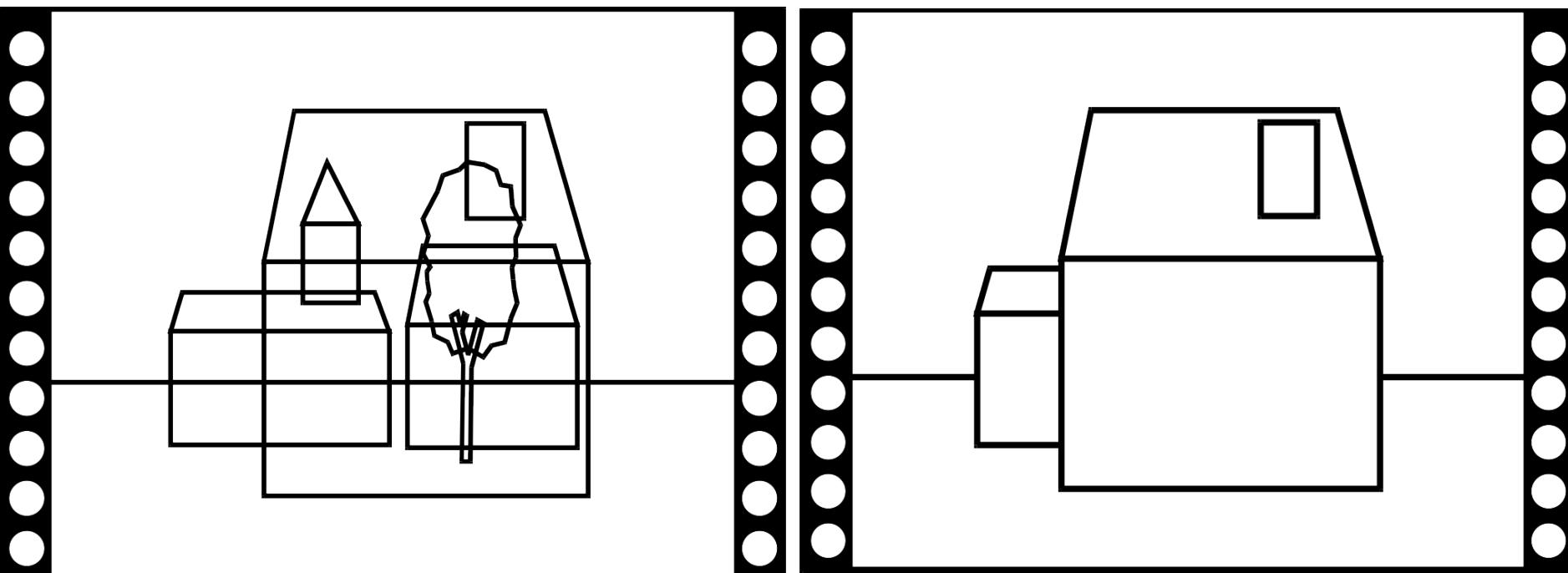
- Perform projection of vertices
- Rasterize triangle: find which pixels should be lit
- Compute per-pixel color
- Test visibility, update frame buffer



# Visibility

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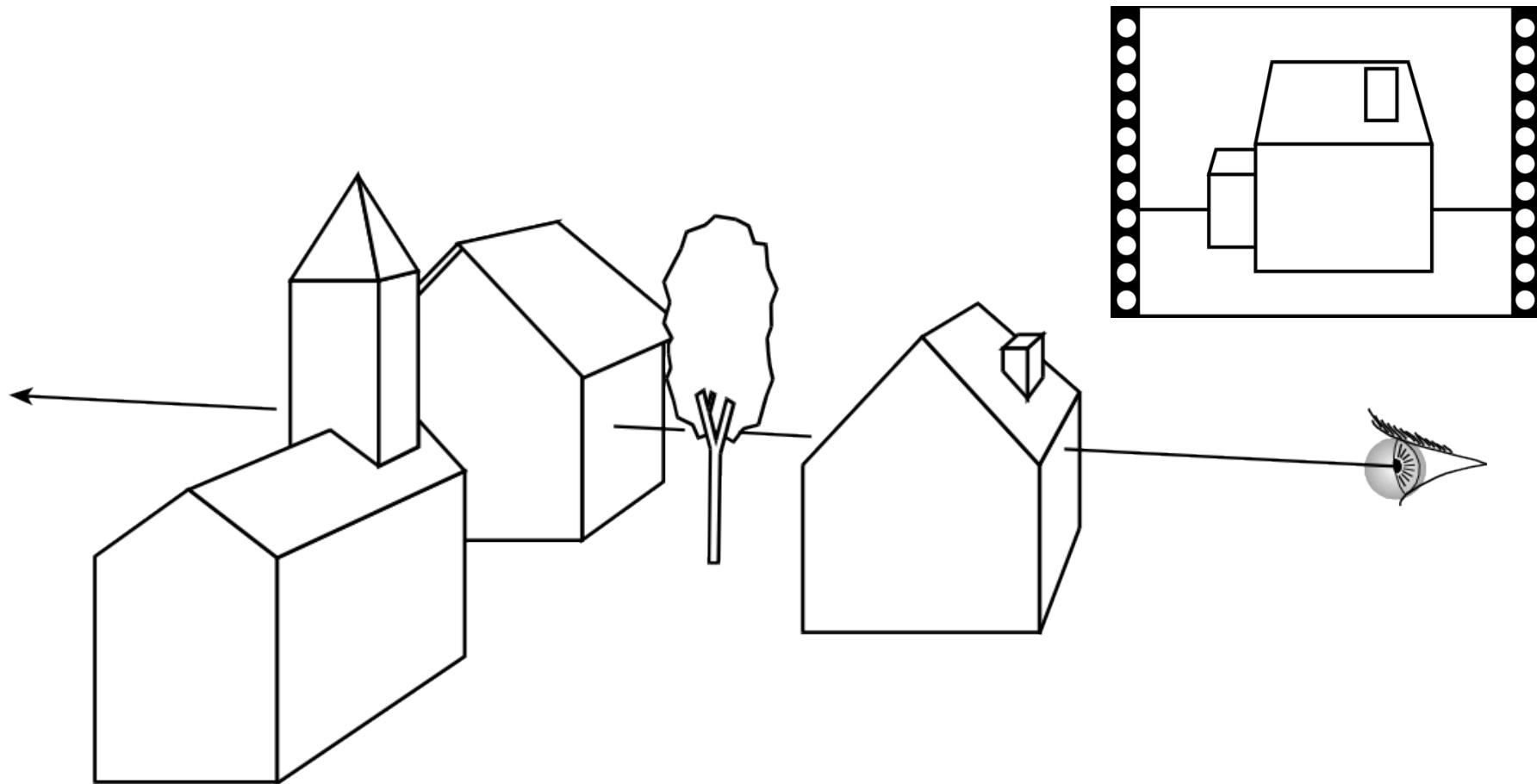
- How do we know which parts are visible/in front?



# Ray Casting

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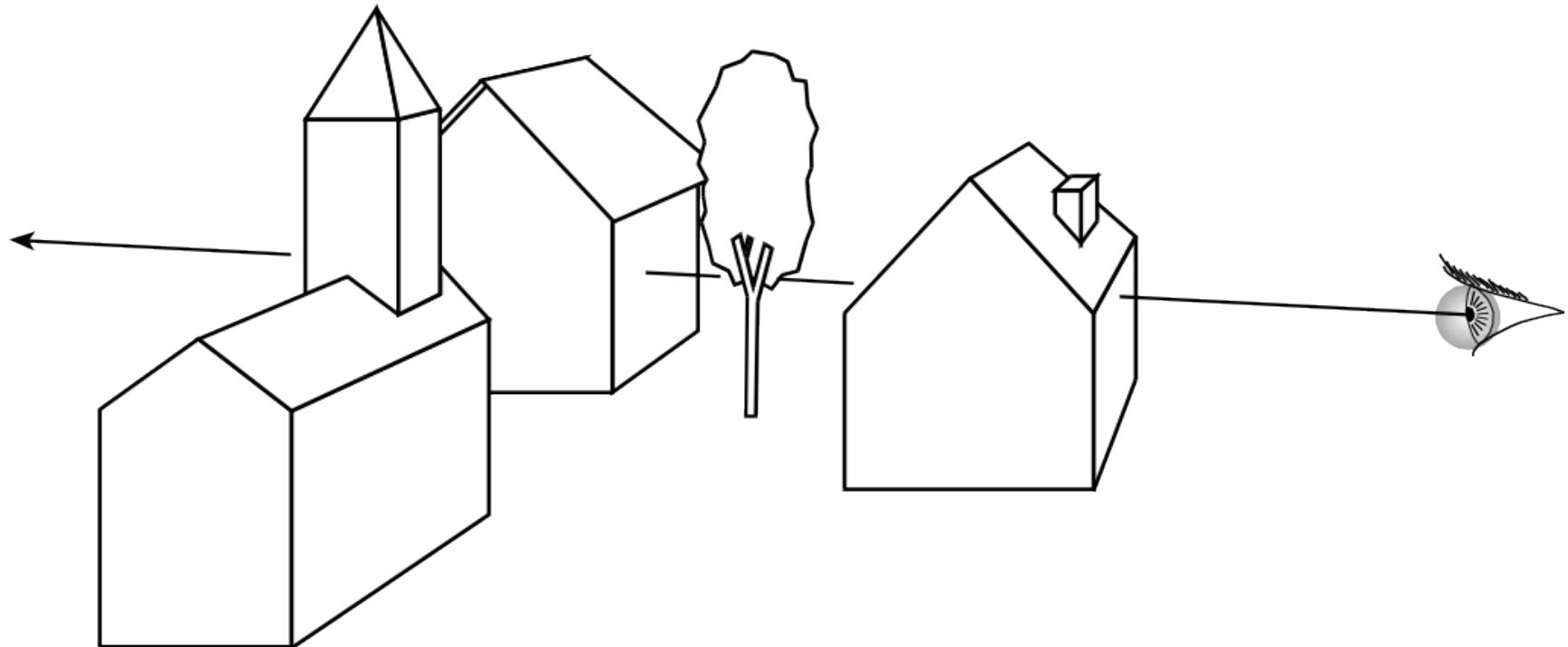
- Maintain intersection with closest object



# Visibility

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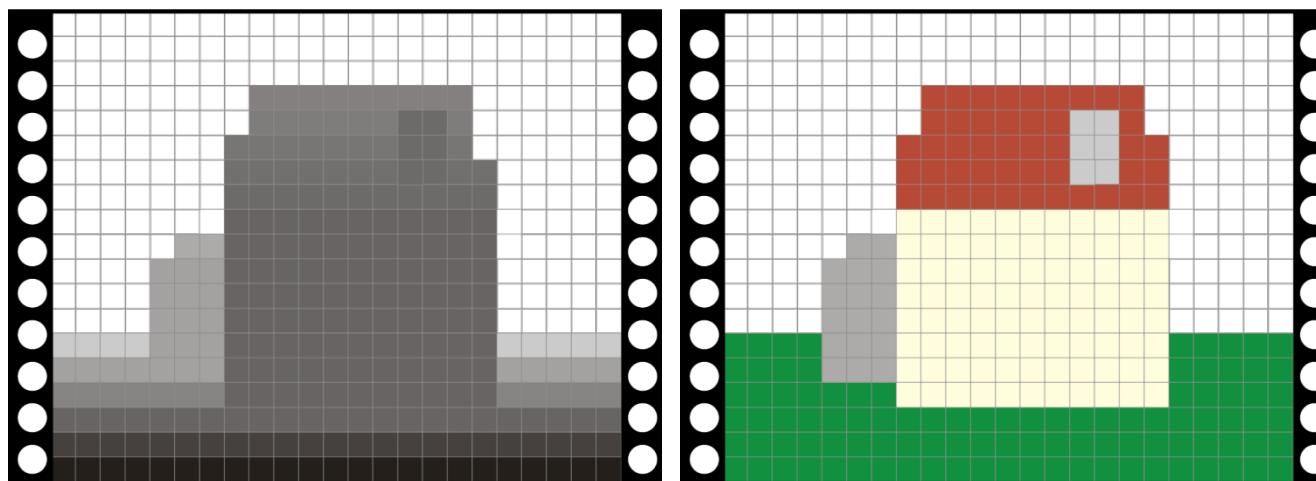
- In ray casting, use intersection with closest  $t$
- Now we have swapped the loops (pixel, object)
- What do we do?



# Z buffer

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- In addition to frame buffer (R, G, B)
- Store distance to camera ( $z$ -buffer)
- Pixel is updated only if  $newz$  is closer than  $z$ -buffer value



# Z-buffer pseudo code

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For every triangle

    Compute Projection, color at vertices

    Setup line equations

    Compute bbox, clip bbox to screen limits

    For all pixels in bbox

        Increment line equations

        Compute currentZ

        Compute currentColor

        If all line equations>0 //pixel [x,y] in triangle

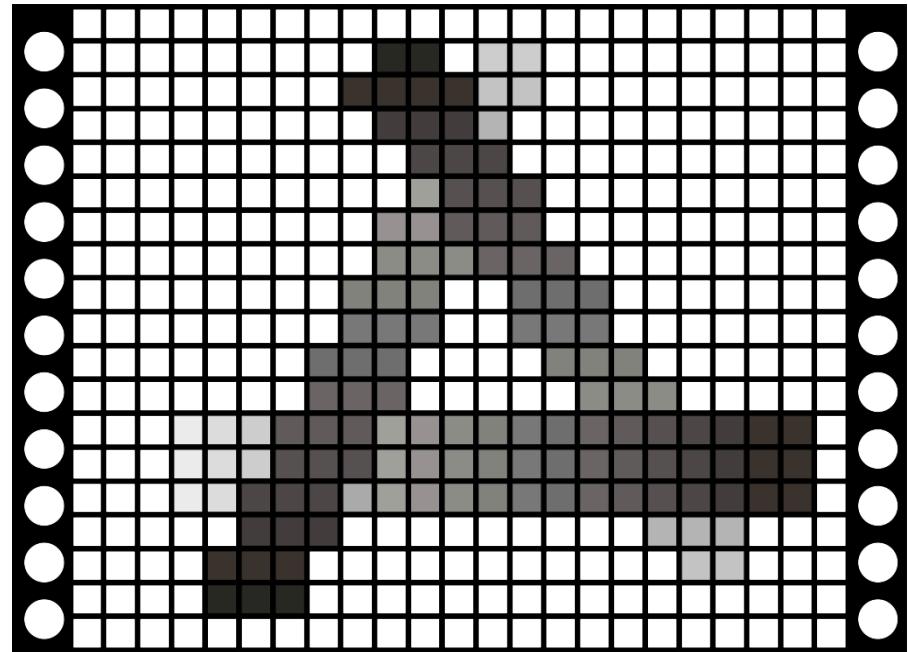
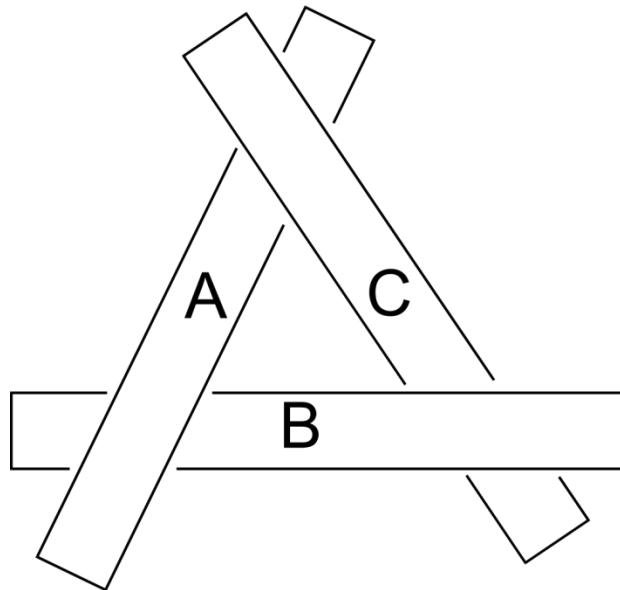
            If currentZ<zBuffer[x, y] //pixel is visible

                Framebuffer[x, y]=currentColor

                zBuffer[x, y]=currentZ

# Works for hard cases!

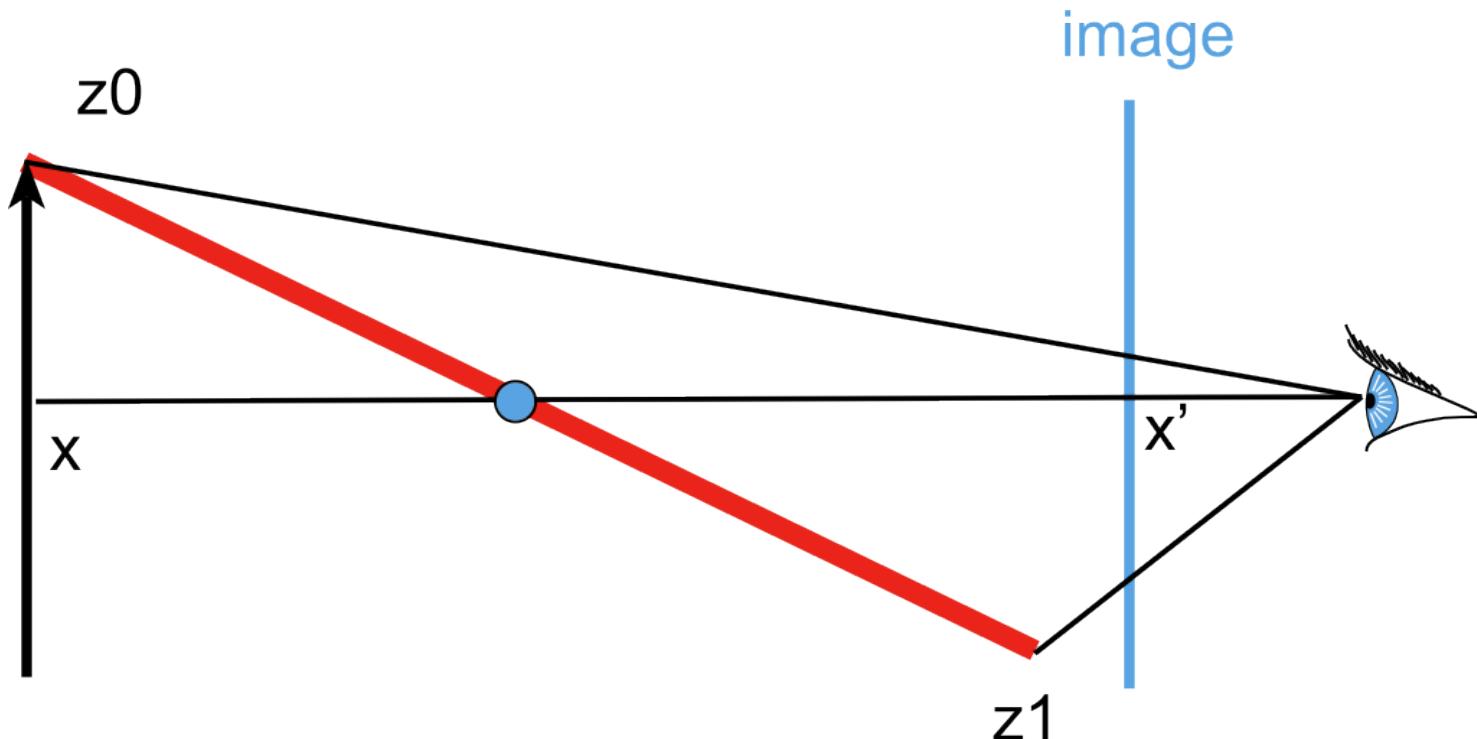
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# More questions for next time

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- How do we get Z?
- Texture Mapping?



# That's All For Today!

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- Next time:  
Screen-space interpolation, visibility, shading