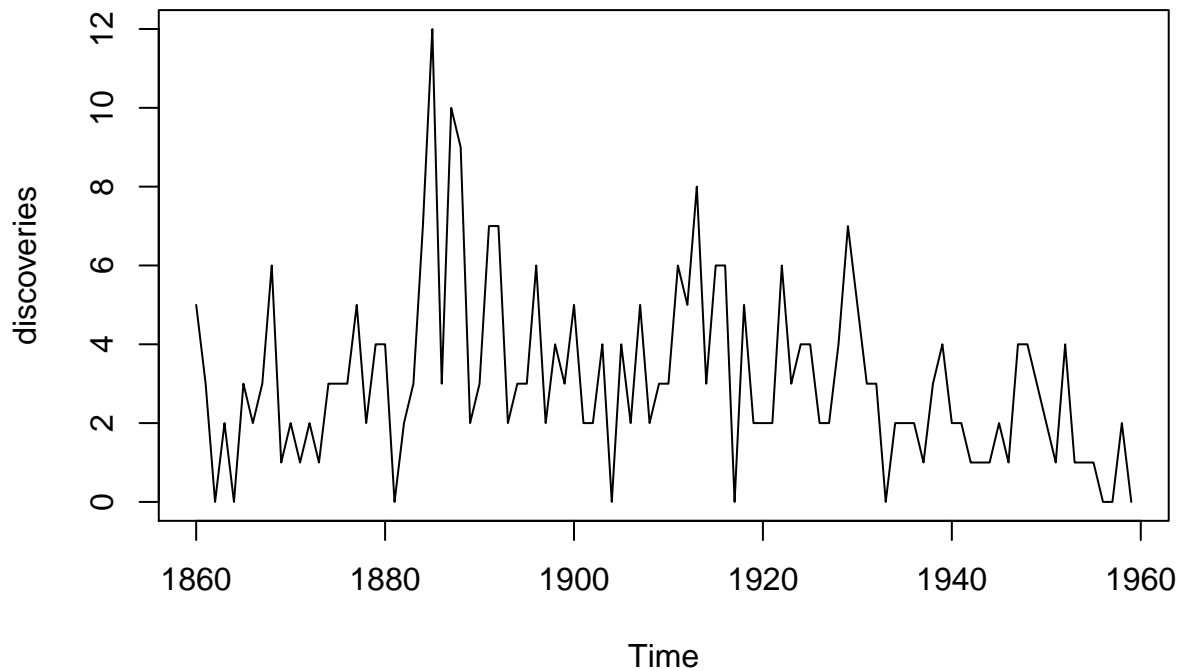


HW4 MA576

```
##3
#a
discover <- discoveries
time <- c(1860:1959)
plot(discoveries)
```



It seems that the rate of discoveries was fluctuating up and down and reaches the peak around 1885. During the recent years, the rate becomes slower.

```
#b
mod1 <- glm(discover~time, family = poisson, data = discover)
mod2 <- update(mod1, .~. + I(time^2))
mod3 <- update(mod2, .~. + I(time^3))
mod4 <- update(mod3, .~. + I(time^4))
mod5 <- update(mod4, .~. + I(time^5))
AIClm <- function(l){
  AIC <- extractAIC(l, k = 2)[2]
  return(AIC)
}
matrix(unlist(lapply(list(mod1, mod2, mod3,mod4,mod5),AIClm)), byrow = TRUE, ncol = 1, dimnames = list(
```

```
##           AIC
## mod1 430.3225
## mod2 407.8451
## mod3 409.7340
## mod4 410.9580
## mod5 410.9930
```

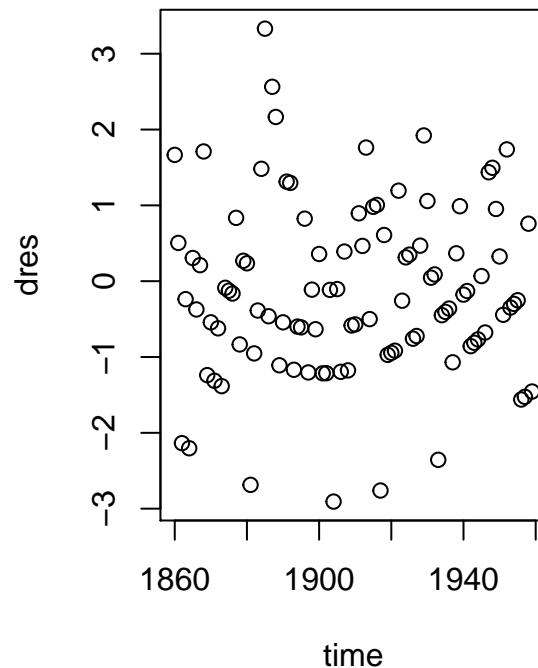
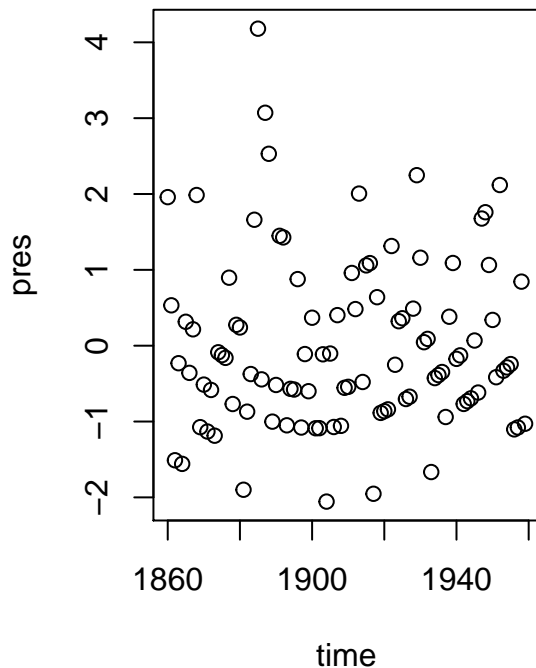
```
summary(mod2)
```

```
##
```

```
## Call:
## glm(formula = discover ~ time + I(time^2), family = poisson,
##      data = discover)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -2.9066  -0.8397  -0.2544   0.4776   3.3303
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -1.482e+03  3.163e+02  -4.685 2.79e-06 ***
## time         1.561e+00  3.318e-01   4.705 2.54e-06 ***
## I(time^2)    -4.106e-04  8.699e-05  -4.720 2.35e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##      Null deviance: 164.68  on 99  degrees of freedom
## Residual deviance: 132.84  on 97  degrees of freedom
## AIC: 407.85
##
## Number of Fisher Scoring iterations: 5
```

Mod2 has the smallest AIC, and order of 2 gives the most parsimonious description of the data. When time is zero, the discover is expected to be $\exp(-1.482e+03)$, very close to zero. As time increases by one year, discoveries will change to $\exp(-1.482e+03)\exp(1.561e+00)\exp(-4.106e-04)$, which is also a very small number and close to zero.

```
#c
pres <- residuals(mod2, type = "pearson")
dres <- residuals(mod2, type = "deviance")
par(mfrow = c(1,2))
plot(time, pres)
plot(time, dres)
```



```
dispersion <- sum((pres^2)/97)
dispersion
```

```
## [1] 1.305649
```

```
with(mod2, cbind(res.deviance = deviance, df = df.residual, p = pchisq(deviance, df.residual, lower.tail=FALSE)))
```

```
##      res.deviance df      p
## [1,]      132.8384 97 0.009204575
```

```
anova(mod2, test = "Chisq")
```

```
## Analysis of Deviance Table
```

```
##
```

```
## Model: poisson, link: log
```

```
##
```

```
## Response: discover
```

```
##
```

```
## Terms added sequentially (first to last)
```

```
##
```

```
##
```

```
##      Df Deviance Resid. Df Resid. Dev Pr(>Chi)
```

```
## NULL      99      164.69
```

```
## time      1    7.3688      98      157.32 0.006637 **
```

```
## I(time^2) 1   24.4774      97      132.84 7.519e-07 ***
```

```
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

residuals plots still presents curved pattern dispersion get from the Residual deviance over 97 df is 1.369485 and my estimate by pearson residuals is 1.305649. They are all very close to 1. However, the p-value of Chi-sq test is less than 0.05. The overdispersion might be significant.

```
##4
```

```
#a
```

```
stc <- read.table("stretch.dat", header = T)
```

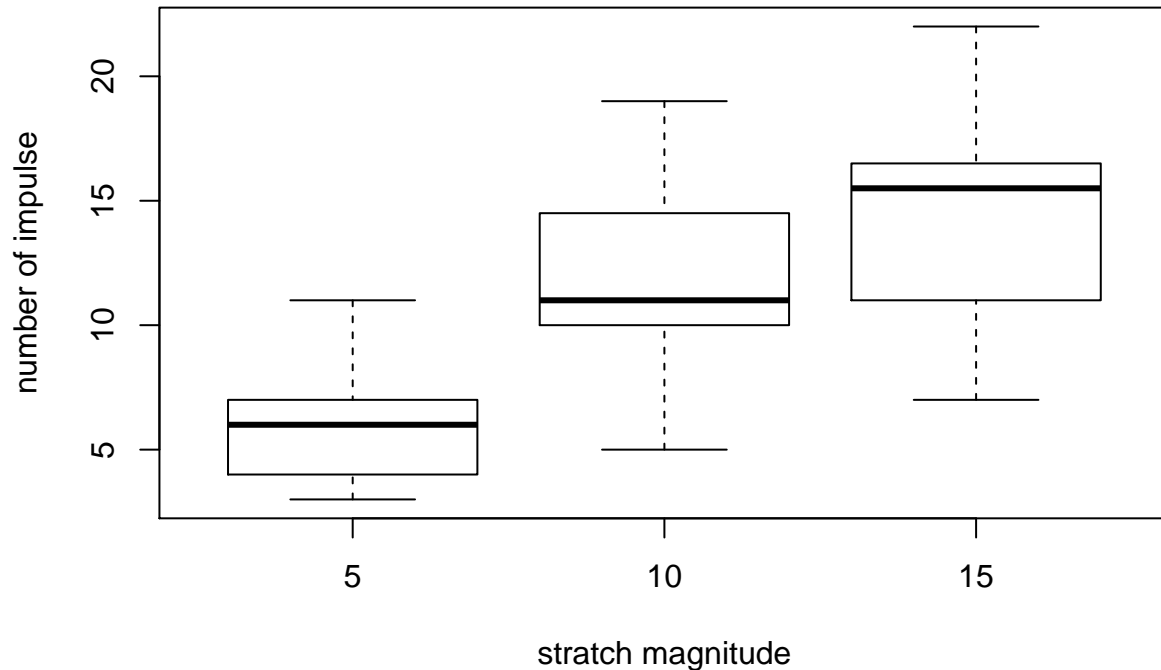
```

attach(stc)
library(dplyr)

## Warning: package 'dplyr' was built under R version 3.4.2
##
## Attaching package: 'dplyr'
## The following objects are masked from 'package:stats':
##
##   filter, lag
## The following objects are masked from 'package:base':
##
##   intersect, setdiff, setequal, union

Stc <- stc %>%
  group_by(Trial) %>%
  summarise(num_trial = length(Trial))
mag <- c(rep(5,20), rep(10,20), rep(15, 20))
boxplot(Stc$num_trial~mag, data = stc, xlab = "stratch magnitude", ylab = "number of impulse")

```

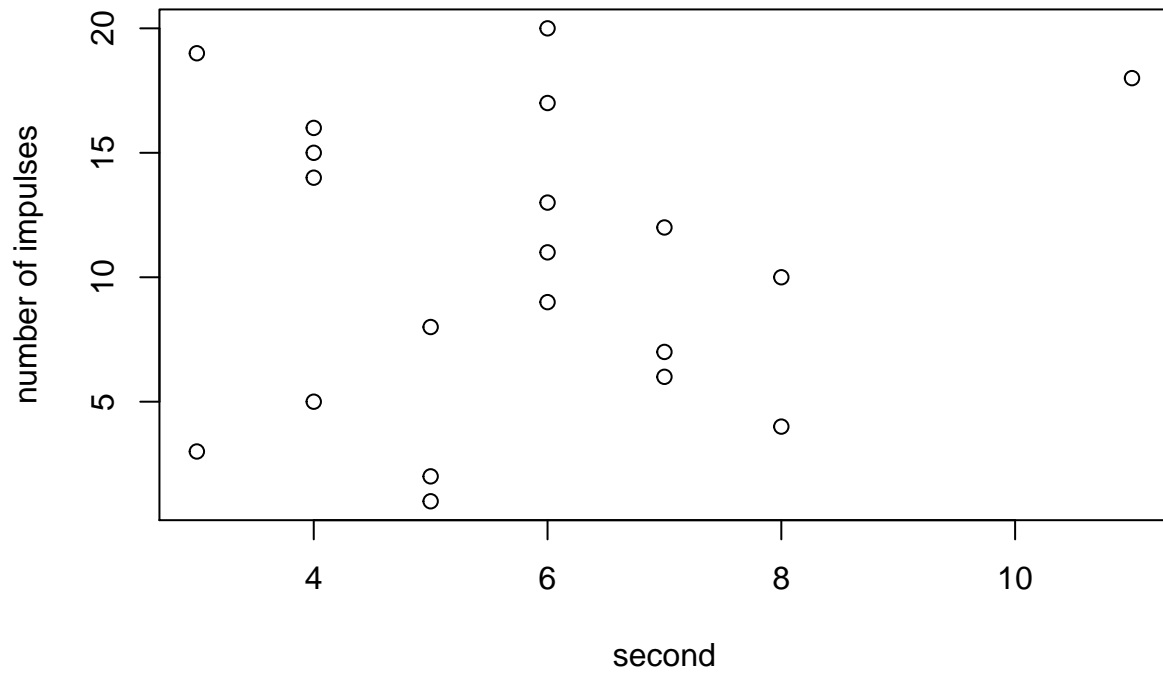


```

secondinterval <- 1:60
plot(Stc$num_trial[1:20], secondinterval[1:20], main = "magnitude 5mm", xlab = "second", ylab = "number

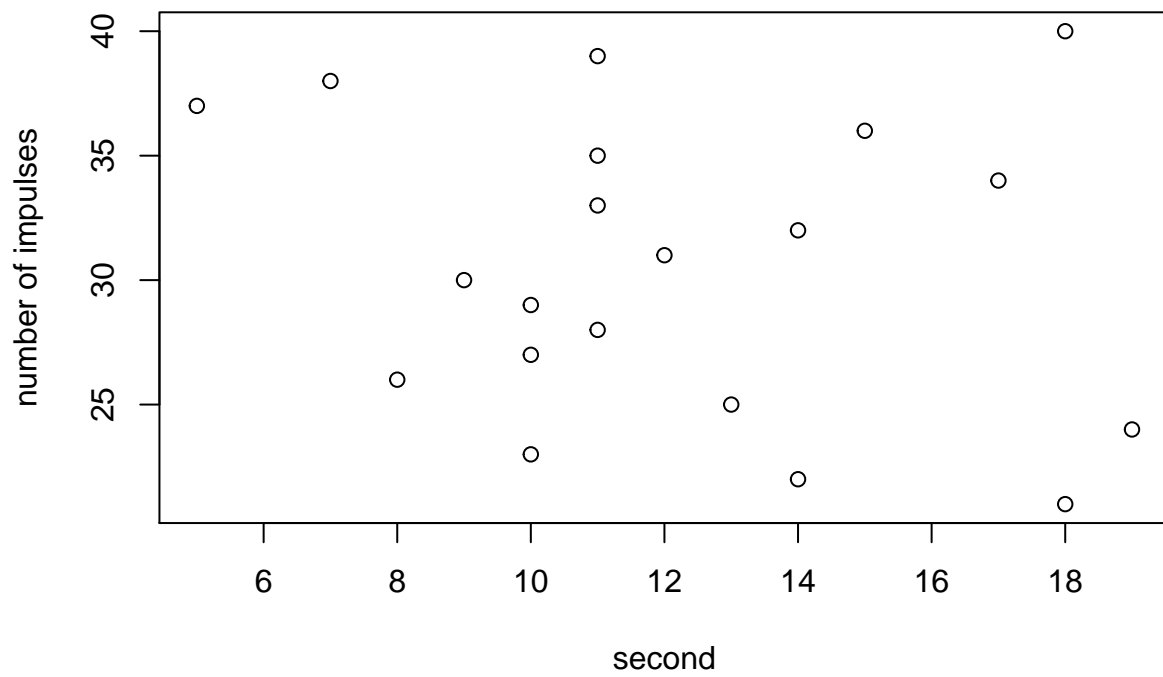
```

magnitude 5mm



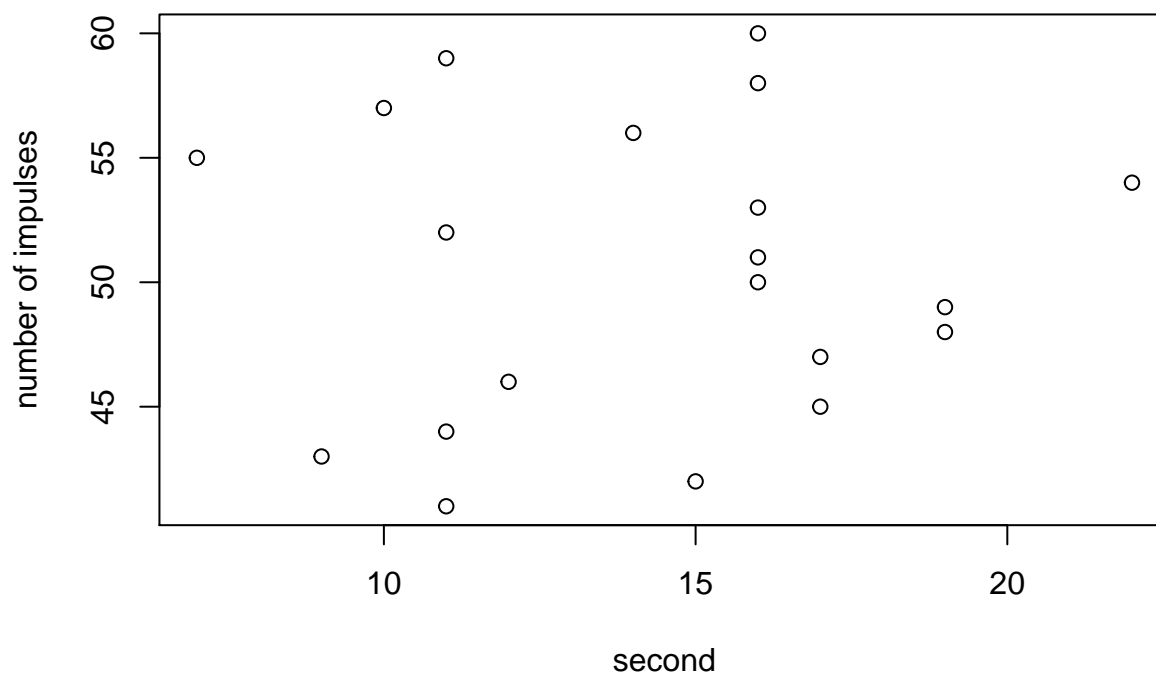
```
plot(Stc$num_trial[21:40], secondinterval[21:40], main = "magnitude 10mm", xlab = "second", ylab = "num
```

magnitude 10mm



```
plot(Stc$num_trial[41:60], secondinterval[41:60], main = "magnitude 15mm", xlab = "second", ylab = "num
```

magnitude 15mm



```
#b
m1 <- glm(Stc$num_trial~mag, family = poisson)
mag_f <- factor(mag, ordered = F)
m2 <- glm(Stc$num_trial~mag_f, family = poisson)
summary(m1)

##
## Call:
## glm(formula = Stc$num_trial ~ mag, family = poisson)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -2.36645  -0.91061  -0.04635   0.44639   2.47577
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  1.50228    0.12055  12.462 < 2e-16 ***
## mag          0.08149    0.01006   8.099 5.53e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##      Null deviance: 135.919  on 59  degrees of freedom
## Residual deviance:  67.589  on 58  degrees of freedom
## AIC: 318.29
##
## Number of Fisher Scoring iterations: 4
```

```
summary(m2)
```

```
##
## Call:
## glm(formula = Stc$num_trial ~ mag_f, family = poisson)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -2.3283  -0.7725   0.0302   0.5075   1.9420
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  1.74920    0.09325  18.758 < 2e-16 ***
## mag_f10      0.74813    0.11319   6.610 3.85e-11 ***
## mag_f15      0.90756    0.11047   8.215 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##      Null deviance: 135.919  on 59  degrees of freedom
## Residual deviance:  55.684  on 57  degrees of freedom
## AIC: 308.39
##
## Number of Fisher Scoring iterations: 4
```

```
anova(m1,m2, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: Stc$num_trial ~ mag
## Model 2: Stc$num_trial ~ mag_f
##      Resid. Df Resid. Dev Df Deviance  Pr(>Chi)
## 1           58      67.589
## 2           57      55.684  1   11.905 0.0005598 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

For model of covariate, all parameters are significant at 95% level. Intercept means if magnitude is zero, the impulse number would be $\exp(1.50228)$ which is about 4. As magnitude increases by one, impulse number would be increase to $\exp(1.50228) * \exp(0.08149)$, which is about 5.

For model of factors, all parameters are significant at 95% level. Intercept means if magnitude is 5mm, the impulse number would be $\exp(1.7492)$ which is about 6. As magnitude goes to 10mm level, impulse number would be $\exp(1.7492) * \exp(0.74813)$, which is about 12. As magnitude goes to 15mm level, impulse number would be $\exp(1.7492) * \exp(0.90756)$, which is about 14.

The analysis of deviance table suggests that factors model is preferred even though both models' parameters are all significant at 95% level. For covariate model, the dispersion is about $67.589 / 58 = 1.165328$ and $55.684 / 57 = 0.9769123$ for the factors model. The covariate model has slight overdispersion and the factor model has little underdispersion, but they are all very close to one. The following is the chi-sq test. Their p-values are all larger than 0.05, the overdispersion and underdispersion might be not significant

```
with(m1, cbind(res.deviance = deviance, df = df.residual, p = pchisq(deviance, df.residual, lower.tail=
```

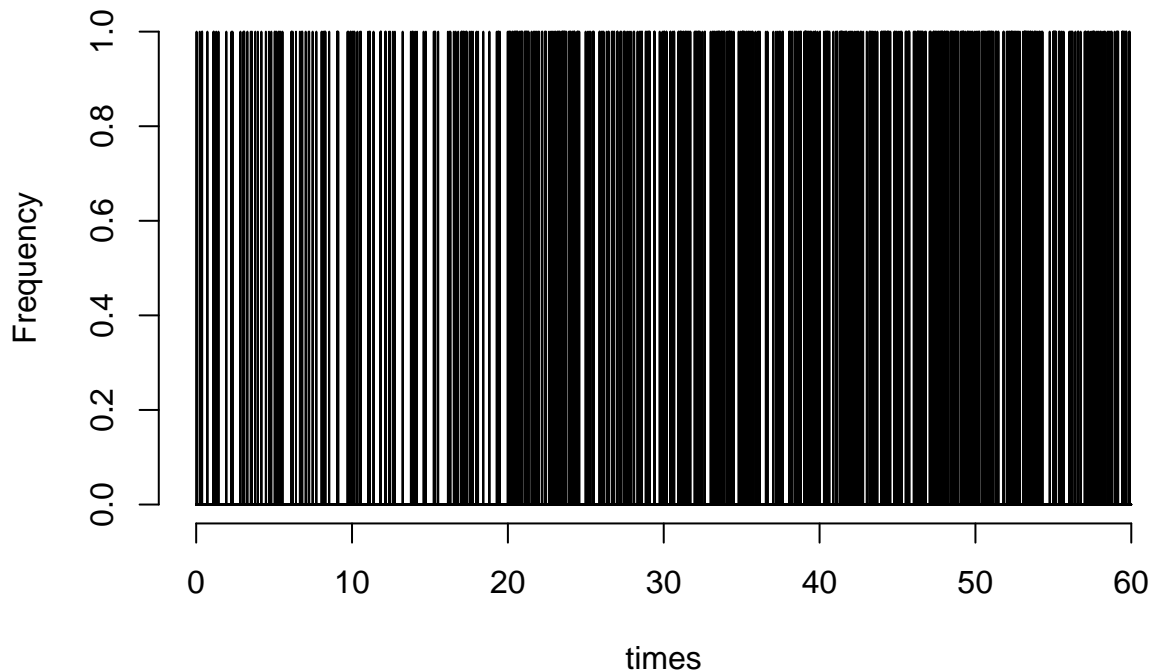
```
##      res.deviance df      p
## [1,]      67.5893 58 0.1822919
```

```
with(m2, cbind(res.deviance = deviance, df = df.residual, p = pchisq(deviance, df.residual, lower.tail=
```

```
##      res.deviance df      p
## [1,]      55.6842 57 0.4754463
```

```
#c
times <- (Trial + SpikeTimes - 1)
mag_tt <- c(rep(5,20000), rep(10,20000), rep(15, 20000))
spike1ms <- hist(times, breaks = seq(0,60,0.001))$counts
bins <- hist(times, breaks = seq(0,60,0.001))$breaks[-1]
```

Histogram of times



```
history <- rep(bins[1:1000],60)
m3 <- glm(spike1ms~mag_tt+history, poisson)
summary(m3)
```

```
##
## Call:
## glm(formula = spike1ms ~ mag_tt + history, family = poisson)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -0.2172  -0.1638  -0.1393  -0.1182   3.0025
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -4.96796    0.13423  -37.011  < 2e-16 ***
## mag_tt       0.08149    0.01006   8.099 5.53e-16 ***
## history     -0.94850    0.13968  -6.791 1.12e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```



```
##
## (Dispersion parameter for poisson family taken to be 1)
##
## Null deviance: 5833.2 on 59999 degrees of freedom
## Residual deviance: 5717.8 on 59997 degrees of freedom
## AIC: 7009.8
##
## Number of Fisher Scoring iterations: 7
with(m3, cbind(res.deviance = deviance, df = df.residual, p = pchisq(deviance, df.residual, lower.tail=

## res.deviance df p
## [1,] 5717.762 59997 0
anova(m3, test = "Chisq")

## Analysis of Deviance Table
##
## Model: poisson, link: log
##
## Response: spike1ms
##
## Terms added sequentially (first to last)
##
##
## Df Deviance Resid. Df Resid. Dev Pr(>Chi)
## NULL 59999 5833.2
## mag_tt 1 68.329 59998 5764.9 < 2.2e-16 ***
## history 1 47.147 59997 5717.8 6.586e-12 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(dp <-sum(residuals(m3,type="pearson")^2)/m3$df.res)

## [1] 0.9795825
summary(m3, dispersion=dp)

##
## Call:
## glm(formula = spike1ms ~ mag_tt + history, family = poisson)
##
## Deviance Residuals:
## Min 1Q Median 3Q Max
## -0.2172 -0.1638 -0.1393 -0.1182 3.0025
##
## Coefficients:
## Estimate Std. Error z value Pr(>|z|)
## (Intercept) -4.967959 0.132850 -37.395 < 2e-16 ***
## mag_tt 0.081492 0.009958 8.183 2.76e-16 ***
## history -0.948497 0.138244 -6.861 6.84e-12 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 0.9795825)
##
## Null deviance: 5833.2 on 59999 degrees of freedom
```

```
## Residual deviance: 5717.8 on 59997 degrees of freedom
## AIC: 7009.8
##
## Number of Fisher Scoring iterations: 7
```

Under 95% confidence level, the rate does not depend on time. Intercept means if magnitude and time is zero, the impulse number would be $\exp(-5.450402)$ which is close 0. As magnitude increases by one, impulse number would increase to $\exp(-5.450402) * \exp(0.099332)$, which is still very close to zero. As time increases by one, impulse number would increase to $\exp(-5.450402) * \exp(-0.004460)$, which is very close to zero. The underdispersion is significant.

```
m4 <- glm(spike1ms[2:60000]~spike1ms[1:59999], poisson)
m5 <- glm(spike1ms[3:60000]~spike1ms[2:59999] + spike1ms[1:59998], poisson)
m6 <- glm(spike1ms[4:60000]~spike1ms[3:59999] + spike1ms[2:59998] + spike1ms[1:59997], poisson)
m7 <- glm(spike1ms[5:60000]~spike1ms[4:59999] + spike1ms[3:59998] + spike1ms[2:59997] + spike1ms[1:59996])
m8 <- glm(spike1ms[6:60000]~spike1ms[5:59999] + spike1ms[4:59998] + spike1ms[3:59997] + spike1ms[2:59996])
AIClm <- function(l){
  AIC <- extractAIC(l, k = 2)[2]
  return(AIC)
}
matrix(unlist(lapply(list(m4, m5, m6,m7,m8),AIClm)), byrow = TRUE, ncol = 1, dimnames = list(c("m4", "m5", "m6", "m7", "m8"), 1:5))
```

```
##          AIC
## m4 7109.361
## m5 7097.332
## m6 7096.320
## m7 7098.175
## m8 7098.966
```

```
summary(m6)
```

```
##
## Call:
## glm(formula = spike1ms[4:60000] ~ spike1ms[3:59999] + spike1ms[2:59998] +
##      spike1ms[1:59997], family = poisson)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -0.1485  -0.1485  -0.1485  -0.1485   2.9571
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)    -4.50790    0.03953  -114.042  <2e-16 ***
## spike1ms[3:59999] -13.79468    225.47746   -0.061    0.951
## spike1ms[2:59998] -13.79468    225.47746   -0.061    0.951
## spike1ms[1:59997]  -0.85963     0.57870   -1.485    0.137
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##      Null deviance: 5833.2 on 59996 degrees of freedom
## Residual deviance: 5802.3 on 59993 degrees of freedom
## AIC: 7096.3
##
## Number of Fisher Scoring iterations: 16
```

```

(dp2 <-sum(residuals(m6,type="pearson")^2)/m6$df.res)

## [1] 0.9679129
summary(m6, dispersion = dp2)

##
## Call:
## glm(formula = spike1ms[4:60000] ~ spike1ms[3:59999] + spike1ms[2:59998] +
##     spike1ms[1:59997], family = poisson)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -0.1485  -0.1485  -0.1485  -0.1485   2.9571
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)    -4.50790     0.03889  -115.917  <2e-16 ***
## spike1ms[3:59999] -13.79468    221.83051   -0.062    0.950
## spike1ms[2:59998] -13.79468    221.83051   -0.062    0.950
## spike1ms[1:59997]  -0.85963     0.56934   -1.510    0.131
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 0.9679129)
##
##      Null deviance: 5833.2  on 59996  degrees of freedom
## Residual deviance: 5802.3  on 59993  degrees of freedom
## AIC: 7096.3
##
## Number of Fisher Scoring iterations: 16
anova(m6, test = "Chisq")

## Analysis of Deviance Table
##
## Model: poisson, link: log
##
## Response: spike1ms[4:60000]
##
## Terms added sequentially (first to last)
##
##
##              Df Deviance Resid. Df Resid. Dev  Pr(>Chi)
## NULL                                59996      5833.2
## spike1ms[3:59999]  1  13.8567     59995      5819.3 0.0001973 ***
## spike1ms[2:59998]  1  14.0076     59994      5805.3 0.0001821 ***
## spike1ms[1:59997]  1   2.9901     59993      5802.3 0.0837762 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

The temporal dependencies exists till lag of three. However, partial auto-correlation also exists with lag of 1 and of 2. The coefficients stand for partial auto-correlation between the impulse count(t) and impulse count(t+h). It suggests that the dispersion I get from previous parts are enlarged because covariance now taken into account.