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An algorithm of determining T-spline classification

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ABSTRACT

T-splines are a new surface modeling technology, whose theoretical framework is not still well founded. Aiming at the problem that how to classify T-splines this paper gives a mathematical analysis, and a sufficient condition of standard T-splines is also given. Then this paper presents an algorithm of determining the T-spline classification, and we can get the inherently mathematical properties of T-splines by the algorithm. At last, the experimental results verify the effectiveness of our algorithm. The results in this paper play an important role in the research on T-spline theory and T-spline modeling algorithms.

1. Introduction

Surface modeling is one of the most important techniques in CAD/CAM systems. By the late 1980s, non-uniform rational B-splines had become an important tool for describing the shape of curves and surfaces. Now classic CAD/CAM systems use NURBS as general expressions of free curves and surfaces, and NURBS is widely applied to the product design of aircrafts, automobiles and ships, etc. (Cohen, Riesenfeld, & Elber, 2001; Piegl & Tiller, 1997). Surface modeling based on non-uniform rational B-splines (NURBS) is a continuous method, but it is not easy to represent complex surfaces. For the surface with more complex topology structure, we often use several NURBS patches to represent the complex surface. As a complex surface defined on an arbitrary topology structure cannot be expressed by one NURBS surface, people began to focus on surface modeling based on subdivision (Doo & Sabin, 1978). Subdivision surfaces are relatively simple and flexible, and they can be defined on any topology structure, but there are some deficiencies for traditional subdivision methods: subdivision surfaces do not have explicit analytical expression, they are not compatible with existing general CAD/CAM systems, etc. For the above reasons, surface modeling based on subdivision has some deficiencies in the practical applications.

Aiming at the deficiencies of NURBS and subdivision surfaces, in 2003 the T-spline was proposed by Sederberg, Zheng, Bakenov, and Nasri (2003). Compared to other modeling methods, the T-spline has brought many significant advantages in the surface merging, local refinement, and data compression, etc. (Sederberg, Cardon, Zheng, & Lyche, 2004; Sederberg et al., 2003). Later on some related application algorithms of T-splines were proposed (Sederberg, Finnigan, Li, Lin, & Ipson, 2008; Uhm & Youn, 2009). Since the introduction of T-splines in 2003, many scholars have conducted extensive research on the T-spline theory and applications, especially in surface modeling based on T-splines and related theories. In 2004, Sederberg etc. posed an problem about T-spline classification (Sederberg et al., 2004). So far these theoretical issues of T-splines have not been solved, which limits the further application of T-splines. In 2010, literature (Buffa, Cho, & Sangalli, 2010) gave a proof of linear independence of the T-spline blending functions associated with some particular T-meshes, after that the equivalent condition of linear independence about the T-spline blending functions has been researched (Li, Zheng, Sederberg, Hughes, & Scott, 2012; Wang & Zhao, 2011), and literature (Li et al., 2012) has proved that any analysis-suitable T-spline has linearly independent blending functions. In recent years Isogeometric analysis using T-splines has been further explored (Ha, Choi, & Cho, 2010; Rypl & Patzak, 2012).

The above literatures enrich the theoretical research of T-spline modeling methods. T-splines have shown us great potential for development. On the one hand, the T-spline allows T-mesh topology structure, which makes it has a greater prospect than NURBS that requires topology structure of rectangular mesh. On the other hand, as an extension of B-spline technique, it is easily compatible with existing CAD/CAM systems, which cannot be achieved by subdivision surface. By our proof of T-spline classification, we can clearly know the inherently mathematical properties of T-splines, and may contribute to the theoretical basis for T-spline modeling methods.

This paper is structured as follows: Section 2 and Section 3 mainly introduce the mathematical properties of T-splines and

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the classification of T-splines. The mathematical analysis of T-spline classification is given in Section 4. In Section 5, we give the algorithm for determining T-spline classification. Several experimental results in Section 6 demonstrate the effectiveness of the determining algorithm. The conclusions are finally given in Section 7.

2. The mathematical properties of T-splines

In 2003, Sederberg, etc. first proposed T-splines that are generalized from NURBS, and introduced the concept of PB-splines. The PB-spline is point based spline whose control points have no topological relationship with each other. So the PB-spline is very flexible, and it is not convenient in practical applications. From literature (Sederberg et al., 2003), we know a T-spline is a PB-spline defined on a T-mesh, the T-spline whose control points satisfy certain topological relations by means of the T-mesh. For NURBS surfaces, all knots on each row and each column of the control mesh are same, unlike NURBS surfaces, the knot sequences of T-splines can be different.

The equation for a T-spline is

$$P(u,v)=\sum_{i=1}^{n}P_iB_i(u,v)$$

where $P_i = (w_ix_i, w_iy_i, w_iz_i, w_i)$ are control points, $B_i(u, v)$ are T-spline blending functions defined by $\mathbf{u}_i \times \mathbf{v}_i$.

$$B_i(u,v)=N[\mathbf{u}_i](u)N[\mathbf{v}_i](v)$$

$$\mathbf{u}_i=[u_{i1},u_{i2},u_{i3},u_{i4},u_{i5}]$$

$$\mathbf{v}_i=[v_{i1},v_{i2},v_{i3},v_{i4},v_{i5}]$$

where $N[\mathbf{u}_i](u)$ and $N[\mathbf{v}_i](v)$ are B-spline functions defined by the knot vectors $\mathbf{u}_i$ and $\mathbf{v}_i$, respectively. T-splines can be any degree, in this paper, we restrict our attention to the common case of cubic T-splines, i.e. the degree = 3, all boundary knots have a multiplicity of no more than 4, all inner knots have a multiplicity of no more than 3. Several relative definitions and properties of T-splines are given as follows.

The least B-mesh of a T-spline (LB-mesh): Given a T-spine, throughout all the T-junctions in its T-mesh, we can get a B-mesh corresponding with the T-mesh, we call the obtained B-mesh as the least B-mesh of the T-spline.

The least B-spline of a T-spline (LB-spline): for a T-spline, we call the B-spline defined on the LB-mesh of the T-spline as the least B-spline of the T-spline.

From the above definitions, we can get each T-mesh corresponds to a unique LB-mesh that is the least rectangle mesh that contains the T-mesh. By the properties of B-splines we have the T-spline blending functions defined on the LB-mesh are linearly independent.

Lemma 1. *For a standard or semi-standard T-spline, if its T-spline blending functions are linearly independent, then $\{\omega_i\}_{i=1}^n$ is unique for $\sum_{i=1}^n\omega_iB_i(u,v)=1$.*

Proof. If $\{\omega_i\}_{i=1}^n$ is not unique, then $\exists\ \{\lambda\}_{i=1}^n\neq\{\omega_i\}_{i=1}^n$ satisfying $\sum_{i=1}^n\lambda_iB_i(u,v)=1$, thus we can get $\sum_{i=1}^n(\lambda_i-\omega_i)B_i(u,v)=0$, $(\lambda_i-\omega_i)1\leqslant i\leqslant n$ not all equal to zero, that is to say the T-spline blending functions are linearly dependent, which contradicts to the known condition. So $\{\omega_i\}_{i=1}^n$ is unique. □

Lemma 2. *The blending functions of a standard T-spline are linearly independent.*

Proof. If the blending functions of the standard T-spline are linearly dependent, then $\sum_{i=1}^n\lambda_iB_i(u,v)=0$, where $\lambda_i(1\leqslant i\leqslant n)$ not all equal to zero. Since the T-spline is a standard T-spline, $\sum_{i=1}^nB_i(u,v)=1$. From above equations, we can get $\sum_{i=1}^n(1+\lambda_i)B_i(u,v)=1$, where $\lambda_i(1\leqslant i\leqslant n)$ not all equal to zero, that is to say the T-spline is a semi-standard T-spline, contradicting to the known condition. Therefore, the blending functions of a standard T-spline are linearly independent. □

3. The classification of T-splines

Literature (Sederberg et al., 2003) gave the definition of standard T-splines, literature (Sederberg et al., 2004) gave the definitions of semi-standard T-splines and non-standard T-splines. Standard, semi-standard, and non-standard describe three categories of T-Splines. Obviously, a B-spline surface is a special standard T-spline. Based on the above literatures, by the view of algebra, this paper gives the mathematical analysis and relative proofs of T-spline classification in Section 4.

For a given T-spline, suppose $B_i(s, t)$ is the blending function corresponding to the ith T-spline control point, the definitions of the three categories of T-Splines are as follows (Sederberg et al., 2003, 2004).

(a) If $\sum_{i=1}^nB_i(s,t)\equiv1$, then the T-spline is a standard T-spline, as is shown in Fig. 1.
(b) If $\sum_{i=1}^n\omega_iB_i(s,t)\equiv1$, where there exists a set of weights ω_i not all equal to 1, then the T-spline is a semi-standard T-spline, as is shown in Fig. 2.
(c) If there does not exist a set of weights ω_i satisfying $\sum_{i=1}^n\omega_iB_i(s,t)\equiv1$, then the T-spline is a non-standard T-Spline, as is shown in Fig. 3.

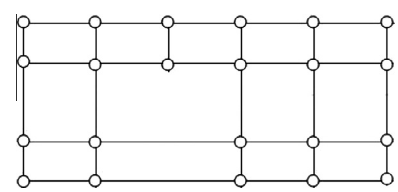


Fig. 1. Standard T-spline.

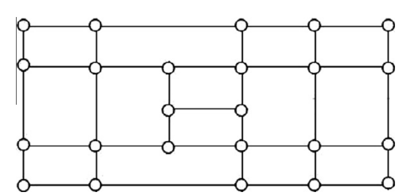


Fig. 2. Semi-standard T-spline.

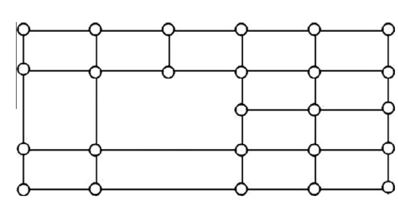


Fig. 3. Non-standard T-Spline.

4. Mathematical analysis of T-spline classification

Given an initial T-spline $P_0(u, v) = \sum_{i=1}^{n0} P_i^0 B_i^0(u, v)$, according to the local refinement algorithm of T-splines (Sederberg et al., 2004), we can get the corresponding LB-spline by successive refinement operation. This paper decomposes the refinement process, which assumes that the initial T-spline becomes its LB-spline by applying m times refinement algorithm (The refinement algorithm we now present has two phases: the topology phase and the geometry phase (Sederberg et al., 2004)). By applying one time refinement algorithm the T-spline becomes $P_1(u, v)$, by applying j times refinement algorithm the T-spline becomes $P_j(u, v) = \sum_{i=1}^{nj} P_i^j B_i^j(u, v)$, where $B_i^j(u, v)$ is the T-spline blending function of $P_j(u, v)$. By applying m times refinement algorithm we obtain its LB-spline $P_m(u, v)$. Let

$$B^j = \begin{bmatrix} B_1^j(u,v) & B_2^j(u,v) & \ldots & B_{nj}^j(u,v) \end{bmatrix}^T \quad (j = 0, \ldots, m) \tag{1}$$

Specifically, for each refinement process which satisfies T-spline local refinement algorithm (Sederberg et al., 2004), this paper has the following constraints: (a) this paper assumes that the sequence B_i^j $(1 \leqslant i \leqslant nj)$ are arranged in row-major order or column-major order, for example, $\cdots, B_{11}^j, B_{12}^j, \cdots, B_{21}^j, B_{22}^j, \cdots$ or $\cdots, B_{11}^j, B_{21}^j, \cdots, B_{12}^j, B_{22}^j, \cdots$. (b) The order of adding knots in this paper agrees with the order of B_i^m $(1 \leqslant i \leqslant nm)$, for example $B^m = [\ldots, B_{11}^m, B_{12}^m, \ldots, B_{21}^m, B_{22}^m, \ldots]^T$, then the knot corresponding to B_{11}^m is added priority than the knot corresponding to B_{12}^m.

For the T-spline $P_j(u, v)$, we can get $P_{j+1}(u, v)$ by adding knots according to the T-spline local refinement algorithm (Sederberg et al., 2004), the blending functions of $P_j(u, v)$ and the blending functions of $P_{j+1}(u, v)$ have the relations as follows:

$$B_c^j(u,v) = B_c^{j+1}(u,v) \tag{2}$$

or

$$B_c^j(u,v) = aB_c^{j+1}(u,v) + \sum_{i=1}^{q} b_{di} B_{di}^{j+1}(u,v) \tag{3}$$

where $a, b_{di} \geqslant 0$, $B_c^j(u, v)$ is the cth element of B^j. According to the knot insertion algorithm, with Eqs. (1)–(3), the relations of T-spline blending functions can be rewritten as

$$B^j = T_{j+1} B^{j+1} \quad (j = 0, \cdots, m-1) \tag{4}$$

where matrix T_{j+1} is a non-negative sparse matrix. With Eq. (4) we can get

$$B^0 = \prod_{j=1}^{m} T_j B^m \tag{5}$$

Let $T = \prod_{j=1}^{m} T_j$, then Eq. (5) can be rewritten as

$$B^0 = T * B^m \tag{6}$$

T is called **transformation matrix** of the initial T-spline $P_0(u, v)$. By the above T-spline refinement operations, we can get $P_m(u, v)$ from the initial T-spline $P_0(u, v)$. With Eq. (1) we have

$$B^0 = \begin{bmatrix} B_1^0(u,v) & B_2^0(u,v) & \cdots & B_{n0}^0(u,v) \end{bmatrix}^T$$

$$B^m = \begin{bmatrix} B_1^m(u,v) & B_2^m(u,v) & \cdots & B_{nm}^m(u,v) \end{bmatrix}^T$$

as $P_m(u, v) = \sum_{i=1}^{nm} P_i^m B_i^m(u, v)$ is a B-spline surface, from the property of B-splines we can get

$$\sum_{i=1}^{nm} B_i^m(s,t) \equiv 1 \tag{7}$$

With Eq. (1) and Eq. (6) we can get

$$\sum_{i=1}^{n0} B_i^0(s,t) = \sum_{i=1}^{n0} (T * B^m)_i \tag{8}$$

Because $P_m(u, v)$ is a B-spline surface, with Eq. (6) we have: the topological relations of the initial T-spline are completely determined by the transformation matrix T. Similarly, $\sum_{i=1}^{n0} B_i^0(s,t)$ is also determined by the properties of the transformation matrix T. By the row transformation of the matrix, the matrix T can be rewritten as

$$T = C\overline{T} \tag{9}$$

where $C = [c_{k,i}]_{n0 \times n0}$ and $\overline{T}$ is the most simplified row matrix. With Eqs. (6) (8) (9) and the properties of matrixes we can get

(i) If $\sum_{k=1}^{n0} \overline{T}(k,l) = 1$ $(l = 1, 2, \cdots, nm)$ and $\sum_{k=1}^{n0} C(k,i) = 1$ $(i = 1, 2, \cdots, n0)$, then $\sum_{i=1}^{n0} B_i^0(u,v) = \sum_{i=1}^{nm} B_i^m(u,v) \equiv 1$, the T-spline $P_0(u, v)$ is a standard T-spline.

(ii) If $\sum_{k=1}^{n0} \overline{T}(k,l) = 1$ $(l = 1, 2, \cdots, nm)$ and $\exists \alpha$ satisfying $\sum_{k=1}^{n0} C(k,\alpha) \neq 1$, then there exists a set of weights ω_i not all equal to 1 for $\sum_{i=1}^{n0} \omega_i B_i^0(u,v) \equiv 1$, that is to say the T-spline $P_0(u, v)$ is a semi-standard T-spline.

(iii) If $\exists \alpha$ satisfying $\sum_{k=1}^{n0} \overline{T}(k,\alpha) \neq 1$, then for $\forall \omega_i$, $\sum_{i=1}^{n0} \omega_i B_i^0(u,v) \neq 1$, that is to say the T-spline $P_0(u, v)$ is a non-standard T-Spline.

By the above conclusions, this paper gives the mathematical analysis of T-spline classification as follows.

Theorem 1. *For a T-spline, its topological relations and the corresponding T-spline classification are completely determined by the transformation matrix T of the T-spline.*

Theorem 2 ((a)). *The initial T-spline is a standard T-spline if and only if each column sum of the matrix $\overline{T}$ and C equals to 1.*

(b) *The initial T-spline is a semi-standard T-spline if and only if each column sum of the matrix $\overline{T}$ equals to 1 and there exists a column sum of the matrix C not equal to 1.*

(c) *The initial T-spline is a non-standard T-spline if and only if there exists a column sum of the matrix $\overline{T}$ not equal to 1.*

Theorem 3. *For a T-spline, it becomes its LB-spline by applying refinement algorithm. If only one knot is inserted for each local refinement process, then the T-spline is a standard T-spline.*

Proof. Suppose that the initial T-spline becomes its LB-spline $P_m(u, v)$ by applying m times local refinement algorithm, then we have $B^0 = \prod_{j=1}^{m} T_j B^m$, $B^j = T_{j+1} B^{j+1}$. As only one knot is inserted for each refinement process, with the property of knot insertion we have $\sum_{k=1}^{nj} T_{j+1}(k,l) = 1$ $(l = 1, 2, \cdots, n(j+1))$. Let $T = \prod_{j=1}^{m} T_j$, according to the property of matrixes, we can get $\sum_{k=1}^{n0} T(k,l) = 1$ $(l = 1, 2, \cdots, nm)$, that is $\sum_{i=1}^{n0} B_i^0(u,v) = 1$. So for a T-spline, it becomes its LB-spline by applying refinement algorithm, if only one knot is inserted for each local refinement process, then the T-spline is a standard T-spline. □

Theorem 4. *For a T-spline, if the T-spline does not have multiple knots, and each T-knot through line does not intersect with other T-knot extension lines, then the T-spline is a standard T-spline (T-knot through line is the line begins at a T-knot and runs through the T-mesh, T-knot extension line is the line begins at a T-knot and extends $\lfloor (\deg ree + 2)/2 \rfloor$ knots in the T-mesh).*

Proof. As each T-knot through line doesn't intersect with other T-knot extension lines, then the T-spline becomes its LB-spline by applying refinement algorithm, and for Eq. (4) each local refinement process inserts only one knot. From Theorem 3, the T-spline is a standard T-spline. □

According to the above conclusions, for a T-spline, this paper gives an algorithm in Section 5 that determines the T-spline classification by the transformation matrix of the T-spline.

5. Detection algorithm of T-spline classification

Based on the above mathematical analysis of T-spline classification, this section gives an algorithm that determines the T-spline classification. By the detection algorithm we can get the inherently mathematical properties of T-splines. Computational complexity of the algorithm is related to the transformation matrix, and if the T-spline has n control points, then the computational complexity of the algorithm is $O(n^3)$. Specifically as follows

5.1. Detection algorithm

Step 1: Input the T-spline, verify whether the T-spline satisfies Theorem 4. If the T-spline satisfies Theorem 4, it is a standard T-spline, or go to Step 2.
Step 2: Applying refinement algorithm to the T-spline corresponding to Section 4, we can get its LB-spline, and obtain the transformation matrix T for the refinement process.
Step 3: According to Theorem 2, this paper detects the T-spline classification by its transformation matrix T. By the row transformation of the matrix, the matrix T can be rewritten as $T = C\overline{T}$: if $\sum_{k=1}^{n0}\overline{T}(k,l) = 1 (l = 1, 2, \cdots, nm)$ and $\sum_{k=1}^{n0}C(k,i) = 1 (i = 1, 2, \cdots, n0)$, then the T-spline is a standard T-spline; if $\sum_{k=1}^{n0}\overline{T}(k,l) = 1$ $(l = 1, 2, \cdots, nm)$ and $\exists\alpha$ satisfying $\sum_{k=1}^{n0}C(k,\alpha)\neq 1$, then the T-spline is a semi-standard T-spline; otherwise the T-spline is a non-standard T-spline.
Step 4: Output the detection results: if the T-spline is a standard T-spline, $\sum_{i=1}^{n0}B_i^0(u,v) \equiv 1$; if the T-spline is a semi-standard T-spline, then $\exists\omega_i$ $(i = 1, 2, \cdots, n0)$ not all equal to 1 for which $\sum_{i=1}^{n0}\omega_i B_i^0(s,t) \equiv 1$; if the T-spline is a non-standard T-spline, then $\forall\omega_i$ $(i = 1, 2, \cdots, n0)$, $\sum_{i=1}^{n0}\omega_i B_i^0(s,t)\neq 1$.

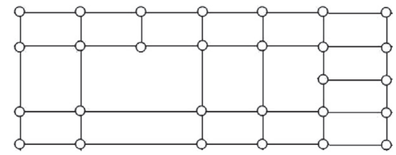
By this algorithm, we can get the T-spline classification and its inherently mathematical properties. Based on the theoretical research, more reasonable T-spline modeling algorithms can be designed for application.

6. Experimental results

For an arbitrary T-spline, the corresponding T-spline classification can be given by the detection algorithm in Section 5. For some special T-splines satisfying Theorem 4, the detection algorithm can easily determine the corresponding T-spline classification. This section applies the detection algorithm to several T-splines and gives the detection results.

Fig. 4 shows an example of determining T-spline classification by our algorithm. Fig. 4a shows the original T-spline 1 which contains 28 control points. Let $B^0 = \left[B_1^0(u,v) \quad B_2^0(u,v)\cdots B_{28}^0(u,v)\right]^T$ be the original T-spline blending functions. According to the detection algorithm, we can get the detection result as Fig. 4b shows.

Fig. 5 is another example, where Fig. 5a shows the T-spline 2 without multiple knots and the T-spline contains 53 control points. Let $B^0 = \left[B_1^0(u,v) \quad B_2^0(u,v) \quad \ldots \quad B_{53}^0(u,v)\right]^T$ be the blending functions of the T-spline 2. By our algorithm we can get the detection result as Fig. 5b shows, that is the T-spline 2 is a non-standard T-spline and its blending functions satisfy $\forall\omega_i$, $\sum_{i=1}^{53}\omega_i B_i^0(u,v)\neq 1$.



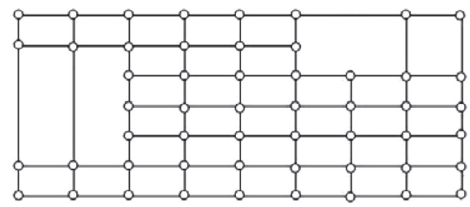
(a) T-spline 1

Standard T-spline

$$\sum_{i=1}^{28} B_i^0(s,t) \equiv 1$$

(b) Detection result of the T-spline 1

Fig. 4. T-spline 1 and its detection result.



(a) T-spline 2

Non-standard T-spline

$$\forall\omega_i \quad \sum_{i=1}^{53}\omega_i B_i^0(u,v) \neq 1$$

(b) Detection result of the T-spline 2

Fig. 5. T-spline 2 and its detection result.

7. Conclusions

In this paper we have given a mathematical analysis about T-spline classification and a sufficient condition of standard T-splines. Finally, we have proposed an algorithm of determining T-spline classification. T-spline classification plays an important role in the T-spline modeling theory, and the results in this paper are meaningful for the T-spline modeling algorithms. With the research of T-spline modeling algorithms, the practical applications of T-splines will be more.

Acknowledgements

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