

# 2

## BASIC GEODESY, COORDINATE SYSTEMS, AND SCALE

**CHAPTER PREVIEW** The Earth is an irregularly shaped body, roughly approximating a sphere but more precisely defined by a reference ellipsoid. The ellipsoid is often part of a map or spatial dataset's datum, which can also describe its origin and type of coordinate system used. Cartesian coordinate geometry provides the underpinnings of eastings and northings used in coordinate systems worldwide. Cartesian coordinate concepts are also incorporated in the geographic grid.

In Chapter 1 it was noted that thematic maps are composed of three main structural elements: the base map, a thematic overlay, and a set of ancillary map elements, such as titles, legends, compilation credits, neatlines, and other objects. Chapters 2 and 3 focus on the issues pertaining to the first element: the base map. Historically, cartographers created their own base maps manually, obtained and associated the attributes to make the thematic overlay, and then created the ancillary elements to complete the thematic map. In the switch from manual to digital map compilation, obtaining source material for the thematic map (the base map and attribute data) is often just a click away on the Web.

Today, there is an ever increasing amount of digital base map data easily accessed in GIS and mapping software. These spatial data often come from the GIS or mapping software manufacturers, or are downloaded from federal, state, local, or other government, educational, or private institutions on the Web. Attribute data are often (but not always) included with the digital base map.

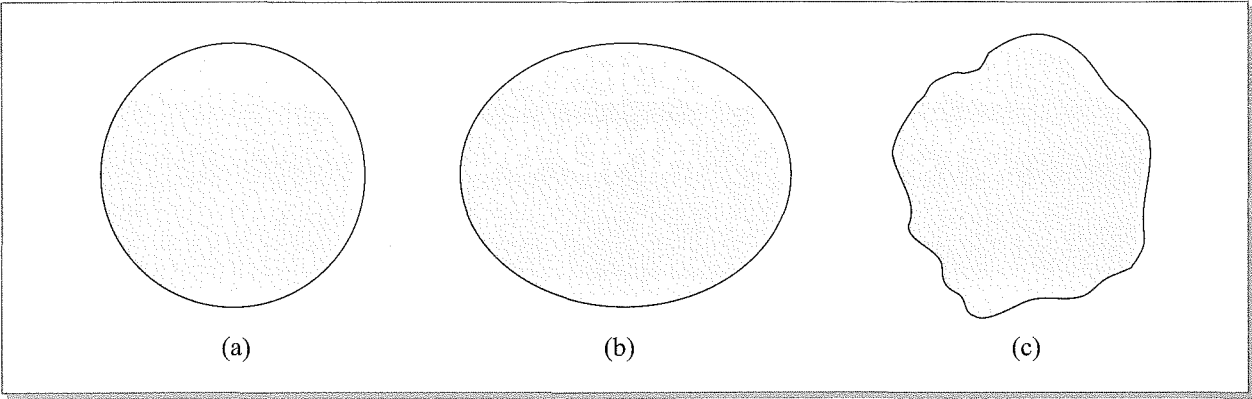
While the attributes determine *what* thematic topic is being mapped, such as population density, agricultural production, per capita income, or precipitation, the base map illustrates *where* those data occur. Are the data local, or are they from a

Latitude and longitude positions may be reported in sexagesimal or DMS format, or as decimal degrees. An understanding of the deeper geometric relationships of the geographic grid can greatly assist the cartographer in choosing the map's projection and other design aspects. Scale and generalization, subjects discussed in Chapter 1, are essential concepts to finding and correctly utilizing the myriads of data that are available on the Web that can be used to design a thematic map. ■

different level, such as state or province, country, continent, or the world? If I download a state map containing counties, will the counties still look good if I zoom in on them? If I overlay a road or river network on top of this map, will the layers still line up? These are questions that are directly impacted by base map concepts that are foci of this chapter: *geodesy*, *datums*, *coordinate systems*, *the geographic grid*, *scale*, and *generalization* (the last two terms were also discussed in Chapter 1). These concepts also play a central role in another important related base map concept, the **map projection**. The map projection is the process by which we obtain our flat two-dimensional map from the three-dimensional Earth surface, and will be dealt with in Chapter 3.

## BASIC GEODESY

**Geodesy** is the science of Earth measurement. Students are often surprised by the fact that there is more to describing the Earth's shape than as simply "round," and that many of the mathematical estimates of the Earth's size and shape can have an impact on the map's production. There are three important approximations of the Earth's shape: the sphere, the ellipsoid,



**FIGURE 2.1** 2-D COMPARISON OF THE SPHEROID (a), ELLIPSOID (b), AND GEOID (c) REPRESENTATION OF THE EARTH. Note that the shape of the ellipsoid and geoid are highly exaggerated for illustrative purposes.

and the geoid (see Figure 2.1). The next section will examine these important shapes and their impact on mapping.

**The Size and Shape of the Earth**

It is not known exactly when the Earth was first thought to be spherical in form, but Pythagoras (sixth century B.C.) and Aristotle (384–322 B.C.) are known to have determined that the Earth was a sphere. Aristotle based his conclusions partly on the idea, then widely held among Greek philosophers, that the sphere was a perfect shape and that the Earth must therefore be spherical. Celestial observations, notably lunar eclipses, also helped him reach this important conclusion. The idea of a spherical Earth soon became adopted by most philosophers and mathematicians.

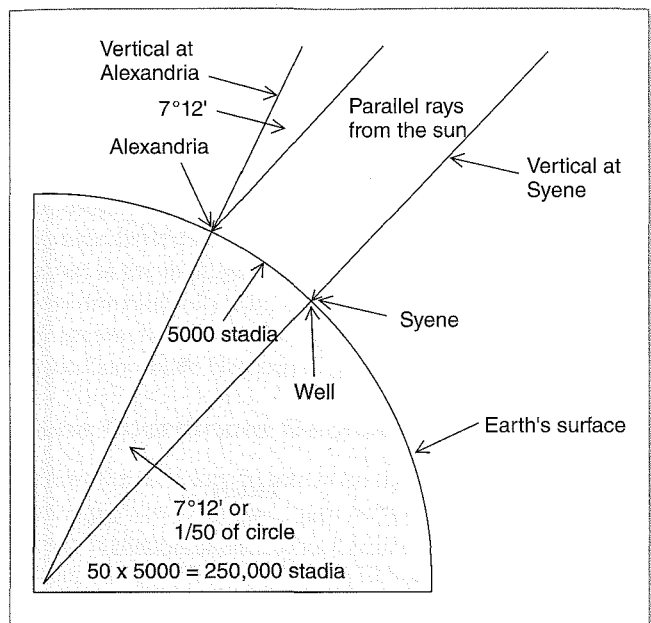
Greek scholars turned their attention to measurement of the Earth. In fact, the Earth’s size was measured quite accurately by the Greek scholar **Eratosthenes** (276–194 B.C.). Living in Alexandria, Egypt, Eratosthenes calculated the equatorial circumference remarkably close to today’s measurement of 40,075 km (24,901 mi) (Campbell 2001).

Eratosthenes’ ingenious method of measuring the Earth employed simple geometrical calculations (see Figure 2.2). In fact, the method is still used today. Eratosthenes noticed on the day of the summer solstice that the noon sun shone directly down a well at Syene, near present-day Aswan in southern Egypt. However, the sun was not directly overhead at Alexandria but rather cast a shadow that was 7°12’ off the vertical. Applying geometrical principles, he knew that the deviation of the sun’s rays from the vertical would form an angle of 7°12’ at the center of the Earth. This angle is 1/50 the whole circumference. The only remaining measurement needed to complete the calculations was the distance between Alexandria and Syene; this was estimated at 5000 stadia (one stadia is the size of an athletic stadium of that time period; the *exact* value is unknown). By multiplying this number by 50, his estimate was 250,000 stadia, which most scholars place within about 15 percent of today’s figure.

Not until the end of the seventeenth century was the notion of an imperfectly shaped Earth introduced. By that time, accurate measurement of gravitational pull was possible. Most notably, Newton in England and Huygens in Holland

**TABLE 2.1** REFERENCE ELLIPSOIDS

Ellipsoid	Equatorial Radius	Polar Radius
	(a)	(b)
	Statute Miles	Statute Miles
Airy (1830)	3,962.56	3,949.32
Austrian Nat’l–South Am. (1969)	3,962.93	3,949.64
Bessel (1841)	3,962.46	3,949.21
*Clarke (1866)	3,962.96	3,949.53
Clarke (1880)	3,962.99	3,949.48
Everest (1830)	3,962.38	3,949.21
*Geodetic Reference System (1980)	3,962.94	3,949.65
International (1924)	3,963.07	3,949.73
Krasovskiy (1940)	3,962.98	3,949.70
World Geodetic System (1972)	3,962.92	3,949.62
*World Geodetic System (1984)	3,963.19	3,949.90
The statute mile is 5,280 feet. *Common in North America		

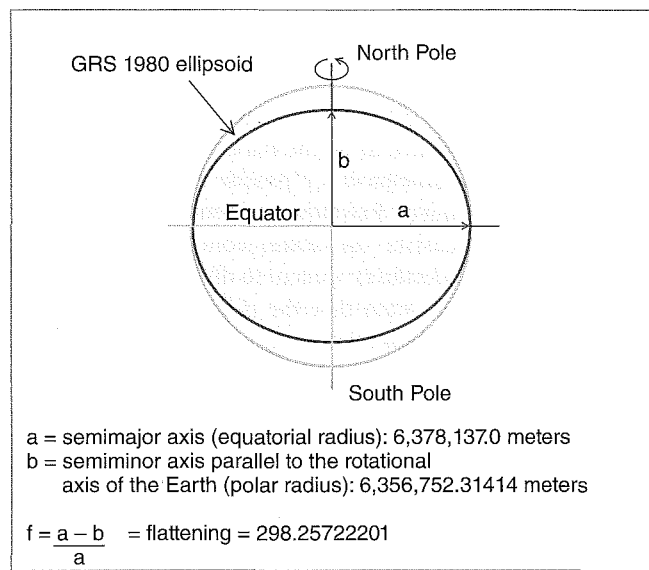


**FIGURE 2.2** ERATOSTHENES' METHOD OF MEASURING THE SIZE OF THE EARTH.

See the text for a complete explanation.

put forward the theory that the Earth was flattened at the poles and extended (bulged) at the equator. This idea was later tested by field observation in Ecuador and Lapland by the prestigious French Academy of Sciences. The Earth was indeed flattened at the poles! It is interesting to note that the first indication of this flattening came from sailors who noticed that their chronometers were not keeping consistent time as they sailed great latitudinal distances. The unequal pull of gravity, caused by the imperfectly shaped Earth, created different gravitational effects on the pendulums of their clocks.

The polar flattening and equatorial bulging of the Earth cause some cartographers to refer to the Earth as an oblate spheroid. The oblate qualities of the Earth can be modeled



**FIGURE 2.3** THE ELLIPTICAL SHAPE OF THE EARTH.

The ellipse is exaggerated for illustrative purposes. The GRS 80 ellipsoid is only one of many possible ellipsoids.

using a reference **ellipsoid** (see Figure 2.3). The geometrical solid is generated by rotating an ellipse around its minor axis and choosing the lengths of the major and minor axes that best fit those of the real Earth. Various reference ellipsoids have been adopted by different countries throughout the world for their official mapping programs, based on the local precision of the ellipsoid in describing their part of the Earth's surface (see Table 2.1). Some ellipsoids, such as the Geodetic Reference System 1980 and the World Geodetic System–1984 (WGS84) are designed for worldwide use. For example, Global Positioning Systems (GPS) technology is based on the WGS84 ellipsoid.

The most precise shape of the Earth is described by geodesists as a **geoid** (meaning Earth-shaped). Satellite

Mean Radius ( $2a + b$ )/3	Ellipticity (Flattening)	
Statute Miles	$f = \frac{a - b}{a}$	Where Used
3,958.15	1/299.32	Great Britain
3,958.50	1/298.25	Australia, South America
3,958.04	1/299.15	China, Korea, Japan
3,958.48	1/294.98	North America (especially NAD27), Central America, Greenland
3,958.49	1/293.46	Much of Africa, some countries of the Middle East
3,957.99	1/300.80	India, Southeast Asia, Indonesia
3,958.51	1/298.26	North America (especially NAD83)
3,958.63	1/297.00	Europe, individual States in South America
3,958.56	1/298.30	Russia (and former socialist states)
3,958.49	1/298.26	NASA; U.S. Department of Defense; oil companies, Russia
3,958.76	1/298.26	Worldwide, Global Positioning System

Sources: Defense Mapping Agency, Hydrographic Center, 1977, 117–20; Snyder, J. 1987, 12; Dana, P. 2003.

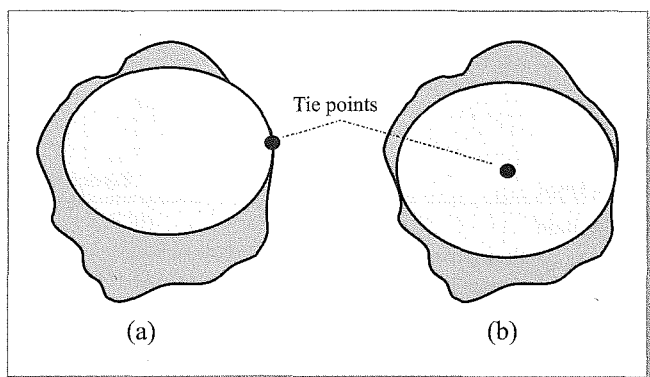
measurements of the Earth since 1958 suggest that, in addition to being flattened at the poles and extended at the equator, the Earth also contains great areas of depressions and bulges (King-Hele 1967, 1976). Of course, these irregularities are not noticeable to us on the Earth's surface. They do, however, have significance for precise survey work and geodetic measurements. For cartographers the most significant aspect of geodesists' use of the geoid is in the fitting of an ellipsoid and a coordinate system to distributed map data.

It is important to note that the differences between a spheroid, an ellipsoid, and the geoid are not going to be visible on small-scale maps and globes. But the differences can be significant at larger scales, where use of an appropriate reference ellipsoid is common (most cartographers do not directly use the actual geoid). Many of today's spatial data sets distributed for use in a GIS incorporate a reference ellipsoid, often as part of a *datum*.

## Datums

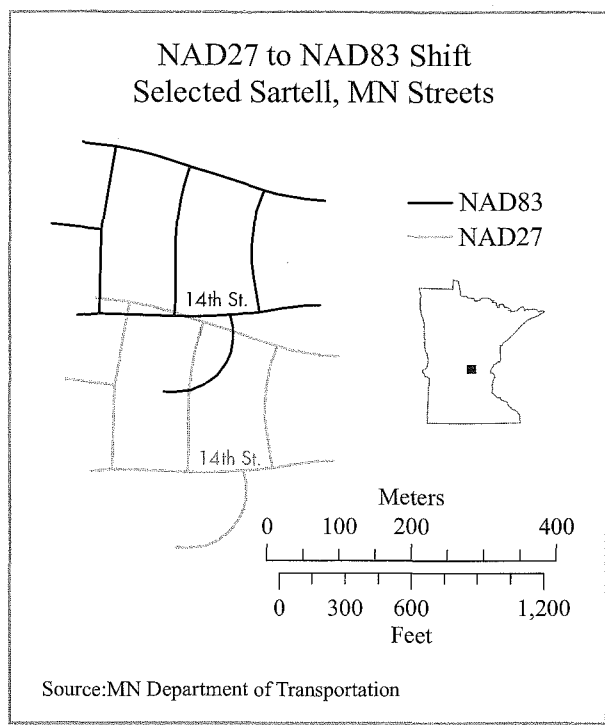
A **datum**, or starting point, gives a context to locations and heights on the Earth's surface. At its most basic, a datum defines the size and shape of the Earth (usually a reference ellipsoid such as those described in Table 2.1) and some sort of tie-point that fixes the ellipsoid to the Earth's surface (see Figure 2.4a) or to the center of the Earth (see Figure 2.4b). For example, the North American Datum of 1927 (NAD27) uses the Clarke 1866 ellipsoid, which has its tie point located at Meades Ranch, Kansas, USA. The North American Datum of 1983 (NAD83) uses the Geodetic Reference System 1980 (GRS80) ellipsoid, which has its tie-point at the center of the Earth.

A datum will also describe the origin and orientation of the coordinate system used (Dana 2003). Coordinate systems will be described in more detail in the next section. Referencing the wrong datum can result in positional errors of hundreds of feet, depending on place and datum used. For example, the



**FIGURE 2.4** APPLYING THE REFERENCE ELLIPSOID TO THE EARTH.

In (a) the ellipsoid is tied to a point on the Earth's surface. In (b) the ellipsoid is tied to the center of the Earth. Again, both ellipsoid and geoid are highly exaggerated for illustrative purposes.



**FIGURE 2.5** NAD27 TO NAD83 POSITIONAL SHIFT IN CENTRAL MINNESOTA.

For larger scale maps and downloaded GIS layers, datum selection can be very important. In this illustration, the y-displacement is over 200 meters (more than 650 feet).

outlines from a section in central Minnesota (see Figure 2.5) illustrate the difference in the shift between NAD27 and NAD83 datums. The visual appearance of referencing the wrong datum is most pronounced at larger scales, but since public data distributed for GIS use are often distributed with a particular ellipsoid or datum, a mismatch in datum can cause a mis-registration among the layers, as in Figure 2.5.

## COORDINATE GEOMETRY FOR THE CARTOGRAPHER

Location was the key idea behind the historical development of the Earth's coordinate geometry. Ancient astronomers were naturally concerned with this question as they delved into debates related to the Earth's size and shape. During the fifteenth, sixteenth, and seventeenth centuries, when exploration flourished, exactness in ocean navigation and location became critical. Death often awaited mariners who did not know their way along treacherous coasts. Naval military operations in the seventeenth and eighteenth centuries also required precise determination of location on the globe, as is the case today.

Today, use of the **Global Positioning System (GPS)** is a common and useful means of determining location. GPS is a system of 24 or more orbiting satellites that transmit timing signals to ground-based receivers. The receivers' computer processor and software then calculate the location. The

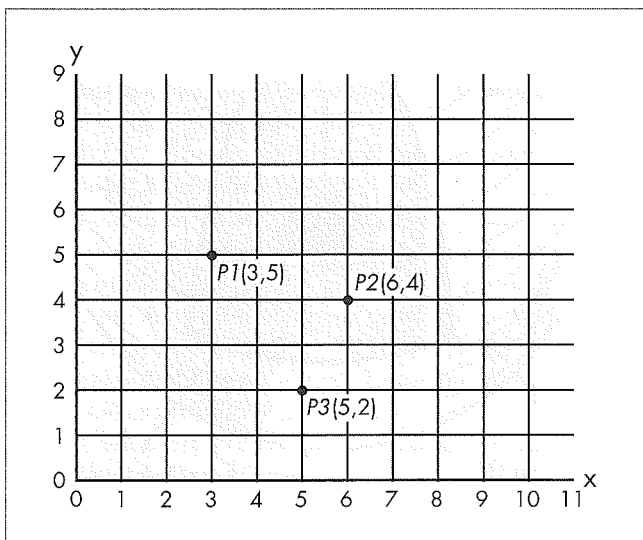
system can pinpoint one's location on the Earth within a few meters (a meter is about 3.28 feet) using consumer-grade receivers. However, land surveyors use higher precision devices and specialized methodologies that can place points within a centimeter (about 0.39 inch) of accuracy.

There are thousands of Earth coordinate systems available for cartographers, although the system of using latitude and longitude (also called the geographic grid, discussed in this section) is the most well known. GIS and mapping software allows for relatively easy conversion among these systems. This section focuses on the structure of coordinate systems in "Plane Coordinate Geometry" and "The Geographic Grid."

## Plane Coordinate Geometry

Perhaps the best way to introduce the major coordinate systems is to examine *plane coordinate geometry*. **Descartes**, a French mathematician of the seventeenth century, devised a system for geometric interpretation of algebraic relationships. This eventually led to the branch of mathematics called analytic geometry (Van Sickle 2004). It is from his contributions that we have **Cartesian coordinate geometry**. This system of intersecting perpendicular lines on a plane contains two principal axes, called the  $x$ - and  $y$ -axes (see Figure 2.6). The vertical axis is usually referred to as the  $y$ -axis and the horizontal as the  $x$ -axis. The intersection of the  $x$ - and  $y$ -axes is referred to as the origin.

The plane of Cartesian space is marked at intervals by equally spaced lines. The position of any point ( $P_{xy}$ ) can be specified by simply indicating the values of  $x$  and  $y$  and plotting its location with respect to the values of the Cartesian plane. In this manner, each point can have its own unique, unambiguous location. Relative location can easily be shown by plotting several points in the space.



**FIGURE 2.6** THE CARTESIAN COORDINATE SYSTEM.

Points ( $P_1$ ,  $P_2$ ,  $P_3$ , and so on) can have their exact locations defined by reference to the grid. Absolute and relative locations are therefore easy to determine.

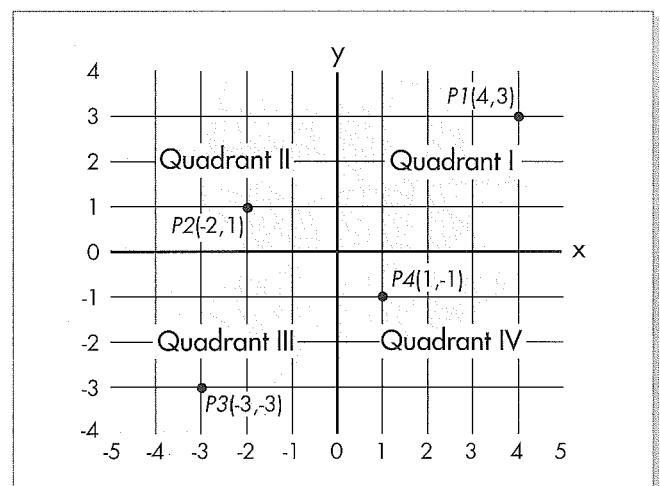
When describing real-world space, it is common among cartographers and land surveyors to refer to this system generically as eastings and northings. That is, a point can be defined by its easting (a measured distance along the  $x$ -axis from the origin) and its northing (a measured distance along the  $y$ -axis from the origin). The time-honored axiom of "read right up" can assist the map reader who is uncomfortable with which coordinate to list first.

In Figure 2.6, all of the coordinates have positive  $x$ - and  $y$ -values. But Cartesian coordinate geometry also allows for negative  $x$ - and/or  $y$ - values as well as positive ones by using quadrants (see Figure 2.7).

Many cartographic coordinate systems are set up so that the origin is always to the south and west of the coordinate space. That is, all  $y$  and  $x$  values will be positive (as in Figure 2.6). When a coordinate system is shifted so that everything is in quadrant one, the origin is sometimes referred to as a *false* origin. Alternatively, an arbitrary number can be added to the  $x$ -value and/or the  $y$ -value so that no values are negative. These numbers are sometimes called a *false* easting or *false* northing.

It is important to note that there are literally thousands of coordinate systems that exist throughout the world, at varying scales, often as part of a local, national, or global datum. Some use feet as their unit of measurement; many more use meters. The underlying principles of locating positions using eastings and northings work essentially the same way for every coordinate system.

There are a number of coordinate systems that are built on *map projections*. Two that are extremely important to cartographers are the State Plane and the Universal Transverse Mercator (UTM) coordinate systems. These are used throughout the United States—and for UTM, throughout much of the world—for storing and distributing spatial data. As these systems have an incorporated projection, most GIS and mapping software tend to include these coordinate systems in their



**FIGURE 2.7** CARTESIAN SYSTEM QUADRANTS.

The four quadrants allow for positive and negative Cartesian coordinate values.

map projection options. Because they are built on specific parameters involved in the map projection process we will discuss these important systems in the next chapter.

## The Geographic Grid

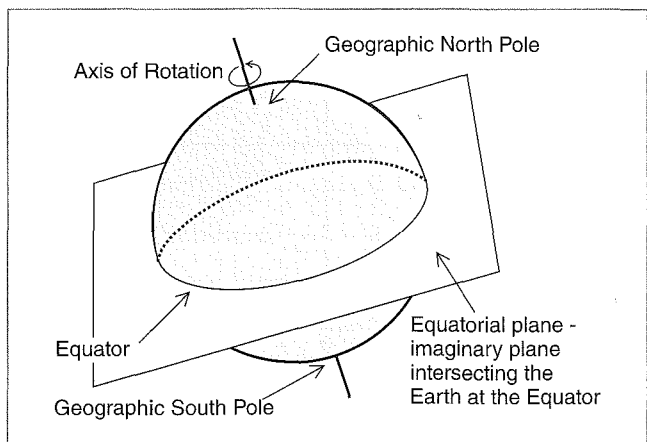
Concepts similar to those used in plane or Cartesian coordinate geometry are incorporated in the Earth's coordinate system, also called the **geographic grid** or the **graticule**. The Earth's geometry is somewhat more complex because of its spherical shape. Nonetheless, it can be easily learned. To specify location on the Earth (or any spherical body), angular measurement must be used in addition to the elements of the ordinary plane system. Angular measurement is based on a **sexagesimal scale**: division of a circle into 360 degrees, each degree into 60 minutes, and each minute into 60 seconds. This method of determining geographic grid values is sometimes referred to as a **DMS format** (for degrees, minutes, and seconds).

Our planet rotates about an imaginary axis, called the **axis of rotation** (see Figure 2.8). If extended, one of the axes approximately points to Polaris, the North Star. The place on Earth where this axis of rotation emerges is referred to as **geographic north** (the North Pole). The opposite, or **antipodal point**, is called **geographic south** or the South Pole. These points are very important because the entire coordinate geometry of the Earth is keyed to them.

If we were to pass an imaginary plane through the Earth perpendicular to and bisecting the axis of rotation, the intersection of the plane with the surface of the Earth would form a complete circle (assuming that the Earth is perfectly spherical). This imaginary circle is referred to as the Earth's **equator** (see Figure 2.8). Since the equator bisects the Earth into two halves, it is referred to as a **great circle**. The North and South Poles and the equator are the most important elements of the Earth's coordinate system.

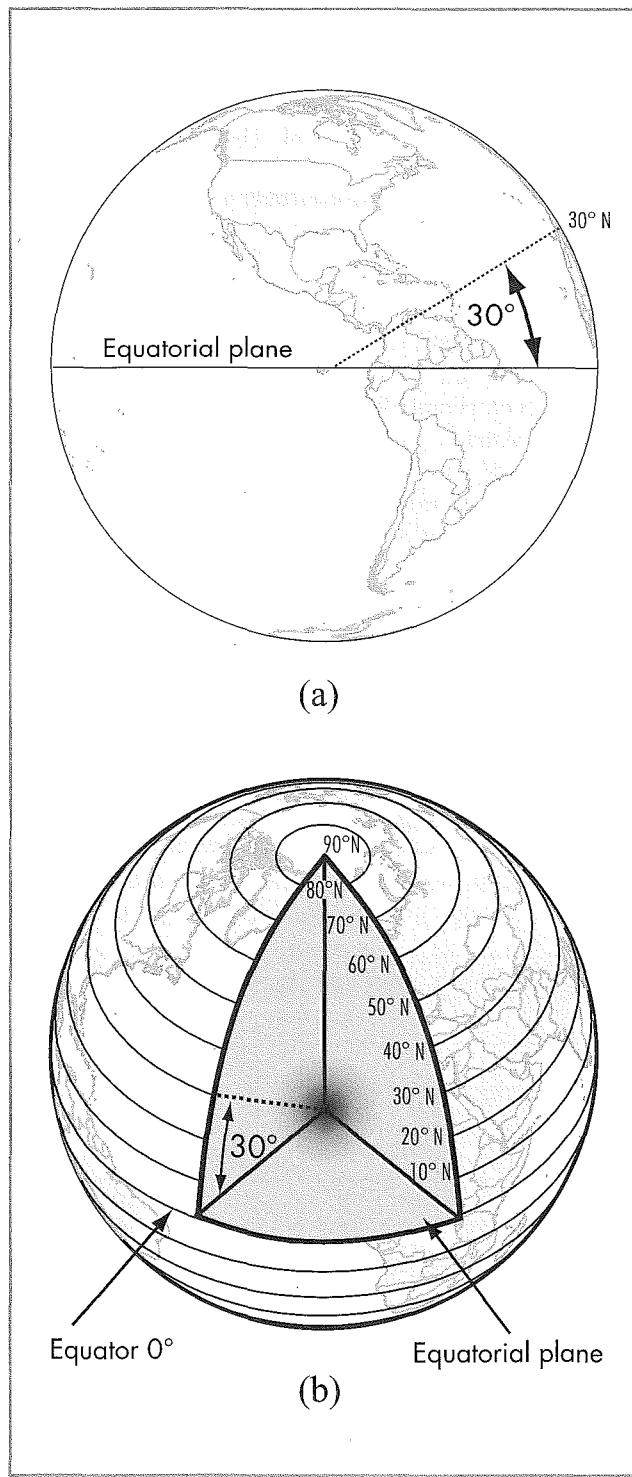
### Latitude Determination

**Latitude** is simply the location on the Earth's surface between the equator and either the North or the South Pole. Latitude determination is easily accomplished; it is a function of the angle between the horizon and the North Star (or some other



**FIGURE 2.8** DETERMINATION OF THE EARTH'S POLES AND THE EQUATOR.

fixed star; see Figure 2.9). As one travels closer to the pole, this angle increases. It can be demonstrated that the angle formed at the center of the Earth between a radius to any point on the Earth's surface and the equator is identical in magnitude to the angle made between the horizon and the North Star at that location. If an imaginary plane is passed through this point



**FIGURE 2.9** DETERMINATION OF LATITUDE ON THE SPHERICAL EARTH.

Two dimensional (a) and three-dimensional (b) illustrations of latitude.

parallel to the equatorial plane, it will intersect the Earth's surface, forming a **small circle**, or a **parallel of latitude** (see Figure 2.9). There are, of course, an infinite number of these parallels, and every place on the Earth can have a parallel.

Latitude is designated in angular degrees, from  $0^\circ$  at the equator to  $90^\circ$  at the poles. It is customary to label with a capital N or S the position north or south of the equator. Angular degrees are subdivided into minutes (60 minutes per degree) and seconds (60 seconds per minute) to provide a more precise latitudinal position. Thus common latitude designations would be  $82^\circ$  N,  $16^\circ 30'$  S,  $47^\circ 15' 47''$  N, and so on. The surface distance for each degree of latitude is about 69.2 miles (111.3 km).

### Longitude Determination

**Longitude** on the Earth's surface has always been more difficult to determine than latitude. It baffled early astronomers and sailors, not so much for its concept, but for the instrumentation required to quantify it. Because the Earth rotates on its axis, there is no fixed point at which to begin counting position. Navigators, cartographers, and others from the fourteenth to the seventeenth centuries knew that in practice they would need a fixed reference point. They also knew that the Earth rotated on its axis approximately every 24 hours. Any point on the Earth would thus move through 360 angular degrees in a day's time, or 15 degrees in each hour. If a navigator could keep a record of the time at some agreed-upon fixed point and determine the difference in time between the local time and the point of reference, this could be converted into angular degrees and hence position.

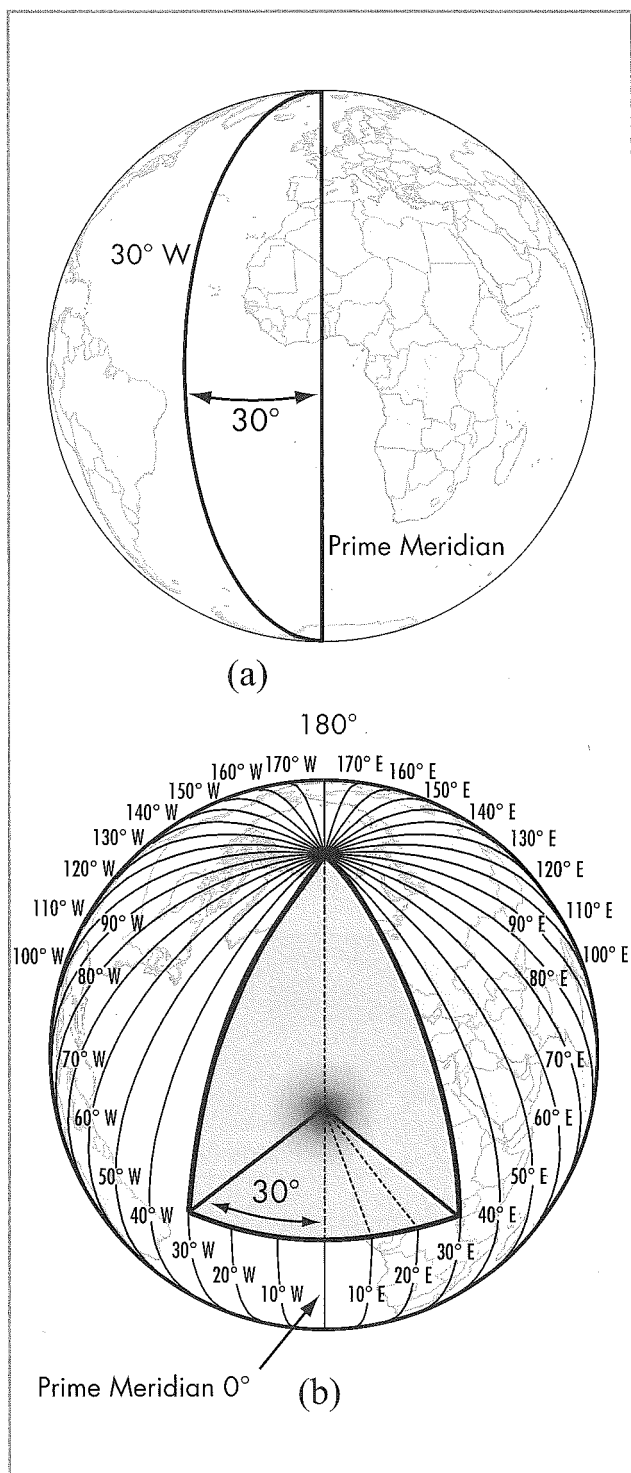
The concept was simple enough, but the technology of measuring time was slow in coming. In 1714, the British Parliament and its newly formed Board of Longitude announced a competition to build and test such an accurate timepiece. John Harrison's famous **marine chronometer** was finally accepted in 1773, after a series of delays and changes in the Board's requirements (Campbell 2001). It was accurate to within 1.25 nautical miles (one nautical mile is 6,076.12 feet, as opposed to the land or statute mile, which is 5,280 feet). The puzzle of longitude was solved (Brown 1959; Sobel and Andrewes 1998).

At first, each country specified some place within its boundaries as the fixed reference point for calculating longitude (historically called "reckoning"). By international agreement, at the International Meridian Conference in 1884, the line passing through the British Royal Observatory at Greenwich, England, called the **prime meridian**, was designated as the origin for longitude.

If an imaginary plane is passed through the Earth so that it intersects the axis of rotation in a line, it will intersect the surface of the Earth as a complete circle. One half of this circle, from pole to pole, is called a **meridian of longitude** (see Figure 2.10). The meridian passing through Greenwich is referred to as the *prime meridian* and has the angular designation  $0^\circ$ . Longitude position is designated as  $0^\circ$  to  $180^\circ$  east or west of the prime meridian for a total of  $360^\circ$ . The  $180^\circ$  meridian provides the basis for the International Date

Line, which is adjusted in places to keep countries entirely on one side or the other of the dateline, such as with the United States (Alaska) and Russia.

Unlike the surface distances between lines of latitude, lines of longitude are unequally spaced. At the equator, each



**FIGURE 2.10** DETERMINATION OF LONGITUDE ON THE SPHERICAL EARTH.

Two dimensional (a) and three-dimensional (b) illustrations of longitude.

degree is 69.2 miles (111.3 km) apart. That distance narrows going away from the equator, until all the meridians converge at the poles. This phenomena is called the **convergence of meridians** (or meridional convergence).

Until the advent of radio after World War I and radar after World War II, navigation and calculating one's position on the Earth were accomplished by "shooting the stars" (celestial navigation which also includes latitude determination), by marine chronometer, compass, and sextant. Today, GPS has become foundational for most navigation.

## The Complete Geographic Grid

With the aforementioned system of parallels and meridians, the Earth's spherical coordinate geometry is complete (see Figure 2.11). Its similarities to the plane Cartesian system should be apparent. By studying Figure 2.11 it should be clear that (1) *latitude is measured by counting the angular degrees north or south of the equator along a meridian* and (2) *longitude is measured by counting the angular degrees east or west of the prime meridian along a parallel*. Thus, the geographic grid is simply a spherical version of the Cartesian coordinate systems, where the equator serves as the  $x$ -axis, and the prime meridian serves the  $y$ -axis. The parallel to the planar coordinate system can be more clearly seen in the increasingly common use of decimal degrees instead of sexagesimal units to report latitude and longitude coordinates.

**Decimal Degrees.** The sexagesimal system described above is very common for teaching the basic principles of the geographic grid. It is also common in historic documents, books, and popular literature, and is sometimes a default setting for consumer-grade GPS units. However, the use of **decimal degrees** is becoming the standard in distributed data sets in

Familiarity with the spherical geographic grid and the characteristics of the arrangement of meridians and parallels is important in estimating graticule distortion on the flat map. The student is encouraged to inspect a globe for this purpose. These important properties should be noted:

1. Scale is the same everywhere on the globe; all great circles have equal lengths; all meridians are of equal length and equal to the equator; the poles are points.
2. Meridians are spaced evenly on parallels; meridians converge toward the poles and diverge toward the equator.
3. Parallels are parallel and are spaced equally on the meridians.
4. Meridians and parallels intersect at right angles.
5. Quadrilaterals that are formed between any two parallels and that have equal longitudinal extent have equal areas.
6. The areas of quadrilaterals between any two meridians and between similarly spaced parallels decrease poleward and increase equatorward.

which the coordinates of country borders, cities, features, and such are reported as latitude and longitude values. Decimal degrees are also used in describing certain aspects of map projections. Thus it is important to also understand the decimal degree format.

With decimal degrees, the degree measure stays the same, but the minutes and seconds are converted to a decimal format. For example, the point  $45^{\circ} 33' 37''$  N,  $94^{\circ} 9' 44''$  W in DMS format is represented as  $-94.162222, 45.560278$  in decimal degrees. Besides the decimals, you may notice two things that happened in the conversion. First, longitude is reported before latitude, since it is following the eastings and northings convention described earlier. Second, there is also a negative sign in front of the 94. Decimal degrees use signed hemispheres instead of a letter designating N, S, E, or W, just like in the signed Cartesian coordinate quadrants described earlier and as shown in Figure 2.7. Figure 2.12 illustrates the sign that each hemisphere takes on in this system: positive numbers for the northern and eastern hemisphere, negative numbers for the southern and western hemisphere.

## Principal Geometric Relationships of the Earth's Geographic Grid

The appropriateness of the final map depends in large measure on how well the cartographer knows the relationships of the elements of the geographic grid. This knowledge can be especially helpful when selecting a *map projection* (defined previously; was first used in Chapter 1, and is the focus of Chapter 3) because: (1) the projection process distorts and deforms the graticule in all sorts of ways—knowing how it really appears can help you assess the distortions in shape, area, distances, and direction that will occur, and (2) the projection process uses geographic grid positions and lines

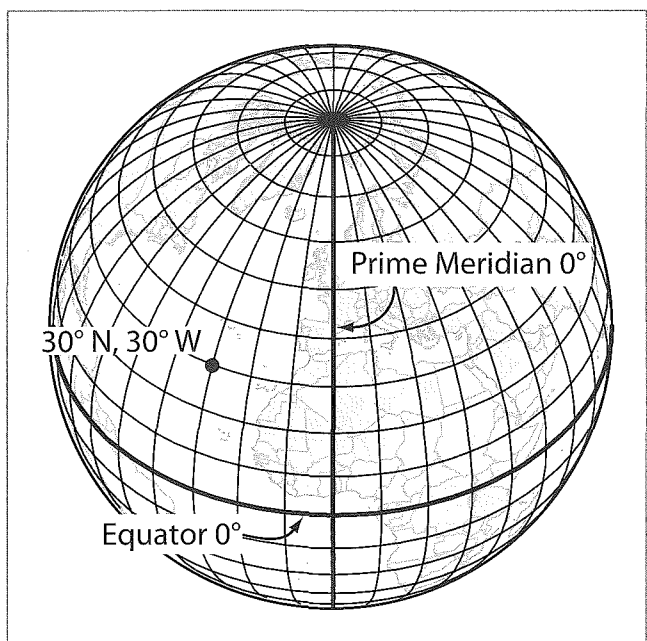
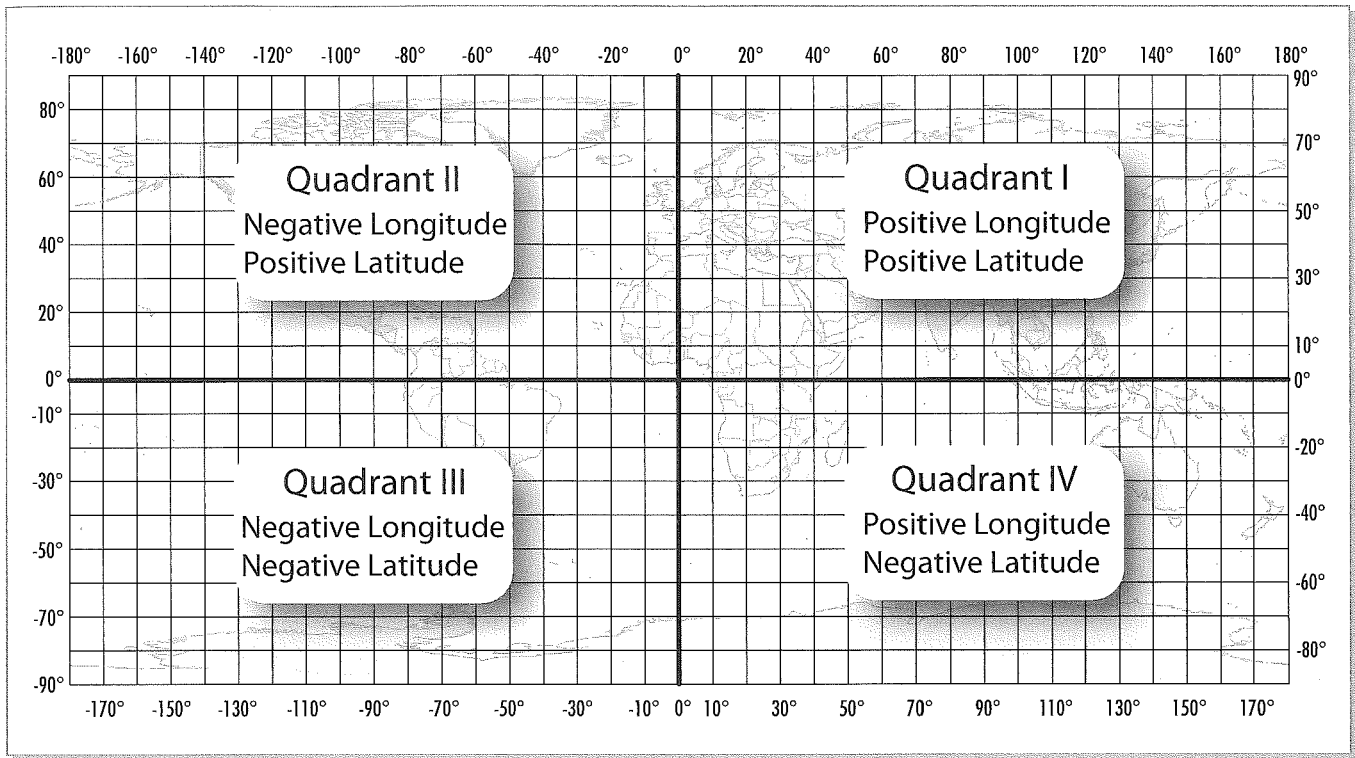


FIGURE 2.11 THE COMPLETE GEOGRAPHIC GRID.





**FIGURE 2.12** SIGNED HEMISPHERES FOR DECIMAL DEGREES.

As with Figure 2.7, the four hemispheres relate to the four Cartesian quadrants. Decimal degrees will be positive in the northern and eastern hemispheres, and negative in the southern and western hemispheres.

to describe some of the projection's qualities. This discussion is intended to provide a more in-depth look at the mathematical relationships of geographic grid.

**Linear.** Lengths of lines of the spherical grid have fixed relations to each other. The most important length is the magnitude of the radius ( $r$ ); from this and easily learned formulas, most other line lengths can be calculated. For example, the diameter is  $2r$ . For the perfect sphere, the polar radius is identical to the equatorial radius. There is only one circumference, and it is equal to  $2\pi r$  ( $\pi = 3.1416$ ). On the real Earth, of course, the equatorial circumference does not equal the meridional circumferences because of flattening at the poles.

Meridional lengths are the easiest to handle. On the perfect sphere, a meridian is one-half the circumference of the globe. Normally, we want to deal only with the length of the degree along a meridian. On the perfect sphere, this length is simply the circumference divided by 360; every degree is equal to every other degree. On the real Earth, however, polar flattening causes the radius of the arc of the meridian to change, fitting it to an ellipse. Consequently, the length of the degree along the meridian is not constant. The meridian is equal to one-half the circumference—or half the length of the equator on a sphere. Knowing this is critical to the evaluation of projection properties.

Parallels of latitude also have fixed linear relationships. No parallel in one hemisphere is equal in length to any other

in the same hemisphere on a perfect sphere. Parallels decrease in length at high latitudes. This relationship has a very definite mathematical expression, namely,

$$\text{length of parallel at latitude } \lambda = (\cosine \text{ of } \lambda) \cdot (\text{length of the equator})$$

The length of the degree of the parallel is determined by dividing its whole extent by 360. The cosine of  $60^\circ$  is 0.5. Thus the length of the degree along the  $60^\circ$  parallel is but one-half that at the equator (see Table 2.2). This information can be used intelligently in assessing map projections and consequent distortions. The lengths of the degree along the parallels on the “Earth” as defined by the GRS80 ellipsoid are listed on the text’s website.

**Angular.** Inspection of a globe printed with a geographic grid will reveal important angular characteristics of the grid’s elements. Most notably, it is easy to see that meridians converge poleward and diverge equatorward. Parallels, by definition, are parallel. What is more subtle is that meridians and parallels intersect at right angles. This is an important characteristic of the grid which can help in the evaluation of projection properties.

One line of note is a **loxodrome**, which has a constant compass bearing. The equator, all meridians, and all parallels are loxodromes. Other loxodromes, called oblique loxodromes, are special, and they too maintain constant compass

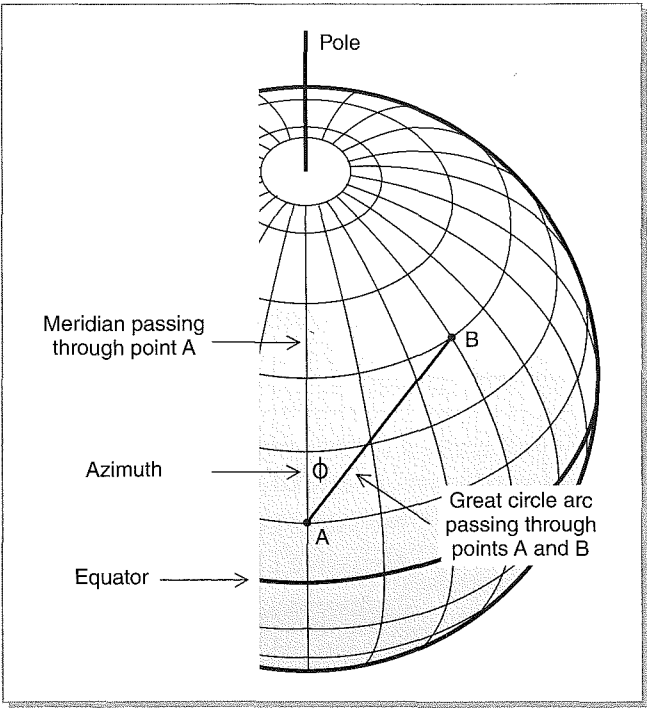
**TABLE 2.2** LENGTH OF PARALLELS ON A PERFECT SPHERE ( $r = 1.0$ ) CIRCUMFERENCE (EQUATOR)  $= 2\pi r$  (OR  $\pi d$ )  $= 6.28318$

Latitude (°)	Length of Parallel	Percent of Equatorial Length
0 (equator)	6.2831	100.00
5	6.2592	99.61
10	6.1877	98.48
15	6.0690	96.59
20	5.9042	93.96
25	5.6945	90.63
30	5.4414	86.60
35	5.1468	81.91
40	4.8132	76.60
45	4.4428	70.71
50	4.0387	64.27
55	3.6039	57.35
60	3.1416	50.00
65	2.6554	42.26
70	2.1490	34.20
75	1.6262	25.88
80	1.0911	17.36
85	.5476	8.71
90	.0000	.00

bearings, because they intersect all meridians at equal angles. Because meridians converge, oblique loxodromes tend to spiral toward the pole, theoretically never reaching it. Throughout history the loxodrome has always been important in sailing; mariners often wish to maintain the same heading throughout much of a journey. Unfortunately, loxodromes do not follow the course of the shortest distance between points on the Earth, which is a **great circle arc**. Navigators would usually approximate a great circle arc by subdividing it into loxodrome segments in order to reduce the number of heading changes during travel. Great circle arcs are followed very closely today, especially in airplane navigation. For example, the route travelled between New York City and London takes the traveler north near Greenland as a part of the great circle route.

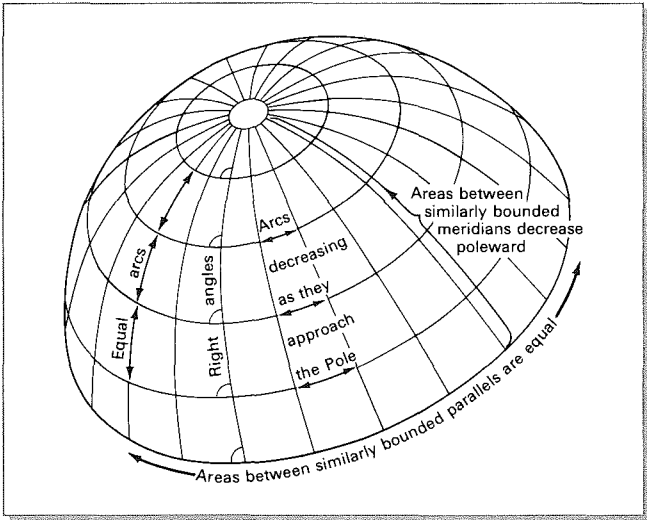
**Azimuth.** Azimuth has a very specific definition in cartography (see Figure 2.13). Azimuth is always defined in reference to two points, for example A and B. *The azimuth from A to B is the angle made between the meridian passing through A and the great circle arc passing from A to B.* It is customary to specify azimuth as an angle counting clockwise from geographic north through 360 degrees. Most azimuths are from geographic north, although possible from the south. The angular measure is followed by either N or S, for example 49° N.

**Area.** The areas of quadrilaterals found between bounding meridians and parallels are important in understanding the



**FIGURE 2.13** THE DETERMINATION OF AZIMUTH.

areal aspects of the Earth’s spherical coordinate system (see Figure 2.14). Between two bounding meridians, for identical latitudinal extents, the quadrilateral areas decrease poleward. Any misrepresentation of this feature during projection will have a profound effect on the appearance of the final land/water areas of the map. The relationship of these areas on a perfect sphere, based on change of latitude, is represented in



**FIGURE 2.14** THE PRINCIPAL GEOMETRICAL RELATIONSHIPS OF THE EARTH’S COORDINATE SYSTEM.

**TABLE 2.3** AREAL RELATIONSHIPS OF QUADRILATERALS USING THE GEOGRAPHIC GRID ON A PERFECT SPHERE ( $r = 1.0$ ) 5° LONGITUDINAL EXTENT

Latitude (°)	Area	Percent of Lowest Quadrilateral
0–5	.007605	100.00
5–10	.007547	99.23
10–15	.007432	97.72
15–20	.007260	95.46
20–25	.007033	92.47
25–30	.006752	88.78
30–35	.006420	84.41
35–40	.006039	79.41
40–45	.005612	73.79
45–50	.005143	67.62
50–55	.004634	60.93
55–60	.004090	53.78
60–65	.003515	46.21
65–70	.002913	38.30
70–75	.002289	30.09
75–80	.001647	21.66
80–85	.000993	13.06
85–90	.000332	4.36

Table 2.3. The decrease in area relative to the lowest quadrilateral is more dramatic at the higher latitudes because of rapid meridional convergence. At lower latitudes, meridians converge slowly; as a result, the change in area is less marked.

**Points.** Perfect spheres are considered to be *allside surfaces*, on which there are no differences from point to point. Every point is like every other point, with the surface falling away from each point in a similar manner everywhere. In dealing with spheres and with the projection of the spherical grid onto a plane surface, we may think of points as having dimensional qualities. That is, they are commensurate figures, though infinitesimally small.

**Circles on the Grid.** Two special circles appear on the spherical grid. A *great circle* is formed by passing a plane through the center of the sphere. This plane forms a perfect circle where it intersects with the sphere's surface. Certain qualities about the circle are worth knowing: (1) The plane forming the great circle bisects the spherical surface. (2) Great circles always bisect other great circles. (3) An arc segment of a great circle is the shortest distance between two points on the spherical surface. Some of the elements of the geographic grid are great circles. All meridians are great circles; the equator is also a great circle.

Circles on the grid that are not great circles are called *small circles*. Parallels of latitude are small circles, except for the equator. On the Earth, to travel along a meridian (N–S) is to go the shortest distance. Traveling along a parallel

(E–W) is *not* the shortest distance! Following the path of the equator, however, is an efficient way of travel.

## SCALE REVISITED

In Chapter 1, we introduced the concept of map scale. These additional comments will also prove useful to a further understanding of scale. In cartography, scale is represented by a ratio of map distance to Earth distance:

$$\text{map scale} = \frac{\text{map distance}}{\text{Earth distance}}$$

It is generally expressed as a representative fraction (RF), and will always contain unity in the numerator. The denominator is referred to the RFD, or representative fraction denominator. The RF scale can be expressed as a traditional fraction (such as  $1/25,000$ ), but is most commonly written as a ratio using a colon, such as  $1:25,000$ . RF scales are to be read, “One unit on the map *represents* so many of the *same* units on the Earth.” The number in the fraction may be in any units, but both numerator and denominator will be in the *same* units. The cartographer should never say, “One unit on the map *equals* so many of the same units on the Earth.” This is incorrect and logically inconsistent.

There are three customary ways of expressing scale on a map: the representative fraction, a graphic or bar scale, and a verbal scale. The RF is perhaps the most important expression of scale for three reasons:

1. It is unit independent: the other methods are designed for expression of scale in specific units. It is mathematically the most precise expression of map to real Earth distances.
2. Most GIS and mapping software allow you to specify a scale or scale range in which map data is to be displayed, which is expressed as an RF. This expression of scale is what cartographers use when interacting with the spatial data via GIS and mapping software.
3. Most distributed maps and map data have suggested scale ranges that are appropriate for display. Sometimes the scale at which the data was originally created is also provided. Provided scales or scale ranges are always in the RF form.

On many maps, a *graphic* (linear) bar scale is included. This bar is usually divided into equally spaced segments and labeled with familiar linear units, such as miles, kilometers, meters, or feet, depending on the scale of the map. This scale is read the same as an RF scale. This form of scale is very useful for the following reasons.

1. It has a fairly high communicative value when compared to the representative fraction. The average map reader probably does not understand what  $1:100,000$  means, but a graphic scale in miles or kilometers makes distances clear.

2. If the map is enlarged or reduced, the bar scale changes in correct proportion to the amount of reduction. For example, a virtual map will change its size from monitor to monitor, and also if displayed on a projector (for example, in a Power Point presentation). If a paper map is enlarged or reduced on a photocopier, the bar scale again changes. In each case, the bar scale will change with the map, but an RF will become incorrect.

Another common expression of scale is the *verbal* scale. This is a simple expression on the face of the map stating the linear relationship. For example, “one inch represents five miles” is an example of a verbal scale. This scale form is easily converted to an RF scale between map and Earth distances (for more on converting between scales and working scale problems, see “Appendix A”), but lacks the visual appeal of the graphic scale, and also is incorrect if the map is enlarged or reduced. This scale also locks you into *specific* units and numbers. For these reasons, some cartographers feel that the verbal scale is somewhat inflexible and therefore of less utility when compared with the RF and graphic scale expressions. Nonetheless, all three scale forms are common. Their inclusion in a thematic map, as with other design decisions, should rest on the purpose of the map, along with consideration of the audience who will view the map. If communication of distances is important to the map, then we recommend the use of the *graphic* (linear) bar scale.

Cartographers use the terms small, medium (or intermediate), and large scales quite often and somewhat casually. In Chapter 1 we saw that larger scales (“zooming in” in popular usage) mean that we have a smaller mapped Earth area, but more information detail with less generalized symbolization and vice versa (see Figure 1.14 in Chapter 1). But what constitutes large, medium, and small scales? We see a qualitative illustration of the basic principal in Color Plate 1.1, but is there a quantitative guide? Actually, there are many suggested ranges published from numerous sources as to what exactly defines small, medium, and large scales. For our purposes, we will use the guidelines of:

- Large Scale—1:30,000 or larger
- Intermediate Scale—1:30,000 to 1:300,000
- Small Scale—1:300,000 or smaller

Although these are relative terms, most texts provide ranges that are somewhat similar. But note that with continental, hemisphere, and world maps, which will all be small scale, one is better off saying “larger or smaller” scales relative to an existing map. For example, a world map in a typical printed atlas may be displayed at 1:100,000,000, but a map of the continental United States might be at a larger scale of 1:15,000,000. Likewise, local maps of varying scales may all fall in the category of large scale—1:1,000 scale mapping is a much larger scale than 1:24,000, but both are still considered large scale maps.

## Scale and Line Generalization

While it has been noted that a majority of thematic mapping is done at smaller scales, we are seeing an upswing in large and intermediate scale thematic mapping. This is due primarily to the increases in availability of state and local data, including both base map and attribute data, which are available on the Web. To find some of these data, places such as the University of Arkansas Libraries web page of state, local, and national information can be a great place to start (University of Arkansas Libraries 2006). Another site that is illustrative of the situation is the Minnesota Data Deli, where large, medium, and small scale data can be found in abundance (Minnesota Department of Natural Resources 2004).

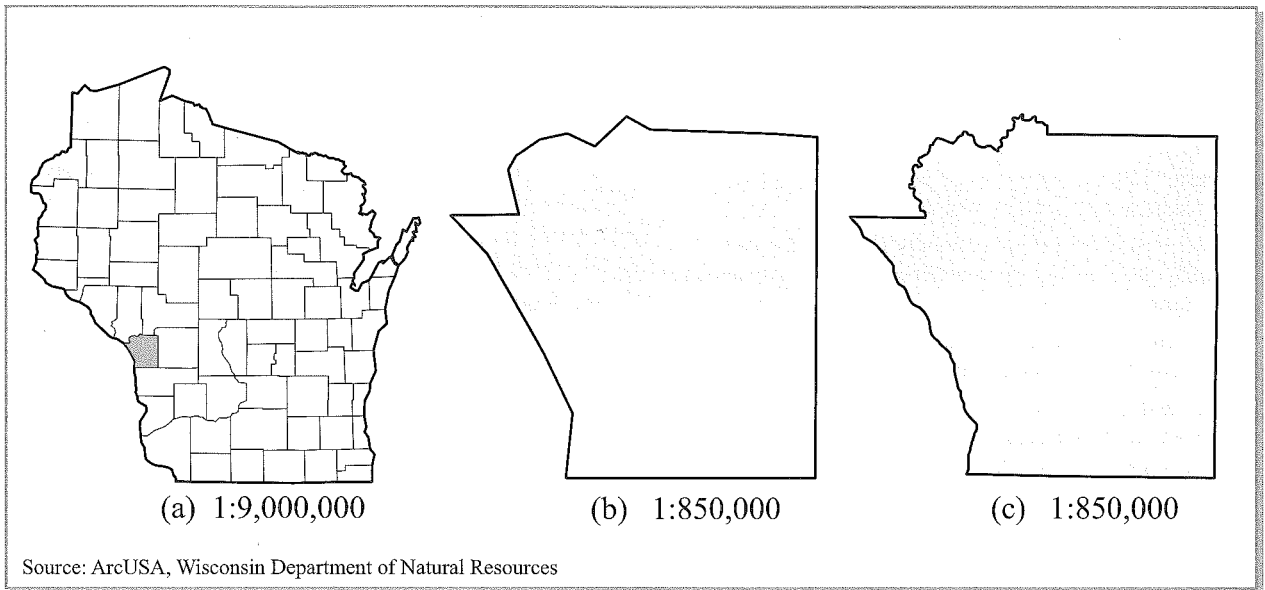
It is precisely this availability of base map and attribute data, created and distributed at multiple scales, that lead us to re-examine generalization levels (introduced in Chapter 1) when compiling base map data from different sources. As we will see, it is usually best to mix and match data from similar scales. In other words, if we create a map that incorporates rivers and roads on a base map at 1:250,000, we are going to want to choose river and road data that was created or designed for 1:250,000 use; not 1:24,000 or 1:15,000,000. For most available spatial data sets, the map will have accompanying **metadata**, or data about data, that will describe the scale of the source data, and at what scale or scale range the data should be displayed.

Cartographers want a detail level that is appropriate for boundaries and other features at a particular scale. In geometric terms, we can say that the vertex density in the line work should be somewhat similar at similar scales. Of course, this will depend on the features being presented. A rectangular county will likely have four vertices for the corners, connected by lines to form the polygon area regardless of scale or generalization level. More complex figures are a different story altogether.

As an example, Figure 2.15a has a vertex density that is appropriate for its scale at 1:9,000,000. But if you zoom into a portion of the same map (see Figure 2.15b) at a larger scale of 1:850,000, the vertex density that worked well at the smaller scale is now too generalized, particularly if it is going to be matched with other data that was generated at 1:850,000. But Figure 2.15c is appropriate, since the data was generated with a higher vertex density that is more appropriate for 1:850,000.

If you are working with relatively large scale maps and data, your sources also have to be large scale and of similar vertex densities. In other words, at 1:850,000 in Figure 2.15b and c, 2.15c is from the better source. You cannot take small scale data (as in Figure 2.15a) and make it appropriate for a large scale map (as is attempted in Figure 2.15b). If you are creating your own data, perhaps by digitizing maps from aerial photographic imagery, the resolution of the imagery must be good enough for you to generate a reasonable vertex density.

If you are working with small scale data, you can either find similar small scale sources, or you can use larger scale data and perform line generalization to remove vertices from



**FIGURE 2.15** LINE GENERALIZATION LEVELS FOR DIFFERENT SCALES.

The first map (a) depicts Wisconsin at 1:9,000,000. The second map (b) shows La Crosse County at 1:850,000. Notice how the vertex density level, while appropriate for (a) is over-generalized for the larger scale. The third map (c) of the same area is also at 1:850,000, but has a vertex density that is appropriate for this zoom level. Notice that the real difference between (b) and (c) occurs in the curved lines.

the line work to make it appropriate for the smaller scale. GIS and mapping software usually provide a procedure to remove extraneous vertices from larger scale map data. In other words, the procedure will keep the most important points that best define the line's shape, and eliminates all others (Douglas and Peucker 1973). How generalized the line work should be will depend on the map's purpose, the medium for which the map is intended, and the audience for which the map is intended.

## REFERENCES

- Brown, L. 1959. *The Story of Maps*. New York: Bonanza Books.
- Campbell, J. 2001. *Map Use and Analysis*. New York: McGraw-Hill.
- Dana, P. 2003. *Geodetic Datum Overview. The Geographer's Craft Project*. Department of Geography, The University of Colorado at Boulder. <http://www.colorado.edu/geography/gcraft/notes/datum/datum.html>.
- Defense Mapping Agency, Hydrographic Center, 1977. *American Practical Navigator* 1. Washington, DC: Defense Mapping Agency.
- Douglas, D., and T. Peucker. 1973. Algorithms for the Reduction of the Number of Points Required to Represent a Digitized Line or Its Caricature. *Canadian Cartographer* 10:2.
- King-Hele, D. 1967. The Shape of the Earth. *Science* 183: 67–76.
- . 1976. The Shape of the Earth. *Science* 192: 1293–1300.
- Minnesota Department of Natural Resources, 2004. *The DNR Data Deli*. <http://deli.dnr.state.mn.us/>
- Snyder, J. 1987. *Map Projections—A Working Manual*. U.S. Geological Survey Professional Paper 1395. Washington, DC: USGPO.

Sobel, D., and W. Andrewes. 1998. *The Illustrated Longitude*. New York: Walker and Company.

University of Arkansas Libraries. 2006. *Starting the Hunt: Guide to Mostly On-Line and Mostly Free U.S. Geospatial and Attribute Data*. Second Edition. <http://libinfo.uark.edu/gis/us.asp>.

Van Sickle, J. 2004. *Basic GIS Coordinates*. Boca Raton, FL: CRC Press.

## GLOSSARY

**antipodal point** point opposite; on the Earth, the North Pole is antipodal to the South Pole; 20° S, 60° E is antipodal to 20° N, 120° W

**axis of rotation** imaginary line around which the Earth rotates

**Cartesian coordinate geometry** system of intersecting perpendicular lines in plane space, useful in analytic geometry and the precise specification of location

**convergence of meridians** spacing of the meridians of longitude becomes less in a poleward direction from the equator

**datum** a starting or reference point that gives a context to things such as the size and shape of the Earth, and for any coordinate system that can be used for positioning (including elevation)

**decimal degrees** means of reporting minutes and seconds of latitude and longitude designations as a decimal (see also DMS format)

**Descartes** French mathematician whose early studies of algebra and geometry led to analytic geometry

**DMS format** the format for reporting sexagesimal latitude and longitude in degrees, minutes, and seconds (see also decimal degrees)

**ellipsoid** a geometrical solid developed by the rotation of a plane ellipse about its minor axis

**equator** imaginary line of the Earth's coordinate system that is formed by passing a plane through the center of the Earth perpendicular to the axis of rotation, midway between the poles

**Eratosthenes** (276–194 B.C.) Greek scholar living in Alexandria who first accurately measured the size of the Earth

**geodesy** the science that measures the size and shape of the Earth; often involves the measurement of the external gravitational field of the Earth

**geographic grid** spherical coordinate system used for the determination of location on the Earth's surface

**geographic north and south** the imaginary line forming the Earth's axis of rotation intersects the Earth's surface at two locations, the North and South Poles, referred to as geographic north or south

**geoid** term used to describe the shape of the Earth; means "Earth-shaped" and does not refer to a mathematical model

**Global Positioning System (GPS)** a system of 24 or more orbiting satellites that transmit timing signals to ground-based receivers which calculate location; used for navigation and mapping

**graticule** meridians of longitude and parallels of latitude on a map

**great circle** circles that result when a plane bisects the Earth into two equal halves (for example, the equator)

**great circle arc** segment of a great circle that is the shortest distance between two points on the spherical surface

**latitude** position north or south of the Earth's equator; designation is by identifying the parallel passing through the position; determined by the angle subtended at the center of a sphere by a radius drawn to a point on the surface

**longitude** position east or west of the prime meridian; designation is by identifying the meridian passing through the position; determined by angular degrees subtended at the center of a sphere by a radius drawn to the meridian and the position in question

**loxodrome** a line on the Earth that intersects every meridian at the same angle; because of meridional convergence, a loxodrome theoretically never reaches the pole; also called a rhumb line

**map projection** the systematic arrangement of the Earth's spherical or geographic coordinate system onto a plane; a transformation process that projects the geographic grid, along with land masses, bodies of waters, and other features onto a flat surface

**marine chronometer** relatively accurate timepiece used to determine longitude; perfected by John Harrison

**meridian (of longitude)** great circle of the Earth's geographic coordinate system formed by passing a plane through the axis of rotation; meridional number designation ranges from 0° to 180° E or W of the prime meridian

**metadata** data about data. For cartographers, information that accompanies spatial datasets that describe the dataset's spatial extent, attributes, lineage, and other information. In the context of scale and generalization, it often includes the scales of the source materials used to create the data set, as well as recommended scale ranges with which to display the data

**parallel (of latitude)** small circle of the Earth's geographic coordinate system formed by passing a plane through the Earth parallel to the equator; parallel number designation ranges from 0° at the equator (a great circle) to 90° at the Pole (either North or South)

**prime meridian** meridian adopted by most countries as the point of origin (0°) for determination of east or west longitude; passes through the British Royal Observatory at Greenwich, England

**sexagesimal** system of numbering that proceeds in increments of 60; for example, the division of a circle into 360 degrees, a degree into 60 minutes, and a minute into 60 seconds. Sexagesimal is also called DMS format

**small circles** any circles on the spherical surface that are not great circles; for example, parallels (except for the equator) are small circles

# 3

## MAP PROJECTIONS

**CHAPTER PREVIEW** One of the most important aspects of thematic mapping is the map projection. The 3-D Earth is transformed to a 2-D flat surface by use of a geometric form, such as a plane, cylinder, or cone, or it may be derived mathematically. Important projection parameters such as standard points and lines, central meridian, latitude of origin, and projection aspect are created in the process. There are nearly an infinite number of possible projections that can be produced, but they generally fall into one of four projection families: azimuthal (planar), cylindrical, conic, and mathematical. Each family has their own characteristic grid and potential applications. The projection process involves map distortion. There are also four projection properties that can be maintained (but not at the same time) that help

define what kind of distortion takes place. Projection distortion can be measured qualitatively by visual inspection and knowledge of a particular projection's parameters and distortion patterns, or it can be quantitatively measured by Tissot's indicatrix. Specific projections can be selected by asking questions about the map purpose, allowable distortion, and area of the world that is being mapped. Although there are thousands of named projections currently supported by most GIS and mapping software, knowledge of a fairly small number of projections and projected coordinate systems can fulfill a variety of mapping needs. When a projection does not exactly meet the requirements for a specific study area, the projection's parameters can often be adjusted to produce a more accurate and aesthetic map. ■

Chapter 2 introduced the concept of a datum, geoid, and reference ellipsoid, examined Earth's spherical grid, and revisited the concept of map scale. This chapter addresses the important related topic of map projections. The concept of a map projection originates from the idea that we have to *project* the imaginary lines of latitude and longitude from the Earth's surface onto a flat surface (for example, a piece of paper) as a grid. Using a map projection is the only practical way to portray the Earth's curvature on a flat surface.

Of course, the Earth can be portrayed using a globe. A globe provides a more accurate picture than a map of the Earth's surface in two important regards. First, on a globe the lines of latitude and longitude are positioned correctly.

Parallels of latitude run parallel to each other; meridians of longitude converge at the poles. It can be said that there is no *deformation* of the graticule on the globe. Second, the relative size and shape of all the continents, oceans, and other area features are true. In addition, distances and directions between points are correct. In other words, the *properties* of area, shape, distance, and direction are true on a globe.

In the map projection process, however, distortion occurs. Distortion can refer to both the deformation of the graticule and to the loss of one or more of the properties (area, shape, distance, direction). The first part of this chapter, "The Map Projection Process," deals with the process of projecting the graticule and the Earth's features (landmasses, countries,

bodies of water, and so on) to a geometric form, such as a plane, cylinder, or cone, or by other mathematical means. As we will see, four basic projection families result from this process, each with their own distinctive characteristics and distortion patterns.

The second part of the chapter, “Employment of Map Projections,” focuses on matching the map projection to a particular application. There are hundreds of map projections within the four basic families, many of which are available in GIS and mapping software. Each one has its advantages and disadvantages. For example, a map projection that may be useful for the entire world may not be the correct projection for a continent, country, or a state. In order to select the optimum projection, it is important to understand not only the basic principles of the *projection properties* but also the *projection parameters*, which define how the projection is centered and the latitude or longitude values where distortion is minimized or eliminated. We will now take a look at the projection process to gain a better understanding of some of these important map projection principles and concepts.

## THE MAP PROJECTION PROCESS

In thematic mapping, as in all cartography, we consider the production of a map a process of representing the Earth (or a part of it) as a *reduced model of reality*. Map projection is the transformation of the spheroid or elliptical surface to a plane surface. In most GIS and mapping software the cartographer simply selects the map projection that best suits the needs of the mapping project at hand. Technically, however, there is a three-step process that occurs. First, a spheroid or ellipsoid model is selected that best fits the geoid. For most distributed

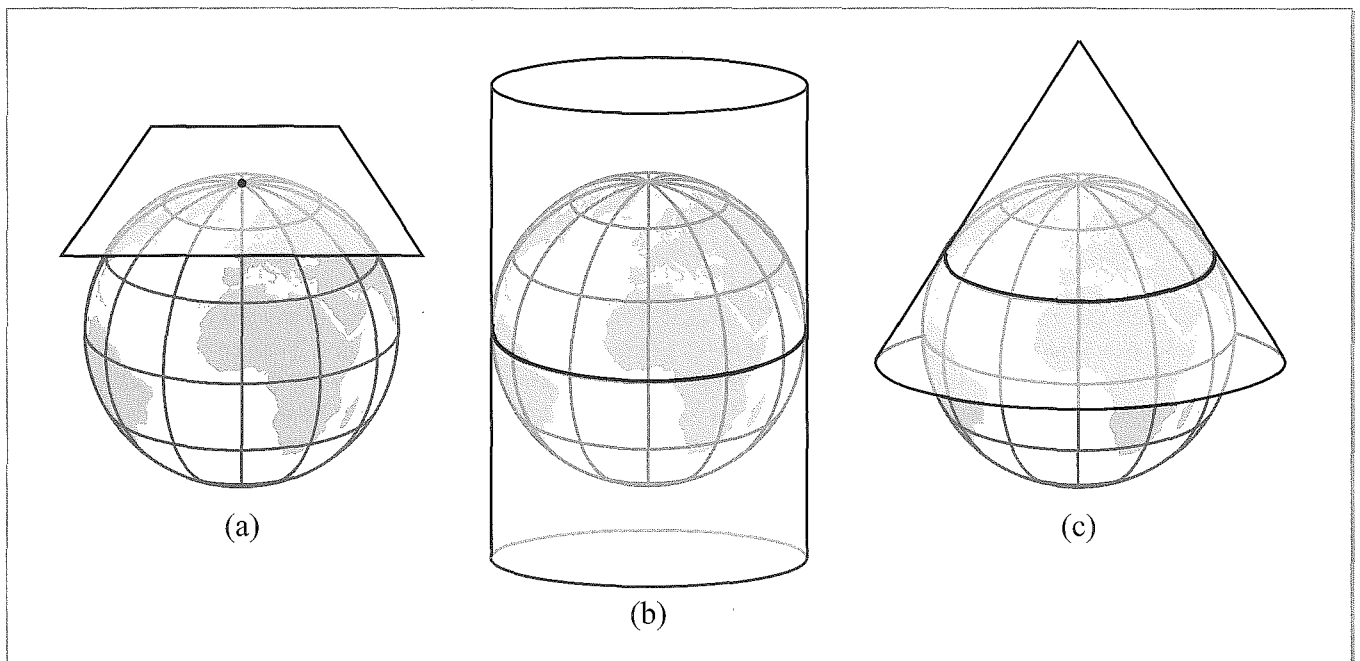
digital data, the incorporation of the ellipsoid or spheroid is already present, often as part of its datum (see Chapter 2 for a discussion of geoids, ellipsoids, and datums). Then there is the transformation of Earth coordinates to plane coordinates, often as eastings and northings, and the reduction in scale from Earth to map size. These last two steps are done mathematically by the GIS or mapping software.

Before computers and GIS were used in the projection process, when map projections were done manually, cartographers sometimes reduced the scale of the ellipsoid or spheroid model to a **reference globe** (also called a *nominal* or *generating globe*), from which the map projection was generated. As we will see, the reference globe is still a useful concept for illustrating basic projection principles.

Any map projection is the systematic arrangement of the Earth's or reference globe's meridians and parallels onto a plane surface. In addition to the graticule, the map projection process includes the transformation of other map features. For example, coastlines and boundaries are also transformed and, as a result, such map objects as landmasses and bodies of water may be distorted.

## Developable Surfaces

The process of transforming and transferring the graticule and its features from a three-dimensional object onto a two-dimensional flat surface is the essence of a map projection. However, neither the Earth nor any of its three-dimensional representations (such as the geoid, ellipsoid, sphere, or globe) are **developable surfaces**. The Earth cannot be pulled or cut apart to lie flat the way a map does. Three geometric forms that have developable surfaces are the plane, the cylinder, and the cone (see Figure 3.1),



**FIGURE 3.1** DEVELOPABLE PROJECTION SURFACES.

The plane (a), cylinder (b), and cone (c) are all geometrically developable surfaces. See the text for an explanation.



although many projections are simply mathematically derived graticules. These give rise to four overall families of map projections: azimuthal, cylindrical, conic, and mathematical. Each family has many individually named projections, where the name often contains one or more of the following: the name of the geometric figure, some of the projection's properties (discussed in the following sections), and the name of the individual identified as the originator of the projection.

Examining the three developable geometric surfaces is particularly useful in illustrating the concept of a map projection. As shown in Figure 3.1, each geometric surface is placed to a point (with the plane) or line of contact (with the cylinder and the cone). It may be helpful to conceptualize a light bulb at the center of the transparent reference globe in Figure 3.1, shining through the globe's surface, projecting the graticule and other features onto the new developable surface. The plane is already in its flat form, but the cone and the cylinder may be "taken off" from the 3-D surface and spread out without tearing, shearing, or distortion of the geometric surface (hence the surface is considered "developable"). However, as we shall see, the resulting maps will be far from distortion free.

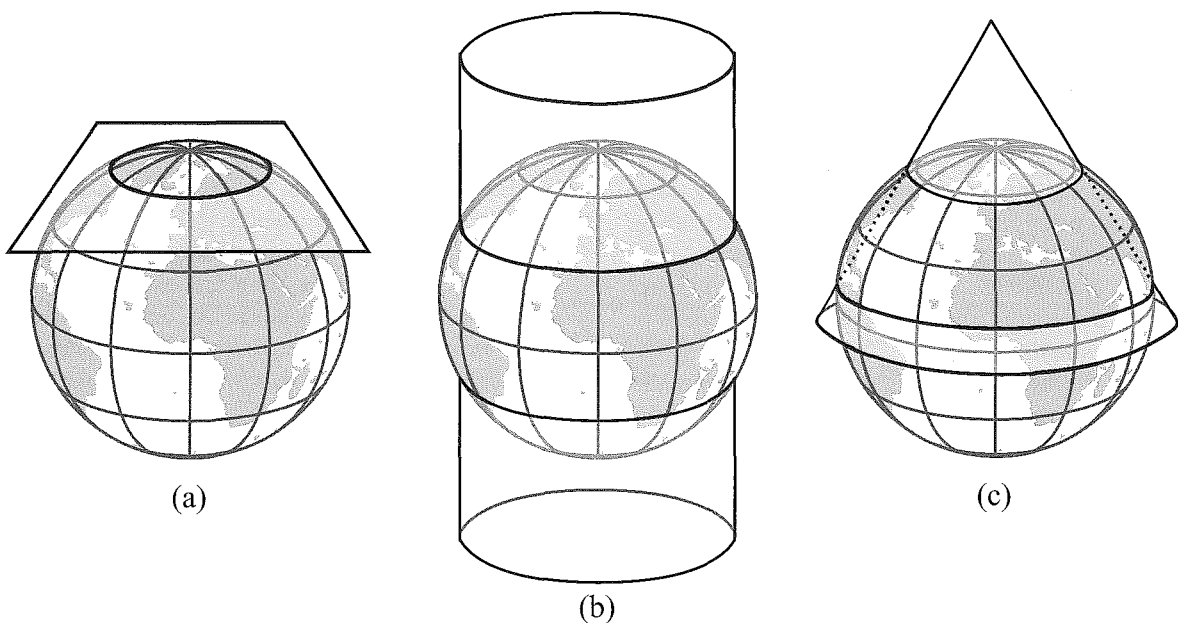
The geometric surfaces' contact with a point or line, as in Figure 3.1, is called tangency, or the **tangent case** (or sometimes *simple form*). In the **secant case** (Figure 3.2), each of the surfaces is placed at a line (with the plane) or two lines (with the cylinder and the cone) of *intersection*. In the secant case, part of the geometric surface intersects and actually goes "into" the reference globe. The light bulb analogy breaks down a bit at this point, as the graticule and the

features are projected up to or down onto the surface depending on the portion of the surface that is being examined.

### Projection Parameters

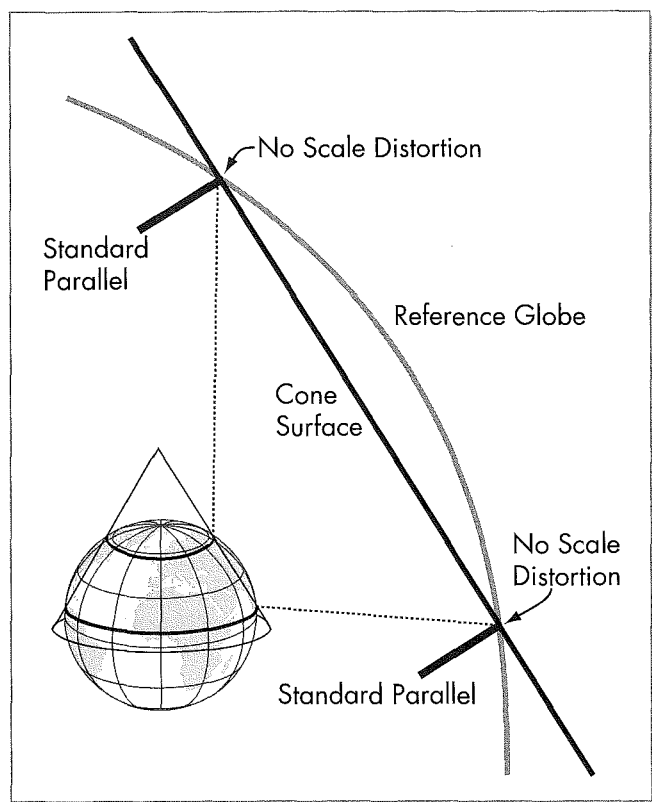
Points and lines of tangency or intersection are called **standard points** and **lines**. If a standard line is also a parallel of latitude then the line is called a *standard parallel*, and if the standard line falls along a meridian of longitude then it is called a *standard meridian*. *Standard* points, lines, parallels, and meridians are one of the most important map **projection parameters**, because those corresponding places on the map will have no scale distortion. That is, the scale of the map along these lines will have the same scale as the reference globe. The farther away from the standard point or line(s), the greater the distortion (or deformation) that occurs. In Figure 3.1, the plane has the least distortion at the North Pole (its standard point); the cylinder has the least distortion at the equator (its standard parallel), and the cone has the least distortion at 30° N. The secant case helps minimize distortion over a larger area by providing additional control (see Figure 3.3).

The positioning of the plane, cylinder, and cone shown in Figures 3.1 and 3.2 are common positions, and are sometimes called their "normal" **projection aspect** (Hilliard *et al.* 1978). The projection aspect is the position of the projected graticule relative to the ordinary position of the geographic grid on the Earth, and can be visualized as the position of the developable geometric surface to the reference globe. In Figures 3.1 and 3.2 the axis of the cylinder and the cone run



**FIGURE 3.2** DEVELOPABLE PROJECTION SURFACES, SECANT CASE.

The plane (a), cylinder (b), and cone (c) intersect the Earth or reference globe. See the text for an explanation.



**FIGURE 3.3** THE SECANT SURFACE.

The amount of scale distortion varies with the distance from the standard lines. In this example, those lines are standard parallels.

through the North and South Poles, and the plane is parallel to the equator. The *normal aspect* is the position that produces the simplest graticule. However, there is no rule that says these figures have to be placed in these particular positions. In particular, with azimuthal and cylindrical projections it is quite common to have other placement positions. Examples of other common projection aspects will be described in the next section.

Two other important *projection parameters* that indicate the projection aspect are the **central meridian** (the meridian that defines the center of the projection) and the **latitude of origin** (latitude of the projection's origin, such as the equator). Most GIS or mapping software, in addition to the selection of a map projection, allow for these parameters to be adjusted as necessary.

In addition, the hypothetical light source (for example, the "light bulb" in our earlier analogy) for any of these figures can be changed as well. In the next section we examine some of the most common projection aspects of the geometric forms and light sources, as well as the typical patterns and distribution of distortion for each of the families.

## Projection Families

Selecting an appropriate projection involves a number of criteria. An understanding of the different projection families,

along with typical **patterns of deformation** (the distribution of distortion over a projection) is one important aspect of this selection process. Although there is great diversity among projections, even within the projection families mentioned here, similarities in construction and appearance yield enough common elements to classify them into a few groups. The approach presented here is a conventional one.

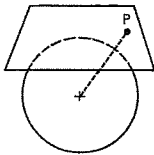
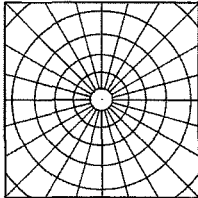
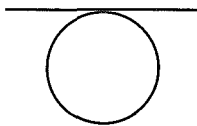
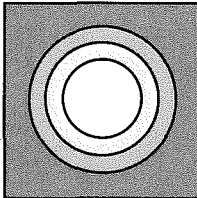
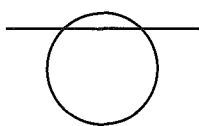
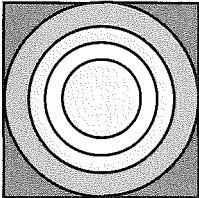
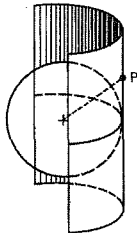
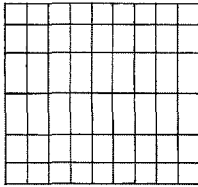
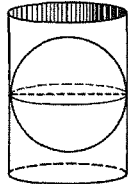
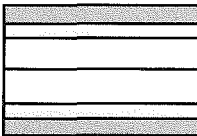
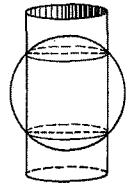

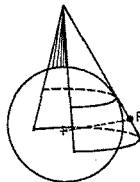
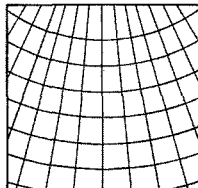
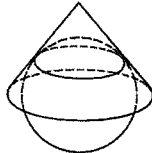
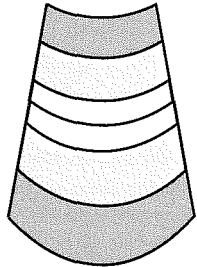
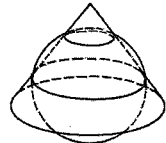
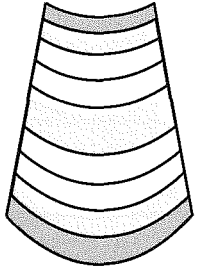
### Azimuthal Family

In the *azimuthal* or *planar* family, the spherical grid is projected onto a plane. This plane can be tangential to the sphere at a standard point (tangent or simple case; see Figure 3.1a), or pass through the sphere, making it intersect along its standard line, which is actually a **small circle** (secant case, see Figure 3.2a). A small circle is the circle that results when a plane intersects the Earth but does not go through its center, effectively dividing the Earth in two unequal parts. Patterns of deformation begin to emerge for the azimuthal class (see Figure 3.4a). Deformation increases with distance from either the standard point (tangent case) or the standard line (secant case). As with all projection families, scale distortion, and hence deformation, is nonexistent at standard points or lines. In the example shown in Figure 3.4a, the deformation increases outward in a series of concentric bands. In the secant case, however, there is also some distortion that occurs toward the center.

The plane may be tangent, of course, at any point on the spherical grid, depending on the projection aspect. Tangency at the pole is a *polar aspect*; at mid-latitude, an *oblique aspect*; and at the equator, an *equatorial aspect* (see Figure 3.5). Normal aspect for this family is the polar position when the plane is tangent at one of the poles, since it produces the simplest graticule. In this case, the meridians are straight lines intersecting the pole, and parallels are concentric circles having the pole as their centers (see Figure 3.4a). Directions to any point from the point of tangency (pole) are held true. All lines drawn to the center are **great circles** (circles that result when a plane bisects the Earth into two equal halves), as is also the case for equatorial and oblique aspects.

Normally, only one hemisphere is shown at a time on these projections. There are a few exceptions, however, in which the entire world is portrayed, but the grid departs radically from what we are accustomed to seeing. Azimuthal (also called *zenithal*) projections became quite popular during World War II, when there was considerable circumpolar air navigation; they have remained so today especially for polar mapping and other applications.

**Light Source Variations.** We mentioned earlier that the hypothetical light source can be from positions other than the center of the globe. Adjustments in light sources are most common in azimuthal projections, although they can occur in other families as well. Figure 3.6 depicts three primary positions for the light source. If the light is emanating from

Family	Grid appearance	Tangent		Secant
Normal aspect				
<div></div> <div>Azimuthal (a)</div>		<div></div> <div></div>	<div></div> <div></div>	
<div></div> <div>Cylindrical (b)</div>		<div></div> <div></div>	<div></div> <div></div>	
<div></div> <div>Conic (c)</div>		<div></div> <div></div>	<div></div> <div></div>	

**FIGURE 3.4** PROJECTION FAMILIES AND PATTERNS OF DEFORMATION.

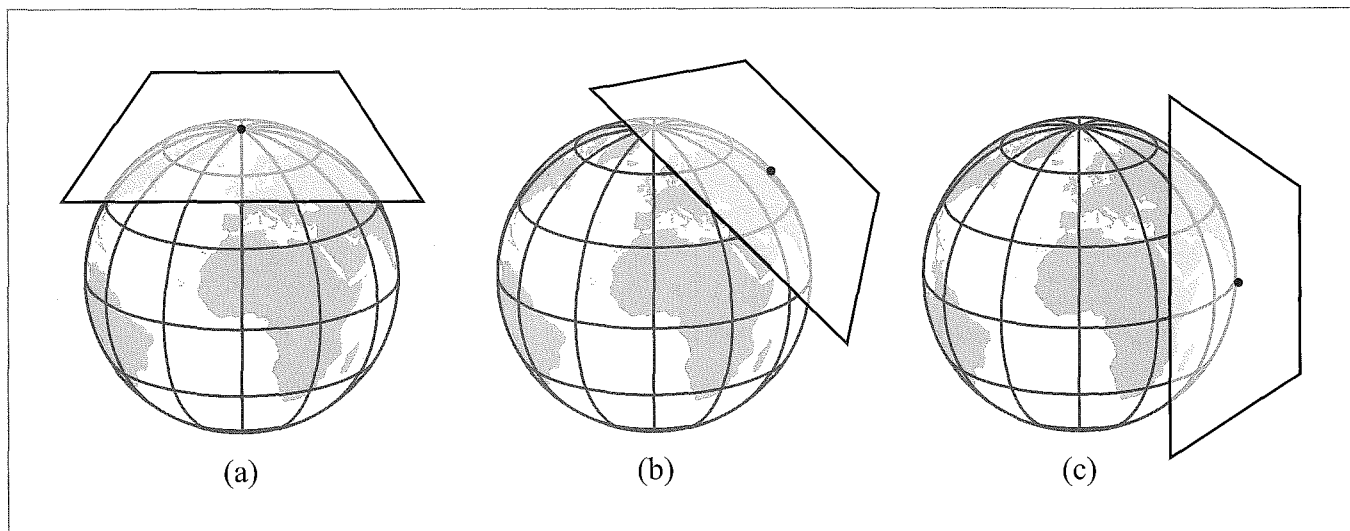
The patterns are clearly related to the way the projection is devised for the azimuthal (a), cylindrical (b), and conic (c) projections. Darker areas represent greater deformation.

the center of the globe, it is a **gnomonic** projection. If the light source is at the point opposite the point of tangency (or antipode position), the projection is **stereographic**; if it is at a theoretical infinity (outside the generating globe, producing parallel light rays), an **orthographic** projection results. An examination of the graticule and continental features

reveals the different deformation patterns that result due to changes in the light source positioning.

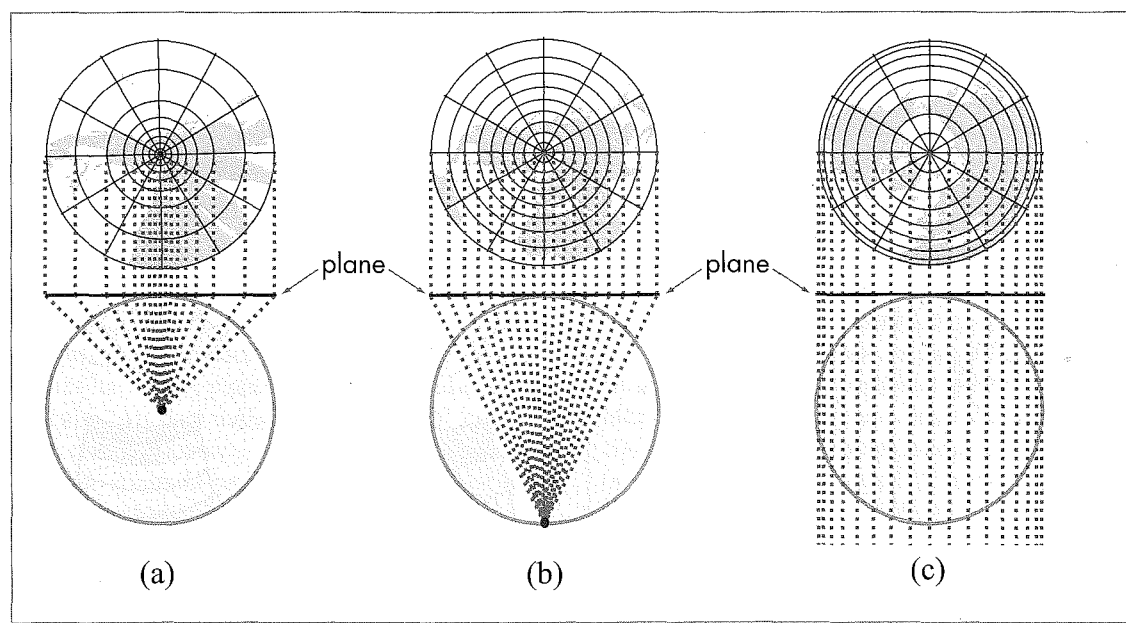
### Cylindrical Family

Cylindrical or rectangular projections are common forms. They are sometimes seen in atlases and other maps



**FIGURE 3.5** COMMON PROJECTION ASPECTS FOR AZIMUTHAL MAPPING.

The polar (a), oblique (b), and equatorial (c) aspects are all common in azimuthal mapping.



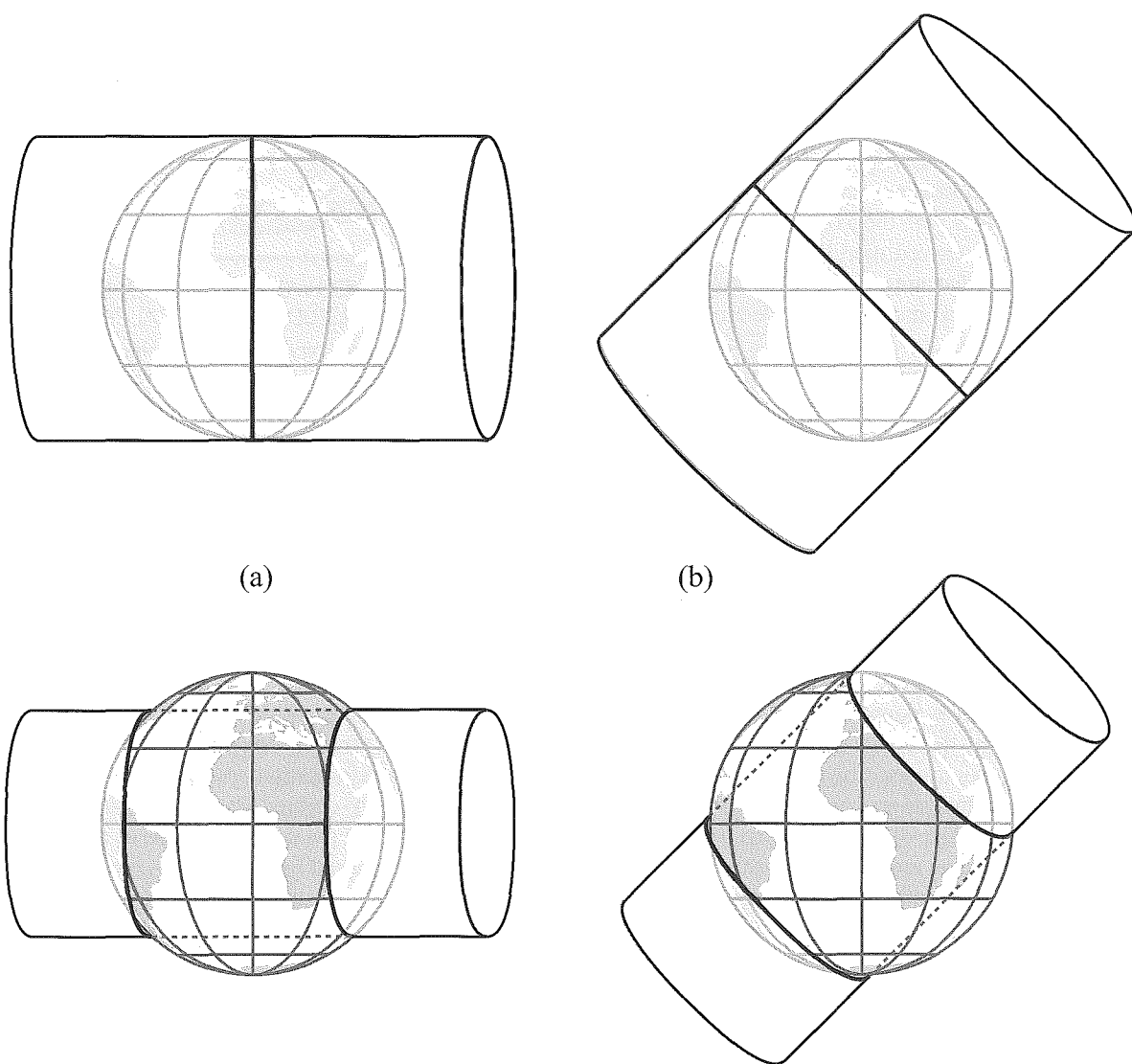
**FIGURE 3.6** SOURCE OF ILLUMINATION VARIATIONS OF AZIMUTHAL MAPPING.

The light source and resulting graticule for the gnomonic (a), stereographic, (b), and orthographic (c) projections.

portraying the whole world, but are perhaps more typically used in medium- and large-scale mapping. The applications of such projections will be discussed in more detail later. They are developed (graphically or mathematically) by wrapping a flat plane or sheet into a cylinder and making it tangent along a line or intersecting two lines (secant case) on the sphere. Points on the spherical grid can be transferred to this cylinder, which is then “un-rolled” into a flat map. The result is a rectangle-shaped

map with parallels and meridians that intersect at  $90^\circ$  angles (see Figure 3.4b).

The normal aspect for these projections is the equatorial aspect, as shown in Figure 3.1b. In the tangent case, the equator is the standard parallel. Here too the standard line is also a great circle. In the secant case, the two standard parallels (*small circles*) will be located north and south of the equator (see also Figure 3.2b). The patterns of deformation in this case are not surprising; areas of least distortion are



**FIGURE 3.7** COMMON ALTERNATIVE PROJECTION ASPECTS FOR CYLINDRICAL MAPPING.

The transverse (a) and oblique (b) aspects in both tangent (above) and secant (below) cases are very common aspect forms for cylindrical mapping.

bands parallel to the standard parallel(s), with increasing exaggeration toward the outer edges of the map plane (see Figure 3.4b). Note that in this aspect, the scale preservation is in the east-west direction, parallel to the axis of the cylinder, and distortion progressively increases in a poleward direction.

Two other common aspects of this projection are the *transverse* (polar) and *oblique* cases (see Figure 3.7). Transverse means that the axis of the cylinder is turned parallel to the equator. In the tangent case, the standard parallel has now become a standard meridian (Figure 3.7a), and is thus preserving the least scale deformation in the north-south

direction. In the secant case of the transverse cylindrical, the small circles produce two standard lines. Not surprisingly, this aspect is popular for enumeration units with elongated north-south directions.

In the oblique aspect, the cylinder is placed at any other position on the globe. It is often placed in such a way that the standard line(s) (a great circle in the tangent case and two small circles in the secant case) are at or near the area that is to be mapped. Remember that scale distortion increases as you move away from any standard lines. Political entities that are elongated in a manner other than a north-south direction, such as Japan, which has a pronounced southwest-northeast

alignment, can be successfully mapped using an oblique cylindrical projection.

### Conic Family

Conic projections are constructed by transferring the graticule from the generating globe to a cone enveloped around the sphere. This cone is then unrolled into a flat plane. The normal aspect is shown in Figure 3.1c. In this aspect, the axis of the cone coincides with the axis of the sphere, which yields either straight or curved meridians that converge on the pole and parallels that are arcs of circles (see Figure 3.4c). With few exceptions, most conic projections are presented in their normal aspect.

In the tangent case of the conic projection, sometimes called the *simple* conic projection, the cone is tangent along a chosen parallel, along which there is no distortion, as illustrated in Figure 3.1c. In the secant case, the cone intersects the globe along two parallels (see Figure 3.2c). This reduces distortion.

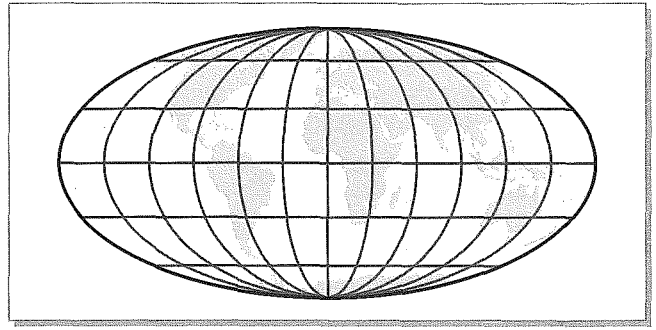
The pattern of deformation includes concentric bands parallel to the standard parallels of the projection (see Figure 3.4c). Secant conics tend to compress scale in areas between the standard lines and to exaggerate scale elsewhere. Conic projections, tangent or secant, are best for mapping Earth areas having greater east-west extent (by the standard parallel[s]) than north-south extent.

One historic form of the conic family is the polyconic projection. This projection uses several cones of development and consequently has several standard lines. Theoretically, each parallel is the base of a tangent cone. Historically, it has been used for mapping areas of great latitudinal extent; the polyconic projection was used by the United States Geological Survey (USGS) in its topographic mapping program until the 1950s.

### Mathematical Family

The purely mathematical projections (those that cannot be developed by projective geometry) are in some cartographers' taxonomies simply classified into the geometric families on the basis of their appearances. A few projections bear striking resemblance to the developable ones but are different enough to be classed as pseudocylindrical, pseudoconic, and pseudoazimuthal. Pseudocylindrical projections are perhaps the most common in the mathematical family, with their meridians that curve toward the poles. A prominent example can be found in the **Mollweide projection** (see Figure 3.8). Other common mathematical projections that will be shown or discussed later in the chapter include the *Sinusoidal* and the *Hammer* projections.

Still others are so different that they are difficult to relate to the three geometric forms at all. As Strebe (2007) notes, sometimes the practice of distinguishing geometric projections from mathematically derived ones is not that important, since all projections can ultimately be mathematically described and are typically projected in a GIS or other mapping software. In the next section we consider the important



**FIGURE 3.8** THE MOLLWEIDE PROJECTION, ONE OF THE MORE COMMON PSEUDOCYLINDRICAL PROJECTIONS.

Note that the meridians curve toward the poles, in contrast to the grid-like appearance of the cylindrical family. See Figure 3.4b for a comparison.

topic of describing types of deformation by examining projection properties.

### Map Projection Properties

In the transformation process from the three-dimensional surface to a plane, some distortion occurs that cannot be completely eliminated. Although designers strive to develop the perfect map, free of error, *all* maps contain errors because of the transformation process, whether geometrically or mathematically derived. It is impossible to render the spherical surface of the reference globe as a flat map without distortion error caused by *tearing*, *shearing*, or *compression* of the surface (see Figure 3.9). The designer's task is to select the most appropriate projection so that there is a measure of control over the unwanted error.

These distortions and their consequences for the appearance of the map vary with scale. One can think of the globe as being made up of very small quadrilaterals. If each quadrilateral were extremely small, it would not differ significantly from a plane surface. For mapping small Earth areas (large-scale mapping), distortion is not a major design problem (although selection of datum can be very significant at larger scales; see Chapter 2 for discussion).

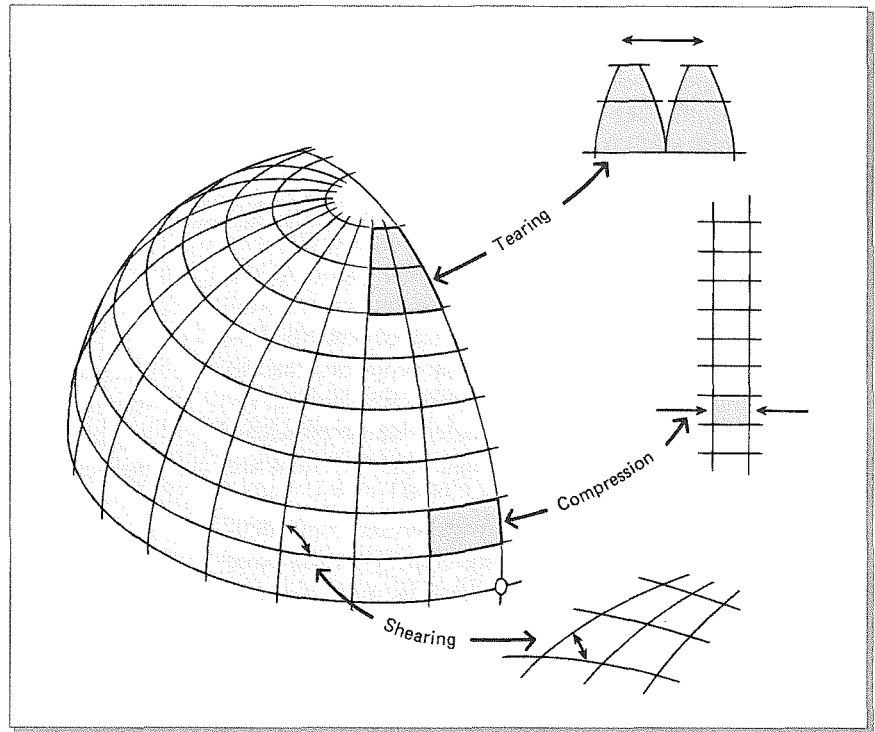
As the mapped area increases to subcontinental or continental proportions, distortion becomes an increasingly significant design problem for the cartographer. In designing maps to portray the whole Earth, surface distortions are maximized. At such scales, the map designer must contend with alterations of area, shape, distance, and direction, which are called a **projection's properties**. No projection of the globe's graticule can maintain all of these properties simultaneously; only on a globe are all four properties maintained.

### Equal Area Mapping

Map projections on which area relationships of all parts of the globe are maintained are called **equal area** (or **equivalent**) **map projections**. The intersections of meridians and

**FIGURE 3.9** DISTORTION CAUSED BY THE PROJECTION PROCESS.

Transformation from sphere to plane may cause distortion brought about by tearing, shearing, or compression.



parallels are not at right angles, but areas of similarly bounded quadrilaterals maintain correct area properties. Linear or distance distortion often occurs in such projections. On equal area projections, therefore, shape is often quite skewed.

Equivalent projections are very important for general quantitative thematic map work, particularly at a global scale. It is usually desirable to retain area properties, particularly when enumeration units are compared or if area is *part* of the data being mapped (see Chapter 4); population density (that is, persons per square mile) is a prime example. Consider Marschner's classic argument for employing equal area projections:

*It is hardly necessary to recapitulate here the arguments used in favor of equal area representation. There is no doubt but that the preservation of a true areal expression of maps, used as a basis for land economic investigation and research, is more important than a theoretical retention of angular values. Of the three geometrical elements that can be considered in mapping, the linear, angular, and areal, the last is the one around which land economic questions usually revolve. They do so for obvious reasons. Man does not inhabit a line, but occupies the area; he does not cultivate an angle of land, but cultivates and utilizes the land area. One of the principal functions of lines and angles is to define the boundary of the area. . . . They provide only the framework for controlling the relative position of features in the area, and with it a means for controlling the areal expression of the map itself (Marschner 1944, 44).*

Today many cartographers still agree that equivalent map projections can often be favorably selected for thematic

mapping, *all other considerations being equal*, particularly at smaller scales. Note that some equal area projections may have deformation patterns that may be unacceptable to the map reader (or the cartographer). Depending on the map scale and purpose, other properties may be deemed important. As we explore the other projection properties, and later in the chapter, assess appropriate thematic map projections for different regions or tasks, we will see cases in which equal area maps are not as common, even for a thematic map. Nonetheless, as a generalization, thematic cartography tends to favor the use of equal area projections.

### Conformal Mapping

Conformal (or orthomorphic) mapping of the sphere means that angles are preserved around points and that the shapes of small areas are preserved. The quality of conformality applies to small areas (theoretically only to points). On **conformal projections**, meridians intersect parallels at right angles, and the scale is the same in all directions about a point. Scale may change from point to point, however. It is misleading to think that shapes over large areas can be held true. Although shapes for small areas are maintained, the shapes of larger regions, such as continents, may be severely distorted. Areas are also distorted significantly at smaller scales.

The shape quality of mapped areas is an elusive element. If we view a continent on a globe so that our eyes are perpendicular to the globe at a point near the center of the continent, we see a shape of that continent. However, the shape of the continent is distorted because the globe's surface is falling away from the center point of our vision. We can view

but one point orthographically at a time. If we select another point, the view changes, and so does our perception of the continent's shape. It becomes difficult for us to compare the shapes we see on a map to those on the globe, and it is safe to say that shapes of large areas on conformal projections should be viewed with caution (Dent 1987).

For large-scale mapping of small Earth areas, distortion is not significant. Indeed, the choice between an equivalent or a conformal projection becomes somewhat moot. At the smallest scales, the selection of the projection is more critical in the design process. Even at these scales, however, it is seldom necessary to specify a conformal projection, except in rare circumstances. Mapping phenomena with circular radial patterns may warrant such a choice. Radio broadcast areas, seismic wave patterns, or average wind directions are possible examples.

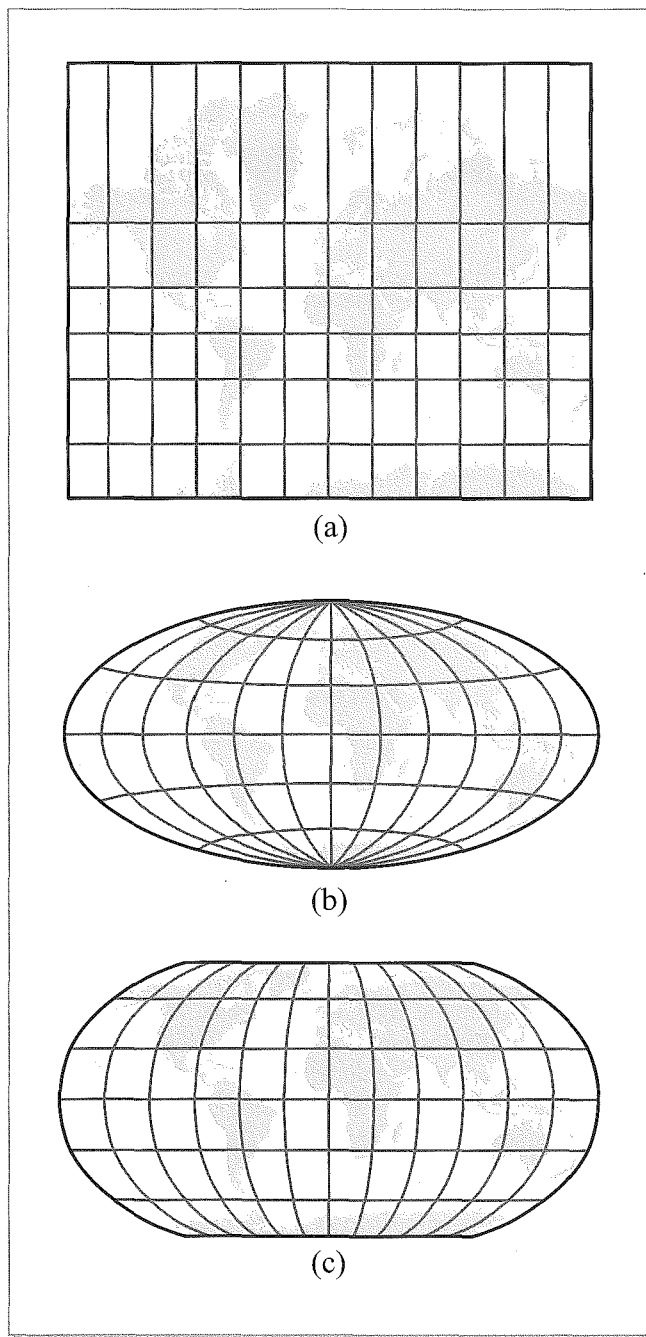
Most cartographers consider equivalence and conformality the two most important property considerations. Note that it is *impossible* for one projection to maintain both equivalency and conformality properties. Sometimes these properties are referred to as the *major* properties, because the properties can exist at all points on certain projections (Campbell 2001). Figure 3.10a and b illustrates the contrast between conformality and equivalence. The **Mercator projection** (3.10a) is the conformal map and distorts area, especially at the higher latitudes. This distortion is minimal near its standard parallel at the equator but increases greatly toward the poles. The **Hammer projection** (Figure 3.10b) is an equal area projection. The areas are equivalent, but conformality is nonexistent. Notice the shape distortion around its margin. The next two properties are considered *minor* properties, because the particular properties are not able to exist everywhere on the map.

### Equidistance Mapping

The property of equidistance on projections refers to the preservation of great circle distances. There are certain limitations: distance can be held true from one to all other points, or from a few points to others, but not from all points to all other points. The distance property is never global. Scale will be uniform along the lines whose distances are true. Projections that contain these properties are called **equidistant projections**. Equidistant projections are sometimes used in general purpose maps in atlases because such projections are neither conformal nor equal area, and often have less distorted-appearing landmasses.

### Azimuthal Mapping (Direction)

On **azimuthal projections**, true directions are shown from one central point to all other points. Directions or azimuths from the central point to other points are accurate, whereas from other (noncentral) points they are not. The quality of azimuthality is not an exclusive projection quality. It can occur with equivalency, conformality, and equidistance.



**FIGURE 3.10** CLASSIC EXAMPLES OF A CONFORMAL, AN EQUAL AREA, AND A COMPROMISE PROJECTION.

The Mercator projection (a) provides an illustration of conformality. The Hammer projection (b), sometimes also referred to as the Hammer-Aitoff projection, illustrates an equal area projection, and the Robinson projection (c) illustrates a compromise or minimum error projection. See text for discussion.

Table 3.1 illustrates projection properties that can be combined. For example, azimuthal and equidistance properties can occur together in a map projection (a “yes” in the table), but conformal and equal area properties cannot exist simultaneously in a map projection (a “no” in the table).



**TABLE 3.1** COMBINATIONS OF PROJECTION PROPERTIES THAT CAN OCCUR IN ONE PROJECTION

	Equal Area	Conformal	Equidistance	Azimuthal
<b>Equal Area</b>	—	no	no	yes
<b>Conformal</b>	no	—	no	yes
<b>Equidistance</b>	no	no	—	yes
<b>Azimuthal</b>	yes	yes	yes	—

Source: After Natural Resources Canada 2007.

### Minimum Error Projections

Over the years, some cartographers have stressed the idea that **minimum error projections** (also called *compromise projections*) are best suited for general geographic cartography. These projections are essentially hybrids that attempt to control or minimize all four map projection properties in varying degrees. Often this is done to try to produce a better and more realistic depiction of the globe or parts of it. These projections are chosen by the designer on much the same basis as a projection having a uniquely preserved property: by selection of those total qualities that best suit the mapping task (Canters 1989). Figure 3.10c depicts the **Robinson projection**, which holds no property as true. When compared with the conformal and equal area (major property) figures, it is easy to see why some designers advocate for projections such as these. Indeed, for a decade the Robinson projection was the official map projection used by the National Geographic Society for their world maps.

### Determining Deformation and its Distribution Over the Projection

Two chief methods are available for determining projection distortion and its distribution over the map. One is to depict a geometrical figure (square, triangle, or circle) or familiar object (such as a person's head) and plot it at several locations on the

projection graticule (see Figure 3.11). Distortion on the projection is readily apparent. This method is very effective and quite sufficient for most general cartographic analyses; its weakness is the lack of a general quantitative index of distortion.

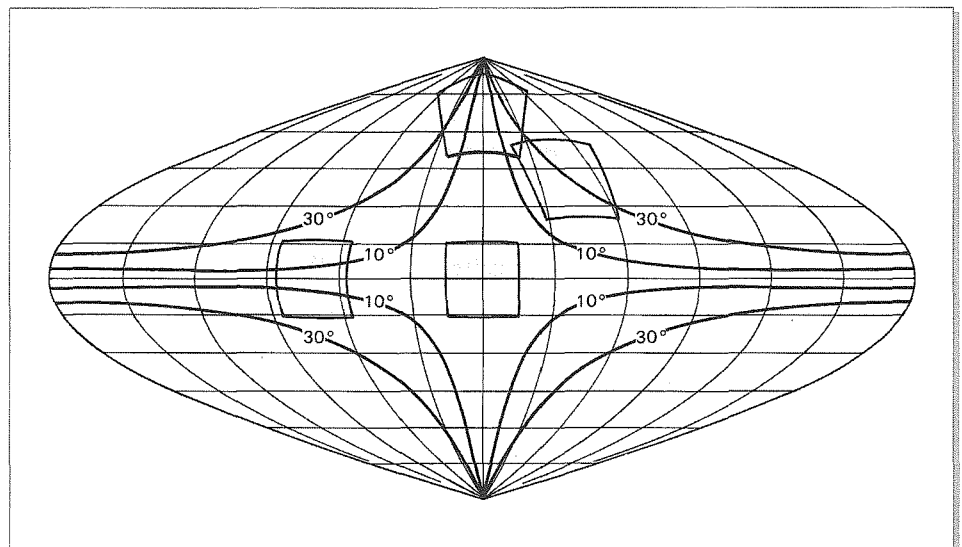
Another method, conceptually and mathematically more complex, uses **Tissot's indicatrix**. Tissot, a French mathematician working in the latter part of the nineteenth century, developed a way to show distortion at points on the projection graticule (Tissot 1881). Following his work, others have computed these indices and mapped them on several projections to show the patterns of distortion. The weakness of Tissot's method is that it is somewhat more complex mathematically than plotting simple geometrical shapes. Its strength lies in its quantitative ability to describe distortion.

The construct of Tissot's indicatrix consists of a very small circle, whose scale is unity (1.0), on the globe's surface. This small circle and two perpendicular radii appear on the plane map surface during transformation as a circle of the same size, a circle of different size, or an ellipse. For equal area mapping, the circle is transformed into an ellipse with the ratio of the new semimajor and semiminor axes such that its area remains the same (unity, or 1.0). Angular properties are not preserved (see Figure 3.12). On conformal projections, the small circle is transformed on the projection as a circle, although its size (area) varies over the map. Because the small circle is accurately projected as a circle, angular relationships are maintained. On some projections,

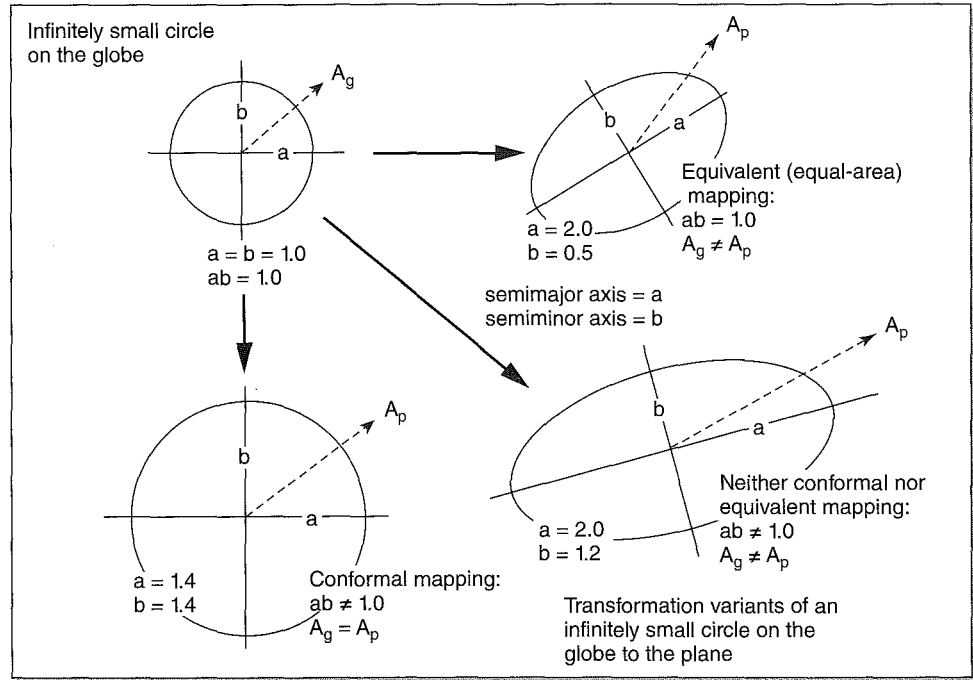
**FIGURE 3.11** SHAPE DISTORTION ON AN EQUAL AREA PROJECTION.

On the globe, the squares in the figure are all the same size. On this Sinusoidal projection, the angular relations (and hence shape) are not preserved. Notice that distortion occurs most severely at the edges of the projection, as shown by the plot of  $2\omega$  values ( $10^\circ$  and  $30^\circ$ ). See text for explanation.

Source: Squares drawn from data in Chamberlin 1947, 97.



**FIGURE 3.12** MAJOR AND MINOR AXES IN TISSOT'S INDICATRIX.



the small circle becomes an ellipse that preserves neither equivalency nor conformality. Such projections are not classified as either equivalent or conformal.

For purposes of computation and explanation, the indicatrix has these qualities:

- Maximum angular distortion =  $2\omega$
- Scale along the ellipse semimajor axis =  $a$
- Scale along the ellipse semiminor axis =  $b$
- Maximum areal distortion =  $S$

Every point on the new surface (map) has values computed for  $2\omega$ ,  $a$ ,  $b$ , and  $S$ . These conditions inform us of the distortion characteristics:

- If  $S = 1.0$ , there is no areal distortion, and the projection is equal area. If  $S = 3.0$ , for example, *local* features are three times their proper areal size.
- If  $a = b$  at all points on the map, it is a conformal projection.  $S$  varies, of course.
- If  $a \neq b$ , it is not conformal, and the amount of angular distortion is represented by  $2\omega$ .

It should be pointed out that no distortion occurs on standard lines, and values of the indicatrix need not be computed along these lines.

Values of  $S$  or  $2\omega$  are usually mapped on the projection graticules to illustrate the patterns of distortion. Values of  $2\omega$  have been plotted on the **Sinusoidal projection** (a mathematical equal area projection) in Figure 3.11.

### Standard Lines and Points, Scale Factor

As noted earlier in the chapter, standard lines are lines (usually meridians or parallels, especially the equator) on a

projection that have identical dimensions to their corresponding lines on the reference globe. For example, if the circumference of the reference globe is 15 inches, the equator on the projection is drawn 15 inches long if it is a standard line. Scale can be thought to exist at points, so every point along a standard line has an unchanging or true scale when compared to the scale of the generating globe. On standard lines, scales are true. At all points on a standard line, using the indicatrix as a guide,  $a = b = 1.0$ ,  $S = 1.0$ , and  $2\omega = 0^\circ$  (see Figure 3.13).

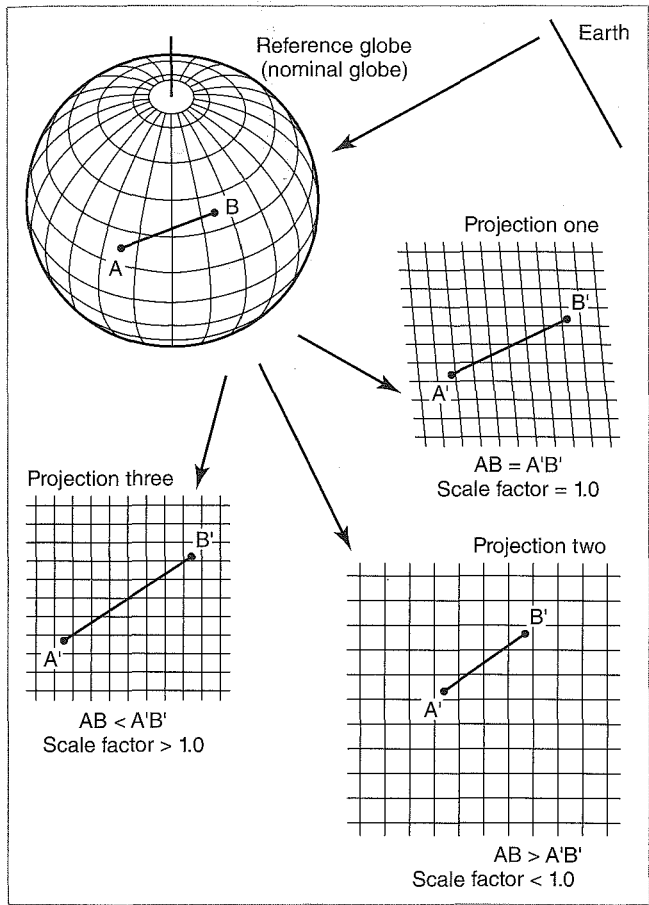
In practice, the lines on the projection can be compared to their corresponding lines on the reference globe by a ratio called **scale factor** (S.F.):

$$\frac{\text{projection scale fraction}}{\text{nominal scale fraction}}$$

In this equation, nominal scale is the scale of the reference globe. If the scale of a reference globe is 1:3,000,000 and the equator on the projection is drafted as a standard line, then

$$\begin{aligned} \frac{\text{projection scale fraction}}{\text{nominal scale fraction}} &= \frac{1}{3,000,000} \\ &= \frac{3,000,000}{3,000,000} \\ &= 1.0 \text{ (S.F.)} \end{aligned}$$

The scale factor of the equator will have a value of 1.0. For another example, suppose the nominal scale is



**FIGURE 3.13** VARIATIONS OF THE STANDARD LINE AND THE DETERMINATION OF SCALE FACTOR.

1:3,000,000 and a line on the projection has a linear scale of 1:1,500,000:

$$\begin{aligned} \frac{\text{projection scale fraction}}{\text{nominal scale fraction}} &= \frac{1}{1,500,000} \\ &= \frac{3,000,000}{1,500,000} \\ &= 2.0 \text{ (S.F.)} \end{aligned}$$

In this example, the line is two times *longer* on the map than it should be, because of stretching. In our final example, the nominal scale is 1:3,000,000 and a line on the projection is drafted at 1:6,000,000:

$$\begin{aligned} \frac{\text{projection scale fraction}}{\text{nominal scale fraction}} &= \frac{1}{6,000,000} \\ &= \frac{3,000,000}{6,000,000} \\ &= 0.5 \text{ (S.F.)} \end{aligned}$$

Here we find the line to be one-half as long as its corresponding line on the globe. *Compression* has taken place in the transformation process.

Standard lines and scale factors are important to the overall understanding of projection distortion. The idea of scale factor can also be used in the assessment of areal scales. In this instance, small quadrilateral areas on the projection can be compared to the corresponding areas on the globe to determine the amount of *areal exaggeration* occurring on the projection. The bottom line is that the least distortion can be achieved not only by selecting the correct projection for the area being studied, but to the extent possible, having the standard points and lines run through the area of interest will produce a more accurate result.

Before we leave this section it is good to note that expressions of scale (for example verbal, graphic, or bar scale, or representative fraction), while often desirable as one of the basic elements of the map (see Chapters 1, 2, and 13 for a discussion of scale as a basic map element), are intrinsically inaccurate for world maps. They will be accurate only in places where there are standard points and lines. The wise designer will generally avoid an expression of scale on thematic world maps, unless there is a pressing need for the scale. If this is the case, a statement of where scale is accurate should accompany the expression of scale.

## EMPLOYMENT OF MAP PROJECTIONS

In the previous section we focused on the map projection process. Considering all of the projection parameters that can be adjusted (geometric figure, aspect, standard points and lines, central meridian, and so on), there is a theoretically infinite number of potential map projections. There are hundreds or even thousands of named projections that currently exist. Indeed, most software today is able to work with many of the most common or popular projections.

In this section we provide some guidelines for selecting a projection from the myriads that are in existence, as well as survey a few of the most common map projections used for regions at different scales. These include ones that are commonly in use today or have had a major influence on modern cartography—even if it is a projection that we do not necessarily recommend for thematic cartography. Of course, no projection survey or list will satisfy every cartographic need or expectation. From an historical perspective, we recommend John Snyder's *Flattening the Earth: 2000 Years of Map Projections* (Snyder 1993) for a comprehensive look at many different map projections. From an applied perspective, we recommend examining the projections that are supported in your GIS or mapping software. Read about them in your software's help section, or better yet, take the (usually provided) world map data (or maps at other scales) and start projecting them. As you examine the projections visually, also try to examine the various projection properties to the extent that your software allows. If

you do not have dedicated GIS or mapping software, there are also several web pages that provide a fairly extensive gallery of map projections. Some of these sites provide a link to free or low-cost projection software that can easily be downloaded to your computer (Anderson 2007; Geocart 2007).

## Essential Questions

Several essential elements must be carefully considered in the selection of a map projection:

1. *Projection properties.* Are the properties of a particular projection suited to the design problem at hand? Are equivalency, conformality, equidistance, or azimuthality needed?
2. *Deformational patterns.* Are the deformational aspects of the projection acceptable for the mapped area? Is linear scale and its variation over the projection within the limits specified in the design goals? Do the characteristics of linear scale over the projection benefit the shape of the mapped area?
3. *Projection center.* Can the projection be centered easily for the design problem? Can the software accommodate experimentation with the re-centering of the projection, such as an adjustment of the projection's central meridian?
4. *Familiarity.* Will the projection and the appearance of its meridians and parallels be familiar to most readers? Will the form of the graticule detract from the main purpose of the map?
5. *Software support.* Is the particular projection supported in your software?
6. *Part of an existing map series or online digital map collection.* Does the map belong to a series that already has a projection? Do you want to continue to match that projection (and especially at larger scales, the datum and coordinate system)?

Again, there are literally hundreds of projections from which the cartographer may choose. Certain ones have proven more useful in mapping particular places (see Table 3.2). The ones included in our discussion are offered only as a guide.

## World Projections

Many world projections may be selected for thematic mapping. World map projection selection can be both debatable and subjective at times. In some cases, they are the most controversial. We have grouped several types of world maps for convenience.

### Mathematical, Equivalent Projections

As suggested earlier, the property of equivalency is often the overriding concern in thematic cartography. Two mathematically derived, pseudocylindrical equal area projections are presented here as good choices when depicting the entire

world on one map. These are the Mollweide (or homolographic) projection mentioned earlier (see Figure 3.8), and the Hammer projection (see Figure 3.10b). Note that no world equal area projection on a single sheet can avoid considerable shape distortion, especially along the peripheries of the map.

The Mollweide projection, named after Carl B. Mollweide who developed it in 1805 (Environmental Systems Research Institute 2007), is widely used for mapping world distributions. Its standard parallels are 40° 44' N and S. The central meridian is one-half the length of the equator and drawn perpendicular to it. Parallels are straight lines parallel to the equator but are not drawn with lengths true to scale, except for the standard parallels. Each parallel, however, is divided equally along its length. The parallels are spaced along the central meridian to achieve equivalency. The elliptical shape of the projection gives a kind of global feel to the projection, which some designers find pleasing as long as one can accept the distortion along the peripheries.

Very similar to the Mollweide is the Hammer projection. The Hammer projection, developed in Germany in 1892, was for many years erroneously called the Aitoff projection (Steers 1962). Nonetheless some software packages refer to this projection as the Hammer-Aitoff projection. The principal difference between this projection and the Mollweide is that the Hammer has curved parallels. This curvature results in less oblique intersections of meridians and parallels at the extremities, and thus reduces shape distortions in these areas. The outline (that is, the ellipses forming the outermost meridians) is identical to the Mollweide.

Hammer's projection also is quite acceptable for mapping world distributions. A comparison of it with the Mollweide shows little difference. Because the parallels are curved, east-west exaggeration at the poles is less on the Hammer than on the Mollweide. This is most notable when comparing the Antarctica landmasses. Africa is less stretched along the north-south axis on the Hammer. Overall, however, these projections are very similar in appearance and attributes.

Also worth mentioning is the Sinusoidal equal area projection, also called the Sanson-Flamsteed projection (see Figure 3.11). This pseudocylindrical projection, with its straight parallels and curved meridians, was popular for mapping worldwide thematic distributions. The top-like shape of the outline makes it distinctive, although there are other similar projections. We have seen a slight falling off of its use in the twenty-first century. The unique shape could be a distraction to the thematic symbolization. There is also a rather extreme shape compression of the polar features, particularly at the North Pole. Cartographers seem to be favoring more elliptically shaped projections such as Mollweide or Hammer when equal area mathematical projections are desired.

*Interrupted Projections.* One solution to minimize distortion is to make an interrupted projection of the world. Many of the previously mentioned world projections can be turned into an interrupted projection. The most famous of the interrupted projections is perhaps the **Goode's Homolosine projection**, a

**TABLE 3.2** GUIDE TO THE EMPLOYMENT OF PROJECTIONS FOR WORLD-, CONTINENTAL-, AND COUNTRY-SCALE THEMATIC MAPS

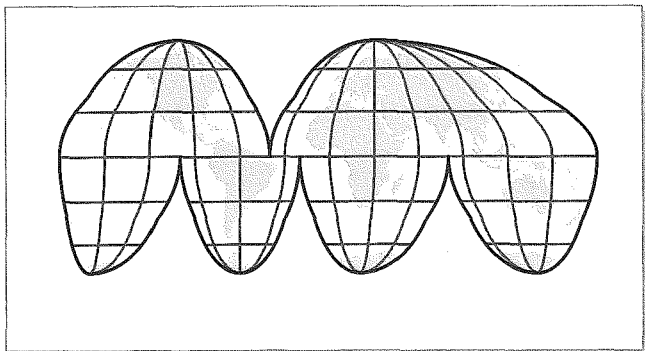
Principal Use	Suitable Projections	Notes
<b>1. Maps of the world</b>		
Equal area	Sinusoidal (Sanson-Flamsteed)	Awkard shape
Equal area	Mollweide	Pleasing shape
Equal area	Hammer	Sometimes called Hammer-Aitoff in software
Compromise	Robinson	Pleasing shape, balances extremes
Compromise	Winkel Tripel	May be most accurate compromise
<b>2. Continental areas</b>		
A. Asia and North America		
Equal area	Bonne*	Considerable distortion in NE and NW corners Bearings true from center
Equal area	Lambert Azimuthal Equal Area	
B. Europe and Australia		
Equal area	Lambert Azimuthal Equal Area*	
	Bonne*	
	Albers Equal Area Conic; ideal for United States	
Conformal	Lambert Conformal Conic	
C. Africa and South America		
Equal area	Lambert Azimuthal Equal Area*	
Equal area	Mollweide*	
Equal area	Sinusoidal*	
Equal area	Homolosine*	
<b>3. Large countries in mid-latitudes</b>		
A. United States, Russia, China		
Equal area	Lambert Azimuthal*	
Equal area	Albers Equal Area Conic	
Equal area	Bonne*	
Conformal	Lambert Conformal Conic	
<b>4. Small countries in mid-latitudes</b>		
Equal area	Albers Equal Area*	
Equal area	Bonne*	
Equal area	Lambert Azimuthal*	
Conformal	Lambert Conformal Conic*	
<b>5. Polar regions</b>		
Equal area	Lambert Azimuthal	
<b>6. Hemispheres and continents</b>		
Visual	Orthographic	View of Earth as if from space; neither equal area nor conformal
*Must take special care to scale (zoom in) and re-center projection parameters to the particular area of interest.		
Sources: Compiled from a variety of sources listed in the references, especially Raisz 1962; Steers 1962; Snyder 1987; Dana 1999; and Environmental Systems Research Institute 2007.		

pseudocylindrical equal area projection created by J. Paul Goode (see Figure 3.14). Goode, the founder of the *Goode's World Atlas* took portions of the Mollweide and Sinusoidal projections and combined them in a manner that created six distinct lobes. Each lobe has its own central meridian, and the distortion that would occur in a continuous projection now is directed over water bodies (there are also versions that keep the water bodies intact and pull apart the land areas). Shape is not as distorted as some of the equal area projections that we have seen so far. The downside to Goode's and other interrupted projections like this is threefold. First, as with the

Sinusoidal projection, the shape of the projection can be distracting. Second, the projection requires the map reader to understand that the gaps along a line of latitude do not exist. Third, if the thematic phenomena to be mapped are spatially continuous, such as climate data, then the projection is wrong for the thematic map type.

### Minimum Error Projections

A number of cartographers strongly believe that minimum error or compromise projections (projections that do not



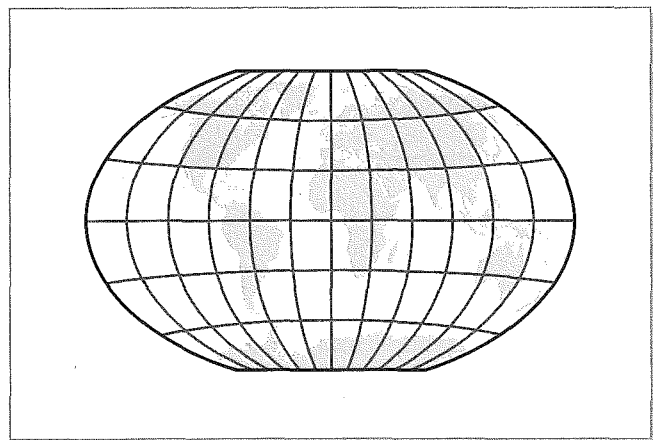
**FIGURE 3.14** THE INTERRUPTED GOODE'S HOMOLOSIONE PROJECTION.

Perhaps the most famous of the interrupted projections, this was used in the Goode's World Atlas series.

retain any specific property but try to minimize the worst distortions) are the best way to go. Two projections that we feel are also quite acceptable for world thematic mapping are the Robinson projection mentioned earlier (see Figure 3.10c) and the **Winkel Tripel projection** (see Figure 3.15).

The Robinson projection was developed in 1961 by Arthur Robinson for Rand McNally's world maps (Robinson *et al.* 1995). It is more famous, however, as being adopted in 1988 by the National Geographic Society for use as a world projection. It was replaced in 1998 by another minimum error projection, the Winkel Tripel projection.

The Winkel Tripel projection was developed in 1921 by Oswald Winkel (Environmental Systems Research Institute 2007). It has slightly less compression in the polar regions than does the Robinson projection. Physicists Goldberg and Gott (2007) have shown quantitatively, using Tissot's indicatrix and other measures, that the Winkel Tripel may be the best (lowest distortion) compromise projection yet.



**FIGURE 3.15** THE WINKEL TRIPLE PROJECTION.

Compromise (minimum error) projection used by the National Geographic Society for world maps.

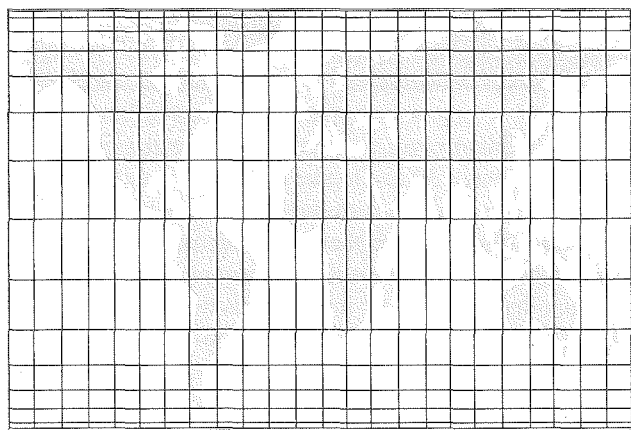
## Cylindrical Projections

This last category in World Projections is more of an acknowledgement that many cylindrical (also called rectangular) projections have been produced and used throughout cartographic history rather than a recommendation or endorsement for their use in world thematic mapping. The most famous projection in this category is doubtlessly the Mercator projection (see Figure 3.10a).

The Mercator projection was developed by Gerardus Mercator in 1569. It was developed for navigational use. All lines of constant compass bearings, called loxodromes or rhumb lines, are shown as straight lines. Navigators could therefore follow a compass heading using these maps (Snyder 1993). Over time, however, the map became popularized as a general reference map of the world. The problem with this use is that landmass areas such as Greenland and Russia (and Antarctica on those Mercator maps that include that area) are extremely distorted in the higher latitudes. Of course, as seen in some of the previous maps in this section, professional cartographers have provided alternatives for years, but the Mercator's popularity as a world projection did not start to wane until the latter half of the twentieth century. It is exactly this resilience of the projection as a world map and the fear that it had imbued a distorted "mental map" in the minds of many people that caused a fairly major controversy in the cartographic community.

**Gall-Peters Projection.** Special consideration is devoted here to what has become known as the **Gall-Peters projection**, primarily to stimulate the cartographic designer to look further into the literature, and partly because of the controversy surrounding its use (see Figure 3.16). In recent times, probably no other map projection has received as much attention in both the scientific and popular literature.

In 1972 Dr. Arno Peters of Germany published what he called a new map projection—the Peters projection (Robinson 1985; Snyder 1988). In fact, this projection had been devised earlier (mid-1880s) by Gall who called it the



**FIGURE 3.16** THE GALL-PETERS PROJECTION.

## RESOLUTION REGARDING THE USE OF RECTANGULAR WORLD MAPS

WHEREAS, the Earth is round with a coordinate system composed entirely of circles, and

WHEREAS, flat world maps are more useful than globe maps, but flattening the globe surface necessarily greatly changes the appearance of the Earth's features and coordinate system, and

WHEREAS, world maps have a powerful and lasting effect on people's impressions of the shapes and sizes of lands and seas, their arrangement, and the nature of the coordinate system, and

WHEREAS, frequently seeing a greatly distorted map tends to make it "look right,"

THEREFORE, we strongly urge book and map publishers, the media, and government agencies to cease using rectangular world maps for general purposes or artistic displays. Such maps promote serious, erroneous conceptions by severely distorting large sections of the world, by showing the round Earth as

having straight edges and sharp corners, by representing most distances and direct routes incorrectly, and by portraying the circular coordinate system as a squared grid. The most widely displayed rectangular world map is the Mercator (in fact a navigational diagram devised for nautical charts), but other rectangular world maps proposed as replacements for the Mercator also display a greatly distorted image of the spherical Earth.

American Cartographic Association  
 American Geographical Society  
 Association of American Geographers  
 Canadian Cartographic Association  
 National Council for Geographic Education  
 National Geographic Society  
 Special Libraries Association, Geography and Map Division

Source: Committee on Map Projections 1989, 223.

orthographic—a form of cylindrical projection. (It may be speculated that Peters had no knowledge of Gall's previous work) (Loxton 1985). In the conventional aspect, the Peters projection is an equal area rectangular projection with standard parallels at 45° North and South. Because the projection is not new, it is referred to as the Gall-Peters projection.

The purpose behind the invention was to counter the Mercator projection's areal exaggeration of the high latitudes and its pervasive use as a general purpose map. Peters believed that the Mercator projection accentuated European dominance over the Third World (Faintick 1986), because the projection's high latitude exaggeration visually minimizes countries located in the tropics. As is evidenced in Figure 3.16, the Gall-Peters projection emphasizes many portions of the tropics dominated by less developed countries. Because of the prevalence of the Mercator projection, Peters stressed that his projection be used exclusively. He also claimed that it portrays distances accurately, which is false (Snyder 1993; Monmonier 2004).

Proponents of the projection argue that the realigning of perceived areas and shapes shakes up our preconceived notions of Third World areas. Others argue that the shape distortion is too great to be of much utility (especially with other available mapping options); it especially distorts shape in the very underdeveloped areas that Peters was trying to portray more fairly (Stocking 2005). The real objection to Peters, however, is with the misconceptions he renders about the projection (the claim that Peters is the only projection that should be used, and that all distances on the projection can be measured correctly). Nonetheless, it has been widely adopted by three United Nations organizations (UNESCO, UNDP, and UNICEF), the National Council of Churches, and Lutheran and Methodist organizations (Loxton 1985).

One good thing has come from the controversy, namely, that the inadequacies of using the Mercator projection for world thematic mapping have finally been noted to the general population. Snyder, for example, has said:

*Nevertheless, Peters and Kaiser (his agent) appear to have successfully accomplished a feat that most cartographers only dream of achieving. Professional mapmakers have been wringing their hands for decades about the misuse of the Mercator Projection, but, as Peters stresses, the Mercator is still widely misused by school teachers, television news broadcasters and others. At least Peters' supporters are rightly communicating the fact that the Mercator should not be used for geographical purposes, and numerous cartographers agree. The Gall-Peters Projection does show many people that there is another way of depicting the world (Snyder 1988, 192).*

One final comment may be made regarding the selection of projections for world mapping. Concern about the use of rectangular projections for world presentations reached the attention of the Committee on Map Projections of the American Cartographic Association, and the resolution it passed has the endorsement of most professional geography and cartography associations (see boxed text). They do not endorse the use of any rectangular world projection. Note that some cartographers feel that this may be an overreaction to the Peters projection. Nonetheless, none of the world projections recommended in this section are of the rectangular type; all have rounded or more globe-like margins.

### Projections for Mapping Continents

There are a number of projections that can be used for mapping continental areas. Two that can be of great utility are

the **Lambert Azimuthal Equal Area projection** and the **Bonne projection**, a favorite in earlier versions of this text. As with most of the projections discussed so far, both of these projections are well supported in GIS and mapping software packages.

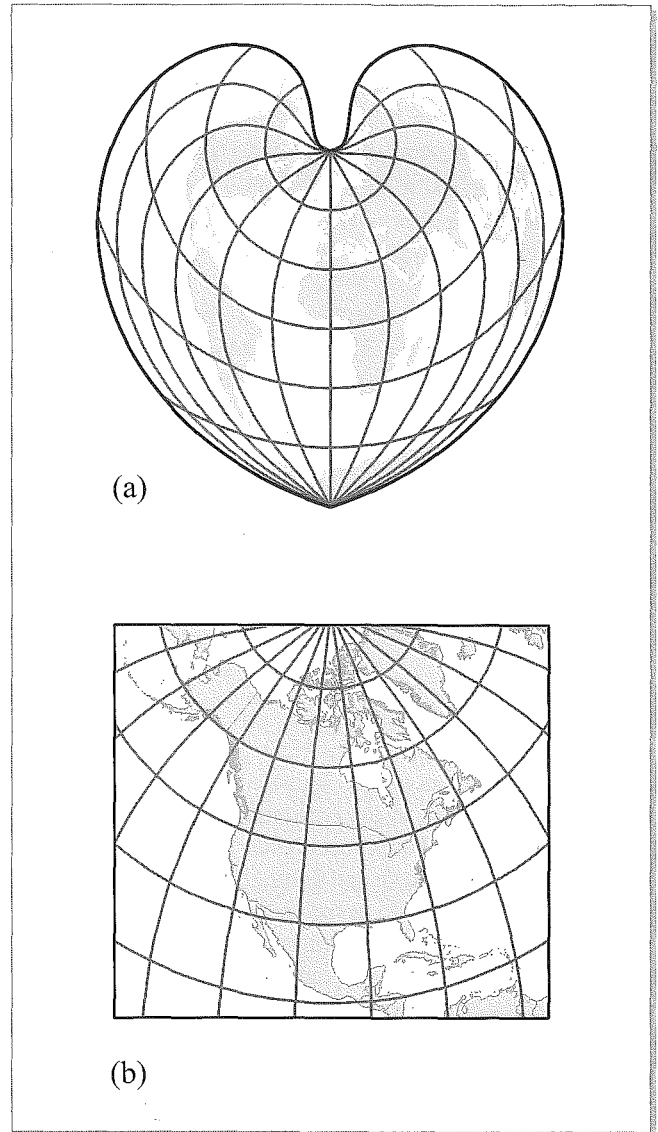
The Lambert Azimuthal Equal Area projection is a versatile projection. In its equatorial aspect, this may be one of the best choices for mapping a hemisphere, and certainly a continent. The fact that its standard point can be placed anywhere (and most GIS and mapping software support oblique positioning for this projection) make it a truly useful projection for mapping continents (and countries, as we will see in a moment; see Figure 3.17). Since the distortion is radial outward from the center of the projection, it is critical that the latitude and longitude of the standard point be placed at the center of the continent or other area of interest.

The **Bonne projection** is named after its inventor, Rigobert Bonne (1727–1795) (Raisz 1962). It is an equal area conical projection, with a central meridian and the cone assumed tangent to a standard parallel (see Figure 3.18). All parallels are concentric circles, with the center of the standard parallel at the apex of the cone. Scale is true at the central meridian and also for each parallel (Environmental Systems Research Institute 2007). If the standard parallel selected is the equator, the projection becomes identical to the Sinusoidal. Map designers select the Bonne projection for a variety of continental mapping cases. It is commonly used to map Asia, North America, South America, Australia, and other large areas. Europe may



**FIGURE 3.17** LAMBERT'S AZIMUTHAL EQUAL AREA PROJECTION, OBLIQUE ASPECT.

This projection can be an excellent choice for continents and countries that extend in all directions. The projection parameters for this particular map of Europe has the central meridian set at 10° E and the latitude of origin set at 30° N.



**FIGURE 3.18** THE BONNE PROJECTION.

The Bonne projection depicting the entire world (a) and more appropriately re-centered and zoomed into North America (b). This projection is suitable for mapping continents, as in (b), but should never be used for mapping complete hemispheres. In (a) notice the severe shape distortion, brought about by shearing, at the northeast and northwest corners of the projection.

also be adequately mapped with the Bonne projection. Caution should be exercised, however; although equivalency is maintained throughout, shape distortion is particularly evident at the northeast and northwest corners. Because of this, the Bonne projection is really best suited for mapping compact regions lying on only one side of the equator (McDonnell 1979). Because shape is best along the central meridian, the distortion becoming objectionable at greater distances from it, the selection of the central meridian relative to the important mapped area (and zooming to that area) is critical.

Some projections that are suitable for world maps are sometimes not the best for mapping continental areas, but



others, such as the Sinusoidal, Mollweide, Goode's Homolosine, or even one of the compromise projections, can also be used effectively for a continent such as Africa or South America, as long as the central meridian is adjusted properly and you are zoomed into the area of interest appropriately. As always, you should let the purpose of the map and the "essential questions" presented earlier in this chapter help guide in your choices. If the projection that you choose for the continent does not meet your needs or hold to your expectations, try another projection.

## Mapping Multiple Size Countries at Mid-Latitudes

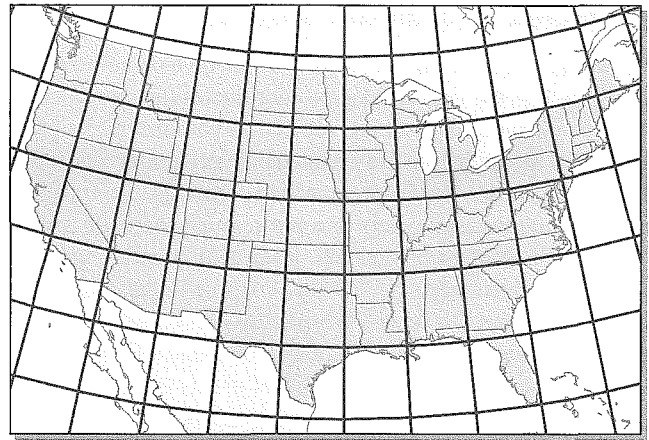
Mapping larger countries at mid-latitudes can be handled in a variety of ways. The Lambert Azimuthal Equal Area or the **Albers Equal Area Conic projection** may be used. If conformality is desired, the **Lambert Conformal Conic projection** can be selected, and depending on scale, the visual appearance will often be somewhat similar to Albers. In general, a conic projection is usually adequate for mapping rather large countries or political units that have an east-west extent. Even the Bonne projection, with adjustments for the correct reference latitude, central meridian, and scale can work for some countries.

For countries or even groups of countries that extend in all directions, the Lambert Azimuthal Equal Area projection can be a great choice. Described in the previous section, the projection should have its standard point placed in the center of the area of interest (the oblique case), such as the center of the country. In addition to the equivalency property, the azimuth of any point on the map (as measured from the center of the projection) is correct. This makes this projection especially useful when mapping phenomena having an important directional relationship to the standard point chosen for the map.

For countries and political entities that have a pronounced east-west orientation, either the Albers Equal Area Conic projection or the Lambert Conformal Conic projection can be great choices. Both projections considered are the secant case, utilizing two standard parallels to lessen scale distortion.

We often recommend the Albers Equal Area Conic for thematic maps in these pronounced east-west cases. Besides maintaining the property of equivalency, the overall scale distortion is nearly the lowest possible for an area the size of the United States (see Figure 3.19). Not surprisingly, it is one of the best choices for mapping the continental United States. The Lambert Conformal Conic projection, however, does not produce appreciably different graticule in maps at this scale, as long as the standard parallels and the central meridian are set appropriately for the study area. Lambert Conformal Conic is used throughout the Atlas of Canada (Natural Resources Canada 2007), as well as in many projected coordinate systems (to be discussed) worldwide and in the United States.

While the Mercator projection is not recommended for world thematic mapping, the **Transverse Mercator** projection



**FIGURE 3.19** THE ALBERS EQUAL AREA CONIC PROJECTION. This projection is frequently used for mapping the continental United States. Angular distortion is minor, nowhere exceeding 2°.

is often used for countries and other political entities that have a pronounced north-south orientation. As described earlier in the section "Cylindrical Family," and illustrated in Figure 3.7a, the cylinder is rotated so that the standard parallel becomes a standard meridian, meaning there is no scale distortion in the north-south direction at the standard meridian. If the secant case is employed, then two standard lines (small circles) straddle the projection's central meridian, increasing the amount of relatively low distortion areas. This would be an appropriate choice for mapping a country such as Chile, and is also employed in projected coordinate systems such as the State Plane and UTM coordinate systems (to be discussed).

It is important to note that as mapped areas become smaller in extent, the selection of the projection becomes less critical; potential scale errors begin to drop off considerably. Using a localized projection and coordinate system might become an important consideration when this happens.

## Mapping at Low Latitudes

Many of the projections already discussed are suitable for mapping countries, large and small, that are on or near the equator. The Lambert Azimuthal Equal Area projection, in its equatorial aspect, is one great choice. Cylindrical projections that have a standard parallel at the equator (see Figure 3.4b) or secant cylinders that have standard parallels near the equator can also be good choices for low latitude mapping. A projection such as Mercator, while best avoided for world and other small scale mapping applications, has no distortion at the equator (in the tangent case). So mapping a country such as Ecuador would not be a problem using Mercator. But the Lambert Azimuthal Equal Area projection in its equatorial aspect could be equally as effective. As with other projections, make sure that you are zoomed into the area of interest appropriately and the central meridian is centered in the area of interest (for example, the center of Ecuador).

## Projected Coordinate Systems

As we have seen so far, all projections involve coordinate systems, such as the decimal degree coordinates that are so common in mapping and setting projection parameters. Another option for mapping is to use a related but yet distinct concept of the projected coordinate system, which combines the projection process with the parameters of a *particular* grid (Iliffe 2000). There are a number of national, state, and county level projected coordinate systems. For this text, we will focus on the **State Plane** and **Universal Transverse Mercator** coordinate systems due to the sheer volume of downloadable data available in these systems. Selecting one of these systems as a projection choice can be effective for many mapping applications, particularly maps with scales at the state level and larger.

### State Plane Coordinate (SPC) System

The original State Plane coordinate system was developed in the early 1930s by the then United States Coast and Geodetic Survey (USCGS), now the National Ocean Service (National Oceanic and Atmospheric Administration 1989). It was devised so that local engineers, surveyors, and others could tie their work into the reference then used, the Clarke ellipsoid of 1866, which was used in the North American Datum (NAD) of 1927 but has since been migrated to the North American Datum (NAD) of 1983. What they desired was a simple rectangular coordinate system on which easy plane geometry and trigonometry could be applied for surveying, because working with spherical coordinates was cumbersome.

Earlier we mentioned that if the area of the Earth being mapped is small enough, virtually no distortion exists. This is the principle behind the SPC system. To ensure stated accuracies of less than one part in 10,000, the states are partitioned into a series of zones (see Figure 3.20). In the continental United States, these zones are elongated either in the north-south direction or the east-west direction. Many states have two or three zones, a few states have only a single zone; Alaska, California, Hawaii, Texas, and Wyoming have four or more zones. Each zone can be referred to by its name; for example, Minnesota has a North, Central, and South zone (see the “Zone Examples” inset in Figure 3.20), or by a FIPS code (Federal Information Processing Standard, discussed in more detail in Chapter 4). Each zone is assigned its own coordinate systems with its own origin and its own projection.

There are three conformal projections used to map the states—the secant case of the *Lambert Conformal Conic* for zones with elongated east-west dimensions, the secant case of the *Transverse Mercator* for zones with elongated north-south dimensions, and the secant case of the *Oblique Mercator* for one section of Alaska. In each case, over small areas these projections essentially project as rectangular grids, with little or no areal and distance distortion. Because they are conformal, no angular distortion between the meridians and parallels is present.

The projection’s parameters are then adjusted for each zone. For example, a zone employing the Lambert Conformal

Conic projection will have its central meridian fit to the center of the zone, and the standard parallels are positioned near the northern and southern portions of the boundary (see Figure 3.21). In a zone employing the secant case of the Transverse Mercator projection (Figure 3.7a), the central meridian is again fit to the center of the zone, and its two standard lines (small circles) will straddle the meridian on either side (although much closer than is depicted in Figure 3.7a), near the western and eastern margins of the zone.

A particular zone’s coordinate system is designed so that surveyors work only with positive (quadrant one) coordinates. The easting and northing measurements are in feet if NAD27 is used and in meters if NAD83 is used. A false origin (see Chapter 2 for discussion of this concept) is placed so that the *x*- and *y*-axis are the south and west of the zone, respectively (see Figure 3.21). The exact placement of the axes can vary greatly between states and zones. A typical configuration for SPC NAD83 would place the *y*-axis 600,000 meters west of the zone’s central meridian in the Lambert Conformal Conic Projection (or 2,000,000 feet west in NAD27), and 200,000 meters of the zone’s central meridian in Transverse Mercator projection (500,000 feet west in NAD27). The *x*-axis position is more arbitrary but is still placed to the south of the zone in all cases (Van Sickle 2004).

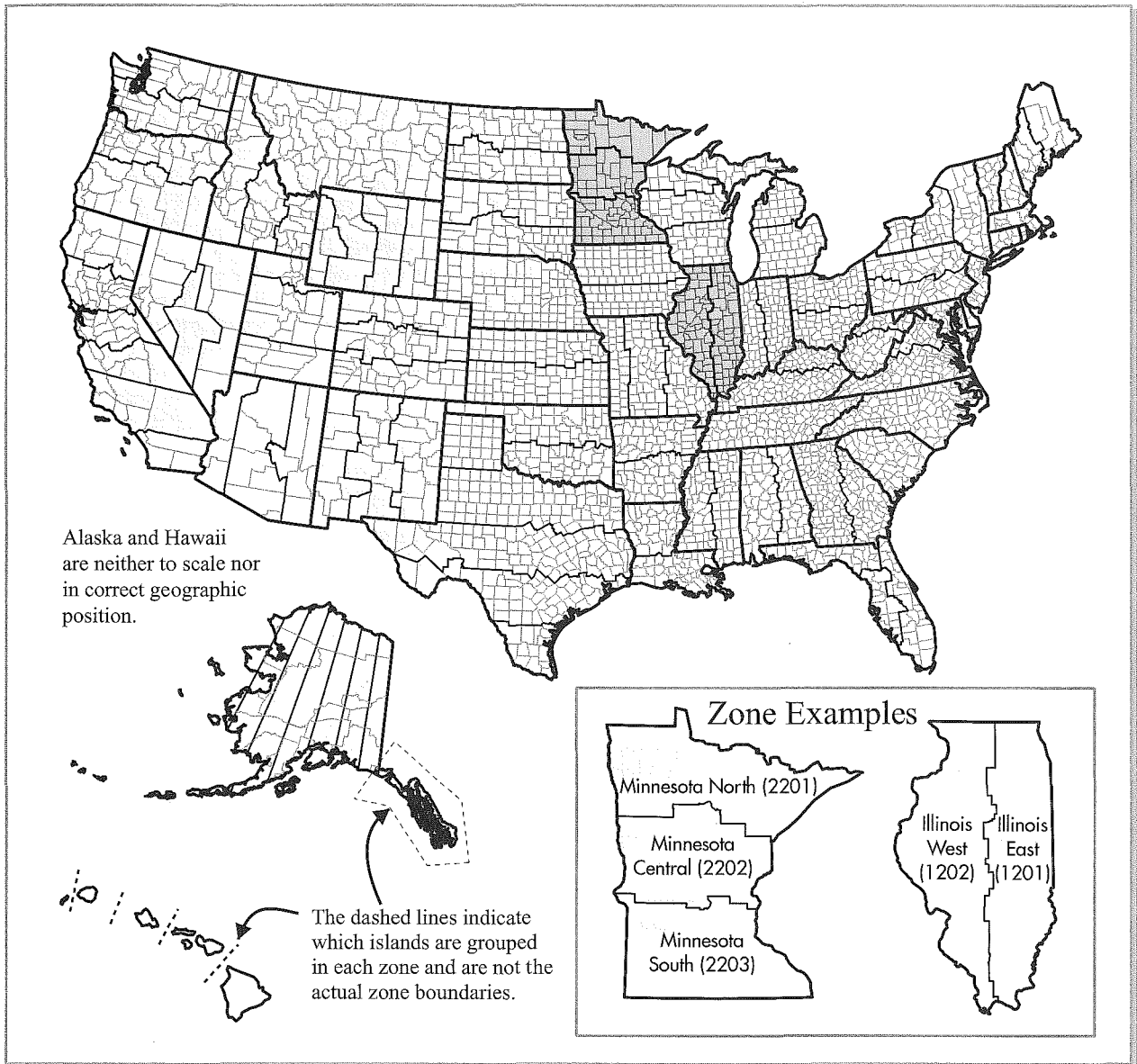
In GIS and mapping software, selecting SPC is usually accomplished in a manner similar to that of other map projections. In many cases, local data are readily available in SPC. Because the distortion is small and the projection is centered, SPC works well for larger scale (up to state level) thematic maps.

If SPC is used to map an entire state, we recommend using as centralized a zone as possible. The final shape is fairly aesthetic (see Figure 3.21). For states with only two north-south elongated zones (such as Illinois, Indiana, or Georgia), we recommend selecting either the western or eastern zones, and then adjusting the central meridian west or east as necessary to provide a centered and balanced appearance to the state.

### Universal Transverse Mercator (UTM) System

The Universal Transverse Mercator (UTM) system, along with the Universal Polar Stereographic System (for polar regions), was created after World War II by several allied nations in order to produce a unified and consistent coordinate system after years of attempting to trade information in disparate coordinate systems. The United States military soon followed with its own adaptation of the UTM system (Van Sickle 2004). We will be following the civilian use of the system.

The UTM system is not quite as accurate as the State Plane system, with accuracies as large as one part in 2,500 (Van Sickle 2004). But the increased coverage and overall acceptance of the system has had a major impact on the mapping community. Many U.S. states, for example, distribute much of their public domain spatial data in UTM format (usually NAD27 or NAD83 datum), which can be easily loaded into GIS and mapping software, where the projection, datum, and/or the projection’s parameters can be adjusted if need be.

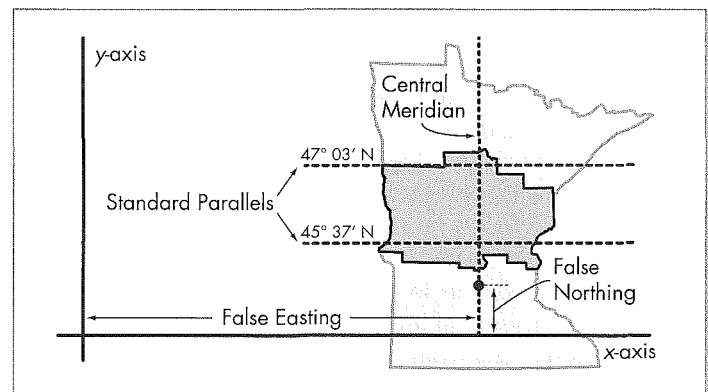


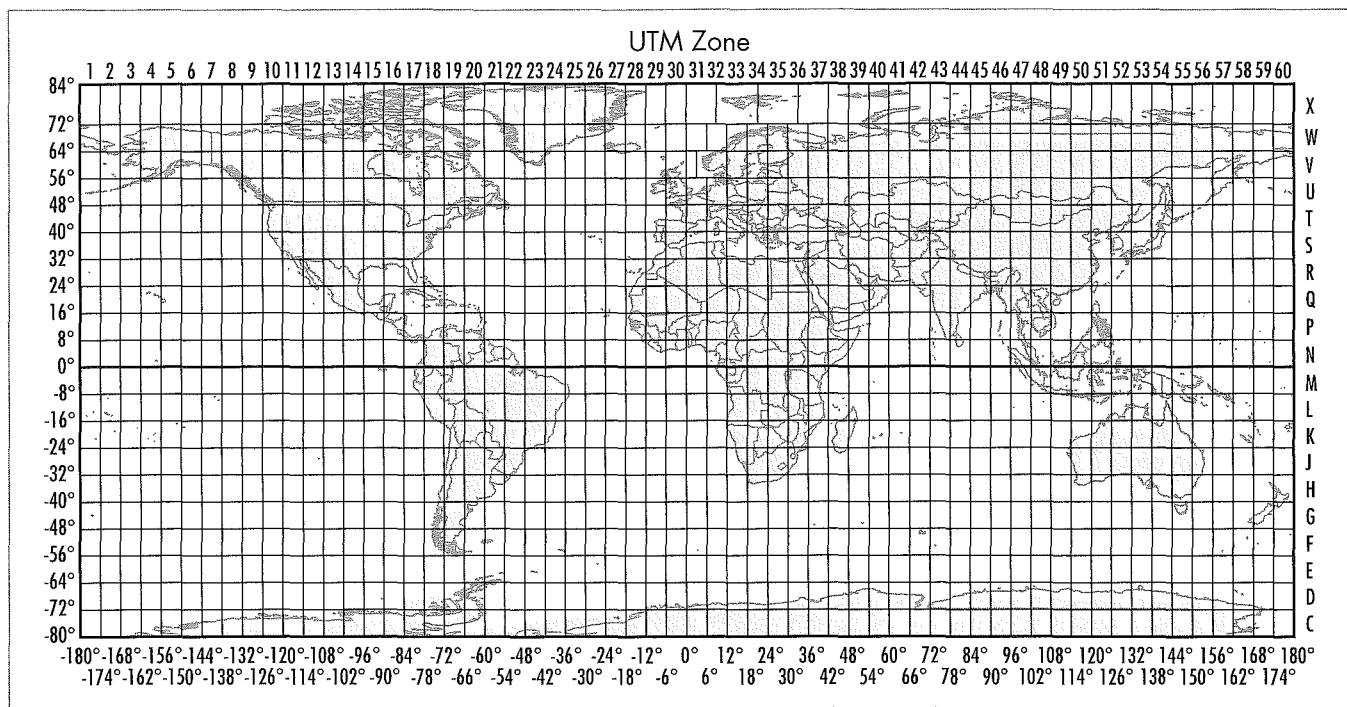
**FIGURE 3.20** THE ZONES OF THE STATE PLANE COORDINATE SYSTEM.

This is the NAD83 configuration of zones for the SPC system, which follow state and county lines for the contiguous United States. The inset of zone examples includes a state with elongated east-west zones (Minnesota) and a state with elongated north-south zones (Illinois).

**FIGURE 3.21** MINNESOTA CENTRAL ZONE OF THE SPC SYSTEM.

Positioning of the Lambert's Conformal Conic standard parallels, and the central meridian, as well as the SPC origin and axes for this zone. The exact positioning of the x- and y-axes varies from state to state and zone to zone, but the axes are always placed so that all coordinates are positive. (See text for discussion.) Note that the projection produces a reasonably aesthetic shape for Minnesota.





**FIGURE 3.22** THE UNIVERSAL TRANSVERSE MERCATOR SYSTEM.

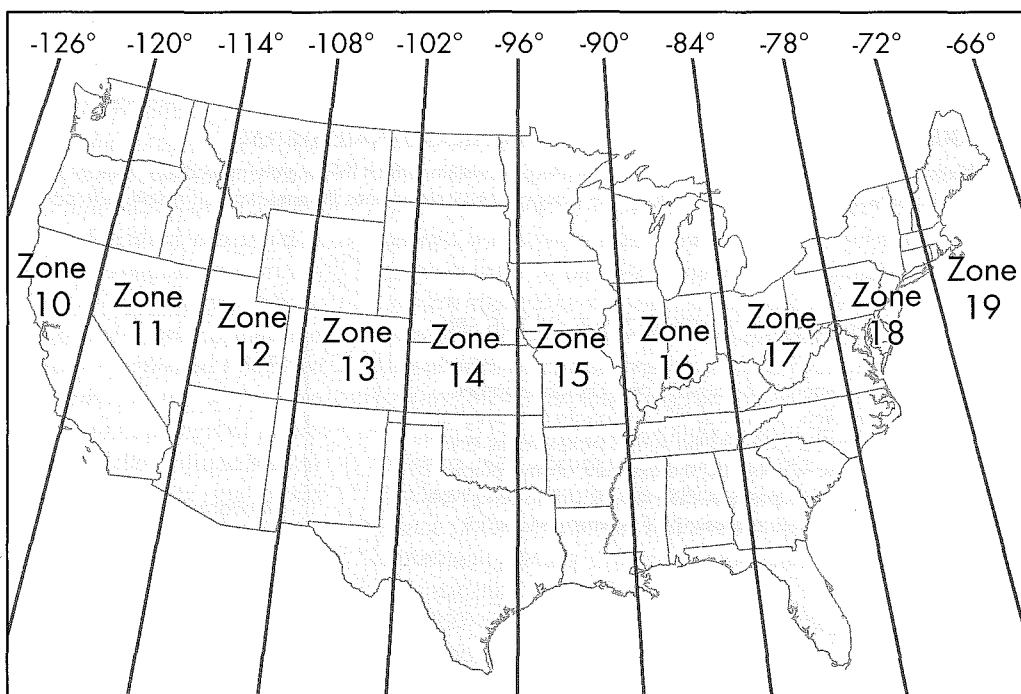
The UTM system is made up of sixty  $6^\circ$  zones that extend from  $80^\circ$  South to  $84^\circ$  North. The numbers on the left and bottom portions of the graphic are latitude and longitude values in decimal degrees. The letters on the right side of the graphic are referred to as zone designators (after Dana 1999). While zone designators are in the official description of the UTM system (Van Sickle 2004), almost all civilian uses of this system simply use the UTM zone(s), along with a designation of N or S to indicate the relevant hemisphere.

The UTM system is a projected coordinate system that covers the entire world from  $80^\circ$  South to  $84^\circ$  North. It is subdivided into 60 six-degree zones that are elongated in the north-south direction (see Figure 3.22). For example,

zone one extends from  $180^\circ$  to  $174^\circ$  W as well as from  $80^\circ$  S to  $84^\circ$  N. The continental United States has ten zones (zones 10–19) that extend from  $126^\circ$  W to  $66^\circ$  W (see Figure 3.23).

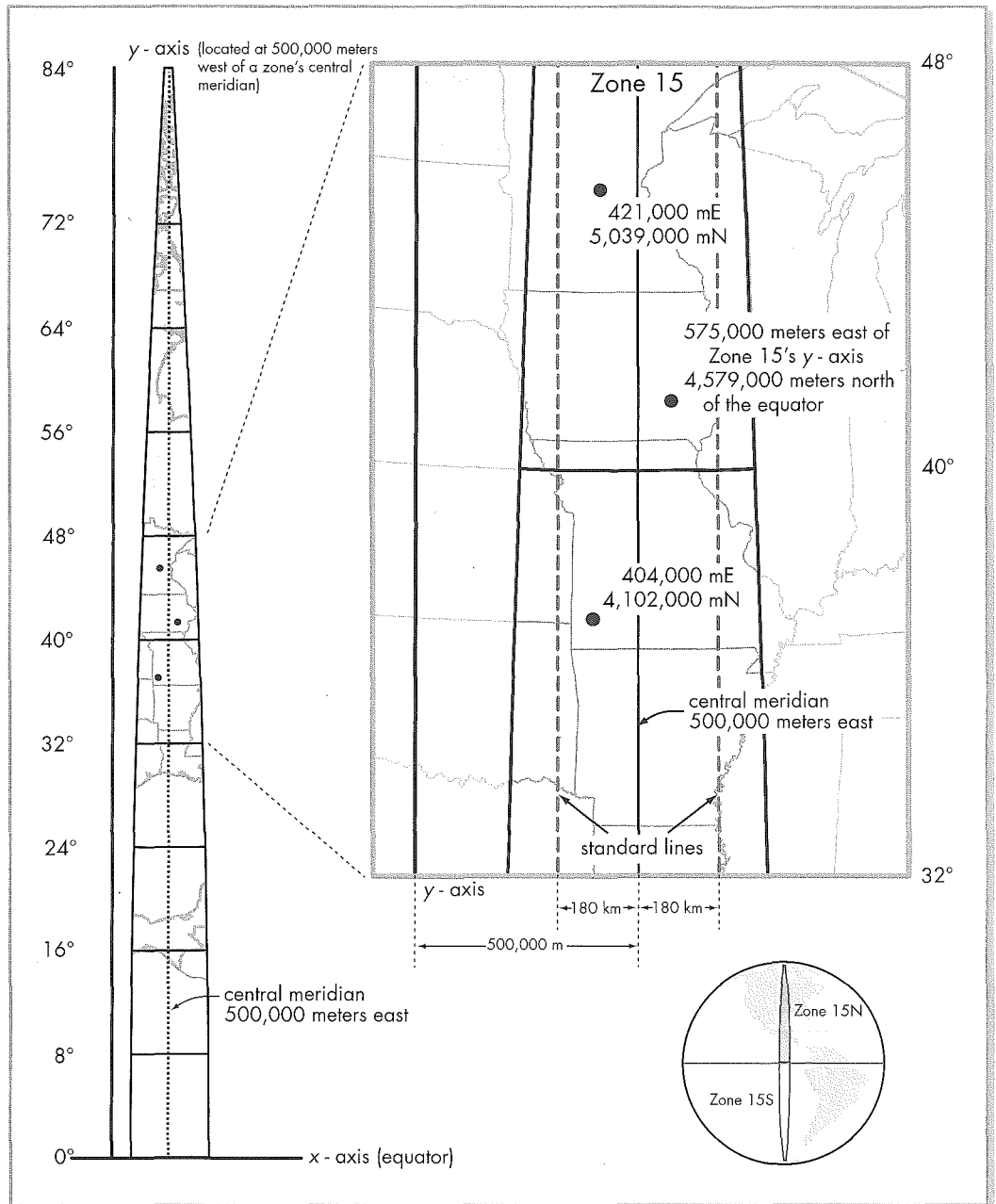
**FIGURE 3.23** THE TEN UTM ZONES FOR THE CONTINENTAL UNITED STATES.

Zones 10–19 are used in the continental United States.



**FIGURE 3.24** UTM ZONE 15 NORTH.

This diagram illustrates some of the most important aspects of a UTM zone. See text for discussion.



The projection for this system is the secant case of the Transverse Mercator (just like the north-south zones of the SPC). The transverse cylinder is positioned such that the central meridian runs north-south through the zone's center. The central meridian for zone one is 177° W. The two standard lines (small circles) will straddle the central meridian at 180 kilometers (111.85 miles) to the meridian's east and west. Relatively low scale distortion is thus maintained in the north-south direction. The scale factor is about 0.9996 along the central meridian, and increases to one at the standard lines (Van Sickle 2004). The cylinder is repositioned 60 times, once for each zone.

The civilian coordinates are based on the equator, which serves as the x-axis. The y-axis is positioned at 500,000 meters west of a particular zone's central meridian. By generating this

false origin, as with the SPC, all coordinates will now be positive. In the northern hemisphere, then, a point is measured as so many meters east (of the y-axis that intersects the false origin) and so many meters north (of the equator) (see Figure 3.24). In the southern hemisphere, the origin is moved south by 10,000,000 meters, which is not too far from the South Pole. But again, by having a false origin to the west and south of a particular zone's area, the coordinates will always be positive to the east and north of a particular zone's false origin.

As with SPC, UTM is often used to map an entire state, since there is relative accuracy of the coordinates, and so much data is readily available in this format. However, in Figure 3.23 it can be observed that many states are not centered or even entirely contained within each zone. Zone boundary

## GIS "PROJECTION ON THE FLY"

Many students new to GIS mapping are unaware that when they load a map and select a map projection in their software, that the projection that they see on the screen may not necessarily be the same projection as the one in their original map data. When there is a difference between the coordinates or projection in the map data and the coordinates and projection being displayed in the GIS software it is known as "projection on the fly" (Environmental Systems Research Institute 2007). For example, if our original world map is in unprojected latitude-longitude coordinates, and we want to see the map in a Winkel Tripel projection, some GIS software allows you to make this projection on the fly without altering the original data's projection or coordinate system. In this case, the cartographer sees the map within the GIS display in the Winkel Tripel projection, but the

original world map data *is still in unprojected latitude-longitude coordinates*.

While most software can effectively translate between projections and coordinate systems, the GIS software must "know" what projection and coordinate system is inherent in the data. In other words, it is not possible for a software to project or otherwise change coordinates for data that does not have its projection/coordinate system properly defined.

As a final note, working across two coordinate systems and projections can be confusing for some cartographers, and in some cases errors result when trying to edit the data across the two systems. Thus, many work environments require their employees to edit within the original projection and coordinate system or enact changes to the projection of the *original* data (and *not* project on the fly) before editing the map in the GIS.

determination was not based on "nice positioning" but rather on the regular 6° increments. States such as California, Montana, Kentucky, and Virginia (among others) are squarely in two zones. When people choose one zone or another, they find that the state's positioning does not look correct. The northern part of the state, for example, may tilt too far east or west. In these cases we usually recommend trying to move the central meridian eastward or westward to make the state look correct. If this is unsuccessful, change to an SPC coordinate system, or choose an appropriate projection in which the standard lines and central meridian can be adjusted to go through the state.

Some states, such as Wisconsin (which is also split by two UTM zones), have developed their own projected coordinate system, precisely to avoid such a split. This system is similar to UTM, except that they designated 90° W as the central meridian. The Wisconsin Transverse Mercator, as it is called, is available in both NAD27 and NAD83 datums (Wisconsin State Cartographer's Office 2004). This is one example among many of how states and other local governmental organizations respond to mapping needs on a more localized level.

A survey of state (or county or country) systems is beyond the scope of the text, but if creating thematic maps at a localized scale (even down to the city level) then it may be prudent to be aware of systems that are standard in that area. A local system may be required if you are making maps as a city or county employee. Even if it is not a *requirement*, if your state or other political unit has a projected coordinate system that is supported in the software being used, examine its properties and parameters. If these are acceptable, then you just may have found another viable projection option for mapping that area.

### Adjustments in Projection Parameters

Many times, cartographers assemble maps that are groups of counties, states, countries, or other levels of areas and

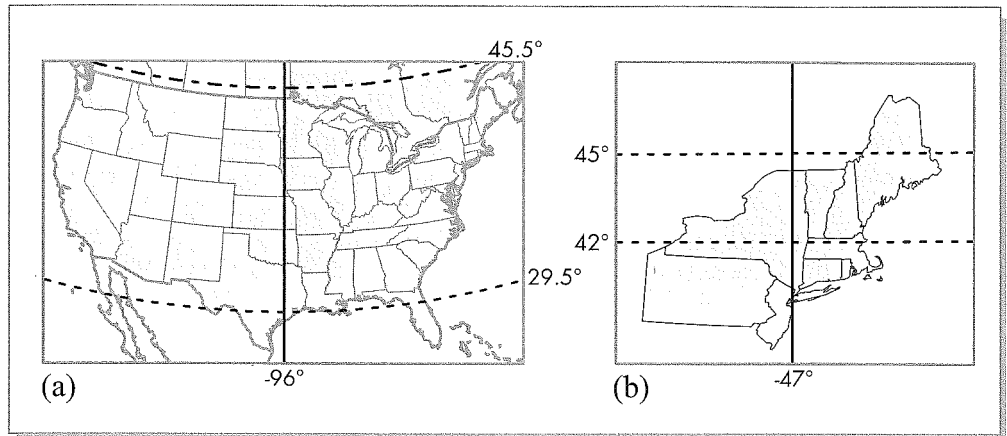
features that are sometimes outside a particular projection or projected coordinate system's "ideal" region. This final brief but important discussion centers on adjusting projection parameters to some of these specialized cases.

Perhaps the most important guideline in map projections is the centering of the projection of the map on the area of interest. As suggested in a number of places throughout the chapter, this means having the standard point, line or lines, as well as the central meridian, run directly through your study area or area of interest. Figure 3.18 is illustrative of this principle. What many students are often unaware of, however, is that if they are using a GIS or mapping software approach to projections, these parameters can be adjusted easily to suit the projection needs of a particular region.

For example, if the cartographer is going to make maps of the Mid-Atlantic and New England states region of the United States (see Figure 3.25a) he or she may start out selecting the Albers Equal Area Conic projection for the continental United States (see Figure 3.20). And this may not always be a bad choice, if the other states are going to be present to give a context to these states. However, if these states are extracted, as they are on so many thematic maps produced today, then the default central meridian (at 96° West or -96° in decimal degrees) and standard parallels (near which is the least scale distortion, at 29.5° and 45.5° North) no longer center the essential projection parameters on the newly created study area. Adjustments can be made to the standard parallels and central meridian, such that the projection appears better centered (on -47°), and will be more accurate by the adjustment of the standard parallels to 45° and 42°, respectively (see Figure 3.25b). For any other region, then, we suggest starting with the projection or projected coordinate system that most closely approximates the ideal projection properties and parameters, and then make the appropriate adjustments.

**FIGURE 3.25**  
REPOSITIONING  
PROJECTION PARAMETERS  
FOR THE MID-ATLANTIC AND  
NEW ENGLAND STATES.

Default Albers Equal Area Conic projection for continental United States (a) can provide a great starting point. But adjusting the standard parallels and central meridian (b) can produce greater accuracy and an improved aesthetic look.



## REFERENCES

- Anderson, P. 2007. *A Gallery of Map Projections*. <http://www.galleryofmapprojections.com>
- Campbell, J. 2001. *Map Use and Analysis*. 4<sup>th</sup> ed. Dubuque, IA: McGraw-Hill.
- Canter, F. 1989. New Projections for World Maps: A Quantitative-Perceptive Approach. *Cartographica* 26: 53–71.
- Chamberlin, W. 1947. *The Round Earth on Flat Paper*. Washington, DC: National Geographic Society.
- Committee on Map Projections. 1989. Technical Notes. *American Cartographer* 16(3): 222–23.
- Dana, P. 1999. *Coordinate Systems Overview*. *The Geographer's Craft Project*. Department of Geography, The University of Colorado at Boulder. [http://www.colorado.edu/geography/gcraft/notes/coordsys/coordsys\\_f.html](http://www.colorado.edu/geography/gcraft/notes/coordsys/coordsys_f.html)
- Dent, B. 1987. Continental Shapes on World Projections: The Design of a Poly-Centered Oblique Orthographic World Projection. *Cartographic Journal* 24: 117–24.
- Environmental Systems Research Institute. 2007. *ArcGIS Desktop Help*. Online Help for ArcGIS 9.2.
- Faintick, M. 1986. The Politics of Geometry. *Computer Graphics World*. May: 101–4.
- Geocart. 2007. *Geocart's Projections List: Simply Professional Map Projections*. <http://www.mapthematics.com>
- Goldberg, D., and J. Gott. 2007. Flexion and Skewness in Map Projections of the Earth. *Cartographica* 42(4): 297–318.
- Hilliard, J., U. Bosoglu, and P. Muehrcke. 1978. *A Projection Handbook*. Madison: University of Wisconsin Cartographic Laboratory.
- Iliffe, J. 2000. *Datums and Map Projections*. Caithness, UK: Whittles Publishing.
- Loxton, J. 1985. The Peters Phenomenon. *The Cartographic Journal* 22: 106–8.
- Marschner, F. 1944. Structural Properties of Medium- and Small-Scale Maps. *Annals of the Association of American Geographers* 34: 44.
- McDonnell, P. 1979. *An Introduction to Map Projections*. New York: Marcel Dekker.
- Monmonier, M. 2004. *Rhumb Lines and Map Wars*. Chicago: University of Chicago Press.
- National Oceanic and Atmospheric Administration. 1989. *State Plane Coordinate System of 1983, Manual NOS NGS 5*. Rockville, MD: National Geodetic Information Center.
- Natural Resources Canada, 2007. *The Atlas of Canada—Map Projections*. [http://atlas.nrcan.gc.ca/site/english/learningresources/cartocorner/map\\_projections.html](http://atlas.nrcan.gc.ca/site/english/learningresources/cartocorner/map_projections.html)
- Raisz, E. 1962. *Principles of Cartography*. New York: McGraw-Hill.
- Robinson, A. 1985. Arno Peters and His New Cartography. Views and Opinions. *American Cartographer* 12: 103–11.
- Robinson, A., J. Morrison, P. Muehrcke, A. Kimerling, and S. Guptill. 1995. *Elements of Cartography*. John Wiley and Sons.
- Snyder, J. 1987. *Map Projections—A Working Manual*. U.S. Geological Survey Professional Paper 1395. Washington, DC: USGPO.
- . 1988. Social Consciousness and World Maps. *The Christian Century*, February 24: 190–92.
- . 1993. *Flattening the Earth: 2000 Years of Map Projections*. Chicago: University of Chicago Press.
- Steers, J. 1962. *An Introduction to the Study of Map Projections*. London: University of London Press.
- Stocking, A. 2005. Everything is Somewhere: Two Views of One Planet. *The American Surveyor* 2 (7): 54–56.
- Strebe, D. 2007. *Map Projection Essentials*. <http://www.mapthematics.com/Essentials/Essentials.html>
- Tissot, A. 1881. *Memoire sur la Representation des Surfaces et les Projections des Cartes Geographiques*. Paris.
- Van Sickle, J. 2004. *Basic GIS Coordinates*. Boca Raton, FL: CRC Press.
- Wisconsin State Cartographer's Office. 2004. *Regional Coordinate Systems*. <http://www.sco.wisc.edu/pubs/wiscoord/regional.php>

## GLOSSARY

**Albers Equal Area Conic projection** secant conical projection having equal area properties; useful for mapping areas of east-west extent

**azimuthal projection** directions from the projection's center to all points are correct; also called a zenithal projection

**Bonne projection** simple conical equal area projection, useful for mapping continent-size areas of the Earth; should not be used for areas of considerable east-west extent

**central meridian** the meridian that defines the center of the projection

**conformal projection** preserves angular relationships at points during the transformation process; cannot be equal-area; also called an orthomorphic projection

**developable surface** geometric form used in the projection process without tearing, shearing, or distortion of the geometric surface

**equal area map projection** no areal deformation; cannot be conformal; also called an equivalent projection

**equidistant projection** preserves correct linear relationships between a point and several other points, or between two points; cannot show correct linear distance between all points to all other points

**equivalent map projection** also the property of equivalence; see equal area map projection

**Gall-Peters projection** rectangular equal area projection emphasizing lower latitude countries; used by several UN organizations for world mapping

**gnomonic projection** the hypothetical light source that projects points onto a plane surface is placed at the center of the globe

**Goode's Homolosine projection** classic interrupted equal area projection of the world

**great circle** circle that results when a plane bisects the Earth into two equal halves (for example, the equator); the great circle arc is a segment of the great circle that is the shortest distance between two points on the spherical surface

**Hammer projection** equal area projection useful for world mapping; also called Hammer-Aitoff

**Lambert Azimuthal Equal Area projection** has equal area properties useful for mapping up to hemisphere areas (in its equatorial aspect) or up to continent-size areas (in its oblique aspect) on the Earth, especially if the area extends in all directions

**Lambert Conformal Conic projection** has conformal properties; used for areas with an east-west extent; one of projections used in the State Plane coordinate system (SPC)

**latitude of origin** latitude of the projections' origin, such as the equator

**Mercator projection** historical conformal map used for navigation; distortion in higher latitudes make it unsuitable for most world mapping applications

**minimum error projection** no equivalency, conformality, azimuthality, or equidistance; chosen for its overall utility and distinctive characteristics

**Mollweide (or homolographic) projection** equal area projection useful for world mapping

**orthographic projection** the hypothetical light source that projects points onto a plane surface is placed at a theoretical infinity

**pattern of deformation** distribution of distortion over a projection; customarily increases away from a standard point or line(s) (see also *standard points, lines, parallels, and meridians*)

**projection aspect** the position of the projected graticule relative to the ordinary position of the geographic grid on the Earth; can be visualized as the position of the developable geometric surface to the reference globe

**projection parameters** latitude and/or longitude values that describe a map projection's standard point, line or lines, its central meridian, and its latitude of origin

**projection properties** properties of a projection that make it an equal area, conformal, equidistant, or azimuthal

**reference globe** the reduced model of the spherical Earth from which projections are constructed; also called a nominal or generating globe

**Robinson projection** classic minimum error (compromise) projection used in world mapping

**scale factor** ratio of the scale of the projection to the scale of the reference globe; 1.0 on standard lines, at standard points, and at other places, depending on the system of projection

**secant case** in a map projection, each of the surfaces is brought to a line (with the plane) or two lines (with the cylinder and the cone) of intersection on the reference globe

**Sinusoidal projection** a mathematical equal area projection for world mapping

**small circle** any circle on the spherical surface that is not a great circle; for example, parallels (except for the equator) are small circles

**standard points, lines, parallels, and meridians** one of the map projection's key parameters; the point, line, or lines on a map that have no scale distortion. In regard to geometric figures, places where the plane, cylinder, or cone touches or intersects the reference globe during the projection process

**State Plane Coordinate (SPC) system** rectangular plane coordinate system applied to states; employs the Lambert Conformal Conic or Transverse Mercator projection for each zone

**stereographic projection** the hypothetical light source that projects points onto a plane surface is placed on the opposite side of the globe with respect to the plane's point of tangency

**tangent case** in a map projection, each of the geometric figures are brought to a point or line of tangency on the reference globe

**Tissot's indicatrix** mathematical construct that yields quantitative indices of distortion at points on map projections

**Transverse Mercator projection** has conformal properties; maintains little or no scale distortion in the north-south direction. One of the projections used in the State Plane coordinate system

**Universal Transverse Mercator (UTM) system** rectangular plane coordinate system applied to north-south zones worldwide; employs the Transverse Mercator projection for each zone

**Winkel Tripel projection** minimum error projection used in world mapping, may be least error in minimum error projections