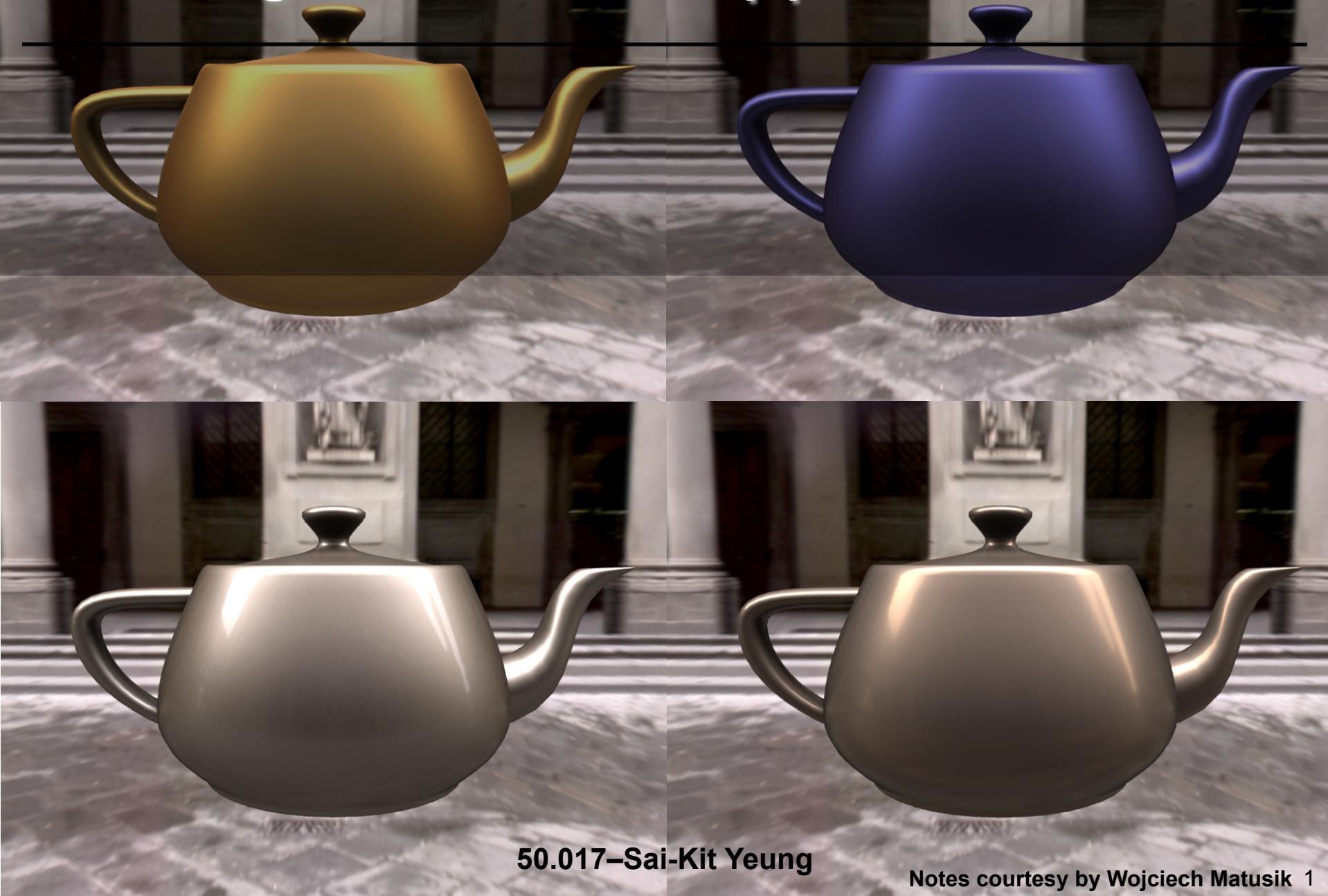


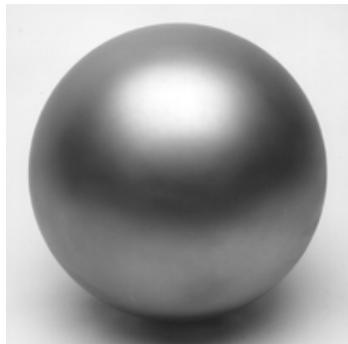
# Shading & Material Appearance



# Lighting and Material Appearance

---

- Input for realistic rendering
  - Geometry, Lighting and Materials
- Material appearance
  - Intensity and shape of highlights
  - Glossiness
  - Color
  - Spatial variation, i.e., texture (next class)



# Radiometry

---

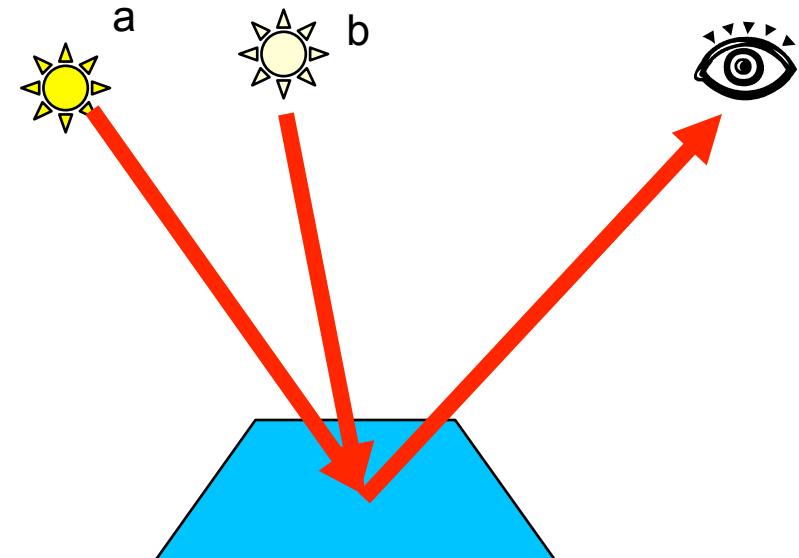
- Techniques for measuring electromagnetic radiation, including visible light
- Unit Issues
  - We will not be too formal in this class
  - Issues we will not really care about
    - Directional quantities vs. integrated over all directions
    - Differential terms: per solid angle, per area
    - Power? Intensity? Flux?
- Color
  - All math here is for a single wavelength only; we will perform computations for R, G, B separately
    - Do not panic, that just means we will perform every operation three times, that is all

# Light Sources

---

- Today, we only consider point light sources
  - Thus we do not need to care about solid angles
- For multiple light sources, use linearity
  - We can add the solutions for two light sources
    - $I(a+b) = I(a) + I(b)$
  - We simply multiply the solution when we scale the light intensity
    - $I(s a) = s I(a)$

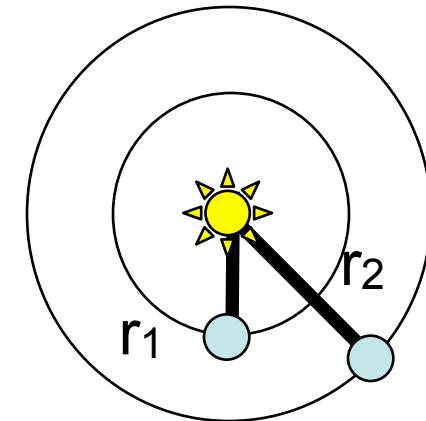
Yet again, linearity  
is our friend!



# Intensity as Function of Distance

---

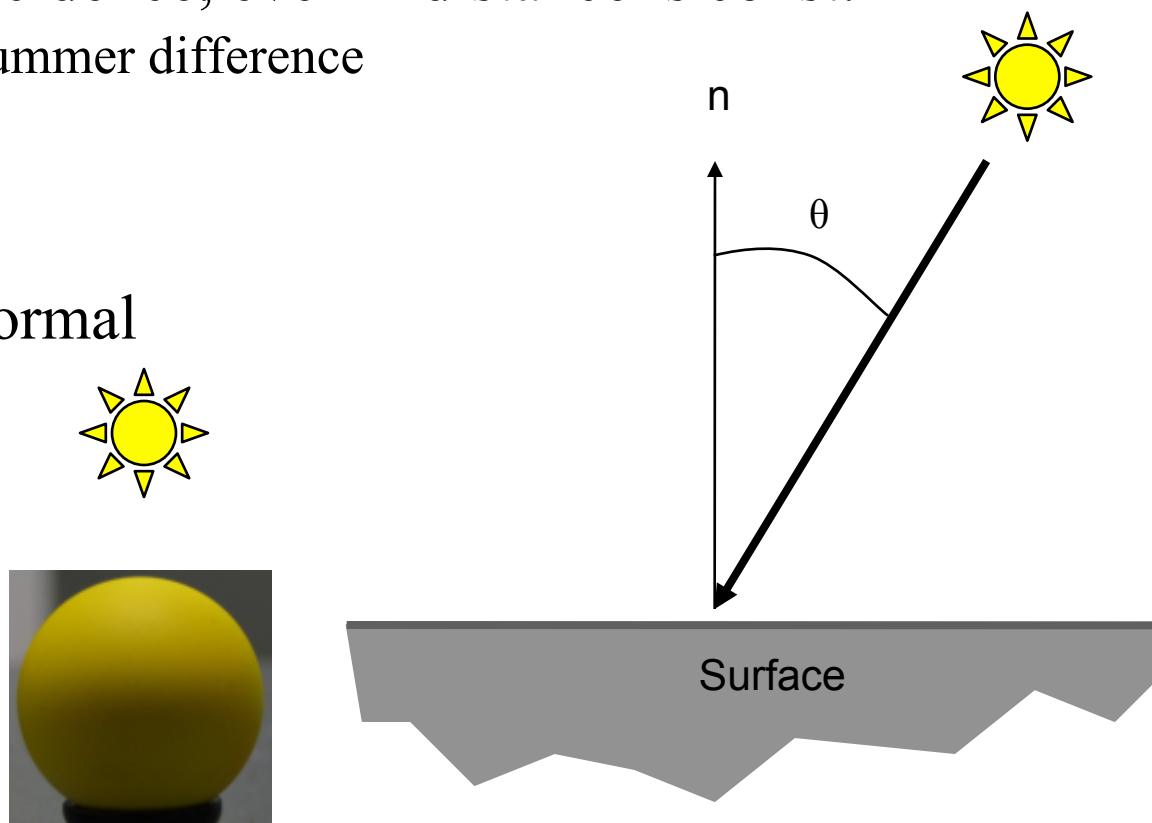
- $1/r^2$  fall-off for isotropic point lights
  - Why? An isotropic point light outputs constant power per solid angle (“into all directions”)
  - Must have same power in all concentric spheres
    - Sphere’s surface area grows with  $r^2 \Rightarrow$  energy obeys  $1/r^2$
- ... but in graphics we often cheat with or ignore this.
  - Why? Ideal point lights are kind of harsh
    - Intensity goes to infinity when you get close – not great!
  - In particular,  $1/(ar^2+br+c)$  is popular



# Incoming Irradiance

---

- The amount of light energy received by a surface
  - Power incident on a surface - *Radiant flux density*
  - depends on incoming angle
  - Bigger at normal incidence, even if distance is const.
    - Similar to winter/summer difference
- How exactly?
  - $\cos \theta$  law
  - Dot product with normal



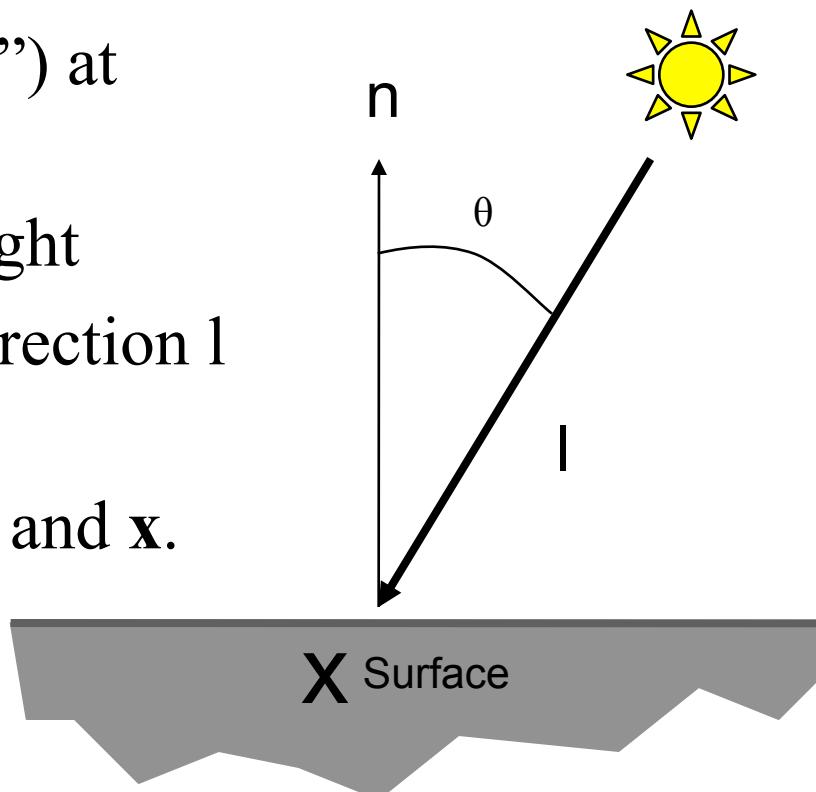
# Incoming Irradiance for Pointlights

---

- Let's combine this with the  $1/r^2$  fall-off:

$$I_{\text{in}} = I_{\text{light}} \cos \theta / r^2$$

- $I_{\text{in}}$  is the irradiance (“intensity”) at surface point  $x$
- $I_{\text{light}}$  is the “intensity” of the light
- $\theta$  is the angle between light direction  $l$  and surface normal  $n$
- $r$  is the distance between light and  $x$ .



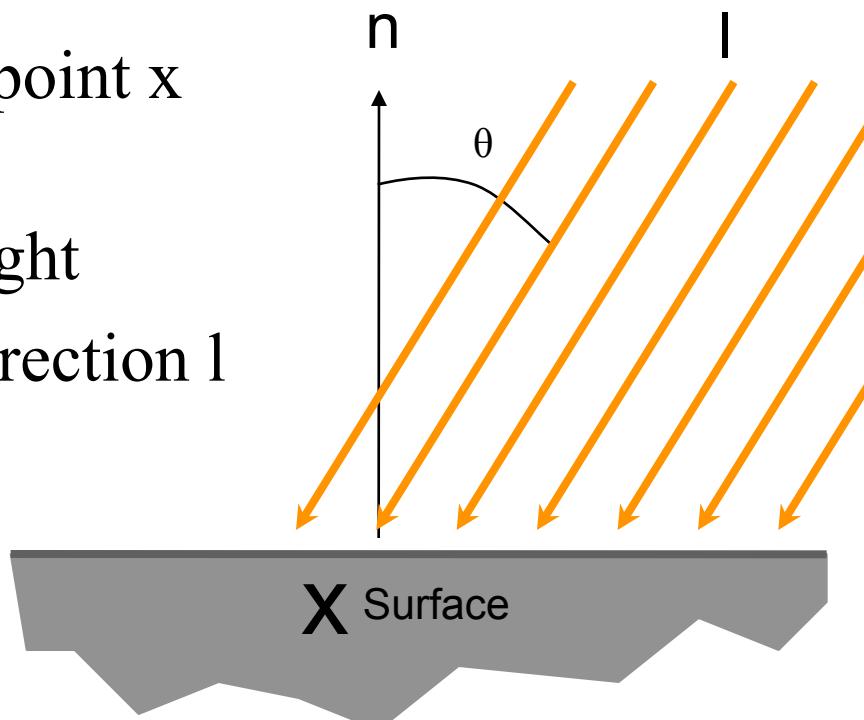
# Directional Lights

---

- “Pointlights that are infinitely far”
  - No falloff, just one direction and one intensity

$$I_{\text{in}} = I_{\text{light}} \cos \theta$$

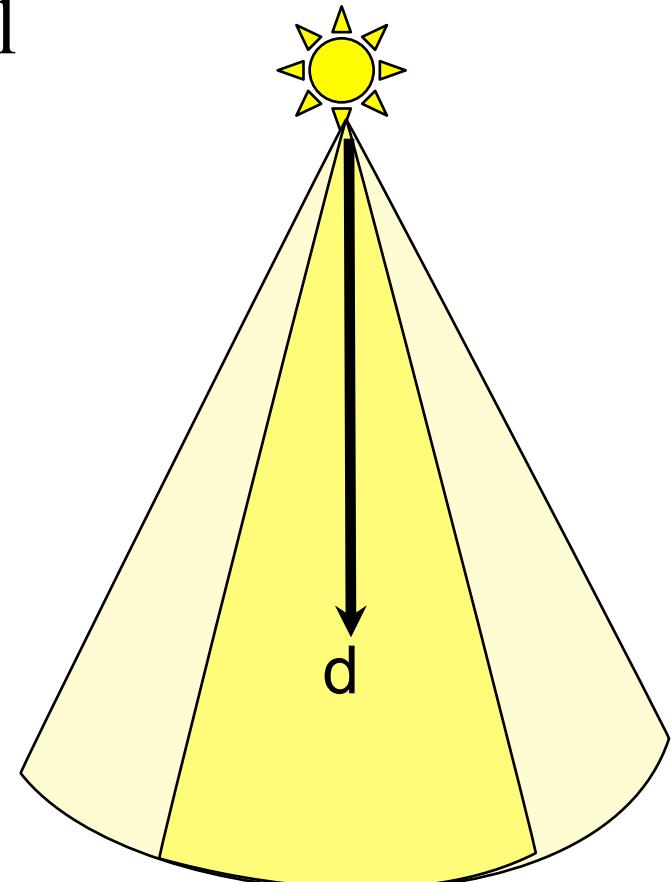
- $I_{\text{in}}$  is the irradiance at surface point  $x$  from the directional light
- $I_{\text{light}}$  is the “intensity” of the light
- $\theta$  is the angle between light direction  $l$  and surface normal  $n$ 
  - Only depends on  $n$ , not  $x$ !



# Spotlights

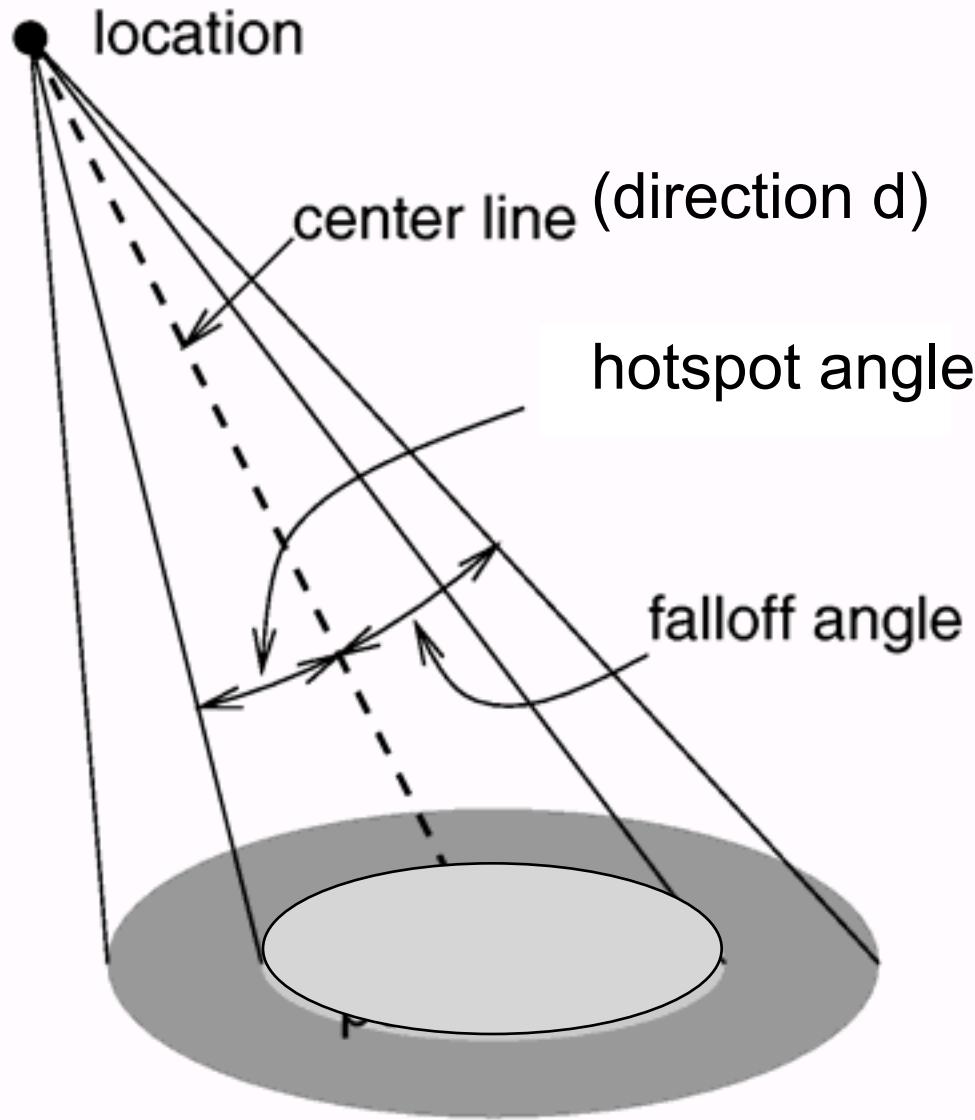
---

- Pointlights with non-uniform directional emission
- Usually symmetric about a central direction  $d$ , with angular falloff
  - Often two angles
    - “Hotspot” angle:  
No attenuation within the central cone
    - “Falloff” angle: Light attenuates from full intensity to zero intensity between the hotspot and falloff angles
- Plus your favorite distance falloff curve



# Spotlight Geometry

---

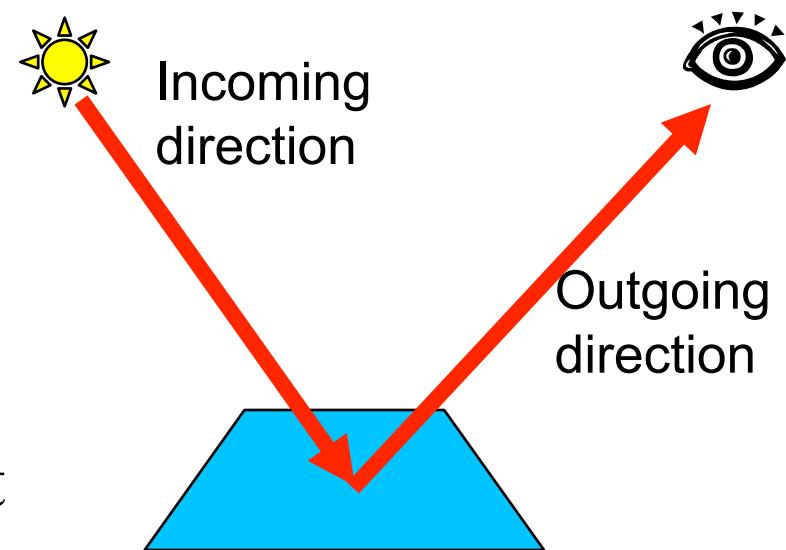


Adapted from  
POVRAY documentation

# Quantifying Reflection – BRDF

---

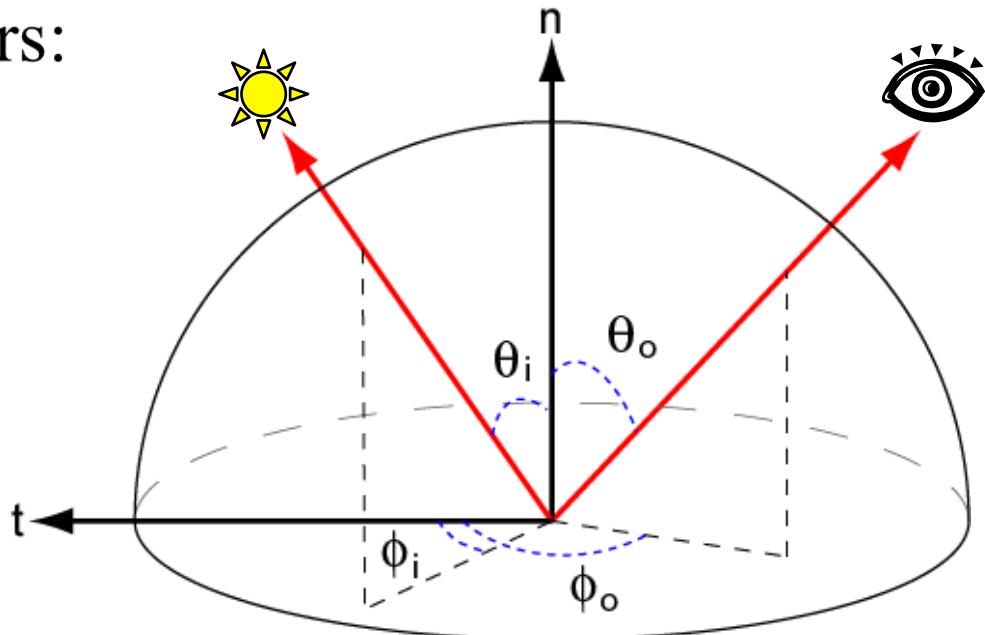
- Bidirectional Reflectance Distribution Function
- Ratio of light coming from one direction that gets reflected in another direction
  - Pure reflection, assumes no light scatters into the material
- Focuses on angular aspects, not spatial variation of the material
- How many dimensions?



# BRDF $f_r$

---

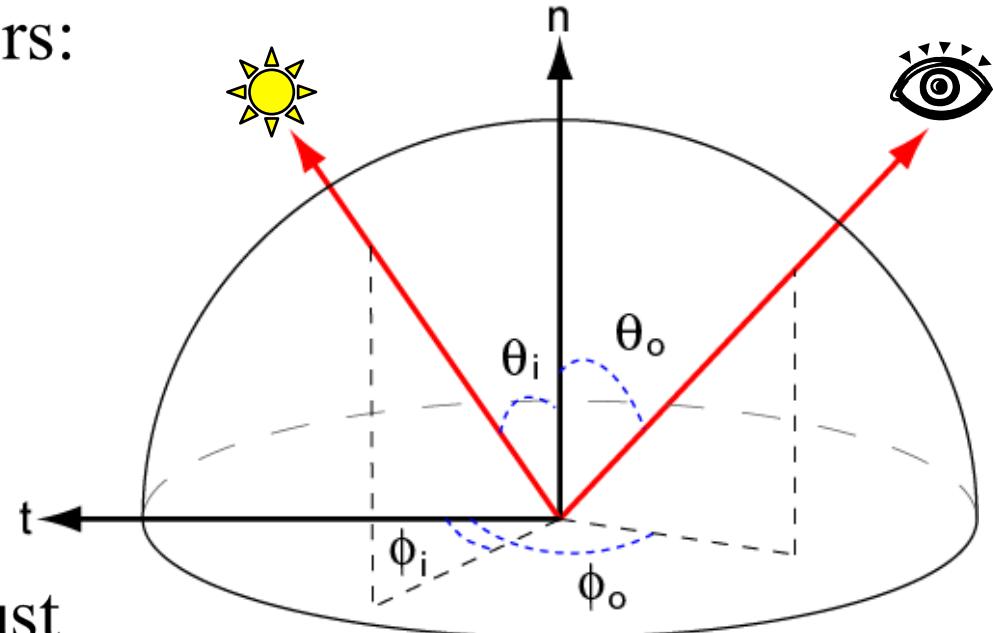
- Bidirectional Reflectance Distribution Function
  - 4D: 2 angles for each direction
  - $\text{BRDF} = f_r(\theta_i, \phi_i; \theta_o, \phi_o)$
  - Or just two unit vectors:  
 $\text{BRDF} = f_r(\mathbf{l}, \mathbf{v})$ 
    - $\mathbf{l}$  = light direction
    - $\mathbf{v}$  = view direction



# BRDF $f_r$

---

- Bidirectional Reflectance Distribution Function
  - 4D: 2 angles for each direction
  - $\text{BRDF} = f_r(\theta_i, \phi_i; \theta_o, \phi_o)$
  - Or just two unit vectors:  
 $\text{BRDF} = f_r(\mathbf{l}, \mathbf{v})$ 
    - $\mathbf{l}$  = light direction
    - $\mathbf{v}$  = view direction
  - The BRDF is aligned with the surface;  
the vectors  $\mathbf{l}$  and  $\mathbf{v}$  must be in a local coordinate system



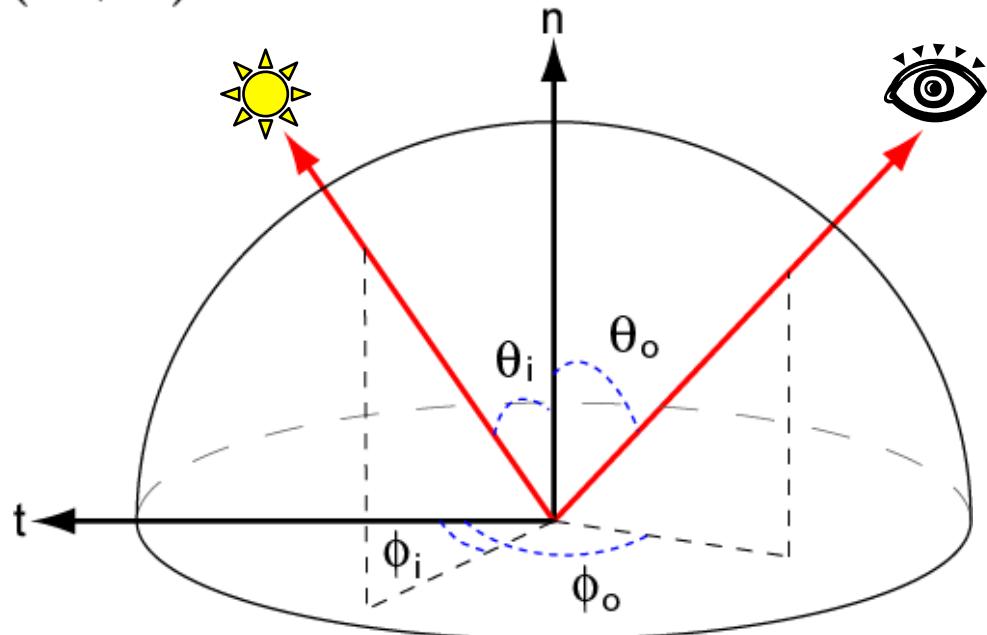
# BRDF $f_r$

---

- Relates incident irradiance from every direction to outgoing light.  
How?

$$I_{\text{out}}(\mathbf{v}) = I_{\text{in}}(\mathbf{l}) f_r(\mathbf{v}, \mathbf{l})$$

$\mathbf{l}$  = light direction  
(incoming)  
 $\mathbf{v}$  = view direction  
(outgoing)



# BRDF $f_r$

---

- Relates incident irradiance from every direction to outgoing light.  
How?

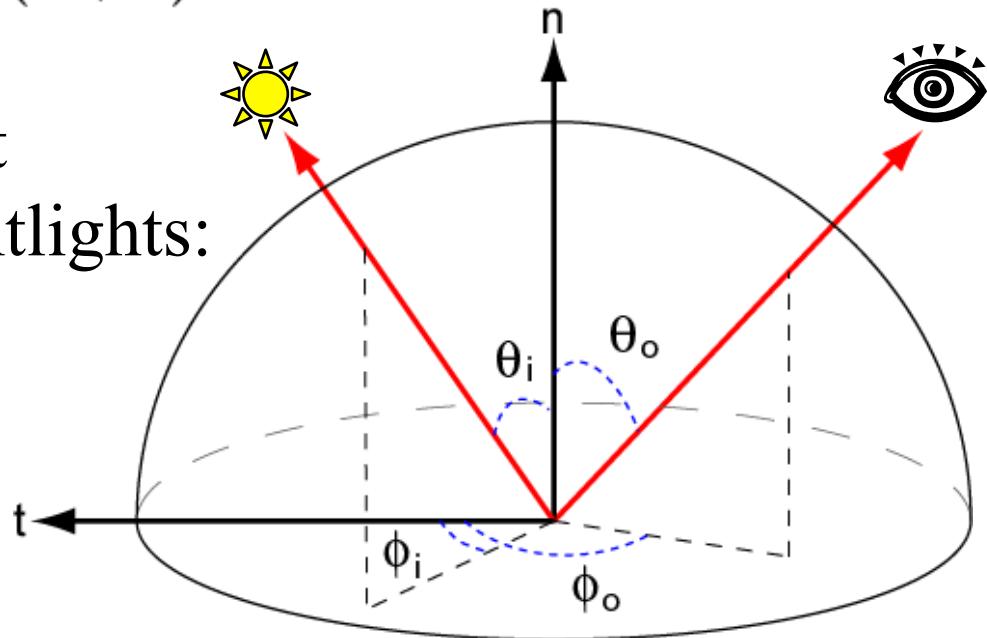
$$I_{\text{out}}(\mathbf{v}) = I_{\text{in}}(\mathbf{l}) f_r(\mathbf{v}, \mathbf{l})$$

- Let's combine with what we know already of pointlights:

$$I_{\text{out}}(\mathbf{v}) =$$

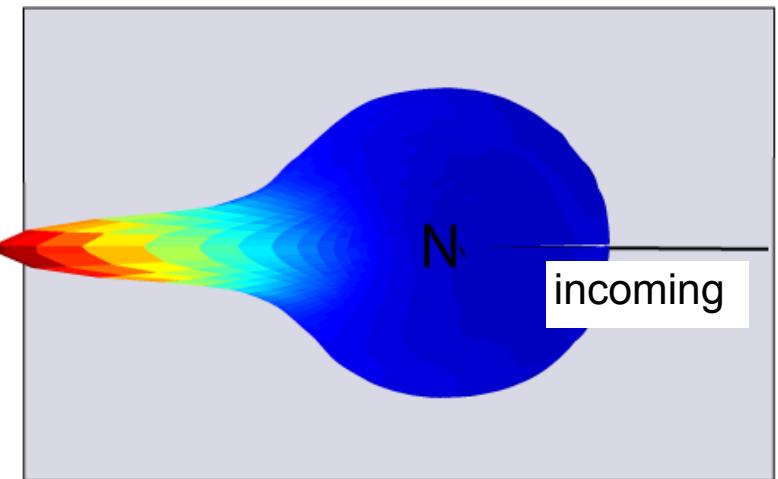
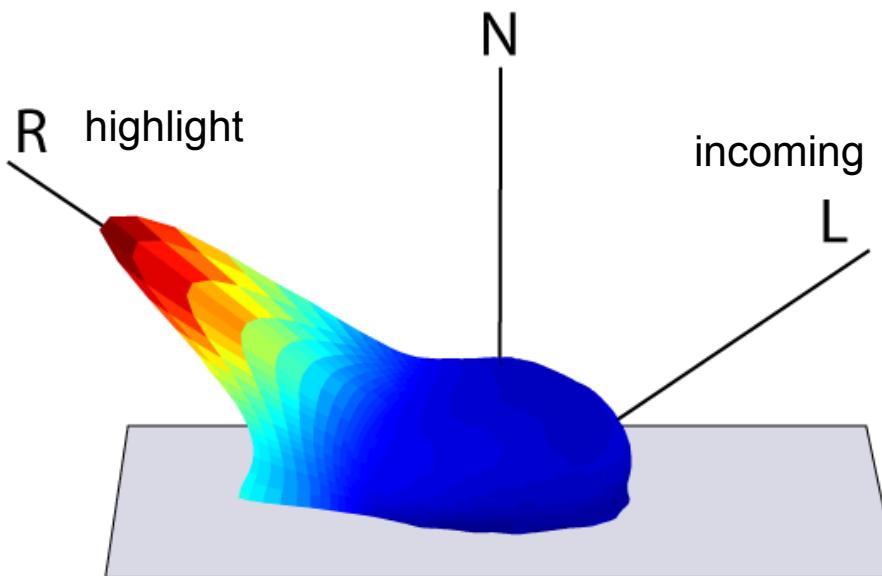
$$\frac{I_{\text{light}} \cos \theta_i}{r^2} f_r(\mathbf{v}, \mathbf{l})$$

$\mathbf{l}$  = light direction  
(incoming)  
 $\mathbf{v}$  = view direction  
(outgoing)



# 2D Slice at Constant Incidence

- For a fixed incoming direction, view dependence is a 2D spherical function
  - Here a moderate specular component



Example: Plot of "PVC" BRDF at 55° incidence

# Isotropic vs. Anisotropic

---

- When keeping  $\mathbf{l}$  and  $\mathbf{v}$  fixed, if rotation of surface around the normal does not change the reflection, the material is called isotropic
- Surfaces with strongly oriented microgeometry elements are anisotropic
- Examples:
  - brushed metals,
  - hair, fur, cloth, velvet

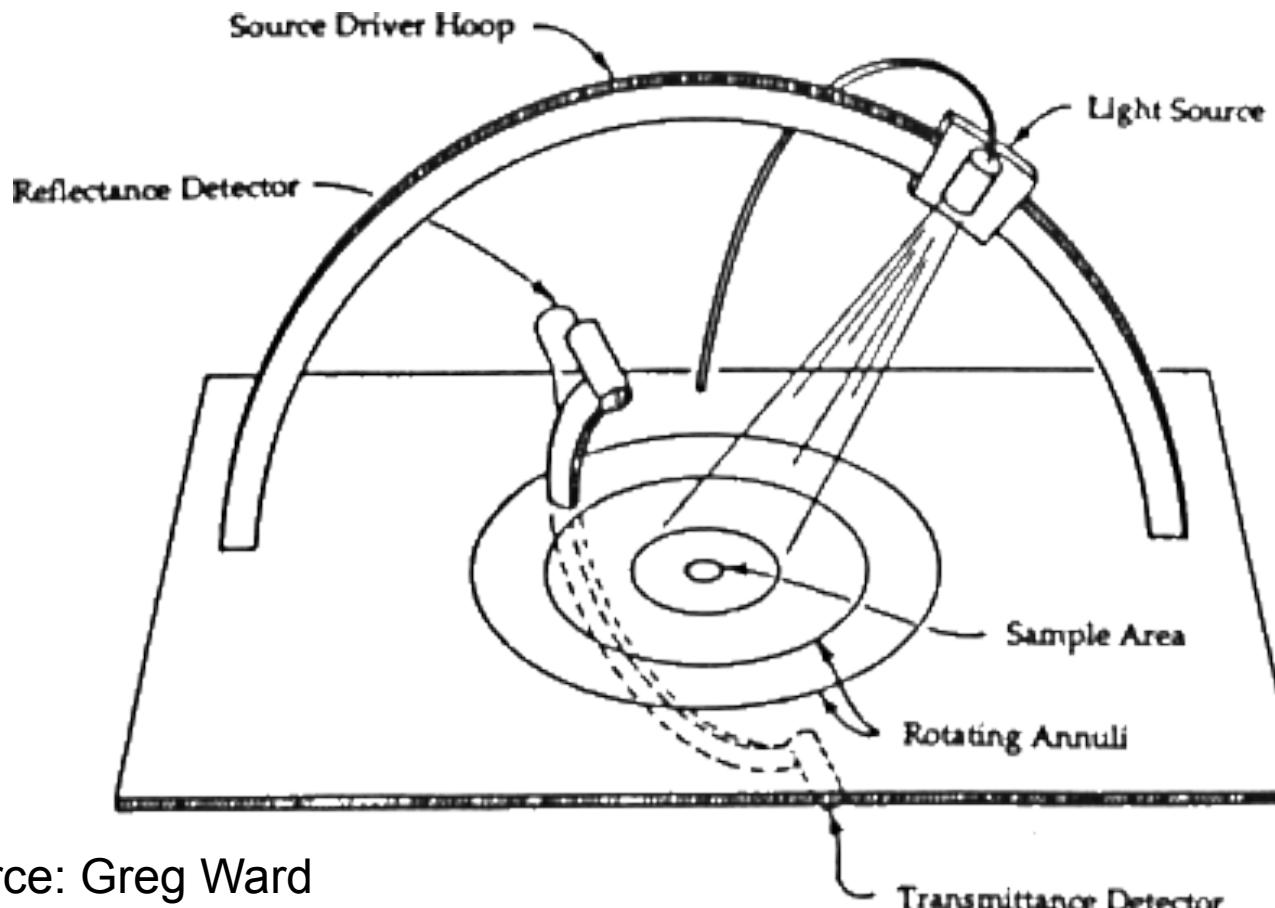


Westin et.al 92

# How do we obtain BRDFs?

---

- One possibility: Gonioreflectometer
  - 4 degrees of freedom



Source: Greg Ward

# How do we obtain BRDFs?

---

- One possibility: Gonioreflectometer
  - 4 degrees of freedom



# How Do We Obtain BRDFs?

- Another possibility: Take pictures of spheres coated with material, rotate light around a 1D arc
  - This gives 3DOF => isotropic materials only

Each image -> 2D slides of the BRDF



# Parametric BRDFs

---

- BRDFs can be measured from real data
  - But tabulated 4D data is too cumbersome for most uses
- Therefore, parametric BRDF models represent the relationship between incident and outgoing light by some mathematical formula
  - The appearance can then be tuned by setting parameters
    - “Shininess”, “anisotropy”, etc.
  - Physically-based or Phenomenological
  - They can model with measured data (examples later)
- Popular models: Diffuse, Blinn-Phong, Cook-Torrance, Lafortune, Ward, Oren-Nayar, ..., etc.

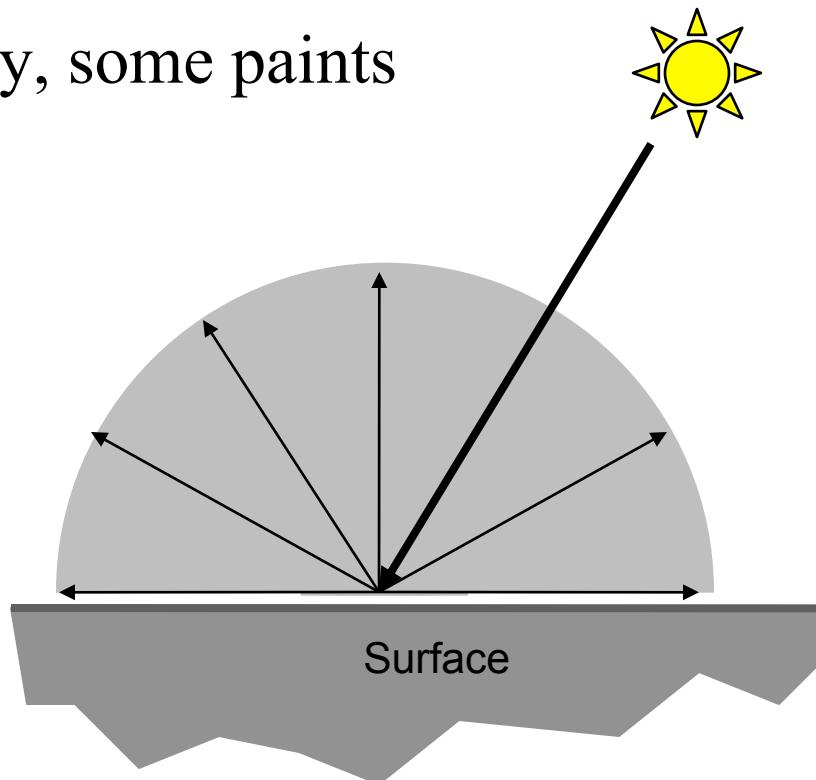
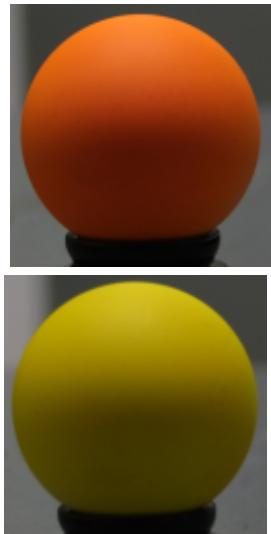
# Questions?

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# Ideal Diffuse Reflectance

---

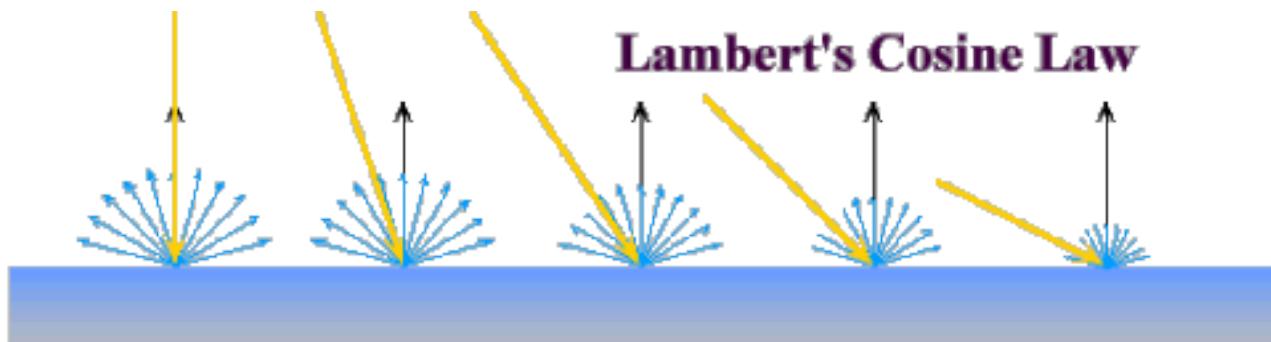
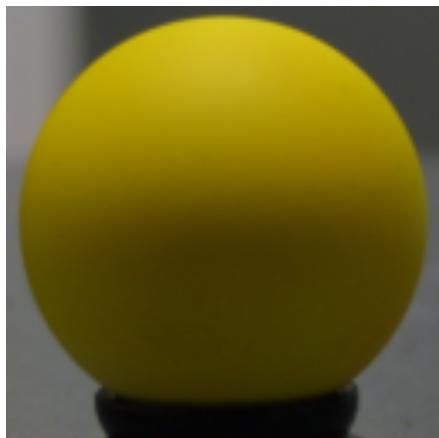
- Assume surface reflects equally in all directions.
- An ideal diffuse surface is, at the microscopic level, a very rough surface.
  - Example: chalk, clay, some paints



# Ideal Diffuse Reflectance

---

- Ideal diffuse reflectors reflect light according to Lambert's cosine law
  - The reflected light varies with cosine even if distance to light source is kept constant



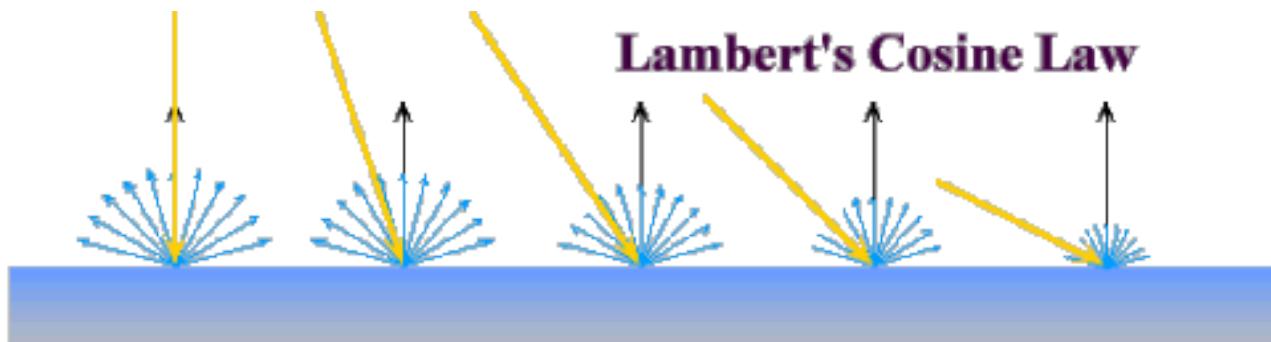
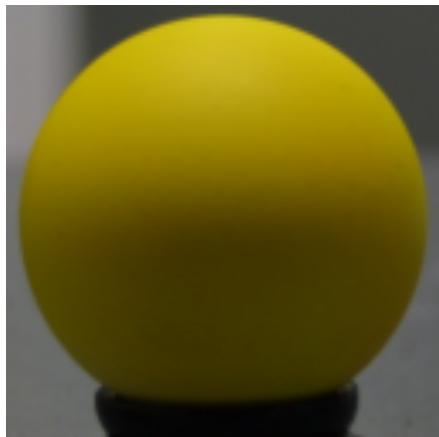
**Lambert's Cosine Law**

# Ideal Diffuse Reflectance

---

- Ideal diffuse reflectors reflect light according to Lambert's cosine law
  - The reflected light varies with cosine even if distance to light source is kept constant

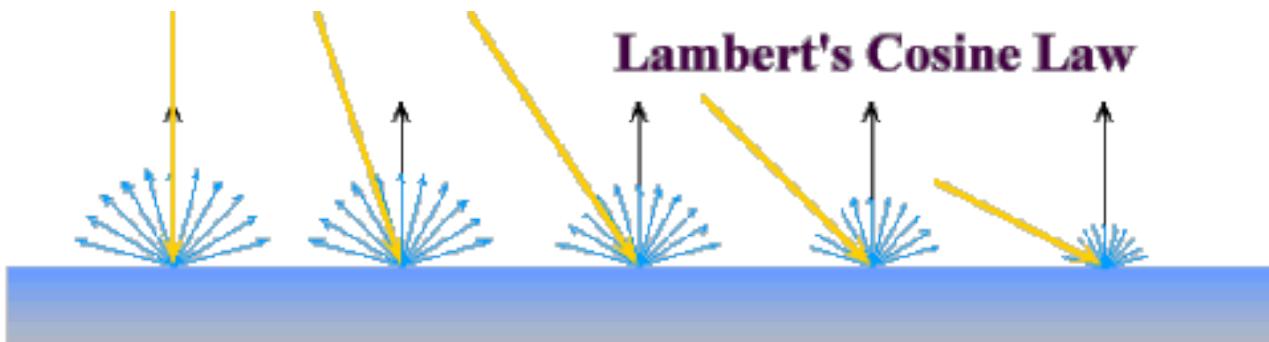
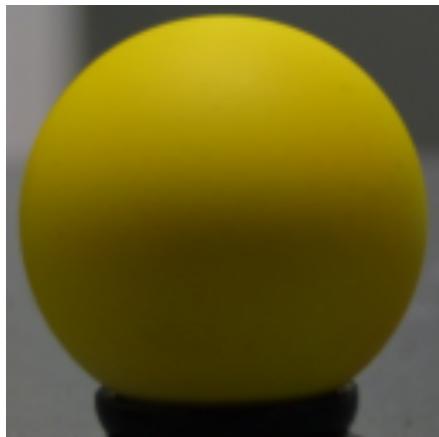
Remembering that incident irradiance depends on cosine, what is the BRDF of an ideally diffuse surface?



# Ideal Diffuse Reflectance

---

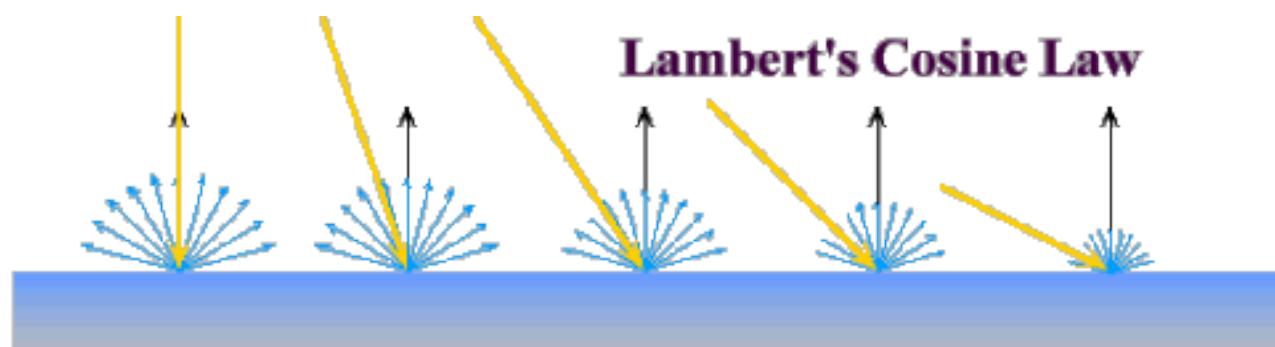
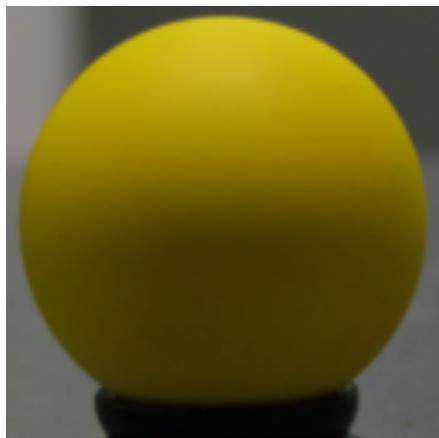
- The ideal diffuse BRDF is a constant  $f_r(\mathbf{l}, \mathbf{v}) = \text{const.}$ 
  - What constant  $\rho/\pi$ , where  $\rho$  is the *albedo*
    - Coefficient between 0 and 1 that says what fraction is reflected
  - Usually just called “diffuse color”  $k_d$
  - You have already (or will be shortly) implemented this by taking dot products with the normal and multiplying by the “color”!



# Ideal Diffuse Reflectance

---

- This is the simplest possible parametric BRDF
  - One parameter:  $k_d$ 
    - (One for each RGB channel)



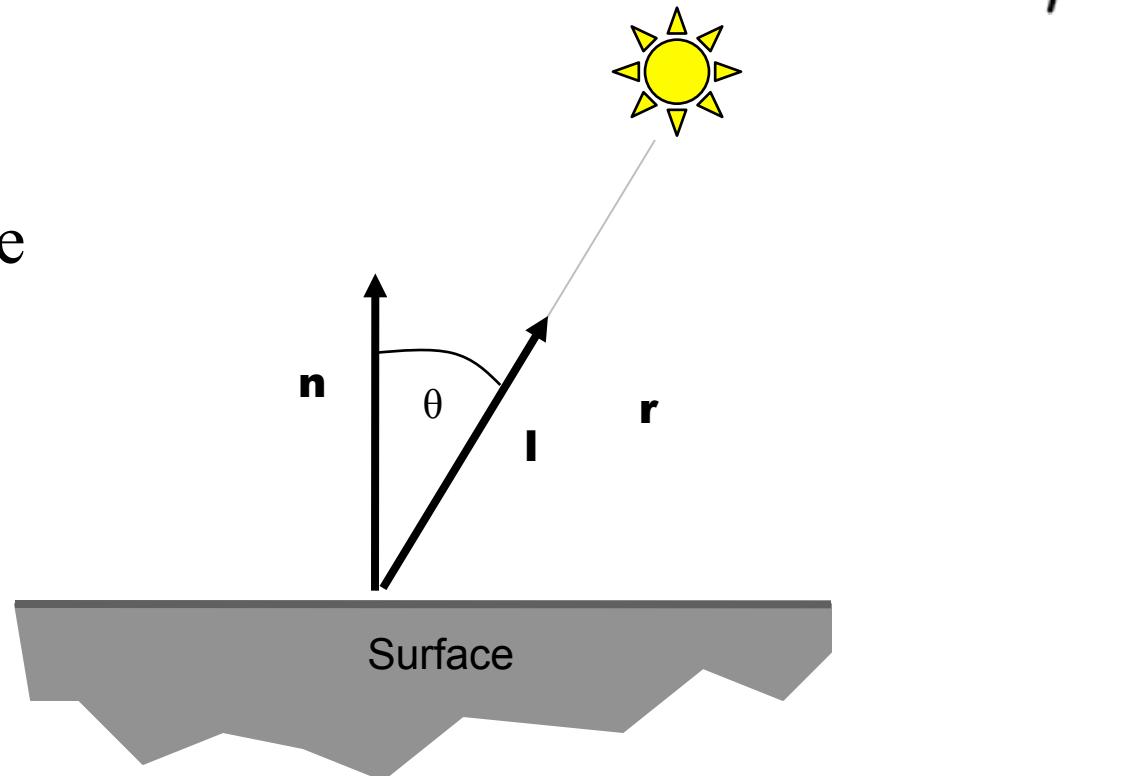
# Ideal Diffuse Reflectance Math

---

- Single Point Light Source

- $k_d$ : diffuse coefficient (color)
- $n$ : Surface normal.
- $l$ : Light direction.
- $L_i$ : Light intensity
- $r$ : Distance to source
- $L_o$ : Shaded color

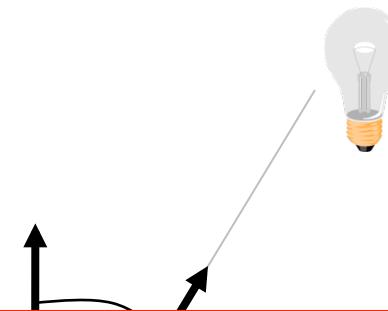
$$L_o = k_d \max(0, \mathbf{n} \cdot \mathbf{l}) \frac{L_i}{r^2}$$



# Ideal Diffuse Reflectance Math

- Single Point Light Source
  - $k_d$ : diffuse coefficient (color)
  - $n$ : Surface normal.
  - $l$ : Light direction.
  - $L_i$ : Light intensity
  - $r$ : Distance to source
  - $L_o$ : Shaded color

$$L_o = k_d \max(0, \mathbf{n} \cdot \mathbf{l}) \frac{L_i}{r^2}$$



Do not forget  
to normalize  
your  $n$  and  $l$ !

We do not want light from below the surface! From now on we always assume (on this lecture) that dot products are clamped to zero and skip writing out the  $\max()$ .

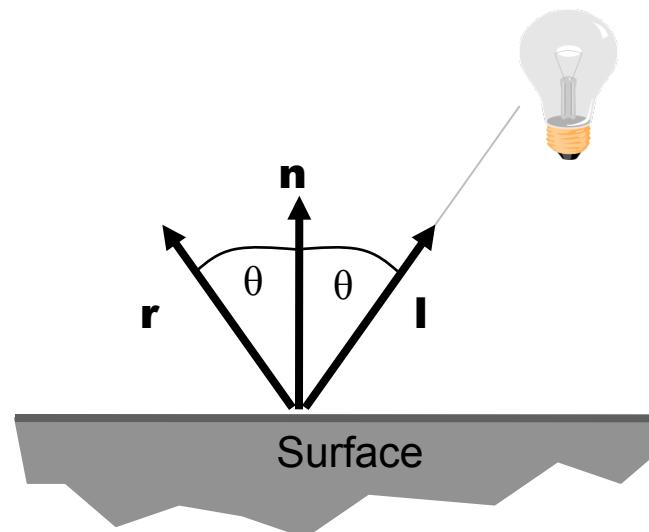
# Questions?

---

# Ideal Specular Reflectance

---

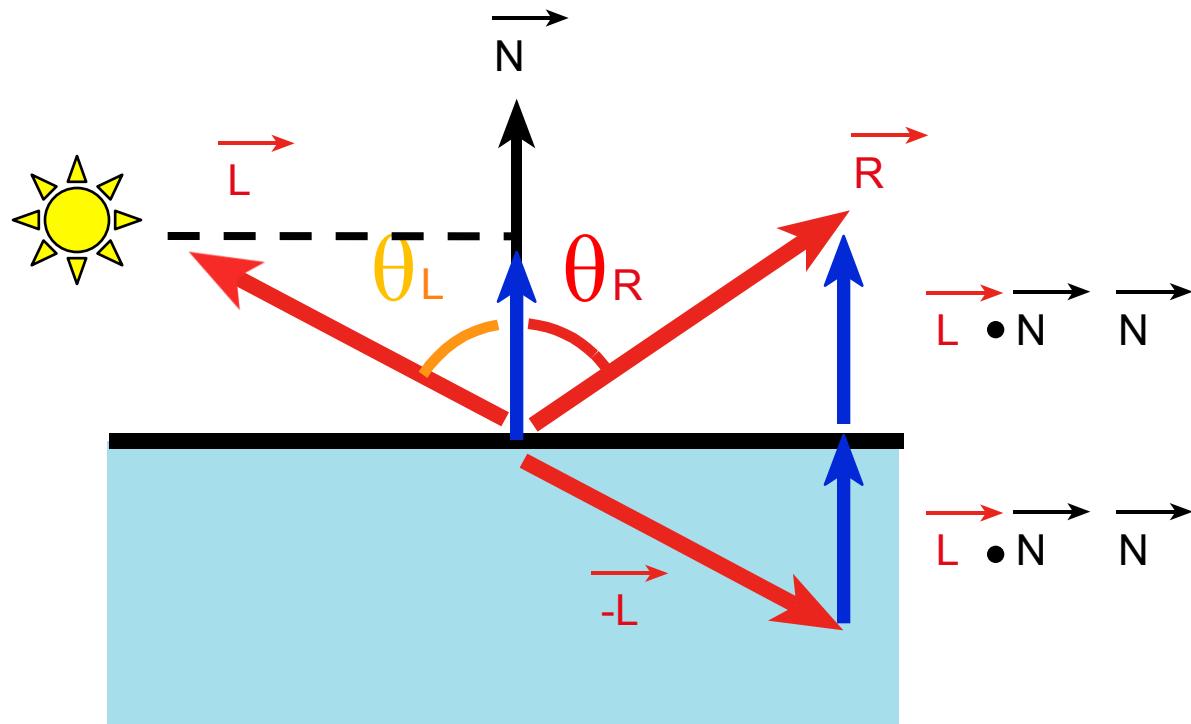
- Reflection is only at mirror angle
- View dependent
  - Microscopic surface elements are usually oriented in the same direction as the surface itself.
  - Examples: mirrors, highly polished metals.



# Recap: How to Get Mirror Direction

---

- Reflection angle = light angle
  - Both R & L have to lie on one plane
- $R = -L + 2(L \cdot N)N$



# Ideal Specular BRDF

---

- Light only reflects to the mirror direction
- A Dirac delta multiplied by a specular coefficient  $k_s$
- Not very useful for point lights, only for reflections of other surfaces
  - Why? You cannot really see a mirror reflection of an infinitely small light!

# Non-ideal Reflectors

---

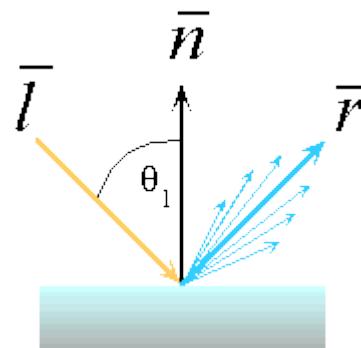
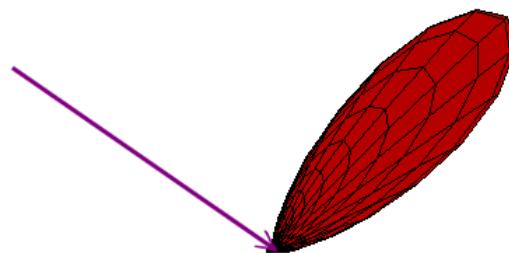
- Real glossy materials usually deviate significantly from ideal mirror reflectors
  - Highlight is blurry
- They are not ideal diffuse surfaces either ...



# Non-ideal Reflectors

---

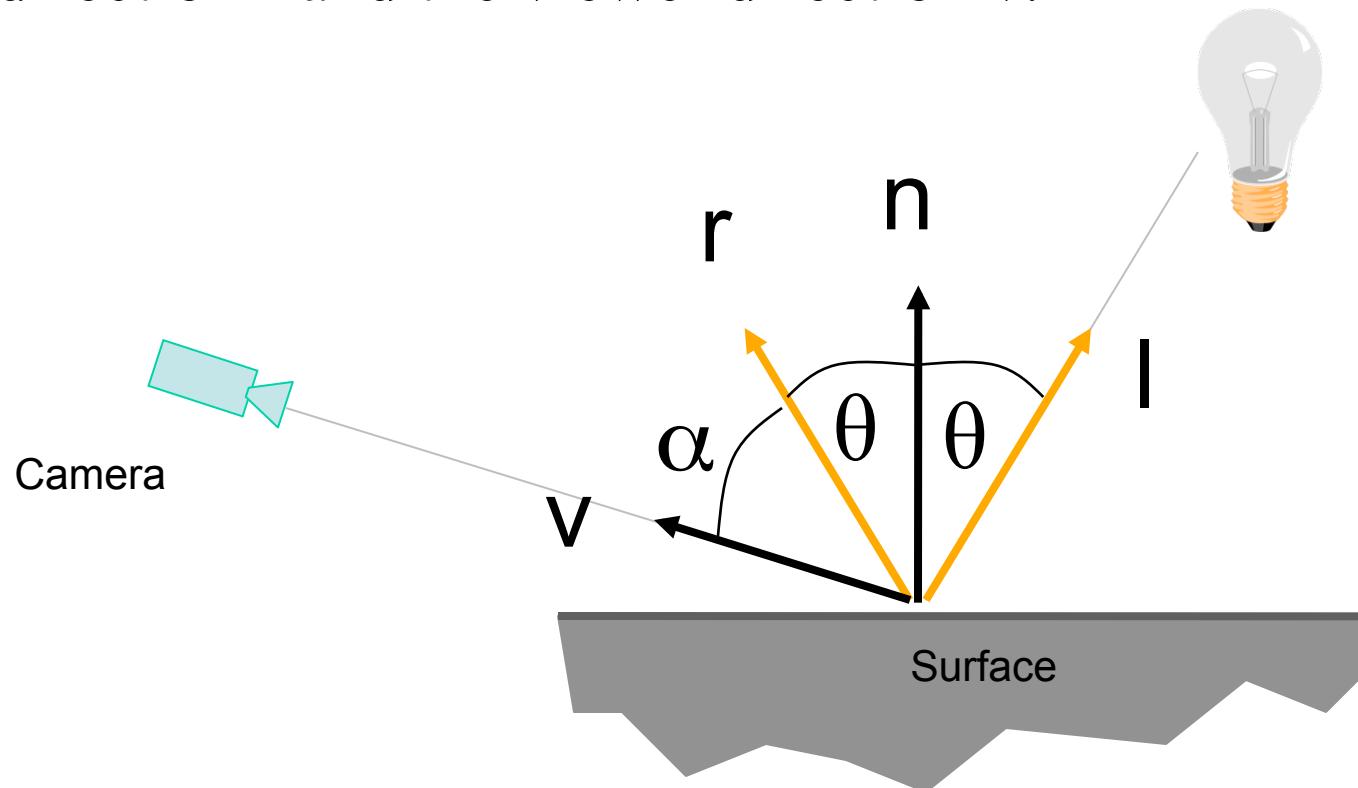
- Simple Empirical Reasoning for Glossy Materials
  - We expect most of the reflected light to travel in the direction of the ideal mirror ray.
  - However, because of microscopic surface variations we might expect some of the light to be reflected just slightly offset from the ideal reflected ray.
  - As we move farther and farther, in the angular sense, from the reflected ray, we expect to see less light reflected.



# The Phong Specular Model

---

- How much light is reflected?
  - Depends on the angle  $\alpha$  between the ideal reflection direction  $r$  and the viewer direction  $v$ .



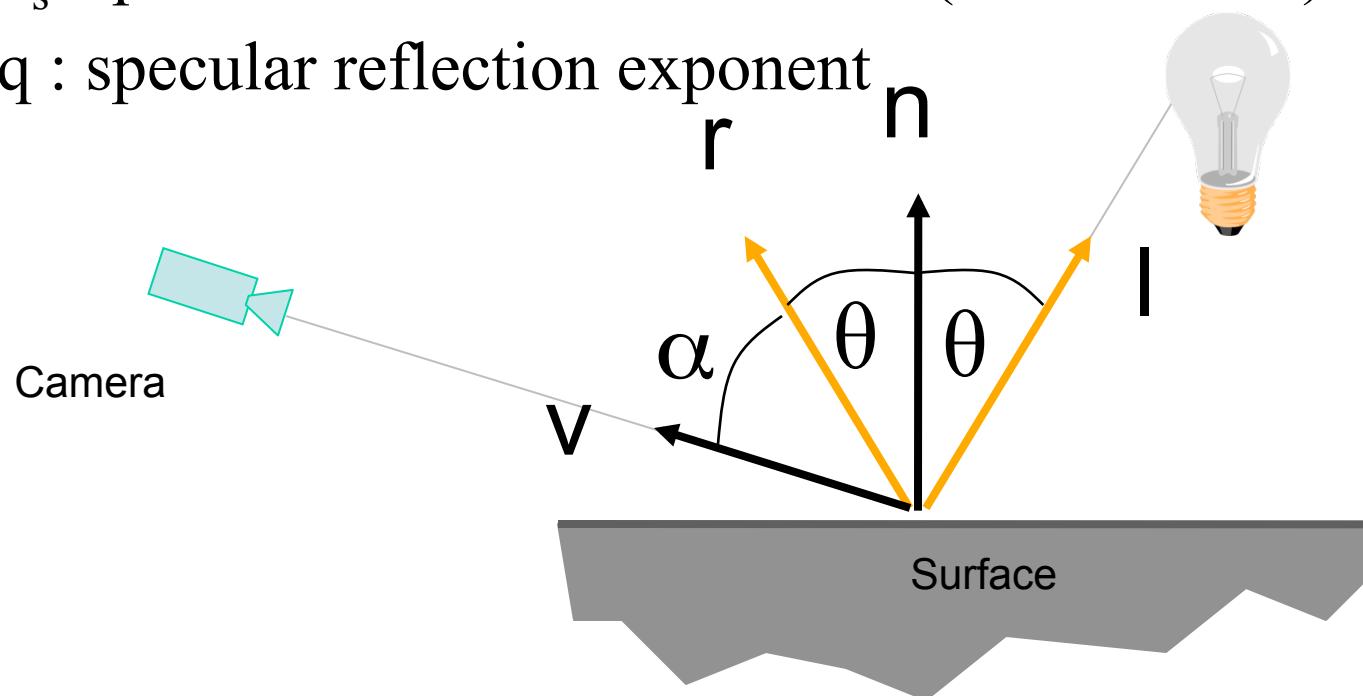
# The Phong Specular Model

---

$$L_o = k_s (\cos \alpha)^q \frac{L_i}{r^2} = k_s (\mathbf{v} \cdot \mathbf{r})^q \frac{L_i}{r^2}$$

- Parameters

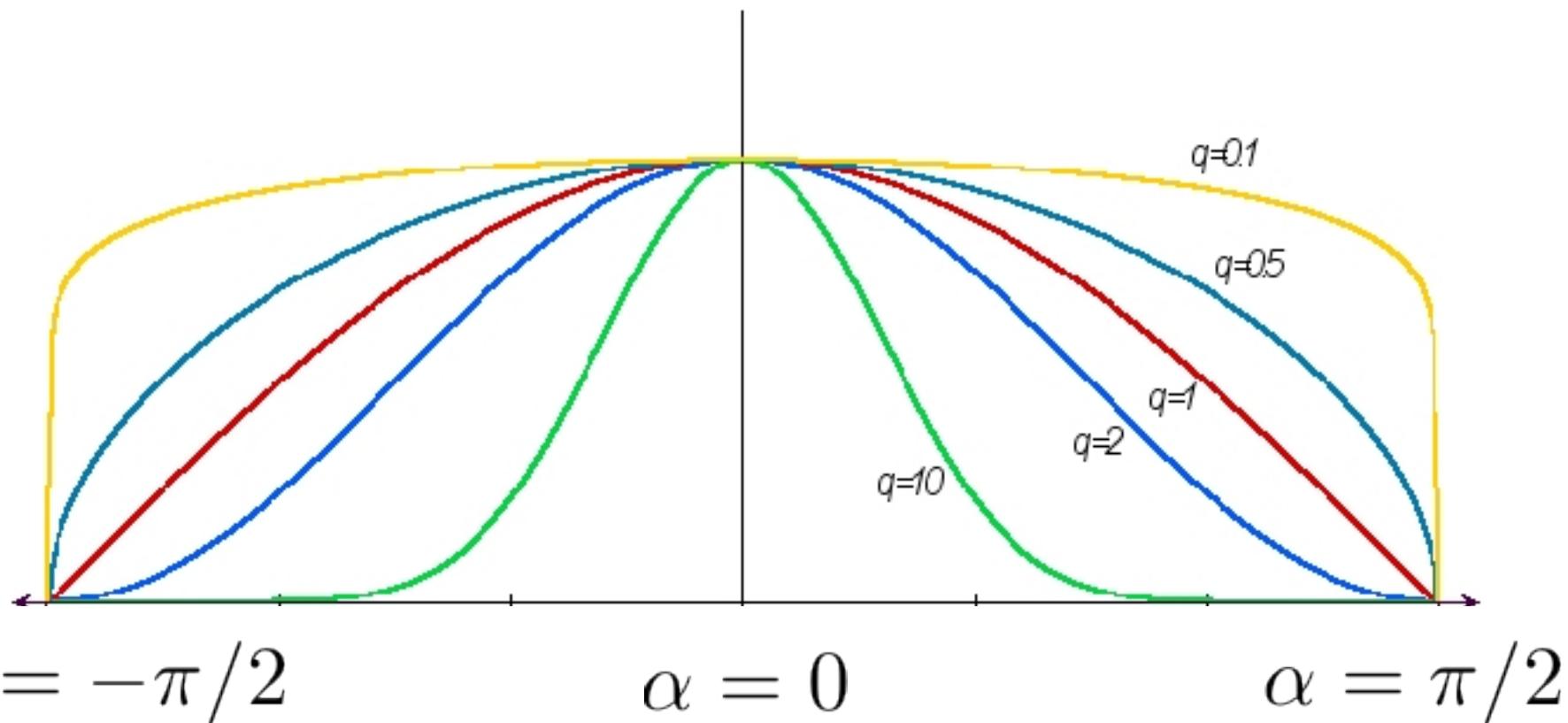
- $k_s$ : specular reflection coefficient (overall scale)
- $q$  : specular reflection exponent



# The Phong Model

---

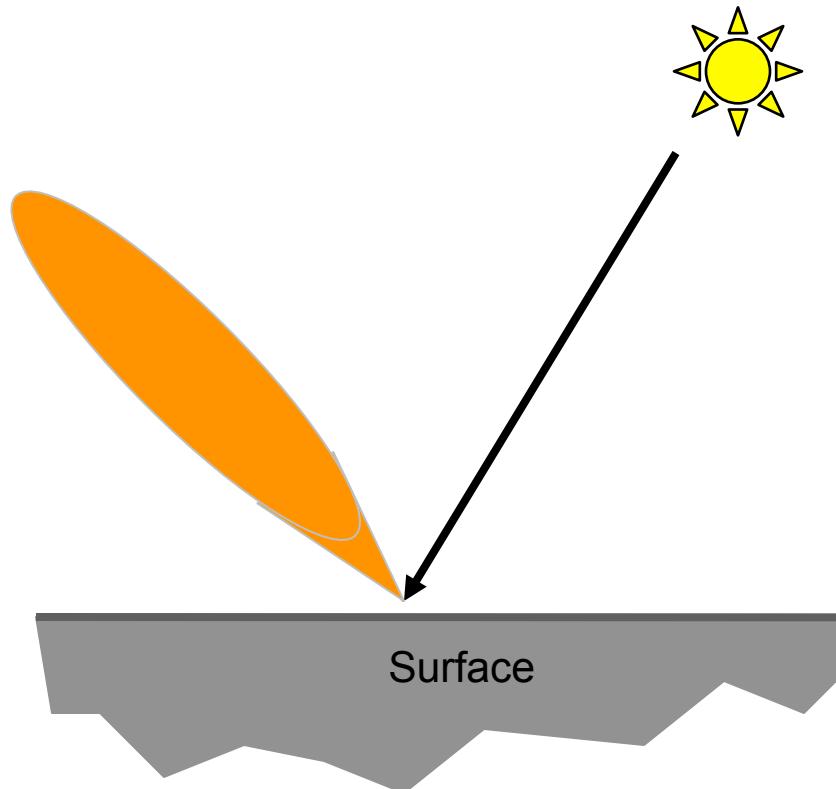
- Effect of  $q$  – the specular reflection exponent



# Terminology: Specular Lobe

---

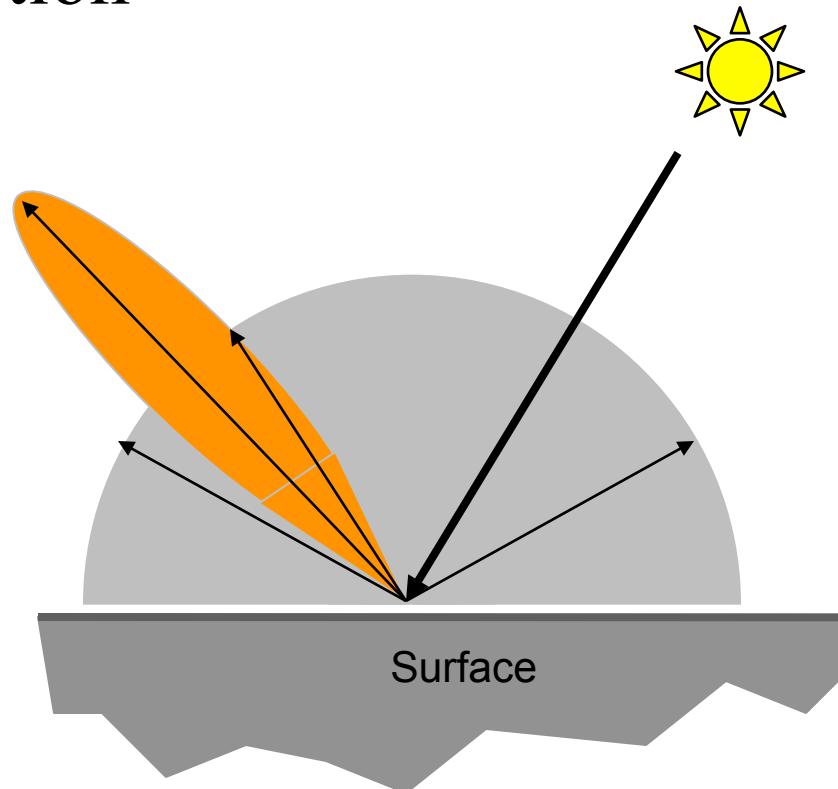
- The specular reflection distribution is usually called a “lobe”
  - For Phong, its shape is  $(r \cdot v)^q$



# The Complete Phong Model

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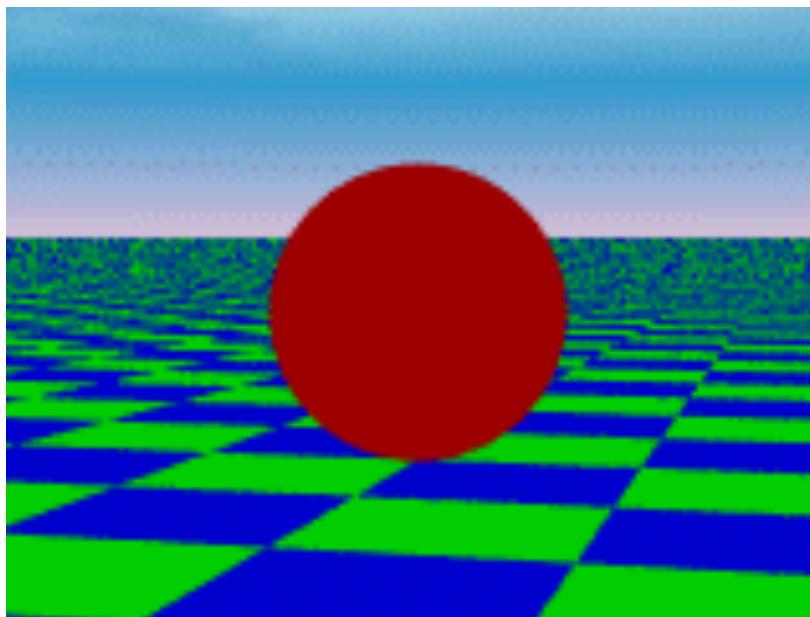
- Sum of three components:  
ideal diffuse reflection +  
specular reflection +  
“ambient”.



# Ambient Illumination

---

- Represents the reflection of all indirect illumination.
- This is a total hack!
- Avoids the complexity of indirect (“global”) illumination

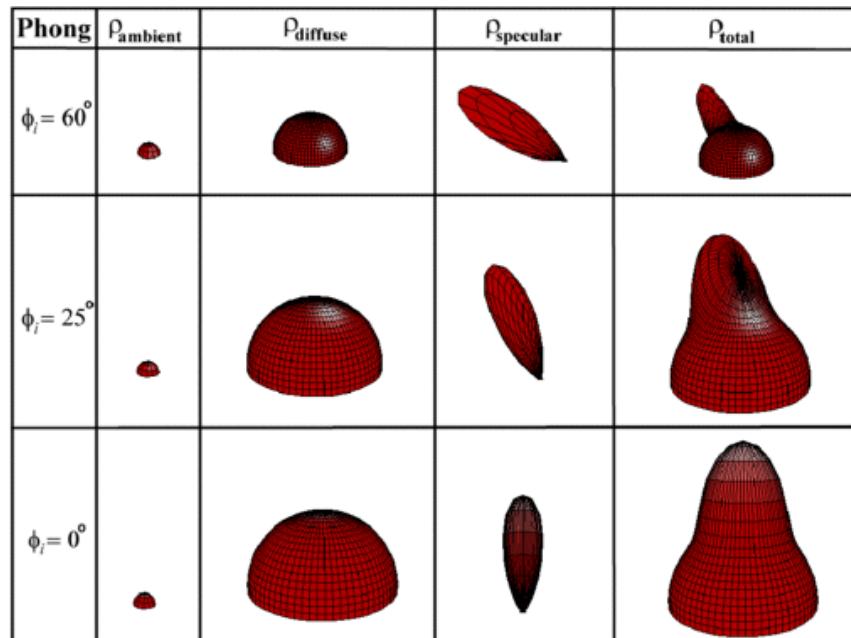


# Putting It All Together

---

- Phong Illumination Model

$$L_o = \left[ k_a + k_d (\mathbf{n} \cdot \mathbf{l}) + k_s (\mathbf{v} \cdot \mathbf{r})^q \right] \frac{L_i}{r^2}$$



# Putting It All Together

---

- Phong Illumination Model

$$L_o = \left[ k_a + k_d (\mathbf{n} \cdot \mathbf{l}) + k_s (\mathbf{v} \cdot \mathbf{r})^q \right] \frac{L_i}{r^2}$$

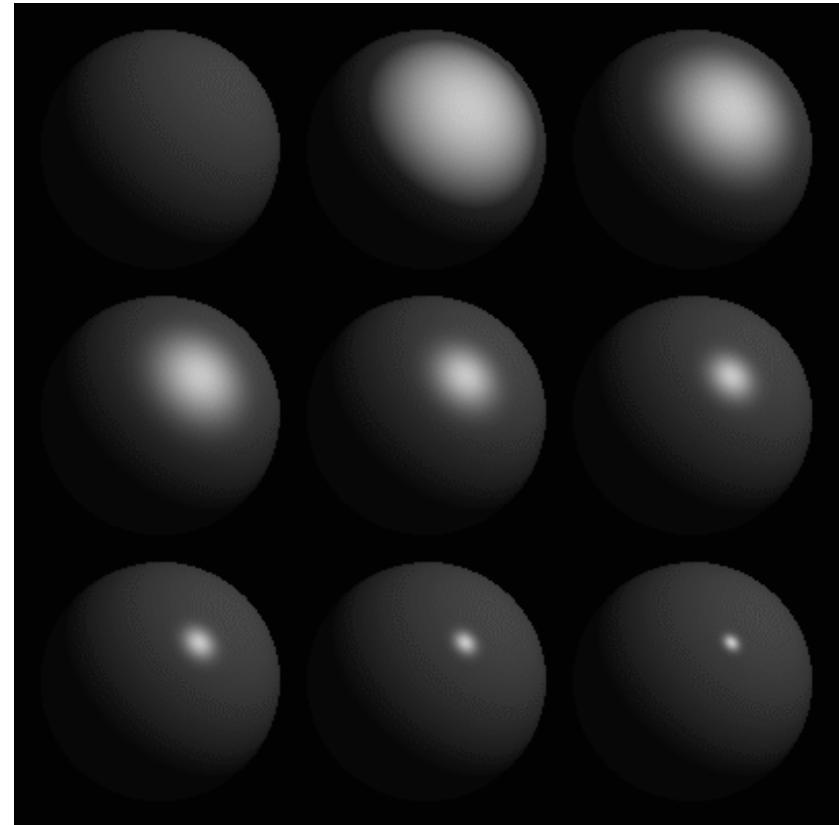
- Is it physically based?

- No, does not even conserve energy,  
may well reflect more energy than what goes in
  - Furthermore, it **does not even conform** to the BRDF model  
directly (we are taking the proper cosine for diffuse, but  
not for specular)
  - And ambient was a total hack

# Phong Examples

---

- The spheres illustrate specular reflections as the direction of the light source and the exponent  $q$  (amount of shininess) is varied.

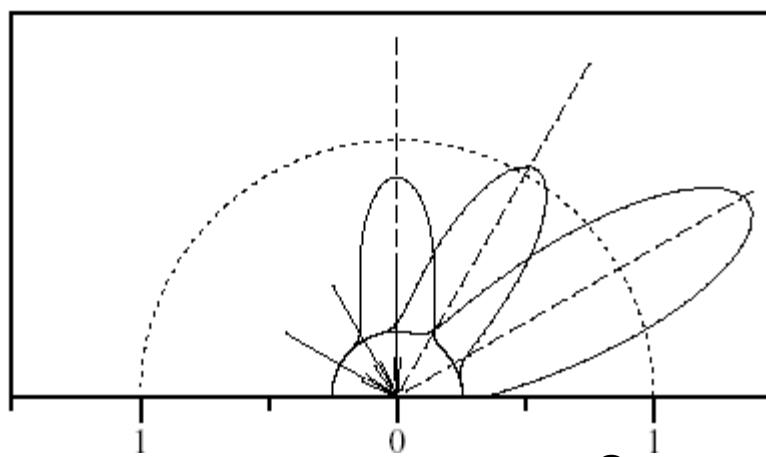


$$L_o = \left[ k_a + k_d (\mathbf{n} \cdot \mathbf{l}) + k_s (\mathbf{v} \cdot \mathbf{r})^q \right] \frac{L_i}{r^2}$$

# Fresnel Reflection

---

- Increasing specularity near grazing angles.
  - Most BRDF models account for this.



Source: Lafourche et al. 97

# Questions?

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# Blinn-Torrance Variation of Phong

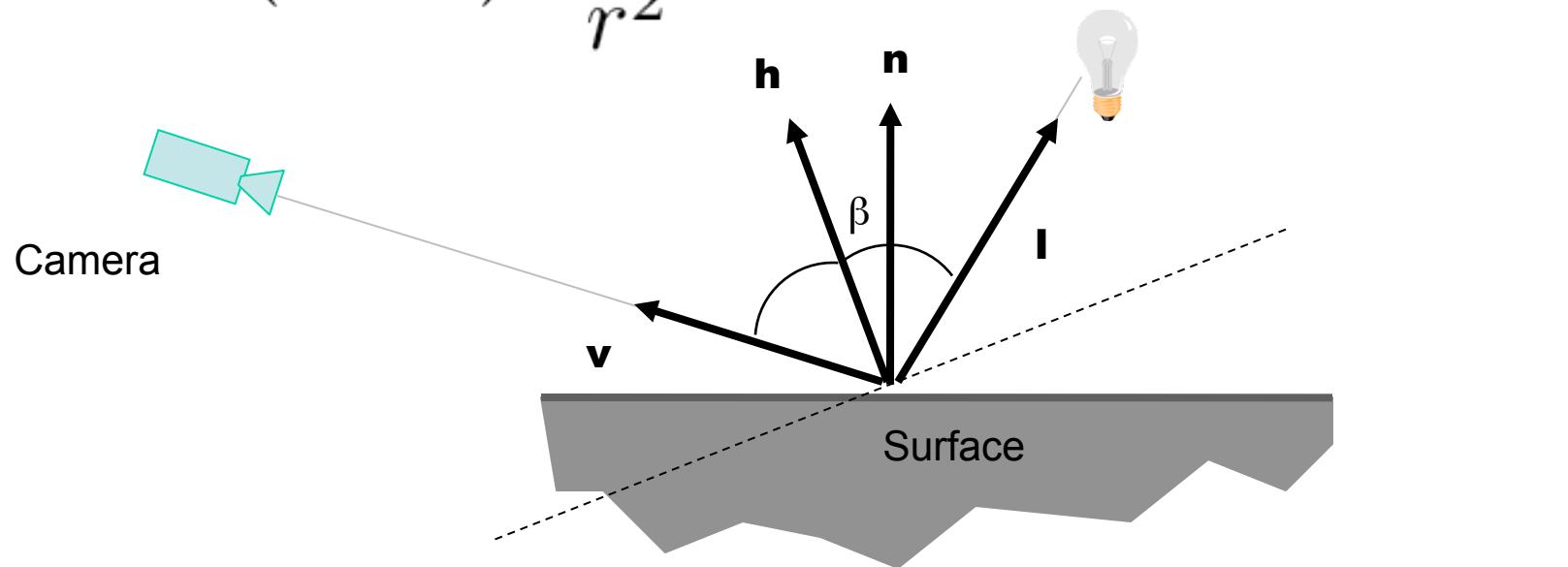
---

- Uses the “halfway vector”  $\mathbf{h}$  between  $\mathbf{l}$  and  $\mathbf{v}$ .

$$L_o = k_s \cos(\beta)^q \frac{L_i}{r^2}$$

$$= k_s (\mathbf{n} \cdot \mathbf{h})^q \frac{L_i}{r^2}$$

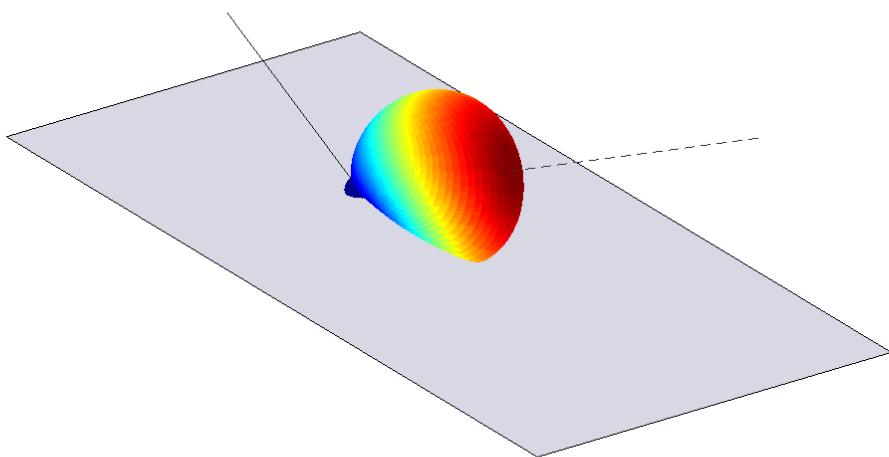
$$\mathbf{h} = \frac{\mathbf{l} + \mathbf{v}}{\|\mathbf{l} + \mathbf{v}\|}$$



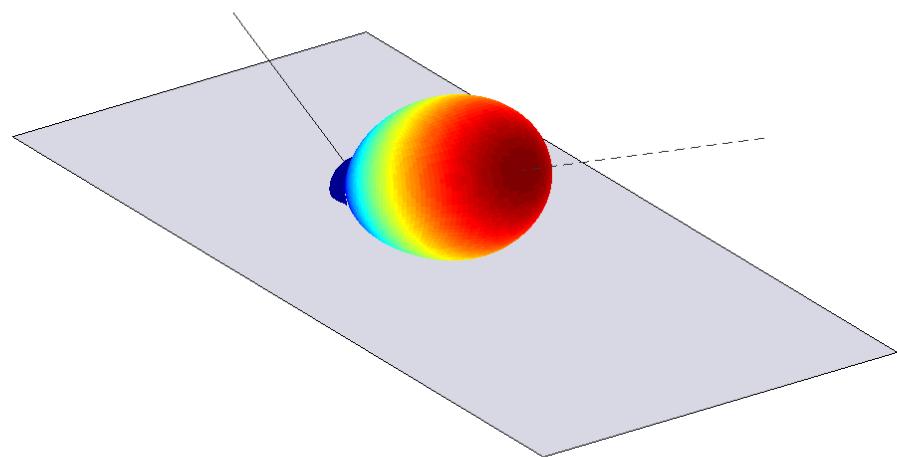
# Lobe Comparison

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- Half vector lobe
  - Gradually narrower when approaching grazing
- Mirror lobe
  - Always circular



Half vector lobe

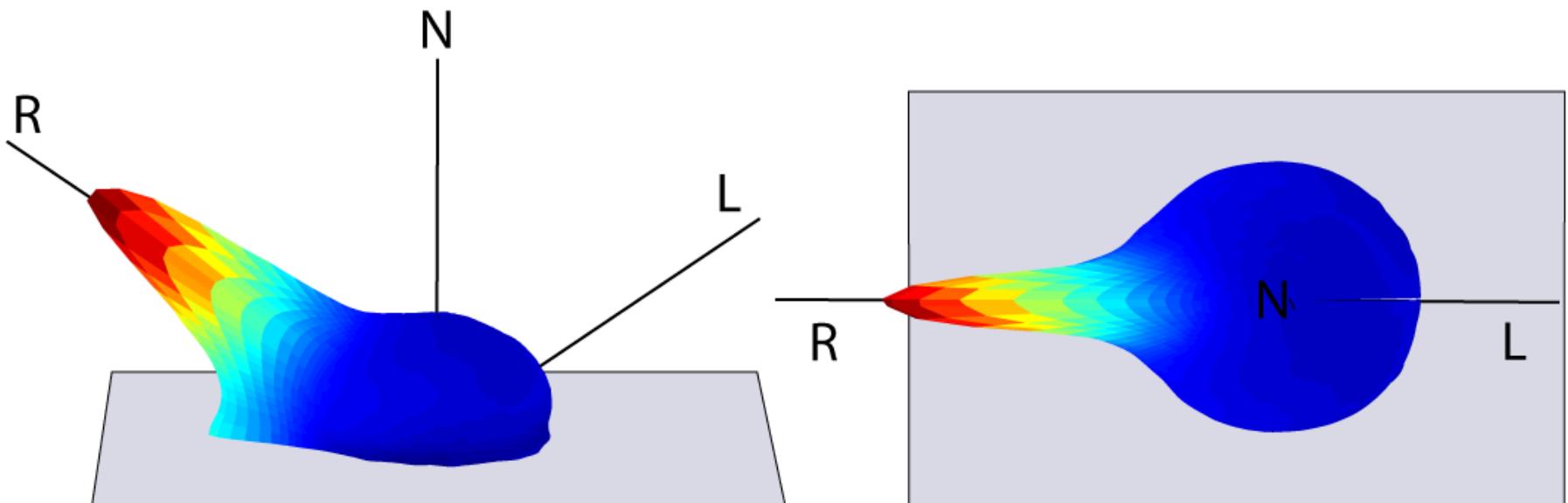


Mirror lobe

# Half Vector Lobe is Better

---

- More consistent with what is observed in measurements (  
Ngan, Matusik, Durand 2005)



Example: Plot of "PVC" BRDF at 55° incidence

# Questions?

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# Microfacet Theory

---

- Example
  - Think of water surface as lots of tiny mirrors (microfacets)
  - “Bright” pixels are
    - Microfacets aligned with the vector between sun and eye
    - But not the ones in shadow
    - And not the ones that are occluded



# Microfacet Theory

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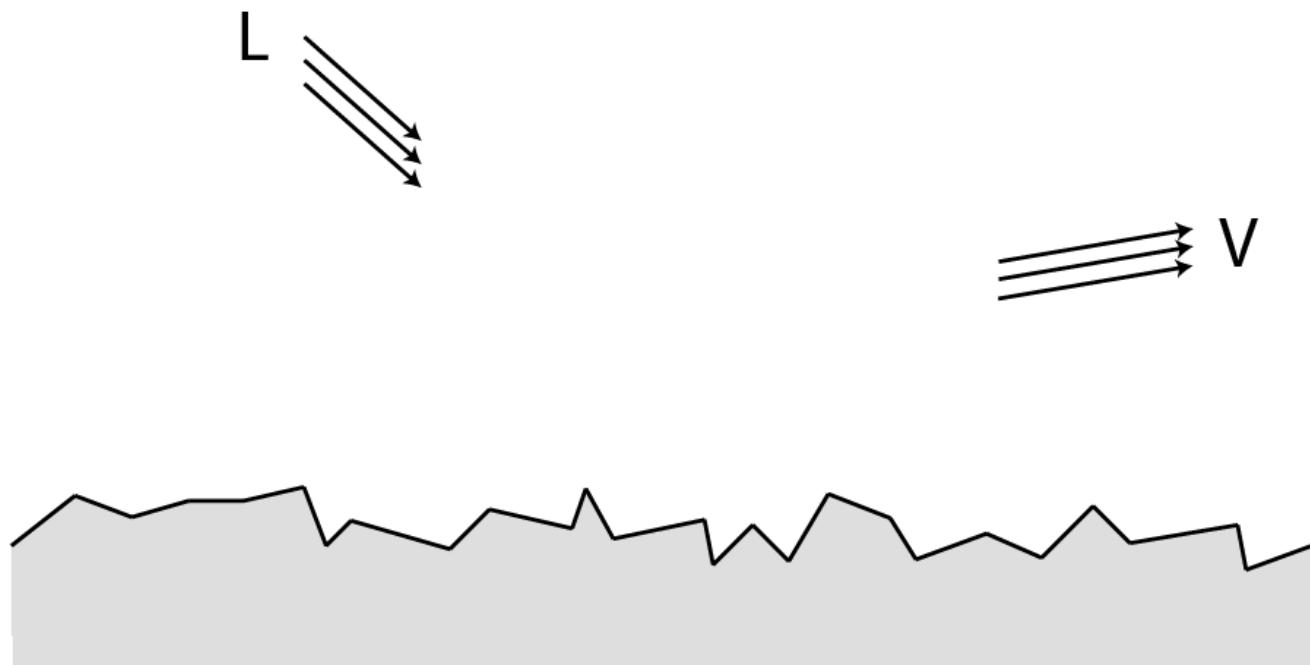
- Model surface by tiny mirrors  
[Torrance & Sparrow 1967]



# Microfacet Theory

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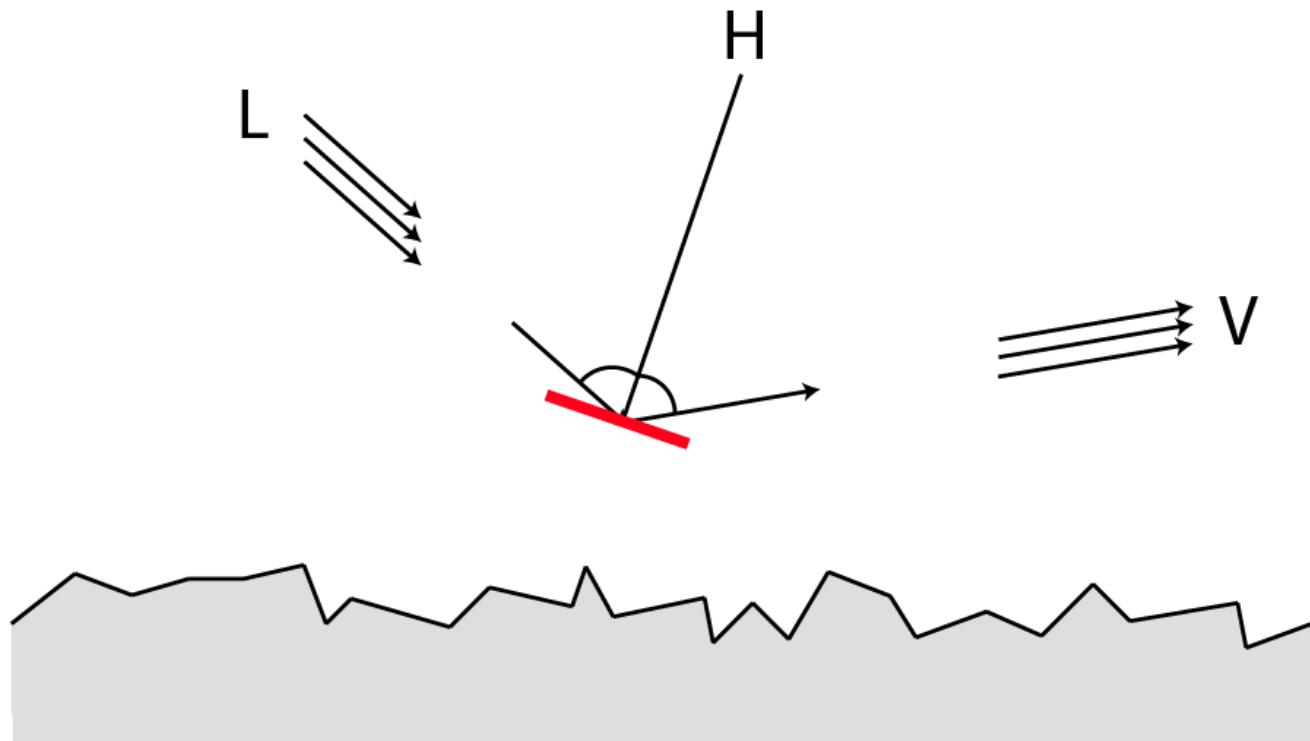
- Value of BRDF at  $(L, V)$  is a product of
  - number of mirrors oriented halfway between  $L$  and  $V$



# Microfacet Theory

---

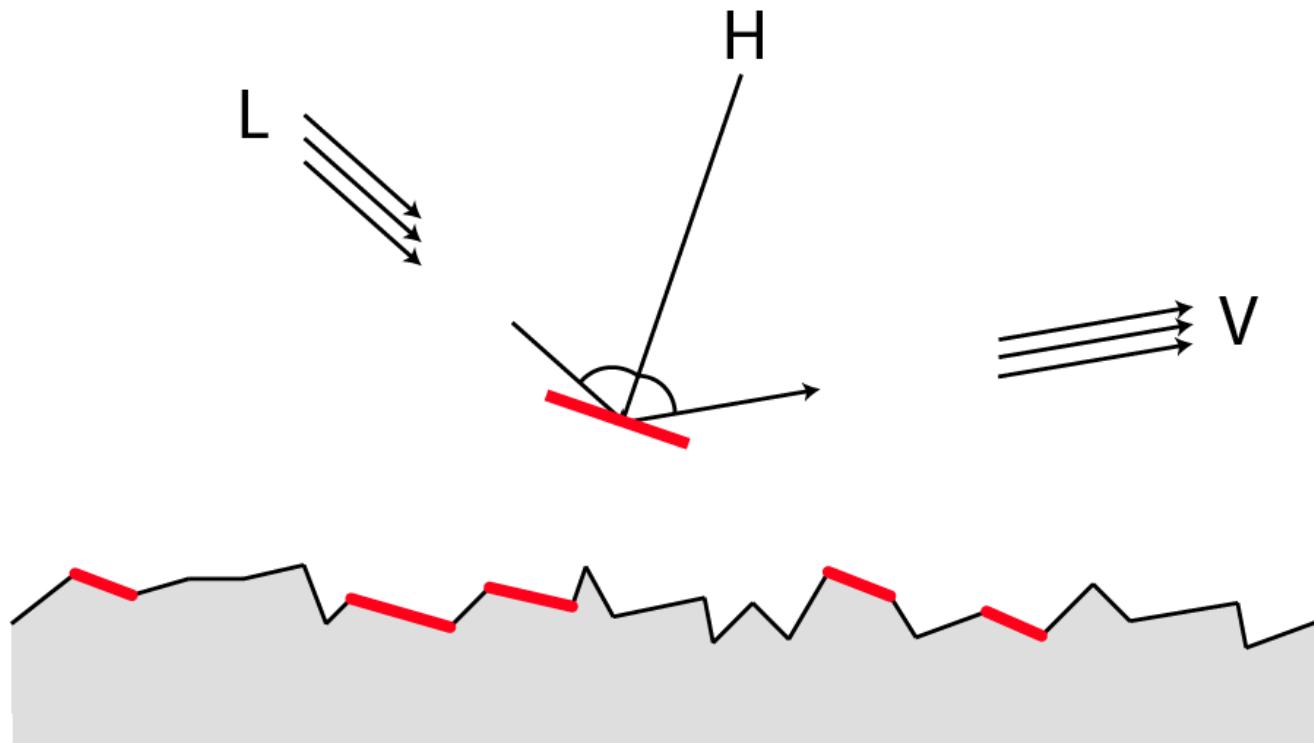
- Value of BRDF at  $(L, V)$  is a product of
  - number of mirrors oriented halfway between  $L$  and  $V$



# Microfacet Theory

---

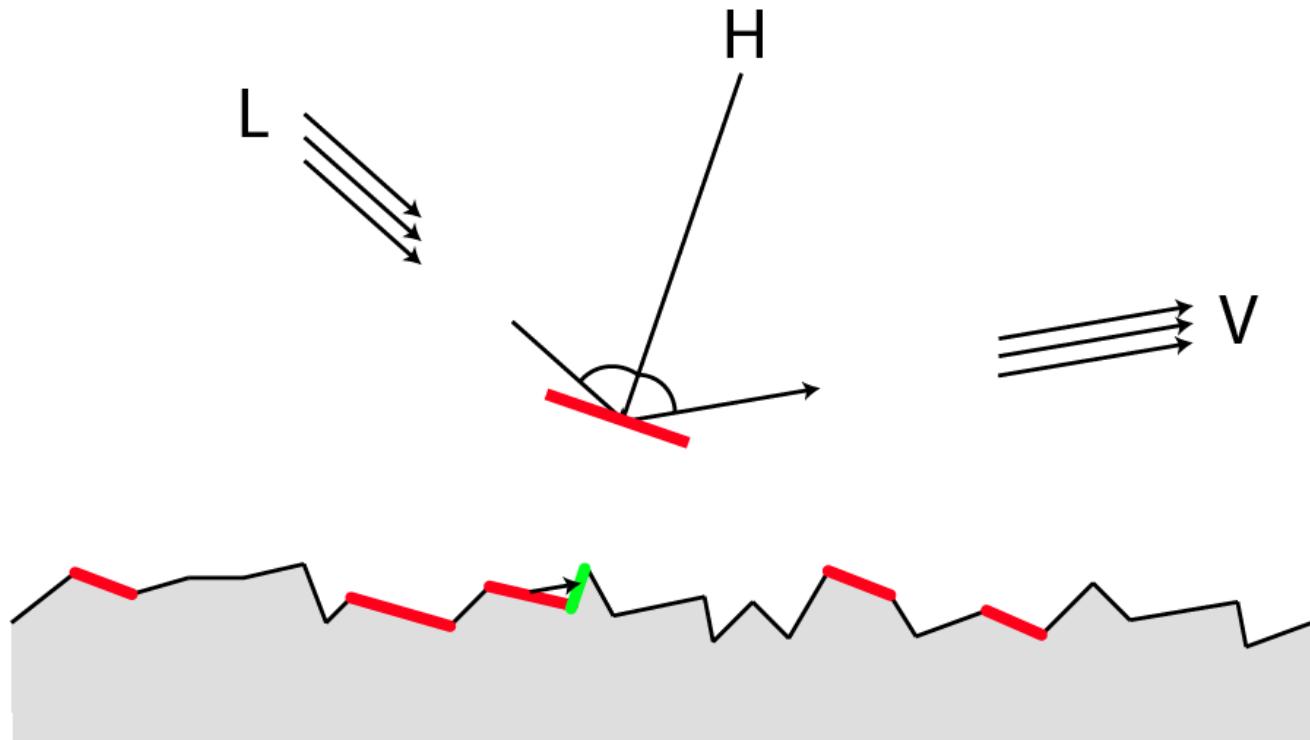
- Value of BRDF at  $(L, V)$  is a product of
  - number of mirrors oriented halfway between  $L$  and  $V$



# Microfacet Theory

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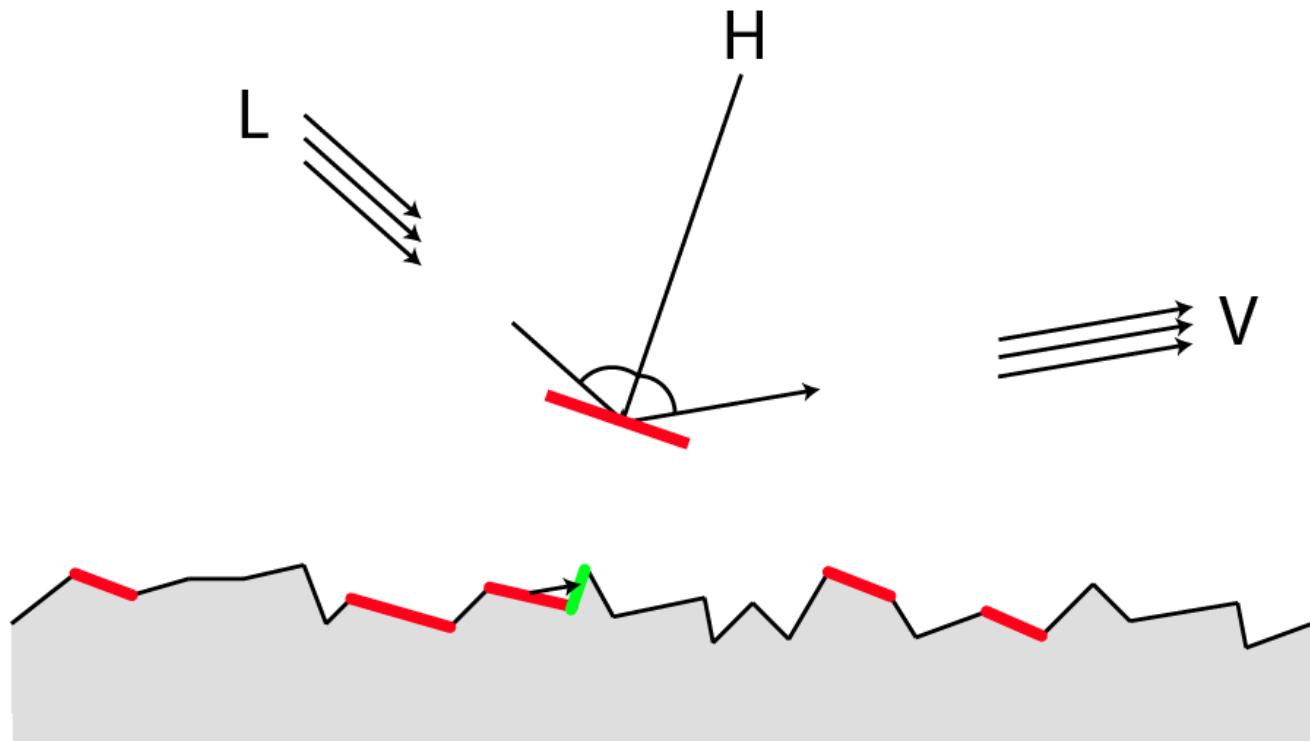
- Value of BRDF at  $(L, V)$  is a product of
  - number of mirrors oriented halfway between  $L$  and  $V$
  - ratio of the un(shadowed/masked) mirrors



# Microfacet Theory

---

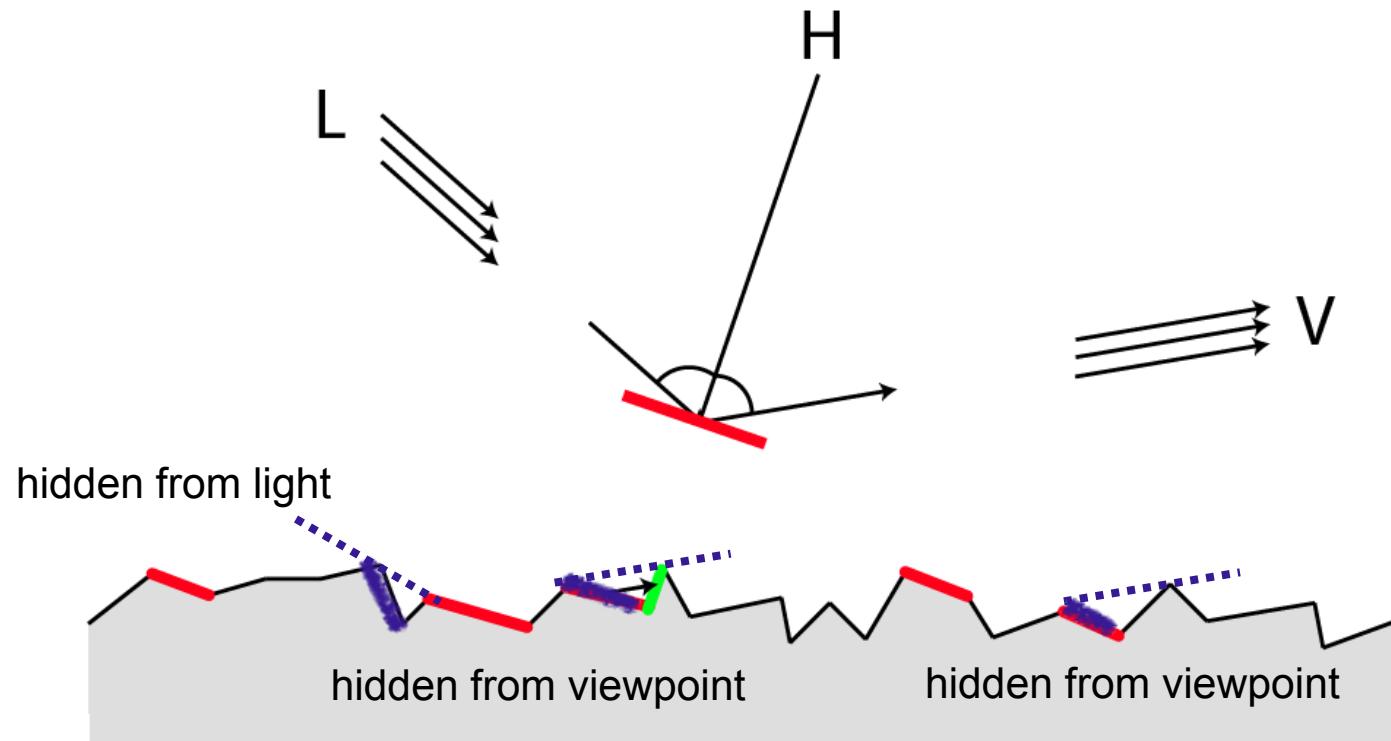
- Value of BRDF at  $(L, V)$  is a product of
  - number of mirrors oriented halfway between  $L$  and  $V$
  - ratio of the un(shadowed/masked) mirrors
  - Fresnel coefficient (depends on index of refraction)



# Shadowing and Masking

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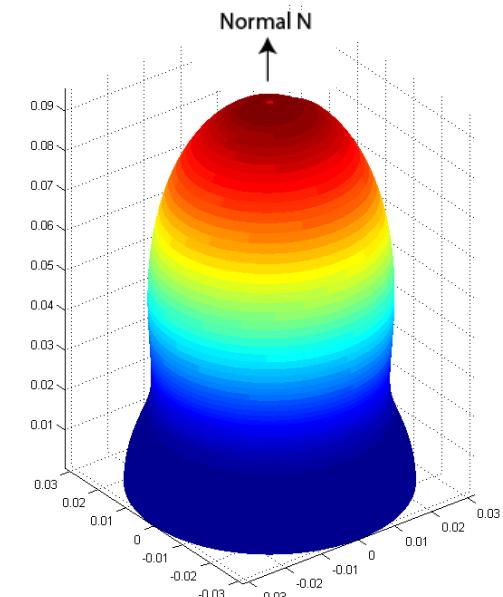
- Some facets are hidden from viewpoint
- Some are hidden from the light



# Microfacet Theory-based Models

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- Develop BRDF models by imposing simplifications  
[\[Torrance-Sparrow 67\]](#), [\[Blinn 77\]](#), [[Cook-Torrance 81](#)], [\[Ashikhmin et al. 2000\]](#)
- Model the distribution  $p(H)$  of microfacet normals
  - Also, statistical models for shadows and masking



spherical plot of a Gaussian-like  $p(H)$

# Full Cook-Torrance Lobe

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- $\rho_s$  is the specular coefficient (3 numbers RGB)
- $D$  is the microfacet distribution
  - $\delta$  is the angle between the half vector  $H$  and the normal  $N$
  - $m$  defines the roughness (width of lobe)
- $G$  is the shadowing and masking term
- Need to add a diffuse term

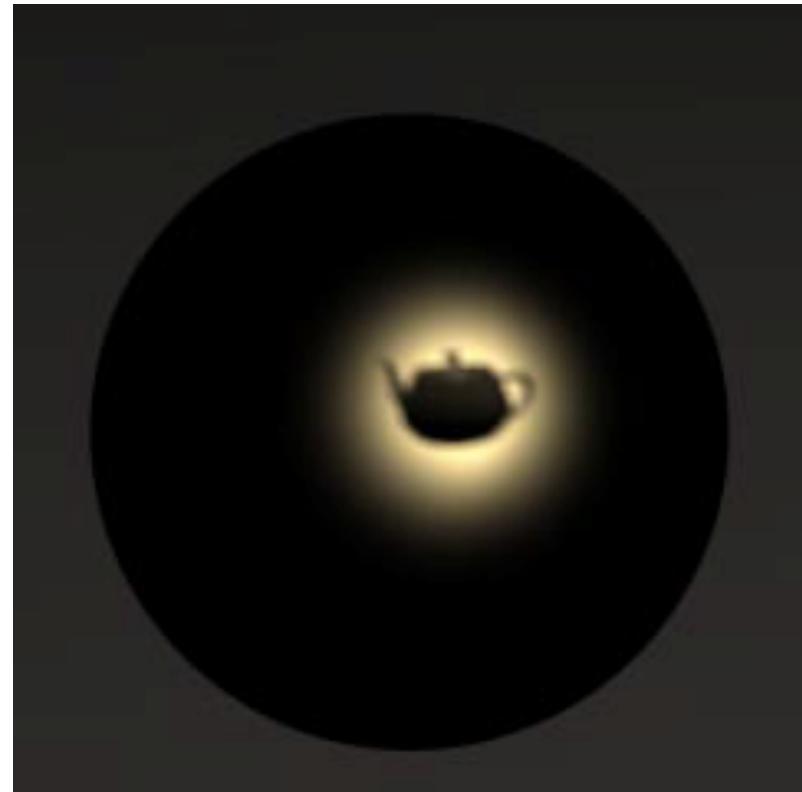
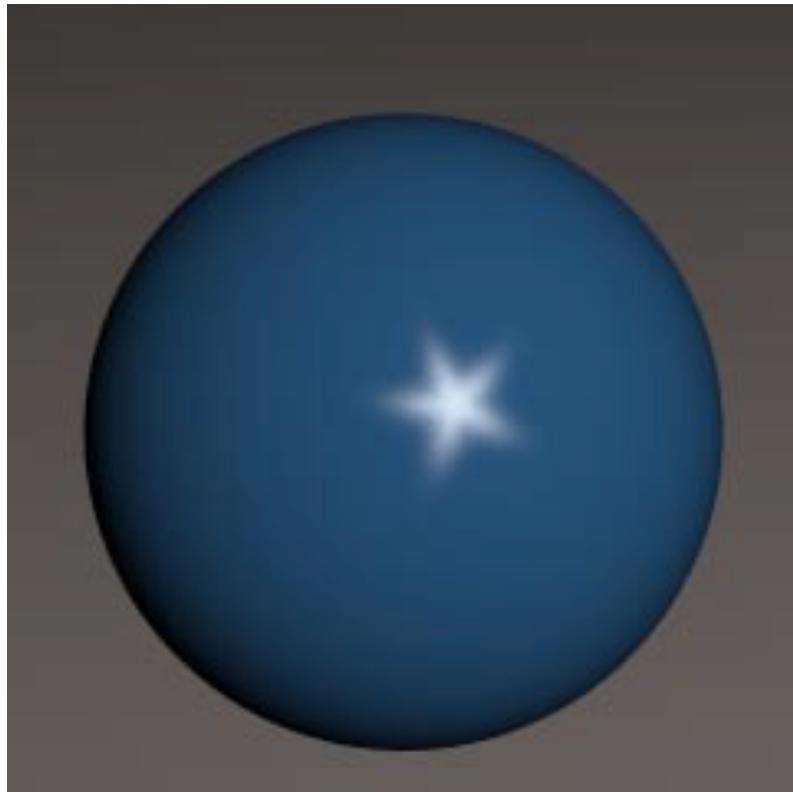
$$K = \frac{\rho_s}{\pi} \frac{DG}{(N \cdot L)(N \cdot V)} Fresnel(F_0, V \cdot H)$$

where  $G = \min\left\{1, \frac{2(N \cdot H)(N \cdot V)}{(V \cdot H)}, \frac{2(N \cdot H)(N \cdot L)}{(V \cdot H)}\right\}$  and  $D = \frac{1}{m^2 \cos^4 \delta} e^{-[(\tan \delta)/m]^2}$

# Questions?

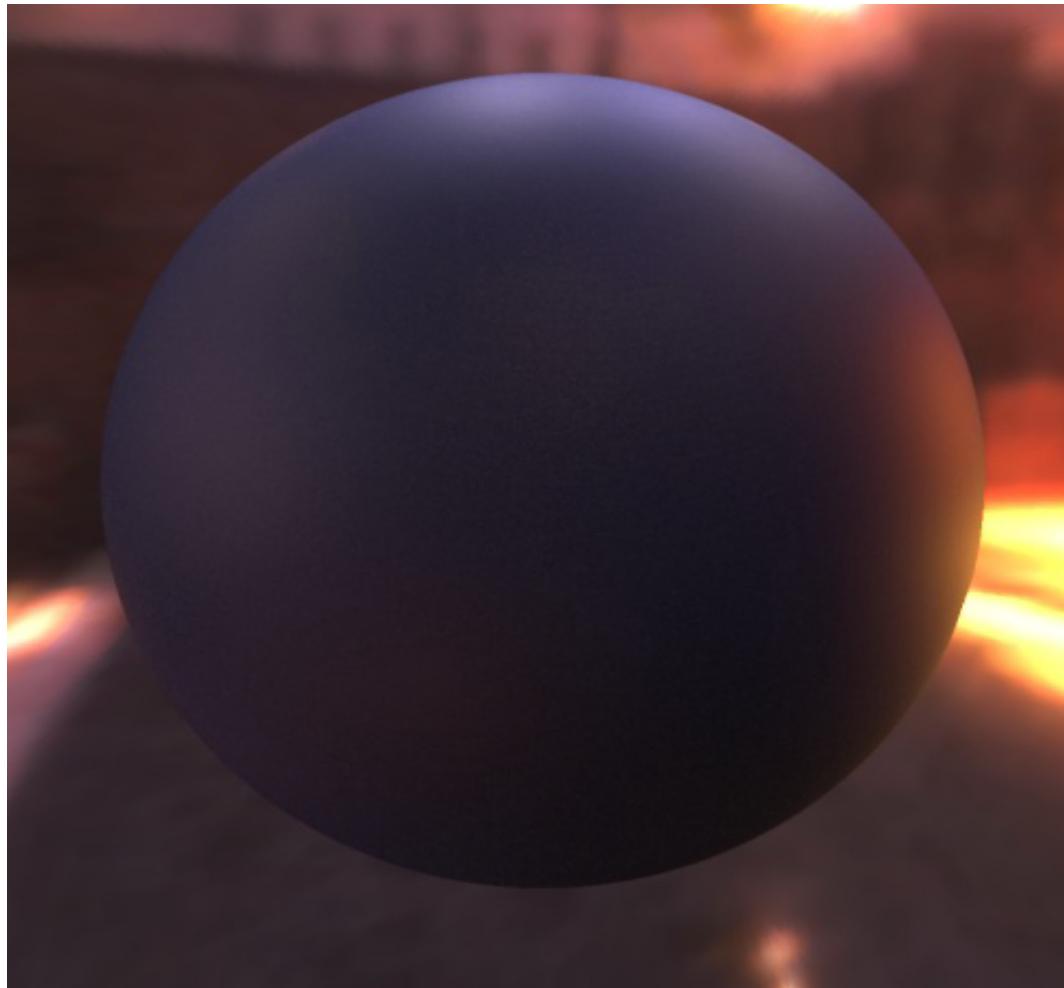
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- “Designer BRDFs” by [Ashikhmin et al.](#)



# BRDF Examples from Ngan et al.

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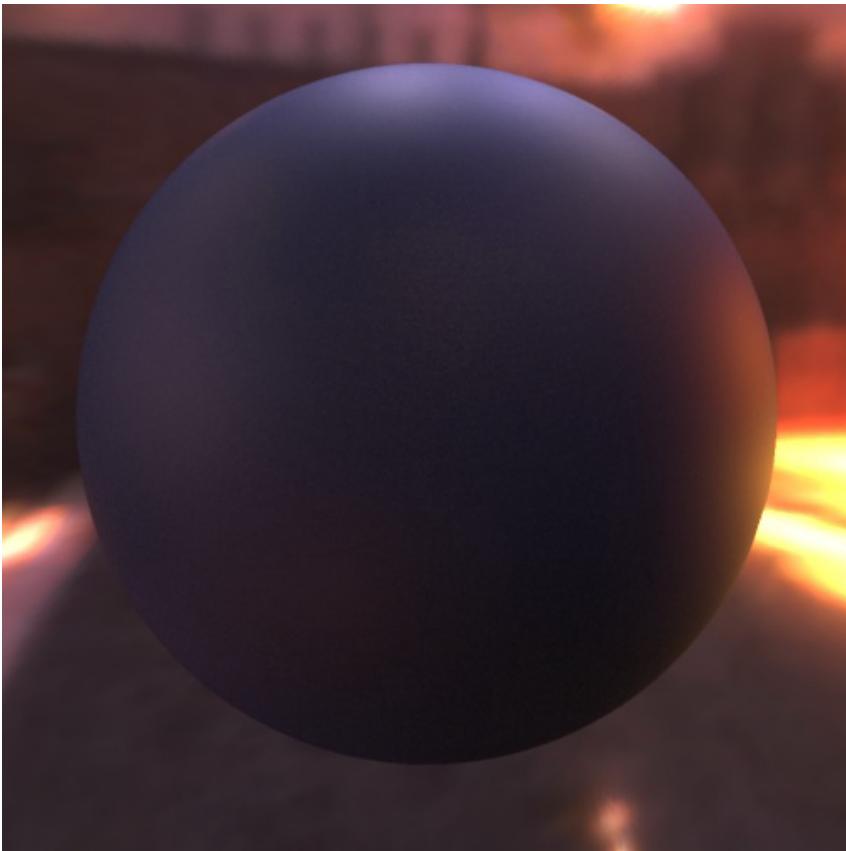
Material – Dark blue paint



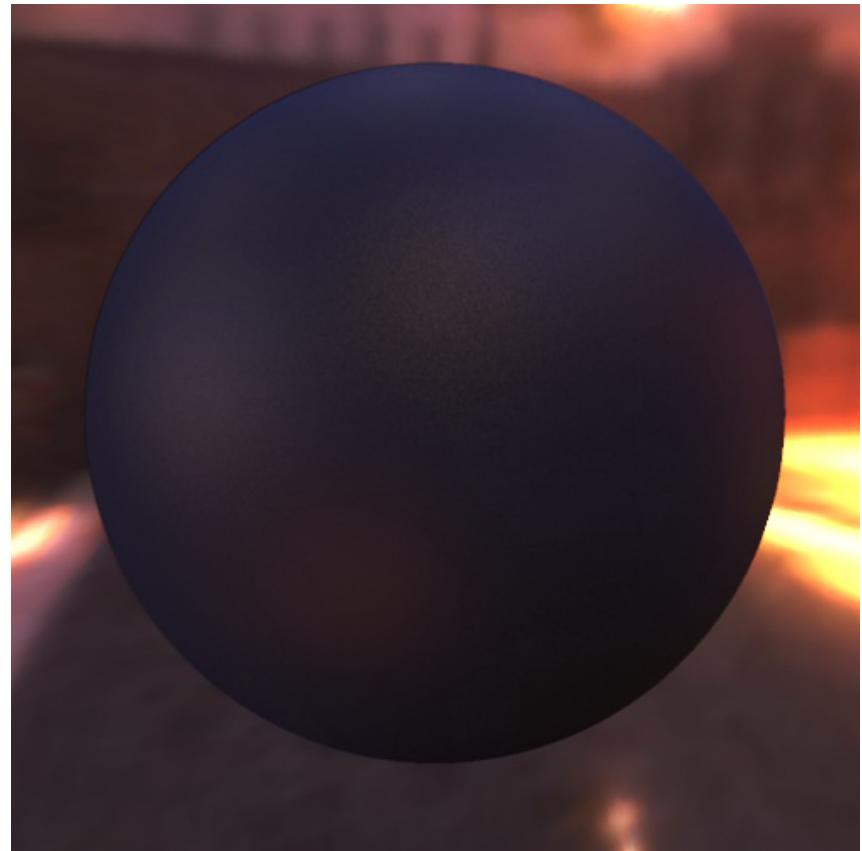
# Dark Blue Paint

---

Acquired data



Blinn-Phong

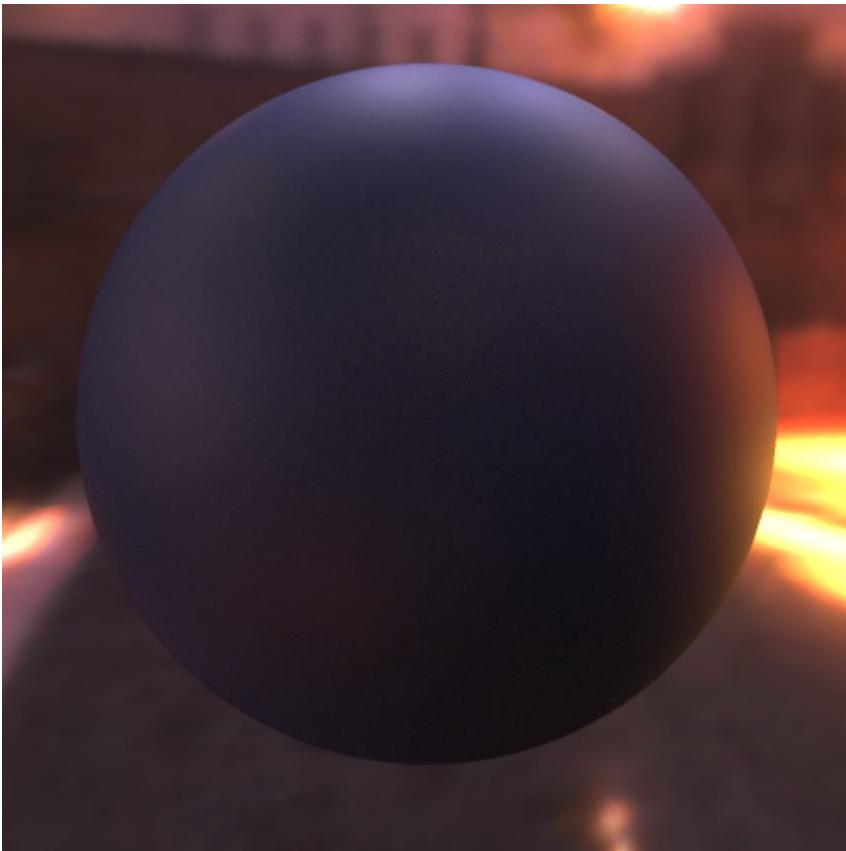


Finding the BRDF model parameters that best reproduce the real material  
Material – Dark blue paint

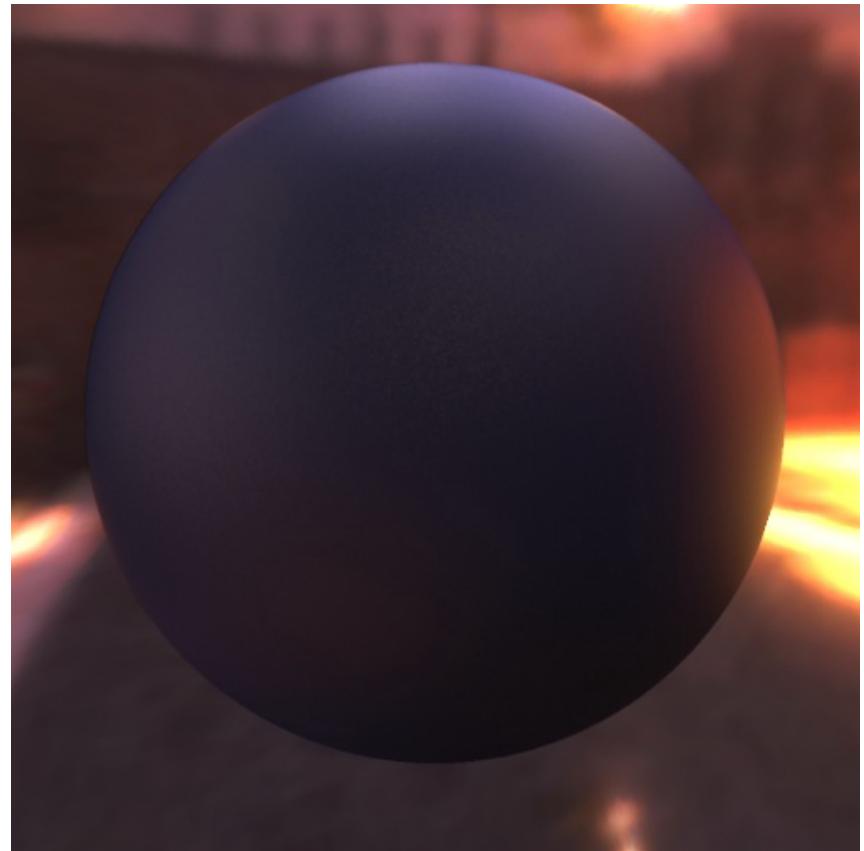
# Dark Blue Paint

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Acquired data



Cook-Torrance



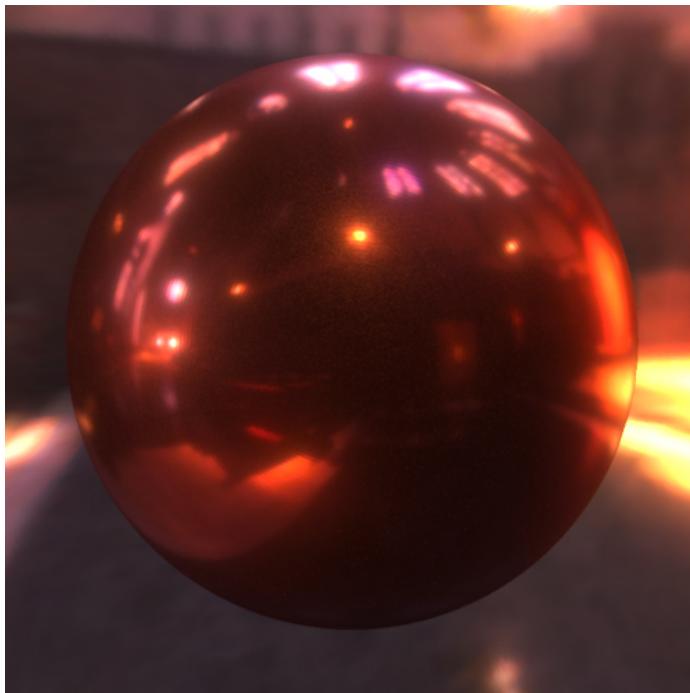
Finding the BRDF model parameters that best reproduce the real material  
Material – Dark blue paint

# Observations

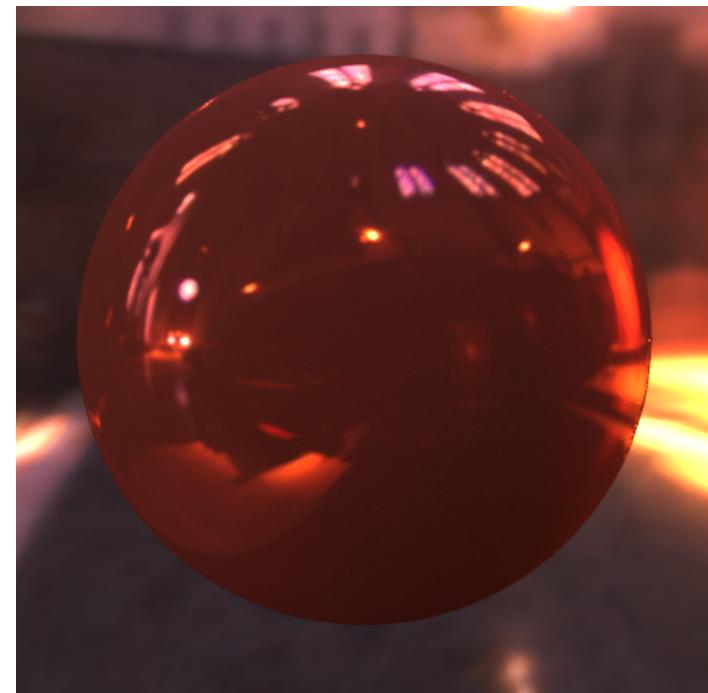
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- Some materials impossible to represent with a single lobe

Acquired data



Cook-Torrance



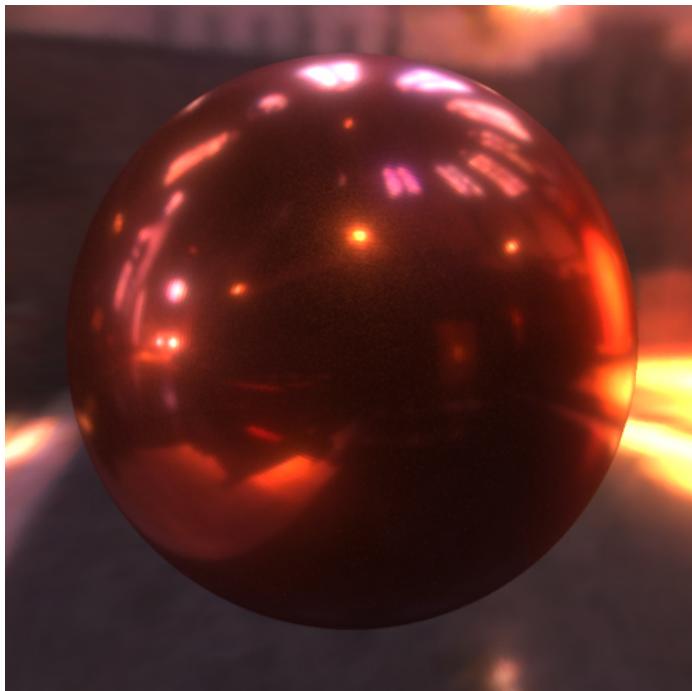
Material – Red Christmas Ball

# Adding a Second Lobe

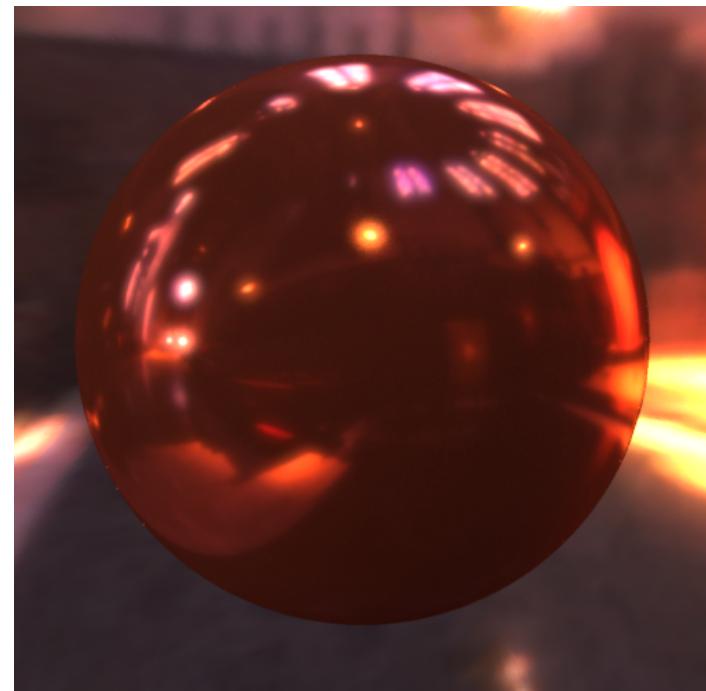
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- Some materials impossible to represent with a single lobe

Acquired data



Cook-Torrance 2 lobes

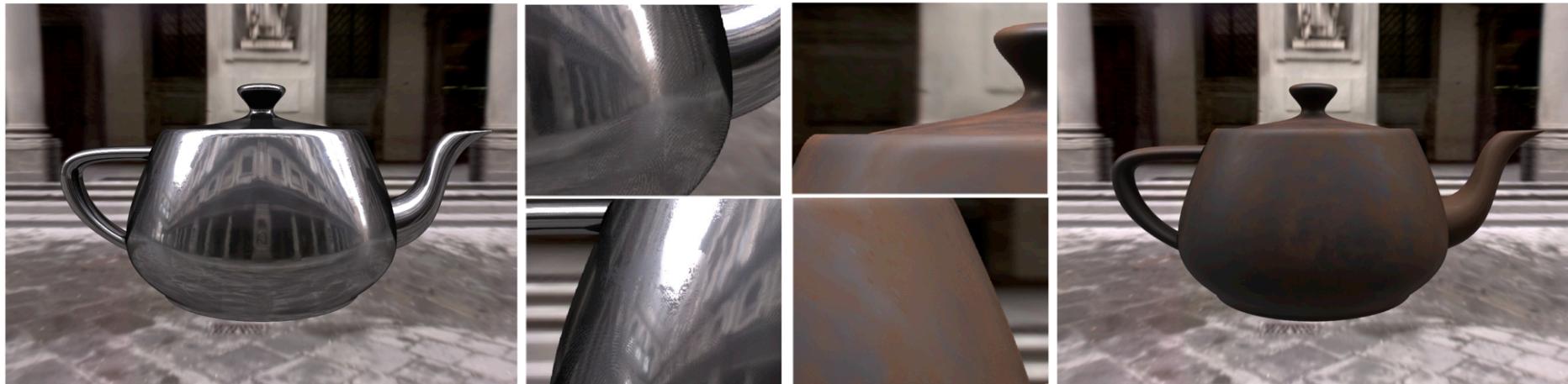


Material – Red Christmas Ball

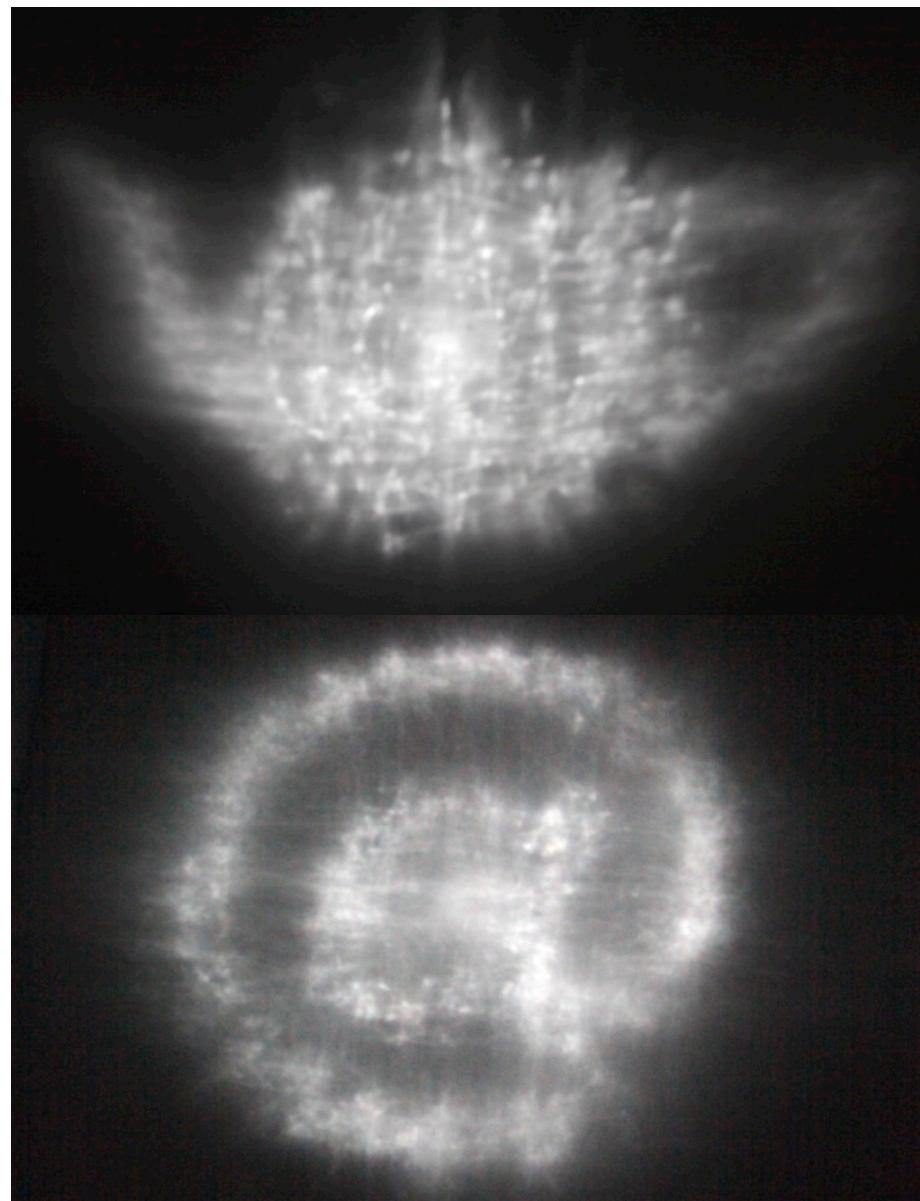
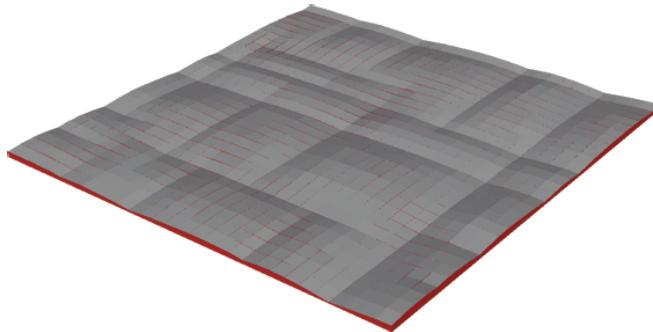
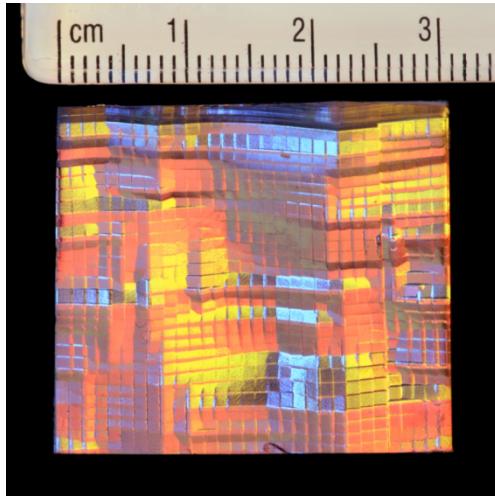
# Image-Based Acquisition

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- A Data-Driven Reflectance Model, SIGGRAPH 2003
  - The data is available [http://people.csail.mit.edu/wojciech/  
BRDFDatabase/](http://people.csail.mit.edu/wojciech/BRDFDatabase/)



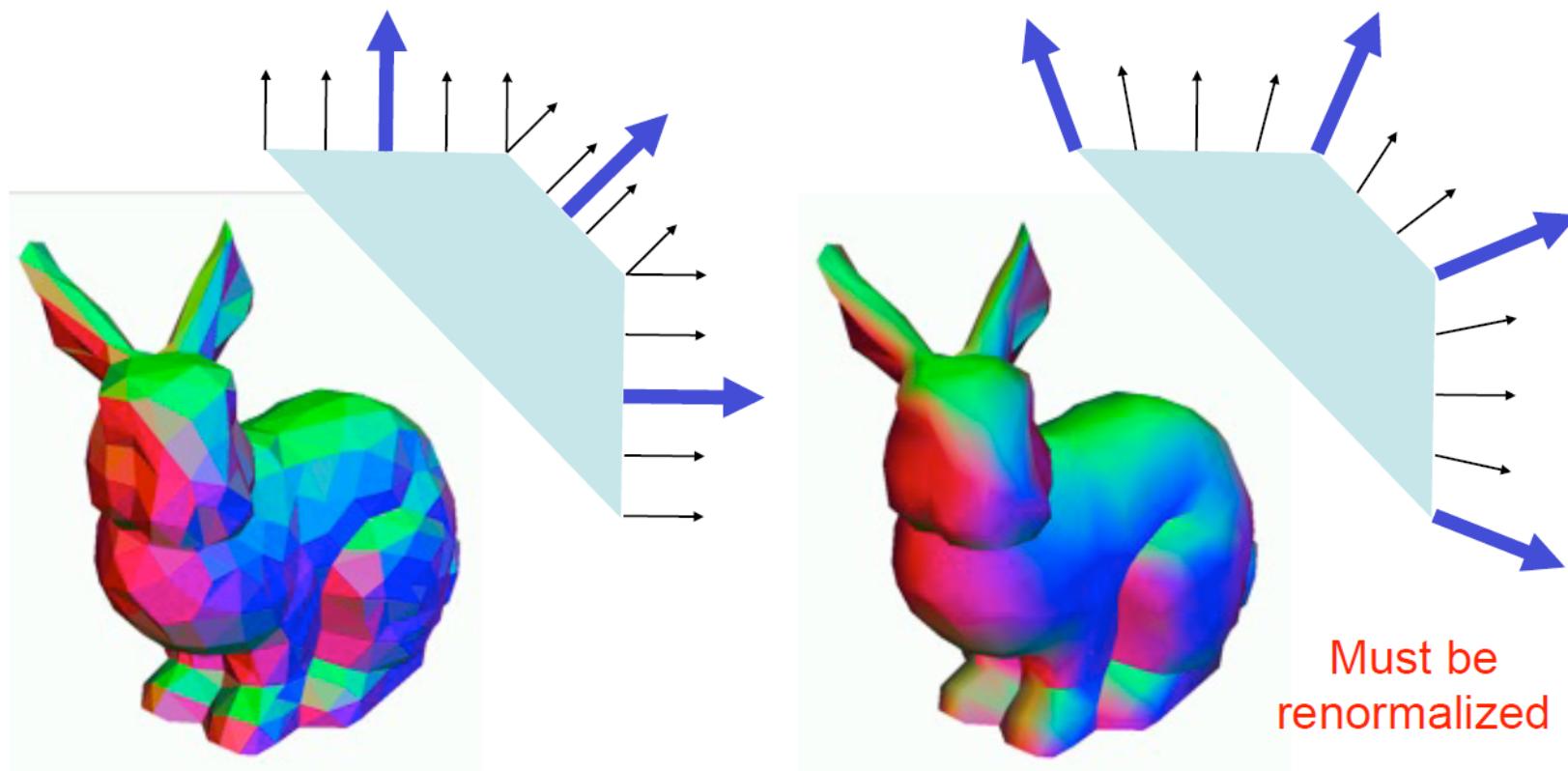
# Questions?



# Phong Normal Interpolation

(Not Phong  
Shading)

- Interpolate the average vertex normals across the face and use this in shading computations
  - Again, use barycentric interpolation!



# That's All for Today!

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# Spatial Variation

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- All materials seen so far are the same everywhere
  - In other words, we are assuming the BRDF is independent of the surface point  $x$
  - No real reason to make that assumption
  - More next time

