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Optimal maintenance policy for multi-component systems under Markovian environment changes

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ABSTRACT

In this paper, we study multi-component systems, which environmental conditions and opportunistic maintenance (OM) involve. Environmental conditions will exert an influence on deterioration processes of the components in the system. For each component, the worse the environmental conditions are, the faster its deterioration speed is. We want to determine when to preventively maintain each component under such environmental influence. Our purpose is to minimize its long-run average maintenance cost. We decompose such a multi-component system into mutually influential single-component systems, and formulate the maintenance problem of each component as a Markov decision process (MDP). Under some reasonable assumptions, we prove the existence of the optimal (n_r, N_r) type policy for each component. A policy iteration method is used to calculate its optimal maintenance policy. Based on the method, we develop an iterative approximation algorithm to obtain an acceptable maintenance policy for a multi-component system. Numerical examples find that environmental conditions and OM pose significant effects on a maintenance policy.

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1. Introduction

The maintenance optimization of multi-component systems has drawn significant attention in recent years because the significance of maintenance decision-making for such complicated system is gradually being recognized. Moreover, there are many multi-component systems in practice that run in a specific environment status and deteriorate gradually with age increasing or operation condition worsening, such as equipment in chemical plants, devices in electric station, aircraft engine, wind turbine, etc. (Eunshin, Ntaimo, & Yu, 2010). Failures of such complicated systems do not lead only to huge production loss, but also to safety issues of personnel and environment. Therefore, it is advisable to conduct a proper maintenance policy to reduce risk of failure.

Maintenance for multi-component systems has been extensively studied in operations research literature for the past few decades. Cho and Parlar (1991) summarized papers published from 1976 to 1991 with five topical categories: machine interference/repair models, group/block/cannibalization/opportunistic models, inventory/maintenance models, other maintenance/replacement models, and inspection/maintenance models. Then Dekker, Wildeman, and Schouten (1997) reviewed the studies of multi-component system maintenance with economic dependence from 1991 to 1997. A more recent review was given by Nicolai and Dekker (2008) which summarized the recent papers with three categories: economic, structural, and stochastic dependence.

However, few papers addressed optimal maintenance policies for multi-component systems subject to economic dependence and environment changes simultaneously. For example, many maintenance-related studies have only considered economic dependence without modeling environment changes. Wang and Lin (2011) studied the periodic preventive maintenance for a serial-parallel system using an improved particle swarm optimization. Gu et al. (2013) proposed maintenance opportunity windows (MOW) algorithms to estimate hidden maintenance opportunities for discrete and complex production lines. Zhang, Wu, Lee, and Ni (2013) proposed a modified iterative aggregation procedure (MIAP) to solve optimal maintenance policies for multi-component systems with failure interaction. Zhang, Wu, Li, and Lee (2013) studied the maintenance policy by decomposing the multi-component system into many mutually influenced single-component systems.

Despite the extensive literature on maintenance problems with multi-component systems, relatively few studies have considered economic dependence when the system is influenced by a time-varying and stochastic environment (Kurt & Kharoufeh, 2010). The related studies all concentrated on a single-component system and thus no economic dependence is considered. For example, Singpurwalla (1995) summarized the methods that involved dynamic environment factor into stochastic deterioration processes from a mathematical perspective. Ozekici (1995) studied

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Nomenclature

Ψ	state space of a component	$c(i)$	operating cost of a component
Φ	state space of the environment	C_{slope}	operating cost rate of a component
S	state space of a multi-component system	L	the component number of a multi-component system
A	action set of a multi-component system	$\mu^k(r)$	occurrence rate of maintenance opportunity of component $k$under environment state r
m	failure state of a component	$\theta^k(i,r)$	steady-state probability of state (i, r) of component k without considering maintenance opportunity
η	worst state of the environment	N_r^k	PM threshold of component $k$under environment state r
$p_{ij}(r)$	deteriorate probability of a component from state i to state j under environment state r	n_r^k	OM threshold of component $k$under environment state r
$q_{r\theta}$	transition probability of the environment from state r to state θ	$\gamma(s,a)$	one step transition cost at state s and action a
$P_{ss'}(a)$	transition probability of a single component system starting from state s to state s' given the action a	$v_\pi(s)$	relative value of state s under policy π
λ	deterioration rate of a component	$T_\pi(s,a)$	policy-improvement test quantity defined in Eq. (9)
χ	transition rate of the environment	$V_n(s,\pi_s)$	total expected maintenance and operating costs over the first n decision epochs when the initial state is s and the policy is π
P	PM cost of a component	PM	preventive maintenance
K	CM cost of a component	CM	corrective maintenance
C_0	set-up cost of the system without maintenance opportunity	OM	opportunistic maintenance
C_1	set-up cost of the system with maintenance opportunity	IFR	increasing failure rate

the maintenance problem of a single-system under environmental process that has a general semi-Markov structure. In Waldmann's study (Waldmann, 1983), the environment was a sequence of randomly occurring shocks. Hu and Yue (2003) extended the maintenance problem to a semi-Markov deterioration single-system under the semi-Markov environment. Kurt and Maillart (2009) studied the threshold type optimal maintenance policy for the single-system which suffered from random shocks. Shocks arrived according to a Markov-modulated Poisson process that represents the environment status. Kurt and Kharoufeh (2010) studied the maintenance optimization problem under a Markov controlled environment. The system's deterioration process depended explicitly on the environment process whereas the environment process did not depend on the current level of deterioration. Therefore, in our paper, we study an optimal maintenance policy for multi-component systems under environment changes, which is more realistic.

Opportunistic maintenance (OM) is one of maintenance strategies widely adopted in industries (Kumar & Maiti, 2012) to makes full use of economic dependence in multi-component systems to reduce maintenance cost. OM basically refers to the maintenance scheme in which preventive maintenance (PM) is carried out at opportunities, either by choice or based on the health condition of the system (Lirong & Haijun, 2006). In this paper, we focus on the situation in which OMs are generated by the failure or PM of individual component. For example, when component A takes PM or fails in the system, component B can be maintained simultaneously if it is more economic. Such a maintenance action for component B is called OM. An advantage of OM is that maintenance combined with other failures or PMs can be used to save set-up costs. Van Der Duyn Schouten and Vanneste (1990) studied (n,N) type policies for two identical units system, and provided a fast computational method to compute the average costs under these (n,N) type policies. Scarf and Deara (1998) studied opportunistic age-based replacement policy for a two-unit series system with failure interaction. Furthermore, Scarf and Deara (2003) extended their study to opportunistic block-based replacement policy.

Because of the environment influence, a (n_r,N_r) type policy is proposed. For each single-component system under the environment state r, the PM threshold is N_r and the OM threshold is n_r. Under the environment state r, if a component's health state is worse than N_r, then PM will be conducted. Furthermore, OM will be carried out if its state is worse than n_r and maintenance opportunity occurs. Note that the maintenance threshold of each single-component system is a state vector, e.g., (n_0,N_0; … ;n_{r-1},N_{r-1}), because there exist one (n_r,N_r) threshold corresponding to each environment state.

Hence, our work differs from the existing maintenance optimization models in three important ways. First, we mathematically formulate the maintenance problem of multi-component systems considering OM and environmental influence simultaneously. Second, we prove the existence of a (n_r,N_r) type optimal policy for single-component systems under reasonable conditions. Finally, we develop an iterative approximation algorithm to obtain an acceptable policy of multi-component systems.

The remainder of the paper is organized as follows. In Section 2, we mathematically describe the model in detail. Subsequently, we develop the MDP formulation and validate the existence of the (n_r,N_r) type optimal policy for each single-component system under some reasonable assumptions. In Section 3, based on the policy iteration method, we develop an iterative approximation method to resolve the acceptable policies for multi-component systems. In Section 4, we provide two numerical examples to study the importance of considering environmental influence in maintenance policy optimization and demonstrate the advantage of (n_r,N_r) type policy as the number of components increases. Conclusions are presented in Section 5.

2. Problem formulation

In this section, we formulate the problem to find the optimal maintenance policy for a single-component in a multi-component system. Then, how to find the maintenance policies for all components in the multi-component system will be discussed using an iterative approximation algorithm in Section 3. For a single component, we mathematically describe the problem with reasonable assumptions, followed by developing MDP formulation and proving the existence of a (n_r,N_r) type optimal policy.

2.1. Model description and assumptions

We assume that the health state of each component can be represented by a finite set of discrete states, $\Psi = \{0, \ldots ,m\}$. State 0 is the initial health state of the component, and states $1,\ldots,m-1$ reflect its deteriorating conditions. If a component deteriorates to the failure state m, then corrective maintenance (CM) which makes the failed component return to initial state 0 is taken. We formulate its state transition as a continuous time Markov chain with a transition rate λ.

Furthermore, every component in the system assumes to be subject to the environmental conditions. The condition of the environment is also described by a finite set of states $\Phi=\{0,\ldots,\eta\}$. State 0 has the least amount of influence to the deterioration process, while state η has the greatest influence. We assume the state transition in this environmental condition follows a continuous time Markov chain and its transition rate is χ. The deterioration process of each component assumes to be Markov modulated process, which has been widely used in reliability model of manufacturing (Kharoufeh & Mixon, 2009). Moreover, it is a common formulation for a dynamic environment in theoretical studies (Kharoufeh, 2004; Kurt & Kharoufeh, 2010; Kurt & Maillart, 2009; Ozekici, 1995; Xiang, Cassady, & Pohl, 2012). The transition probability from state r to state θ in the embedded Markov chain is denoted by $q_{r\theta}$, $r,\ \theta\in\Phi$. Then, the condition of the environment directly affects the evolution of the system's deterioration process by accelerating (or decelerating) its movement toward more degraded states.

Under a specific environment state r, the transition probability of a component from state i to state j in the embedded Markov chain can be denoted by $p_{ij}(r)$, $i,j\in\Psi, r\in\Phi$. The transition probability $q_{r\theta}$ and $p_{ij}(r)$ can be estimated from a set of measurement data. The procedure of finding probability transition matrices from a set of industrial data via a HMM is provided in Liu (2008) in detail. The transition matrix $\{q_{r\theta}\}$ can be derived from historical environmental condition data and that of $\{p_{ij}(r)\}$ can be derived from historical component state data under a specific environment condition r. The Hidden Markov Model (HMM) is a method that enables us to stochastically relate available measurements to the component states or environment conditions. Moreover, several examples of maintenance models which were applied to real case studies are found in Kurt and Kharoufeh (2010), Nicolai and Dekker (2008), Chen (2011).

For a maintenance policy, the economic dependence of multi-component system has been considered via an opportunistic maintenance approach. The maintenance opportunity for component i means that the occurrence of other components' failures or PMs can provide a PM opportunity for component i. Since all the components assume to follow the deterioration process of continuous time Markov chains, the occurrence of maintenance opportunity from other components should be a Poisson process with rate $\mu(r), r\in\Phi$.

We assume that the operating cost $c(i)$ increases along with deterioration and is a linear function of state i.

$$c(i)=C_{\text{slope}}i,\quad C_{\text{slope}}>0,\ 0\leqslant i<m \tag{1}$$

If a component is repaired without maintenance opportunities, the set-up maintenance cost is denoted by C_0, and the CM cost and the PM cost are equal to C_0+K and $C_0+P(K\geqslant P)$, respectively. If a component is repaired under maintenance opportunities, the set-up maintenance cost is denoted by $C_1(\leqslant C_0)$, and the opportunistic corrective maintenance (OCM) cost and the opportunistic preventive maintenance (OPM) cost are C_1+K and C_1+P, respectively.

2.2. Mathematical model of a single component in a multi-component system

With the above assumptions, we can model this maintenance problem as an infinite horizon MDP so as to minimize its long-run maintenance average cost. The state space of a single-component system under the environmental influence is defined by

$$S=\{(i,r,z)|0\leqslant i\leqslant m;0\leqslant r\leqslant \eta;z\in\{0,1\}\},$$

where variable i denotes the state of a component, variable r denotes the state of the environment, and variable z denotes the occurrence of maintenance opportunity given by:

- $z=0$: no maintenance opportunity occurrence
- $z=1$: maintenance opportunity occurrence

We define a decision epoch as a time point when the state of a single-component system transits. At each decision epoch, a maintenance action a will be determined among the possible actions $A=\{0,1\}$ where $a=0$: no maintenance and $a=1$: conducting maintenance.

Given action a, the transition probability of the *embedded* Markov chain starting from state $s\in S$ to state $s'\in S$ is defined by $P_{ss'}(a)$. The transition probabilities are expressed for all $z=0,1$ as follows:

$$\begin{aligned}
P_{(i,r,z)(j,r,0)}(a)&=\begin{cases}\lambda(\mu(r)+\lambda+\chi)^{-1}p_{ij}(r) & a=0,i,j\in\Psi,r\in\Phi\\ \lambda(\mu(r)+\lambda+\chi)^{-1}p_{0j}(r) & a=1,i,j\in\Psi,r\in\Phi\end{cases}\\
P_{(i,r,z)(j,\theta,0)}(a)&=\begin{cases}\chi(\mu(r)+\lambda+\chi)^{-1}q_{r\theta} & a=0,i=j\in\Psi,r,\theta\in\Phi\\ \chi(\mu(r)+\lambda+\chi)^{-1}q_{r\theta} & a=1,i\in\Psi,j=0,r,\theta\in\Phi\\ 0 & \text{else}\end{cases}\\
P_{(i,r,z)(j,r,1)}(a)&=\begin{cases}\mu(r)(\mu(r)+\lambda+\chi)^{-1} & a=0,i=j\in\Psi,r\in\Phi\\ \mu(r)(\mu(r)+\lambda+\chi)^{-1} & a=1,i\in\Psi,j=0,r\in\Phi\\ 0 & \text{else}\end{cases}
\end{aligned} \tag{2}$$

When action a is taken in state $s\in S$, we use $\gamma(s,a)$ to denote the one step transition cost which involves operating and maintenance cost.

$$\gamma((i,r,1),a)=\begin{cases}c(i) & a=0,\ \text{if}\ i\neq m\\ C_1+P & a=1,\ \text{if}\ i\neq m\\ \text{N/A} & a=0,\ \text{if}\ i=m\\ C_1+K & a=1,\ \text{if}\ i=m\end{cases} \tag{3}$$

This completes the specification of the basic elements of the Markov decision process. Let $v_{\pi^*}(s)$ denote the relative value of state s under the optimal policy π^* and we can derive the following optimality equations,

$$\begin{aligned}
&v_{\pi^*}(i,r,z)=\min_{a\in A}\left\{\gamma((i,r,z),a)-g_{\pi^*}(\lambda+\mu(r)+\chi)^{-1}+\sum_{s'\in S}P_{(i,r,z)s'}(a)v_{\pi^*}(s')\right\},\\
&i\neq m,z\in\{0,1\}\,v_{\pi^*}(m,r,0)=C_0+K+v_{\pi^*}(0,r,0)\,v_{\pi^*}(m,r,1)=C_1+K+v_{\pi^*}(0,r,0),\\
&r\in\Phi
\end{aligned} \tag{4}$$

The solution of Eq. (4) will provide the optimal maintenance policy given the problem framework. We will demonstrate that the optimal maintenance policy is represented in the form of (n_r,N_r) type under the environment state r in the following sections.

In (n_r,N_r) type maintenance policy, PM under environment state r is conducted when health state i deteriorates to or above N_r, but smaller than m, $N_r\leqslant i<m$. OM under environment state r will be performed when health state i deteriorates to $n_r\leqslant i<N_r$ under the condition that maintenance opportunity from other components exists. When $i<n_r$, no maintenance action will be taken. When $i=m$, CM will be taken. Next we will prove the existence of (n_r,N_r) type optimal policy for a single component in multi-component systems.

2.3. Optimal maintenance policy (n_r,N_r)

Before stating the existence theorems, we begin our analysis by imposing the following definition and assumptions that are commonly employed in the maintenance optimization literature (Ghasemi, Yacout, & Ouali, 2007; Kurt & Kharoufeh, 2010; Kurt & Maillart, 2009).

Definition 1. A transition probability matrix $\{x_{ij}\}$ is said to be an increasing failure rate (IFR) if

$$\sum_{j=k}^{N} x_{ij} \text{ is non-decreasing in state } i \text{ for all } k = 0,\ldots,N$$

Assumption 1. The transition probability matrix $\{p_{ij}(r)\}$, $i, j \in \Psi$, $r \in \Phi$ of deterioration process under the environment state r is IFR.

$$\sum_{l=k}^{m} p_{il}(r) \leqslant \sum_{l=k}^{m} p_{jl}(r) \quad 0 \leqslant i \leqslant j \leqslant m, \quad k \in \Psi, \ r \in \Phi$$

Assumption 2. The operation cost and maintenance costs satisfy the following condition

$$c(i) \leqslant C_1 + P, \quad 0 \leqslant i < m, \ r \in \Phi.$$

Assumption 1 asserts that for an arbitrary environment $r \in \Phi$, the deterioration speed of a component does not decrease as its health state becomes worse. Assumption 2 presents that the one step transition cost of state i taken action $a = 1$ is not smaller than that taken action $a = 0$. In other words, operation cost is cheaper than maintenance cost. Subsequently, we can introduce Lemmas 1 and 2 that are immediately derived from these two assumptions. They will be the foundation in the proof of existence of a (n_r,N_r) type optimal policy.

Lemma 1. *For arbitrary $r \in \Phi$ and $z \in \{0,1\}$, one step transition cost $\gamma((i, r, z), \pi_{(i,r,z)})$ under threshold type policy $\pi = (0,\ldots,0,1,\ldots,1)$ is non-decreasing on i.*

Proof. For arbitrary $r \in \Phi$ and $z \in \{0,1\}$, maintenance policy $\pi_{(i,r,z)}$ is a function of state i. A threshold type policy poses the fact that there exists a certain state j such that it satisfies the following:

$$\pi_{(i,r,z)} = \begin{cases} 0 & \text{for } 0 \leqslant i < j \\ 1 & \text{for } j \leqslant i \leqslant m \end{cases}$$

Then, we can denote the threshold type policy as $\pi = (0,\ldots,0,\underset{j}{1},\ldots,1)$.

$$\begin{aligned} &\gamma((i,r,z),0) = c(i) && \text{for } 0 \leqslant i < j \\ &\gamma((i,r,z),1) = C_1 + P && \text{for } j \leqslant i < m \\ &\gamma((i,r,z),1) = C_1 + K && \text{for } i = m \end{aligned}$$

For $0 \leqslant i < j$, $\gamma((i,r,z),0) = c(i)$ is non-decreasing (from Eq. (1)). For $j \leqslant i < m$, $\gamma((i,r,z),1) = C_1 + P$ is constant. In addition, we know that $c(i) \leqslant C_1 + P$ from Assumption 2. For the failure state m, $\gamma((i,r,z),1) = C_1 + K (> C_1 + P)$. Thus for arbitrary $r \in \Phi$ and $z \in \{0,1\}$, one step transition cost $\gamma((i,r,z),\pi_{(i,r,z)})$ under a threshold type policy $\pi = (0,\ldots,0,1,\ldots,1)$ is non-decreasing on i. □

Lemma 1 states that under a threshold type policy, the one step transition cost in the given Markov decision process is monotonically non-decreasing as the health of a component deteriorates.

Lemma 2. *If $f(i)$ be a non-decreasing function in state i, and transition probability matrix $\{p_{ij}(r)\}$, $r \in \Phi$ is IFR, then*

$$\sum_{l=k}^{m} p_{il}(r) f(l) \leqslant \sum_{l=k}^{m} p_{jl}(r) f(l), \quad 0 \leqslant i \leqslant j \leqslant m, r \in \Phi, k \in \Psi \tag{5}$$

$$\sum_{l=k}^{m} P^{(t)}_{(i,r,z)(l,\theta,z')}(\pi) \leqslant \sum_{l=k}^{m} P^{(t)}_{(j,r,z)(l,\theta,z')}(\pi),$$
$$0 \leqslant i \leqslant j \leqslant m, r, \theta \in \Phi, k \in \Psi, z, z' \in \{0,1\}, t = 1,\ldots \tag{6}$$

Proof. We can directly derive Eq. (5) based on Assumption 1 and the Proposition 8.1.2 of Ross (1996). Next, we prove Eq. (6) by the mathematical induction on t. When $t = 1$, Eq. (6) is true as follows:

$$\sum_{l=k}^{m} P_{(i,r,0)(l,r,0)}(0) = \lambda(\mu(r)+\lambda+\chi)^{-1} \sum_{l=k}^{m} p_{il}(r) \leqslant \lambda(\mu(r)+\lambda+\chi)^{-1}$$
$$\sum_{l=k}^{m} p_{jl}(r) = \sum_{l=k}^{m} P_{(j,r,0)(l,r,0)}(0)$$
$$P_{(i,r,0)(i,\theta,0)}(0) = \chi(\mu(r)+\lambda+\chi)^{-1} q_{r\theta} = P_{(j,r,0)(j,\theta,0)}(0)$$
$$P_{(i,r,0)(i,r,1)}(0) = \mu(r)(\mu(r)+\lambda+\chi)^{-1} = P_{(j,r,0)(j,r,1)}(0)$$
$$\sum_{l=k}^{m} P_{(i,r,0)(l,r,0)}(1) = \lambda(\mu(r)+\lambda+\chi)^{-1} \sum_{l=k}^{m} p_{0l}(r) = \sum_{l=k}^{m} P_{(j,r,0)(l,r,0)}(1)$$
$$P_{(i,r,0)(0,\theta,0)}(1) = P_{(0,r,0)(0,\theta,0)}(0) = \chi(\mu(r)+\lambda+\chi)^{-1} q_{r\theta} = P_{(j,r,0)(0,\theta,0)}(1)$$
$$P_{(i,r,0)(0,r,1)}(1) = P_{(0,r,0)(0,r,1)}(0) = \mu(r)(\mu(r)+\lambda+\chi)^{-1} = P_{(j,r,0)(0,r,1)}(1).$$

Suppose that Eq. (7) holds for $t = \omega$

$$\sum_{l=k}^{m} P^{(\omega)}_{(i,r,z)(l,\theta,z')}(\pi) \leqslant \sum_{l=k}^{m} P^{(\omega)}_{(j,r,z)(l,\theta,z')}(\pi) \tag{7}$$

Then for $t = \omega + 1$,

$$\begin{aligned} \sum_{l=k}^{m} P^{(\omega+1)}_{(i,r,z)(l,\theta,z')}(\pi) &= \sum_{l=k}^{m} \left[\sum_{(i_1,r_1,z_1)\in S} \left(P^{(\omega)}_{(i,r,z)(i_1,r_1,z_1)}(\pi) P_{(i_1,r_1,z_1)(l,\theta,z')}(\pi) \right) \right] \\ &= \sum_{(i_1,r_1,z_1)\in S} \left[P^{(\omega)}_{(i,r,z)(i_1,r_1,z_1)}(\pi) \sum_{l=k}^{m} P_{(i_1,r_1,z_1)(l,\theta,z')}(\pi) \right] \\ &= \sum_{z_1=0}^{1} \sum_{r_1\in\Phi} \sum_{i_1\in\Psi} \left[P^{(\omega)}_{(i,r,z)(i_1,r_1,z_1)}(\pi) \sum_{l=k}^{m} P_{(i_1,r_1,0)(l,\theta,z')}(\pi) \right] \\ &\leqslant \sum_{z_1=0}^{1} \sum_{r_1\in\Phi} \sum_{i_1\in\Psi} \left[P^{(\omega)}_{(j,r,z)(i_1,r_1,z_1)}(\pi) \sum_{l=k}^{m} P_{(i_1,r_1,0)(l,\theta,z')}(\pi) \right] \\ &= \sum_{l=k}^{m} P^{(\omega+1)}_{(j,r,z)(l,\theta,z')}(\pi) \end{aligned}$$

Note that given r_1, $\sum_{l=k}^{m} P_{(i_1,r_1,0)(l,\theta,z')}(\pi)$ is a non-decreasing function of i_1. Using Eq. (5) we obtain the above inequality. Thus we prove Eq. (6) by the mathematical induction on t. □

Lemma 2 extends the IFR concept. In inequality (5), transition probability from state i multiplies a non-decreasing function in state i to obtain a new matrix. Inequality (5) indicates that for any environment state, if a deterioration matrix is IFR, then the new matrix is also IFR. Inequality (6) indicates that the t-step transition matrix of the single-component system is also IFR.

Next, we present the two main theorems of this paper. First, we prove that the relative value under a threshold type policy is non-decreasing in the order of deterioration in any combination of environment and opportunity status.

Theorem 1. *The relative value $v_\pi(i,r,z)$, $(i,r,z) \in S$ under a threshold type policy $\pi = (0,\ldots,0,1,\ldots,1)$ is non-decreasing in state i for all $r \in \Phi$, $z \in \{0,1\}$.*

Proof. Referring to Tijms (1994), we know

$$v_\pi(j,r,z) - v_\pi(i,r,z) = \lim_{n\to\infty}\{V_n((j,r,z),\pi_{(j,r,z)}) - V_n((i,r,z),\pi_{(i,r,z)})\}, \quad \text{for } \forall i,j, 0 \leqslant i \leqslant j \leqslant m \tag{8}$$

where $V_n((i,r,z),\pi_{(i,r,z)}) = \sum_{t=0}^{n-1}\sum_{l=0}^{m}\sum_{r'\in\Phi,z'\in\{0,1\}}P^{(t)}_{(i,r,z)(l,r',z')}(\pi)\gamma((l,r',z'),\pi_{(l,r',z')})$ is the total expected maintenance and operating costs over the first n decision epochs when the initial state is (i,r,z) and policy π is used. Based on Lemmas 1 and 2, we have

$$\begin{aligned} V_n((i,r,z),\pi_{(i,\omega)}) &= \sum_{t=0}^{n-1}\sum_{r'\in\Phi,z'\in\{0,1\}}\sum_{l=0}^{m}P^{(t)}_{(i,r,z)(l,r',z')}(\pi)\gamma((l,r',z'),\pi_{(l,r',z')}) \\ &\leqslant \sum_{t=0}^{n-1}\sum_{r'\in\Phi,z'\in\{0,1\}}\sum_{l=0}^{m}P^{(t)}_{(j,r,z)(l,r',z')}(\pi)\gamma((l,r',z'),\pi_{(l,r',z')}) \\ &= V_n((j,r,z),\pi_{(j,r,z)}) \end{aligned}$$

Consequently, we prove that

$$v_\pi(j,r,z) - v_\pi(i,r,z) = \lim_{n\to\infty}\{V_n((j,r,z),\pi_{(j,r,z)}) - V_n((i,r,z),\pi_{(i,r,z)})\} \geqslant 0.$$

□

Now we can introduce the policy-improvement test quantity $T_\pi(s,a)$ to prove Theorem 2.

$$T_\pi(s,a) \equiv \gamma(s,a) - g_\pi(\lambda+\mu)^{-1} + \sum_{s'\in S}P_{ss'}(a)v_\pi(s'), \quad a \in A. \tag{9}$$

Note that $T_\pi(s,\pi_s) = v_\pi(s)$ and this quantity can be interpreted as the relative value when selecting action a in the initial state s and using policy π. Then, based on the aforementioned analysis, we can present the proof of existence of a (n_r,N_r) type optimal policy.

Theorem 2. *There exists a (n_r,N_r) type optimal policy with the threshold states $(N_r,0)$, $(n_r,1)$ and $n_r \leqslant N_r$ for environment state $r \in \Phi$.*

Proof. We will first show that an optimal policy has thresholds $(N_r,0)$, $(n_r,1)$ for environment state $r \in \Phi$. Sequentially, $n_r \leqslant N_r$ will be proved. Suppose that an optimal policy for the case of $z = 0$, $r \in \Phi$ is $\bar\pi = \left(\ldots,\underset{\substack{\downarrow\\ i}}{1},\ldots,\underset{\substack{\downarrow\\ j}}{0},1,1,\ldots,1\right)$ where $\bar\pi_{(i,r,z)} = 1$ and $\bar\pi_{(j,r,z)} = 0$ for any $j(>i)$. We want to show that a form of $\bar\pi$ cannot be optimal by demonstrating that if $\pi_{(j,r,0)} = 0$ in $\pi = \left(\ldots,\underset{\substack{\downarrow\\ i}}{0},\ldots,\underset{\substack{\downarrow\\ j}}{0},1,1,\ldots,1\right)$, then $\pi_{(i,r,0)} = 0$ is a better policy than $\pi_{(i,r,0)} = 1$. Under policy π, we calculate

$$\begin{aligned} T_\pi((j,r,0),1) &= \gamma((j,r,0),1) - g_\pi(\lambda+\mu(r)+\chi)^{-1} \\ &\quad + \sum_{s'\in S}P_{(j,r,0)s'}(1)v_\pi(s') = C_0 + P - g_\pi(\lambda+\mu(r)+\chi)^{-1} \\ &\quad + \sum_{s'\in S}P_{(0,r,0)s'}(0)v_\pi(s')(\because P_{(j,r,0)s'}(1) = P_{(0,r,0)s'}(0)) \end{aligned}$$

$$v_\pi(j,r,0) = c(j) - g_\pi(\lambda+\mu(r)+\chi)^{-1} + \sum_{s'\in S}P_{(j,r,0)s'}(0)v_\pi(s')$$

From $\pi_{(j,r,0)} = 0$ and Theorem 3.2.1 of Tijms (1994), we have

$$\begin{aligned} 0 &\leqslant T_\pi((j,r,0),1) - v_\pi(j,r,0) \\ &= C_0 + P - c(j) + \sum_{s'\in S}P_{(0,r,0)s'}(0)v_\pi(s') - \sum_{s'\in S}P_{(j,r,0)s'}(0)v_\pi(s') \\ &< C_0 + P - c(i) + \sum_{s'\in S}P_{(0,r,0)s'}(0)v_\pi(s') - \sum_{s'\in S}P_{(i,r,0)s'}(0)v_\pi(s') \\ &= T_\pi((i,r,0),1) - v_\pi(i,r,0) \end{aligned} \tag{10}$$

The last inequality in (10) comes from the following inequalities, based on Eq. (1), Assumption 2 and Lemma 2:

$$\begin{aligned} &c(i) < c(j) \\ &\sum_{s'\in S}P_{(i,r,0)s'}(0)v_\pi(s') < \sum_{s'\in S}P_{(j,r,0)s'}(0)v_\pi(s') \quad \text{for } i<j \text{ under } \pi \end{aligned}$$

Thus we have $\pi_{(i,r,0)} = 0$, which indicates that a policy $\pi_{(i,r,0)} = 0$ is more cost-saving than $\pi_{(i,r,0)} = 1$. Therefore $\bar\pi$ cannot be the optimal. Since a CM policy $(0,\ldots,0,1)$ is one of possible optimal policies and $\bar\pi$ cannot be a form of an optimal policy, we can conclude that an optimal policy for $z = 0$ should be a threshold type. Similarly, the existence of thresholds $(n_r,1)$ for $z = 1$ can be shown.

Next, we will show that $n_r \leqslant N_r$, $r \in \Phi$. Suppose the optimal policy is π^* with thresholds $(N_r,0)$, $(n_r,1)$. Then the following inequalities hold

$$\begin{cases} T_{\pi^*}((i,r,0),0) < T_{\pi^*}((i,r,0),1) & \text{if } i<N_r \\ T_{\pi^*}((i,r,0),0) \geqslant T_{\pi^*}((i,r,0),1) & \text{if } i \geqslant N_r \end{cases} \quad \begin{cases} T_{\pi^*}((i,r,1),0) < T_{\pi^*}((i,r,1),1) & \text{if } i<n_r \\ T_{\pi^*}((i,r,1),0) \geqslant T_{\pi^*}((i,r,1),1) & \text{if } i \geqslant n_r \end{cases}$$

It is easy to understand that the following relationships exist,

$$T_{\pi^*}((N_r,r,0),0) \geqslant T_{\pi^*}((N_r,r,0),1)$$

$$T_{\pi^*}((N_r,r,0),0) = T_{\pi^*}((N_r,r,1),0)$$

$$T_{\pi^*}((N_r,r,0),1) \geqslant T_{\pi^*}((N_r,r,1),1)$$

Then, we have the relationship $T_{\pi^*}((N_r,r,1),0) \geqslant T_{\pi^*}((N_r,r,1),1)$. According to the structure of threshold n_r, we obtain the inequality $n_r \leqslant N_r$. Thus the theorem is proved. □

Theorem 2 provides the (n_r,N_r) form of an optimal policy for a single component in a multi-component system subjected to environmental influence. This solution can be calculated through the policy iteration method. In next section, we present an approximation algorithm to calculate near optimal policies for all components in a multi-component system.

3. The iterative approximation algorithm for multi-components

In this section, we will provide an iterative approximation algorithm for multi-component systems. This algorithm is based on the policy iteration method for a single-component system (Tijms, 1994). First, the algorithm initializes PM threshold N_r, OM threshold n_r, and maintenance opportunity $\mu(r)$ for each component. Second, suppose the algorithm has been running for the Tth iteration. Based on the PM threshold, OM threshold, and maintenance opportunity obtained at the $(T\text{-}1)$th iteration, a new maintenance policy can be computed by the policy iteration method for each component. Using this renewed policy, the algorithm updates the PM threshold, OM threshold and maintenance opportunity again. If the renewed policy is identical to the one calculated at the $(T\text{-}1)$th iteration, then the algorithm is terminated. Otherwise, conduct Step 2 again. Here we present the iterative approximation algorithm for multi-component systems.

Step 1. For each environment r, initialize PM threshold N^k_r, $r \in \Phi$, OM threshold $n^k_r = N^k_r$, $r \in \Phi$ and maintenance opportunity $\mu^k(r)$, $r \in \Phi$ for each component $k = 1,\ldots,L$.

Step 2. For the Tth iteration, obtain the $\left(n_r^k, N_r^k\right)$ type policy by the policy iteration method for each component $k = 1, \ldots, L$. Then update their maintenance opportunistic rates. For each component $k = 1, \ldots, L$, obtain its steady-state probability distribution $\theta^k(i,r)$ from the embedded Markov chain without maintenance opportunities consideration. Then calculate the followings:

– maintenance rate : $\alpha_k(r) = \lambda_k \dfrac{\sum_{i=N_r^k}^{m_k} \theta^k(i,r)}{\sum_{i=0}^{m_k} \theta^k(i,r)}$

– maintenance opportunistic rate : $\mu^k(r) = \sum_{l \neq k} \alpha_l(r)$

Step 3. Check whether the renewed maintenance policy is the same with the calculated policy at the (T-1)th iteration. If so, the algorithm is terminated. Otherwise, go to Step 2.

The main idea of this iterative approximation algorithm is adapted from the decomposition method used in Federgruen, Groenevelt, and Tijms (1984). Policy iteration method is a classical way to solve MDP which is introduced in detail in any MDP literature (Puterman, 1994; Tijms, 1994). Consequently, we directly use this method to renew the maintenance policy for each single-component. Based on renewed policies during iteration procedures, we update maintenance opportunistic rates for all single components in a system. In this study, maintenance opportunity comes from the other components' failures and PMs. We introduce the maintenance rate $\alpha_k(r)$, which represents an occurrence rate of maintenance action (including CM and PM) from component k under environment r in its long-run operation. The maintenance rate is related to the sum of steady-state probabilities of being in either failure state or PM states. Thus, the maintenance opportunistic rate $\mu^k(r)$ of a component k equals to the sum of all the maintenance rates from other components.

Next, we use numerical examples to confirm the feasibility of this algorithm and analyze the maintenance model for multi-component systems.

4. Numerical examples

We provide two numerical examples to validate the feasibility of the proposed algorithm for multi-component systems and to demonstrate the advantage of the obtained maintenance policy with a relatively large-scale problem. Throughout all the examples in this section, we assume that degradation process of each component can be modeled by a Markov model with six states and the environmental condition can be modeled by another Markov model with four states.

4.1. Numerical case study I for OM justification

Consider a multi-component system consisting of 16 identical components (C1 ~ C16) with their transition probability matrices for degradation processes (A_i) and cost parameters listed in Tables 1 and 2, respectively. Furthermore, transition probability matrices for environmental changes (B_i) are listed in Table 3. The four matrices from A_1 to A_4 correspond to degradation processes under four environment states. For instance, if the environment state r is equal to 2, then components under this environmental condition will degrade according to the transition probability matrix of A_2. The deterioration speed of the transition probability matrix assumes to become faster as the condition of environment worsens, as shown in Table 1.

Table 2
Parameter settings for numerical case study I.

C_0	C_1	K	P	C_{slope}	λ	χ	$\{p_{ij}(r)\}$	$\{q_{ij}\}$
400	100	8000	4000	200	0.02	0.0083	A_r	B

In this numerical case study I, we want to illustrate three findings on the proposed $\left(n_r^k, N_r^k\right)$ policy. First, environmental influence should be considered when it comes to determining a maintenance policy. Second, OM is necessary to reduce average maintenance cost especially for multi-component systems. Third, the cost savings from OM become significant as the scale of the problem increases. In order to demonstrate such findings, we design the following four scenarios:

Scenario 1: OM with no environmental influence is considered.
Scenario 2: OM is ignored. Under this scenario, if a component health state is worse than N, PM is performed.
Scenario 3: Both environmental influence and OM are considered.
Scenario 4: The policy obtained from Scenario 1 is applied to Scenario 3 in which the environmental influence exists.

The system in the first scenario assumes to have no environmental influence to components' deterioration processes by setting

$$\{p_{ij}(0)\} = \{p_{ij}(1)\} = \{p_{ij}(2)\} = \{p_{ij}(3)\} = A_1. \tag{11}$$

As we can see in Eq. (11), we force all the transition probability matrices for deterioration in the first scenario to be the same despite different environmental conditions. In the second scenario, in order to realize non-OM in the given framework, we set $C_1 = C_0$, $\mu(r) = 0$, $r \in \Phi$ for each component, so that we assures the probability of OM occurrence is zero. For the third scenario, all the parameters are set as listed in Tables 1–3. Furthermore, in the last scenario, we apply the maintenance policy obtained in the first scenario to the third scenario, and then calculate its long-run average maintenance cost.

Then, we apply the proposed algorithm in Section 3 to calculate the optimal average maintenance cost g_π and to find optimal maintenance policy for each scenario. Scenario 1 does not have environmental influence, so for each environment state $r = 0,1,2,3$ the maintenance threshold is the same. We denote its one single optimal OM threshold n and optimal PM threshold N as $\pi_1 = (n,N)$ for all identical components. However, there is no OM in Scenario 2, its optimal policy just involves PM threshold for each environment state. We denote its optimal PM threshold under environment state r as $\pi_2 = (N_0,N_1,N_2,N_3)$. For Scenario 3, we denote the optimal policy as $\pi_3 = (n_0,N_0;n_1,N_1;n_2,N_2;n_3,N_3)$. The results are listed in Table 4 and the following findings can be discussed.

Table 1
Transition probability matrices for degradation processes.

A_1	A_2	A_3	A_4
$\begin{bmatrix} 0.8 & 0.1 & 0.1 & 0 & 0 & 0 \\ 0 & 0.8 & 0.1 & 0.1 & 0 & 0 \\ 0 & 0 & 0.8 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0.7 & 0.3 & 0 \\ 0 & 0 & 0 & 0 & 0.7 & 0.3 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.7 & 0.3 & 0 & 0 & 0 & 0 \\ 0 & 0.6 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0.4 & 0.6 & 0 & 0 & 0 \\ 0 & 0 & 0.4 & 0.6 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 0.7 & 0 \\ 0 & 0 & 0 & 0 & 0.3 & 0.7 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.3 & 0.7 & 0 & 0 & 0 & 0 \\ 0 & 0.2 & 0.8 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0.9 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0.9 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

Table 3
Transition probability matrices for environmental changes.

Normal environment B	Good environment B_1	Not- bad environment B_2	Bad environment B_3
$\begin{bmatrix} 0 & 0.1 & 0.3 & 0.6 \\ 0.1 & 0 & 0.3 & 0.6 \\ 0.1 & 0.2 & 0 & 0.7 \\ 0.6 & 0.2 & 0.2 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0.6 & 0.3 & 0.1 \\ 0.6 & 0 & 0.3 & 0.1 \\ 0.7 & 0.2 & 0 & 0.1 \\ 0.6 & 0.2 & 0.2 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0.1 & 0.6 & 0.3 \\ 0.1 & 0 & 0.6 & 0.3 \\ 0.7 & 0.2 & 0 & 0.1 \\ 0.6 & 0.2 & 0.2 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0.1 & 0.6 & 0.3 \\ 0.1 & 0 & 0.6 & 0.3 \\ 0.1 & 0.7 & 0 & 0.2 \\ 0.2 & 0.6 & 0.2 & 0 \end{bmatrix}$

Table 4
Optimal policy π and g_π for different scales.

# of components	Scenario 1 Non-environmental influence & OM	Scenario 2 Environmental influence & non-OM		Scenario 3 Environmental influence & OM		Scenario 4 Environmental influence & OM with π_1	Cost saving from environmental influence (%)	Cost saving from OM (%)
	$\pi_1 = (n,N)$	$\pi_2 = N_r$	g_{π_2}	$\pi_3 = (n_r,N_r)$	g_{π_3}	g'_{π_1}	$\left(g'_{\pi_1} - g_{\pi_3}\right)/g_{\pi_3}$	$(g_{\pi_2} - g_{\pi_3})/g_{\pi_3}$
2 (C1-C2)	(3,3)	(3,3,4,4)	20.3626	(3,3;3,3;3,4;4,4)	20.3595	21.2883	4.56	0.02
3 (C1-C3)	(2,3)	(3,3,4,4)	20.8465	(3,3;3,3;3,4;4,4)	20.8375	22.1383	6.24	0.04
4 (C1-C4)	(2,3)	(3,3,4,4)	21.3305	(3,3;3,3;3,4;4,4)	21.3131	22.6338	6.20	0.08
5 (C1-C5)	(2,3)	(3,3,4,4)	21.8144	(3,3;3,3;3,4;3,4)	21.7846	23.0757	5.93	0.14
6 (C1-C6)	(2,3)	(3,3,4,4)	22.2983	(3,3;3,3;3,4;3,4)	22.2399	23.4776	5.57	0.26
7 (C1-C7)	(2,3)	(3,3,4,4)	22.7822	(2,3;3,3;3,4;3,4)	22.6776	23.8489	5.17	0.46
8 (C1-C8)	(2,3)	(2,3,4,4)	23.3742	(2,3;2,3;3,4;3,4)	23.0699	24.1961	4.88	1.32
9 (C1-C9)	(2,3)	(2,3,4,4)	23.9143	(2,3;2,3;3,4;3,4)	23.4353	24.5239	4.64	2.04
10 (C1-C10)	(2,3)	(2,3,3,4)	24.2894	(2,3;2,3;3,4;3,4)	23.7906	24.8358	4.39	2.10
11 (C1-C11)	(2,3)	(2,2,3,4)	24.8261	(2,3;2,3;3,4;3,4)	24.1375	25.1345	4.13	2.85
12 (C1-C12)	(2,3)	(2,2,3,3)	26.1079	(2,3;2,3;3,4;3,4)	24.4774	25.4221	3.86	6.66
13 (C1-C13)	(2,3)	(2,2,3,3)	26.4758	(2,3;2,3;3,4;3,4)	24.8112	25.7004	3.58	6.71
14 (C1-C14)	(2,3)	(2,2,3,3)	26.8438	(2,3;2,3;3,4;3,4)	25.1400	25.9705	3.30	6.78
15 (C1-C15)	(2,3)	(2,2,2,3)	27.8285	(2,3;2,3;2,3;3,4)	25.7706	26.4303	2.56	7.99
16 (C1-C16)	(2,3)	(2,2,2,3)	28.1738	(2,3;2,3;2,3;3,4)	26.0745	26.6961	2.38	8.05

1. $g'_{\pi_1} \geqslant g_{\pi_3}$ for all the scales
The ignorance of environmental impact on degradation processes increases the average cost. Note that we do not directly compare the optimal average cost g_{π_1} calculated in Scenario 1, and g_{π_3}, because these two costs involve the effect of different transition matrices of the environment. In order to eliminate this effect, we apply the optimal policy obtained in Scenario 1 to Scenario 3, and then obtain the average cost g'_{π_1}. Thus the discrepancy between g_{π_3} and g'_{π_1} is completely from the ignorance of the environmental influence.
2. $g_{\pi_2} \geqslant g_{\pi_3}$ for all the scales
When OM is performed, the optimal average cost of each component can be saved. This is because $C_1 \leqslant C_0$, which infers that maintenance opportunity provides a relatively cheaper chance to preventively maintain a component in a multi-component system.
3. $\left[\frac{(g_{\pi_2} - g_{\pi_3})}{g_{\pi_3}}\right]_{k=i} \leqslant \left[\frac{(g_{\pi_2} - g_{\pi_3})}{g_{\pi_3}}\right]_{k=j}$ for $i \leqslant j$
As the scale of the problem (i.e., the number of components) increases, the cost savings from OMs become significant. Since maintenance opportunity comes from other components' failures and PMs, OM becomes more beneficial when the proposed maintenance policy is applied to a multi-component system.

4.2. Numerical case study II for environmental influence and model analysis

In this case study, we perform the sensitivity analysis to study how cost parameters affect the maintenance cost saving rate from environmental influence. Consider 16 identical components with the cost parameters and transition probability matrices, listed in Tables 1 and 2. We use the parameters in Table 2 as a baseline, and vary only one of the cost parameters (C_0, C_1, K, and P) at each experiment. We calculate the optimal policy for the baseline, and denote it as $\pi_B = (2,3;2,3;3,4;3,4)$. This baseline policy π_B is applied to all the experiments. Then, we calculate the cost saving rates from different values of cost parameters with three different environment transition matrices of B_1, B_2, and B_3 (see Table 3). The value of cost saving rate in each condition is listed from Tables 5–8. The cost saving rate is derived from the discrepancy between the scenario without environmental influence and the scenario with environmental influence.

From Tables 5–8, it is evident that from column B_1 to column B_3 the cost saving rate increases. A harsh environment condition will accelerate degradation processes of all components, and thus

Table 5
Cost saving rates under different environment transition matrices and values of K.

CM cost, K	Cost saving rate from environmental influence (%)		
	B_1	B_2	B_3
$K = 11000$	1.2828	1.9715	2.1356
$K = 10000$	1.2828	1.9715	2.1356
$K = 9000$	1.2828	1.9715	2.1356
$K = 8000$	1.2828	1.9715	2.1356

Table 6
Cost saving rates under different environment transition matrices and values of P.

PM cost, P	Cost saving rate from environmental influence (%)		
	B_1	B_2	B_3
$P = 5500$	3.1297	4.6979	4.9895
$P = 5000$	2.5961	3.9043	4.1613
$P = 4500$	1.9863	3.0052	3.2185
$P = 4000$	1.2828	1.9715	2.1356

Table 7
Cost saving rates under different environment transition matrices and values of C_0.

Setup cost without maintenance opportunities, C_0	Cost saving rate from environmental influence (%)		
	B_1	B_2	B_3
$C_0 = 550$	1.3049	2.0078	2.1591
$C_0 = 500$	1.2976	1.9957	2.1513
$C_0 = 450$	1.2902	1.9836	2.1435
$C_0 = 400$	1.2828	1.9715	2.1356

Table 8
Cost saving rates under different environment transition matrices and values of C_1.

Setup cost with maintenance opportunities, C_1	Cost saving rate from environmental influence (%)		
	B_1	B_2	B_3
$C_1 = 100$	1.2828	1.9715	2.1356
$C_1 = 75$	1.2486	1.9219	2.0812
$C_1 = 50$	1.2142	1.8721	2.0266
$C_1 = 25$	1.1795	1.8220	1.9717

Table 9
Average slopes of the cost parameters under different environment status.

Cost parameters	Average slope of cost saving rate (10^{-4})		
	B_1	B_2	B_3
K	0	0	0
P	12.31	18.18	19.03
C_0	1.47	2.42	1.57
C_1	13.77	19.93	21.85

require more intelligent maintenance actions. The benefit of employing the proposed maintenance policy has clearly been shown in all the sensitivity analysis.

Next we calculate the average slopes of the cost parameters (C_0, C_1, K,and P) under different environment status in Table 9. For cost parameter K, its variance has no effect on cost saving rate from environmental influence in Table 5. This is because in both the scenario with environmental influence and the scenario without environmental influence, CM is hardly taken. Thus, the variance of K does not change the discrepancy between their optimal policies. The variances of parameters P and C_1 significantly affect the cost saving rate because opportunistic preventive maintenance (OPM) is often conducted for a 16-component system. On the other hand, the impact of the variance of C_0 is small because PMs are performed but not as frequently as OPM.

5. Conclusion

In this paper, we study the optimal maintenance policy for multi-component systems, which environmental conditions and opportunistic maintenance (OM) involve. The deterioration of each component is formulated as a continuous Markov chain and influenced by their surrounding environmental conditions. As the environmental condition turns to be worse, the deterioration speed of each component accelerates. OM is also considered when some other components fail or take PM. We decompose the multi-component system into many single-component systems and formulate each of them as a MDP. Under some reasonable maintenance assumptions, it was shown that there exist (n_r,N_r) type optimal policies. Furthermore, based on the policy iteration method, we develop an iterative approximation algorithm to obtain an acceptable policy for the entire multi-component system. Finally, through numerical examples, we demonstrate that environmental conditions and OM pose significant impacts on how to determine an optimal maintenance policy, and that how cost parameters affect the optimal average costs and maintenance policies.

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