

Supplementary File for “ATM-R: An Adaptive Tradeoff Model with Reference Points for Constrained Multiobjective Evolutionary Optimization”

Bing-Chuan Wang, Yunchuan Qin, Xian-Bing Meng, Zhi-Zhong Liu

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TABLE S-1
THE IGD VALUES OF NSGAII-CDP, PPS, CTAEA, CCMO, ToP, AND ATM-R ON THREE SETS OF BENCHMARK TEST FUNCTIONS.

Test Functions	NSGAII-CDP mean IGD (std)	PPS mean IGD (std)	CTAEA mean IGD (std)	CCMO mean IGD (std)	ToP mean IGD (std)	ATM-R mean IGD (std)
MW1	4.0545e-2 (1.02e-1) -	2.3190e-2 (4.01e-2) -	2.1884e-3 (9.96e-4) -	1.8990e-3 (1.42e-3) +	NaN (NaN) -	2.1748e-3 (1.70e-3)
MW2	2.3926e-2 (7.65e-3) -	4.2401e-2 (3.25e-2) -	1.7953e-2 (6.74e-3) ≈	2.1515e-2 (8.20e-3) -	2.3108e-1 (1.89e-1) -	1.9130e-2 (9.76e-3)
MW3	7.4318e-2 (2.31e-1) -	7.5935e-3 (9.94e-4) -	5.4804e-3 (4.86e-4) ≈	5.2178e-3 (4.41e-4) ≈	5.9698e-1 (2.78e-1) -	5.3646e-3 (4.08e-4)
MW4	5.5780e-2 (2.97e-3) -	5.3955e-2 (1.73e-3) -	4.6413e-2 (4.99e-4) -	4.1285e-2 (3.48e-4) ≈	NaN (NaN) -	4.1255e-2 (3.45e-4)
MW5	4.2761e-1 (3.35e-1) -	1.4507e-1 (1.97e-1) -	1.5758e-2 (3.38e-3) -	4.6474e-3 (7.30e-3) -	NaN (NaN) -	4.0638e-3 (1.06e-2)
MW6	8.0099e-2 (1.51e-1) -	1.0037e-1 (1.61e-1) -	1.1188e-2 (6.68e-3) ≈	5.2473e-2 (1.26e-1) -	1.0872e+0 (1.81e-1) -	1.5369e-2 (8.69e-3)
MW7	1.0205e-1 (1.93e-1) -	2.5520e-2 (1.88e-2) -	7.2156e-3 (5.22e-4) -	4.8994e-3 (4.80e-4) +	4.7226e-1 (2.39e-1) -	5.2004e-3 (4.73e-4)
MW8	6.1793e-2 (8.78e-3) -	7.4112e-2 (2.69e-2) -	5.5531e-2 (2.47e-3) -	4.9189e-2 (1.58e-2) ≈	9.5949e-1 (2.01e-1) -	4.6368e-2 (5.74e-3)
MW9	2.1737e-1 (3.10e-1) -	7.4032e-2 (1.81e-1) -	8.8691e-3 (9.23e-4) ≈	5.1927e-2 (1.79e-1) -	NaN (NaN) -	9.9563e-3 (2.88e-3)
MW10	2.3341e-1 (2.36e-1) -	1.3321e-1 (1.48e-1) -	1.7599e-2 (1.22e-2) ≈	4.2867e-2 (2.54e-2) -	NaN (NaN) -	2.7242e-2 (2.33e-2)
MW11	4.7335e-1 (3.24e-1) -	1.3565e-2 (2.15e-2) -	1.6564e-2 (2.80e-3) -	6.3416e-3 (5.30e-4) ≈	9.2141e-1 (1.37e-1) -	6.1791e-3 (2.34e-4)
MW12	8.2766e-2 (2.23e-1) -	2.9552e-2 (1.20e-1) +	8.0645e-3 (6.84e-4) +	3.0553e-2 (1.40e-1) ≈	NaN (NaN) -	7.8769e-2 (2.24e-1)
MW13	2.0642e-1 (2.77e-1) -	1.3735e-1 (6.32e-2) -	3.8211e-2 (2.66e-2) ≈	8.2172e-2 (4.41e-2) -	8.3328e-1 (5.33e-1) -	5.2527e-2 (3.23e-2)
MW14	1.2974e-1 (1.27e-2) -	2.5313e-1 (9.28e-2) -	1.1279e-1 (6.91e-3) +	9.8349e-2 (2.41e-3) +	4.9059e-1 (5.81e-1) -	1.1492e-1 (4.72e-2)
CTP1	8.1699e-2 (6.62e-2) -	1.9234e-2 (1.80e-2) -	1.8672e-2 (3.64e-2) -	4.4317e-3 (1.05e-3) -	3.9400e-3 (1.48e-4) -	3.2367e-3 (7.38e-5)
CTP2	2.4408e-3 (1.89e-3) -	3.7003e-3 (6.96e-4) -	4.6860e-2 (1.25e-2) -	1.6836e-3 (1.65e-4) -	4.4453e-3 (1.08e-3) -	1.4735e-3 (5.94e-5)
CTP3	6.2833e-2 (9.95e-2) -	3.1094e-2 (4.07e-3) -	5.8093e-2 (5.79e-3) -	2.2180e-2 (2.24e-3) -	3.2847e-2 (6.99e-3) -	1.0066e-2 (1.60e-3)
CTP4	2.4494e-1 (1.29e-1) -	1.4930e-1 (1.84e-2) -	1.5350e-1 (1.92e-2) -	1.3538e-1 (2.18e-2) -	1.8414e-1 (3.29e-2) -	7.9556e-2 (1.13e-2)
CTP5	7.2574e-3 (2.92e-3) -	1.8168e-2 (6.11e-3) -	1.8209e-2 (4.61e-3) -	7.6639e-3 (1.76e-3) -	1.2167e-2 (3.45e-3) -	3.3142e-3 (4.13e-4)
CTP6	1.1404e-2 (4.04e-4) -	1.3061e-2 (7.81e-4) -	3.8535e-2 (5.23e-3) -	1.0141e-2 (3.56e-4) -	1.5214e-2 (2.78e-3) -	9.7103e-3 (3.13e-4)
CTP7	1.6882e-3 (1.39e-3) -	1.6825e-3 (7.14e-5) -	1.6364e-3 (1.31e-4) -	1.1669e-3 (4.52e-5) ≈	1.5176e-3 (5.54e-5) -	1.1599e-3 (4.62e-5)
CTP8	1.2019e-1 (1.45e-1) -	1.1932e-2 (5.26e-3) -	3.4505e-2 (4.79e-3) -	5.5516e-3 (6.49e-4) -	8.0925e-2 (1.38e-1) -	4.7357e-3 (2.32e-4)
LIRCOP1	2.6010e-1 (8.10e-2) -	1.1024e-1 (3.40e-2) -	3.7900e-1 (1.66e-1) -	2.0503e-1 (6.82e-2) -	1.2547e-1 (1.36e-1) -	3.5295e-2 (1.26e-2)
LIRCOP2	1.9890e-1 (7.22e-2) -	7.3024e-2 (2.79e-2) -	1.2324e-1 (6.12e-2) -	1.1419e-1 (3.19e-2) -	6.8227e-2 (5.38e-2) -	3.1146e-2 (9.36e-3)
LIRCOP3	2.4894e-1 (8.38e-2) -	1.7697e-1 (5.80e-2) -	3.4751e-1 (1.14e-1) -	2.0960e-1 (7.98e-2) -	3.6351e-1 (5.72e-2) -	2.3380e-2 (1.06e-2)
LIRCOP4	2.3080e-1 (6.23e-2) -	1.4996e-1 (5.59e-2) -	2.7661e-1 (1.38e-1) -	1.9069e-1 (7.18e-2) -	3.2442e-1 (5.76e-2) -	2.3928e-2 (1.11e-2)
LIRCOP5	7.3176e-1 (4.81e-1) -	8.4362e-2 (2.44e-2) -	1.3918e-1 (4.41e-2) -	1.4046e-2 (8.21e-3) ≈	1.2091e-1 (3.48e-1) -	1.3635e-2 (6.48e-3)
LIRCOP6	5.7447e-1 (4.57e-1) -	9.5258e-2 (6.31e-2) -	1.3633e-1 (1.13e-1) -	1.1357e-2 (7.75e-3) ≈	6.4593e-3 (3.45e-4) +	5.7790e-2 (1.51e-1)
LIRCOP7	1.7441e-2 (1.32e-2) ≈	5.6488e-2 (5.84e-2) -	2.5246e-2 (9.12e-3) -	1.1404e-2 (6.38e-3) ≈	8.6357e-3 (2.52e-4) +	1.2400e-2 (5.03e-3)
LIRCOP8	3.6946e-2 (4.51e-2) -	6.6479e-2 (7.05e-2) -	3.6096e-2 (6.64e-2) -	9.1531e-3 (5.01e-3) ≈	8.6820e-3 (4.53e-4) +	9.4267e-3 (3.85e-3)
LIRCOP9	5.3564e-1 (1.24e-1) -	1.4063e-1 (8.94e-2) ≈	1.1622e-1 (5.42e-2) ≈	3.4398e-2 (3.91e-2) +	2.4115e-1 (1.73e-1) -	1.1216e-1 (7.48e-2)
LIRCOP10	3.6496e-1 (9.66e-2) -	8.2848e-3 (1.47e-2) -	6.0919e-2 (6.50e-2) -	5.4399e-3 (3.36e-4) +	5.4878e-3 (2.21e-4) +	6.9018e-3 (6.27e-4)
LIRCOP11	2.4114e-1 (1.80e-1) -	8.1119e-3 (7.48e-3) -	1.3778e-1 (3.83e-2) -	2.4538e-3 (8.89e-5) +	1.2447e-1 (6.37e-2) -	5.3691e-3 (1.45e-2)
LIRCOP12	1.5180e-1 (8.66e-2) -	1.5216e-2 (2.43e-2) ≈	3.1152e-2 (1.68e-2) -	4.6113e-3 (2.58e-3) ≈	2.9104e-2 (5.29e-2) ≈	7.8014e-3 (7.28e-3)
LIRCOP13	2.3757e-1 (3.69e-1) -	1.1968e-1 (3.45e-3) -	1.0834e-1 (3.97e-4) -	9.3972e-2 (1.13e-3) -	1.2450e-1 (3.78e-3) -	9.3120e-2 (9.31e-4)
LIRCOP14	2.0248e-1 (2.93e-1) -	1.1859e-1 (3.97e-3) -	1.1126e-1 (7.98e-4) -	9.5773e-2 (7.40e-4) -	1.1883e-1 (4.04e-3) -	9.4848e-2 (7.79e-4)
+/-/≈	0/35/1	1/33/2	2/27/7	6/19/11	4/25/1	

TABLE S-2
THE HV VALUES OF NSGAII-CDP, PPS, CTAEA, CCMO, ToP, AND ATM-R ON THREE SETS OF BENCHMARK TEST FUNCTIONS.

Test Functions	NSGAII-CDP mean HV (std)	PPS mean HV (std)	CTAEA mean HV (std)	CCMO mean HV (std)	ToP mean HV (std)	ATM-R mean HV (std)
MW1	4.5445e-1 (8.09e-2) -	4.6529e-1 (3.63e-2) -	4.8849e-1 (2.03e-3) -	4.8927e-1 (3.04e-3) +	NaN (NaN) -	4.8853e-1 (3.60e-3)
MW2	5.4798e-1 (1.15e-2) -	5.2241e-1 (4.44e-2) -	5.5765e-1 (1.14e-2) \approx	5.5199e-1 (1.30e-2) -	3.2482e-1 (1.46e-1) -	5.5635e-1 (1.54e-2)
MW3	5.0168e-1 (1.37e-1) -	5.4398e-1 (4.88e-4) +	5.4413e-1 (6.14e-4) +	5.4368e-1 (7.81e-4) +	1.2745e-1 (1.27e-1) -	5.4292e-1 (7.86e-4)
MW4	8.2309e-1 (5.63e-3) -	8.2478e-1 (2.49e-3) -	8.3814e-1 (4.04e-4) -	8.4116e-1 (4.35e-4) +	NaN (NaN) -	8.4001e-1 (7.93e-4)
MW5	1.7725e-1 (9.80e-2) -	2.5212e-1 (6.86e-2) -	3.1449e-1 (2.61e-3) -	3.2205e-1 (5.38e-3) -	NaN (NaN) -	3.2214e-1 (6.45e-3)
MW6	2.8267e-1 (4.89e-2) -	2.5928e-1 (6.08e-2) -	3.1251e-1 (9.93e-3) \approx	2.9009e-1 (5.16e-2) -	1.2194e-2 (2.75e-2) -	3.0911e-1 (1.20e-2)
MW7	3.7706e-1 (6.78e-2) \approx	4.0647e-1 (2.09e-3) -	4.0868e-1 (1.03e-3) -	4.1205e-1 (5.95e-4) +	1.9015e-1 (7.70e-2) -	4.1019e-1 (9.75e-4)
MW8	4.9733e-1 (2.20e-2) -	4.7275e-1 (5.64e-2) -	5.2198e-1 (1.16e-2) -	5.2798e-1 (3.48e-2) -	4.6501e-2 (7.81e-2) -	5.3338e-1 (1.72e-2)
MW9	2.6792e-1 (1.71e-1) \approx	3.4455e-1 (1.00e-1) -	3.9100e-1 (2.43e-3) +	3.7160e-1 (1.01e-1) -	NaN (NaN) -	3.8287e-1 (4.60e-3)
MW10	3.1175e-1 (1.18e-1) -	3.5982e-1 (7.57e-2) -	4.3564e-1 (1.30e-2) \approx	4.1378e-1 (1.88e-2) -	NaN (NaN) -	4.2764e-1 (1.94e-2)
MW11	3.2816e-1 (8.07e-2) -	4.4157e-1 (9.48e-3) -	4.4127e-1 (1.39e-3) -	4.4609e-1 (2.03e-3) -	2.2321e-1 (4.19e-2) -	4.4746e-1 (2.05e-4)
MW12	5.4172e-1 (1.81e-1) -	5.8181e-1 (1.06e-1) +	6.0052e-1 (7.80e-4) +	5.8415e-1 (1.10e-1) \approx	NaN (NaN) -	5.4377e-1 (1.82e-1)
MW13	4.0153e-1 (5.63e-2) -	4.1137e-1 (4.30e-2) -	4.6130e-1 (1.23e-2) \approx	4.3974e-1 (2.53e-2) -	2.3054e-1 (1.15e-1) -	4.5371e-1 (1.66e-2)
MW14	4.5123e-1 (5.66e-3) -	4.2008e-1 (2.54e-2) -	4.6575e-1 (3.90e-3) \approx	4.7246e-1 (1.53e-3) +	3.4138e-1 (1.53e-1) -	4.6217e-1 (1.49e-2)
CTP1	3.5920e-1 (1.97e-2) -	3.7510e-1 (5.31e-3) -	3.7588e-1 (1.03e-2) -	3.8065e-1 (3.93e-4) -	3.8036e-1 (1.15e-4) -	3.8106e-1 (1.09e-4)
CTP2	4.3083e-1 (1.66e-3) -	4.2928e-1 (7.86e-4) -	3.9367e-1 (8.22e-3) -	4.3073e-1 (2.94e-4) -	4.2689e-1 (1.40e-3) -	4.3128e-1 (2.45e-4)
CTP3	3.7267e-1 (5.45e-2) -	3.8376e-1 (4.03e-3) -	3.5588e-1 (7.25e-3) -	3.9219e-1 (2.18e-3) -	3.8119e-1 (6.97e-3) -	4.0507e-1 (1.75e-3)
CTP4	2.2838e-1 (6.88e-2) -	2.5406e-1 (1.86e-2) -	2.4611e-1 (2.00e-2) -	2.7265e-1 (2.41e-2) -	2.2246e-1 (2.88e-2) -	3.3430e-1 (1.20e-2)
CTP5	3.9329e-1 (2.59e-2) -	3.8822e-1 (4.53e-3) -	3.5629e-1 (9.01e-3) -	3.9643e-1 (2.52e-3) -	3.8643e-1 (5.29e-3) -	4.0665e-1 (1.86e-3)
CTP6	4.6359e-1 (3.75e-4) -	4.6198e-1 (6.48e-4) -	4.4896e-1 (2.68e-3) -	4.6381e-1 (3.03e-4) -	4.6034e-1 (1.83e-3) -	4.6468e-1 (2.22e-4)
CTP7	5.6701e-1 (2.23e-3) -	5.6676e-1 (9.22e-4) -	5.6637e-1 (4.11e-4) -	5.6745e-1 (1.69e-4) \approx	5.6692e-1 (1.86e-4) -	5.6721e-1 (1.62e-3)
CTP8	3.4937e-1 (2.45e-2) -	3.6598e-1 (2.71e-3) -	3.5213e-1 (3.79e-3) -	3.6932e-1 (8.68e-4) -	3.5503e-1 (2.42e-2) -	3.7069e-1 (4.25e-4)
LIRCOP1	1.3114e-1 (2.17e-2) -	1.9042e-1 (1.06e-2) -	1.0593e-1 (3.85e-2) -	1.4954e-1 (1.82e-2) -	1.8833e-1 (4.61e-2) -	2.2304e-1 (6.36e-3)
LIRCOP2	2.5580e-1 (2.95e-2) -	3.2332e-1 (1.35e-2) -	2.9229e-1 (3.67e-2) -	2.9325e-1 (2.05e-2) -	3.2282e-1 (2.82e-2) -	3.4702e-1 (3.44e-3)
LIRCOP3	1.1697e-1 (2.61e-2) -	1.4007e-1 (1.86e-2) -	9.9083e-2 (2.08e-2) -	1.2942e-1 (2.47e-2) -	9.1646e-2 (1.42e-2) -	1.9947e-1 (4.24e-3)
LIRCOP4	2.1773e-1 (2.72e-2) -	2.4241e-1 (3.29e-2) -	1.8974e-1 (4.86e-2) -	2.3242e-1 (3.14e-2) -	1.8379e-1 (2.31e-2) -	3.0693e-1 (3.76e-3)
LIRCOP5	8.9983e-2 (1.05e-1) -	2.4475e-1 (1.22e-2) -	2.4215e-1 (1.32e-2) -	2.8700e-1 (5.09e-3) \approx	2.6214e-1 (8.89e-2) -	2.8657e-1 (5.90e-3)
LIRCOP6	8.6641e-2 (5.68e-2) -	1.7269e-1 (1.24e-2) -	1.4582e-1 (3.86e-2) -	1.9402e-1 (3.37e-3) \approx	1.9677e-1 (1.59e-4) +	1.8377e-1 (3.56e-2)
LIRCOP7	2.8752e-1 (6.87e-3) \approx	2.6957e-1 (2.12e-2) -	2.8582e-1 (3.18e-3) -	2.9114e-1 (4.01e-3) \approx	2.9389e-1 (1.55e-4) +	2.8957e-1 (4.05e-3)
LIRCOP8	2.8236e-1 (1.47e-2) -	2.6950e-1 (1.80e-2) -	2.8424e-1 (1.40e-2) -	2.9321e-1 (3.40e-3) \approx	2.9387e-1 (2.04e-4) \approx	2.9235e-1 (3.25e-3)
LIRCOP9	3.5772e-1 (8.15e-2) -	5.2821e-1 (2.63e-2) \approx	4.9955e-1 (2.81e-2) -	5.5712e-1 (8.54e-3) +	4.9582e-1 (5.19e-2) -	5.3708e-1 (2.00e-2)
LIRCOP10	5.1193e-1 (6.34e-2) -	7.0668e-1 (5.59e-3) +	6.7264e-1 (2.74e-2) -	7.0659e-1 (3.77e-4) +	7.0755e-1 (1.24e-4) +	7.0630e-1 (4.01e-4)
LIRCOP11	5.3737e-1 (1.32e-1) -	6.9062e-1 (4.88e-3) -	6.4145e-1 (1.47e-2) -	6.9392e-1 (7.61e-5) \approx	6.1686e-1 (4.29e-2) -	6.9393e-1 (5.71e-5)
LIRCOP12	5.5161e-1 (4.68e-2) -	6.1582e-1 (9.12e-3) \approx	6.0522e-1 (7.22e-3) -	6.1952e-1 (1.29e-3) \approx	6.0839e-1 (2.46e-2) -	6.1811e-1 (3.04e-3)
LIRCOP13	4.7950e-1 (1.63e-1) -	5.3426e-1 (4.14e-3) -	5.4704e-1 (3.37e-4) -	5.5421e-1 (1.49e-3) -	5.1626e-1 (3.56e-3) -	5.5578e-1 (1.23e-3)
LIRCOP14	4.9121e-1 (1.36e-1) -	5.3744e-1 (4.86e-3) -	5.4656e-1 (7.33e-4) -	5.5357e-1 (1.22e-3) -	5.2944e-1 (4.28e-3) -	5.5604e-1 (1.16e-3)
+/-/ \approx	0/33/3	3/31/2	3/28/5	7/20/9	3/32/1	

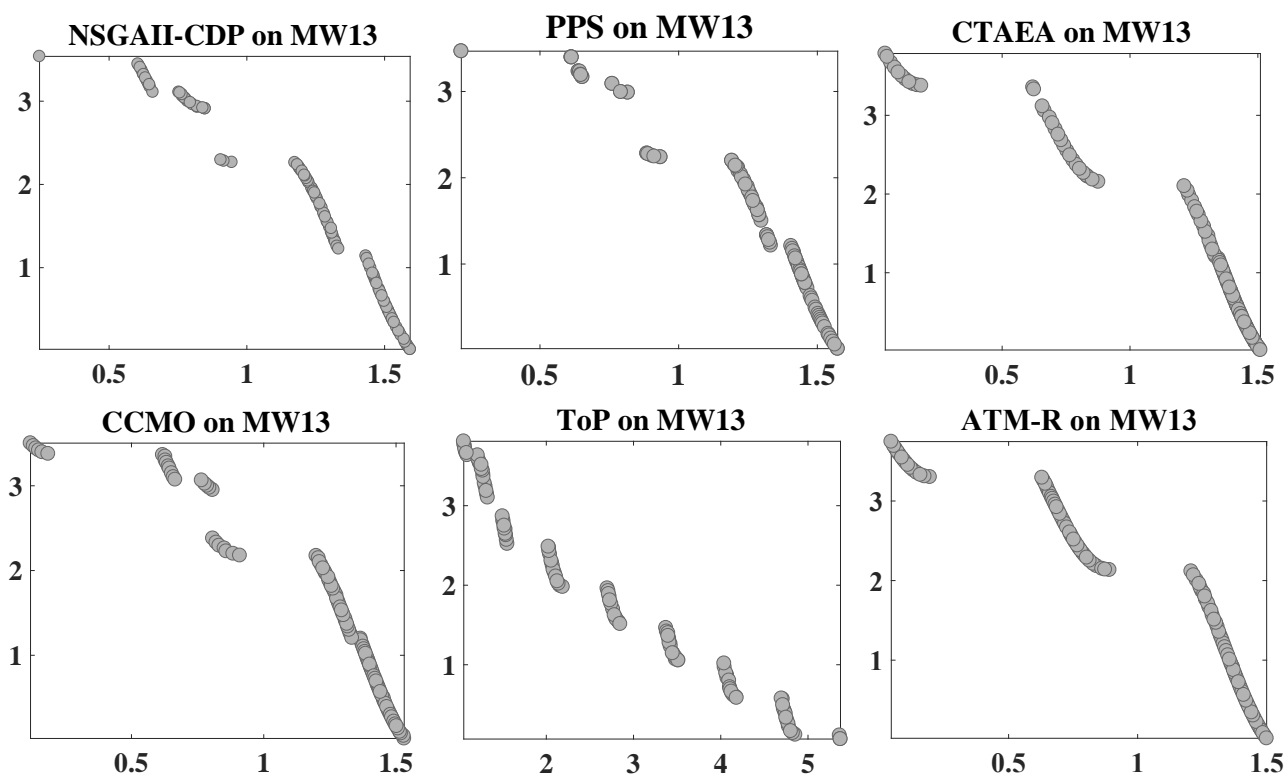


Fig. S-1. The constrained Pareto front with median value among 30 runs obtained by NSGAII-CDP, PPS, CTAEA, CCMO, ToP, and ATM-R on MW13.

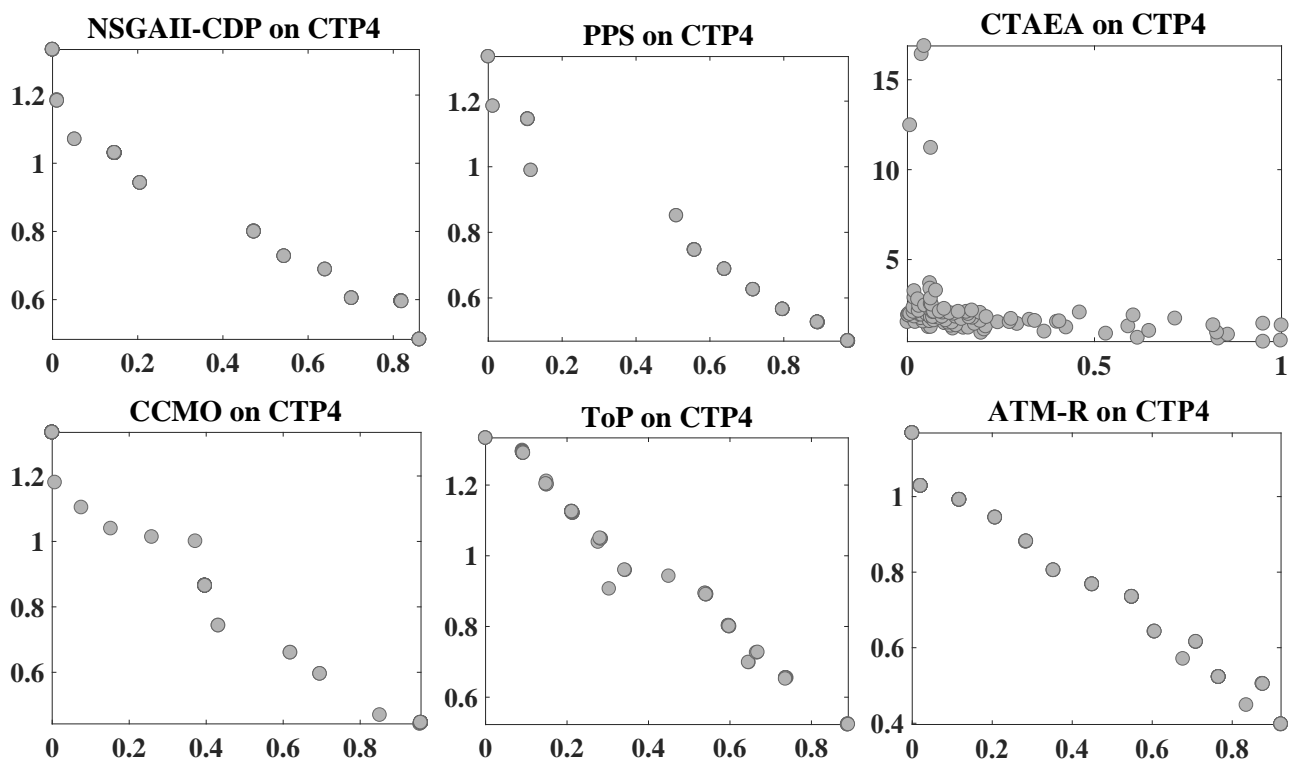


Fig. S-2. The constrained Pareto front with median value among 30 runs obtained by NSGAII-CDP, PPS, CTAEA, CCMO, ToP, and ATM-R on CTP4.

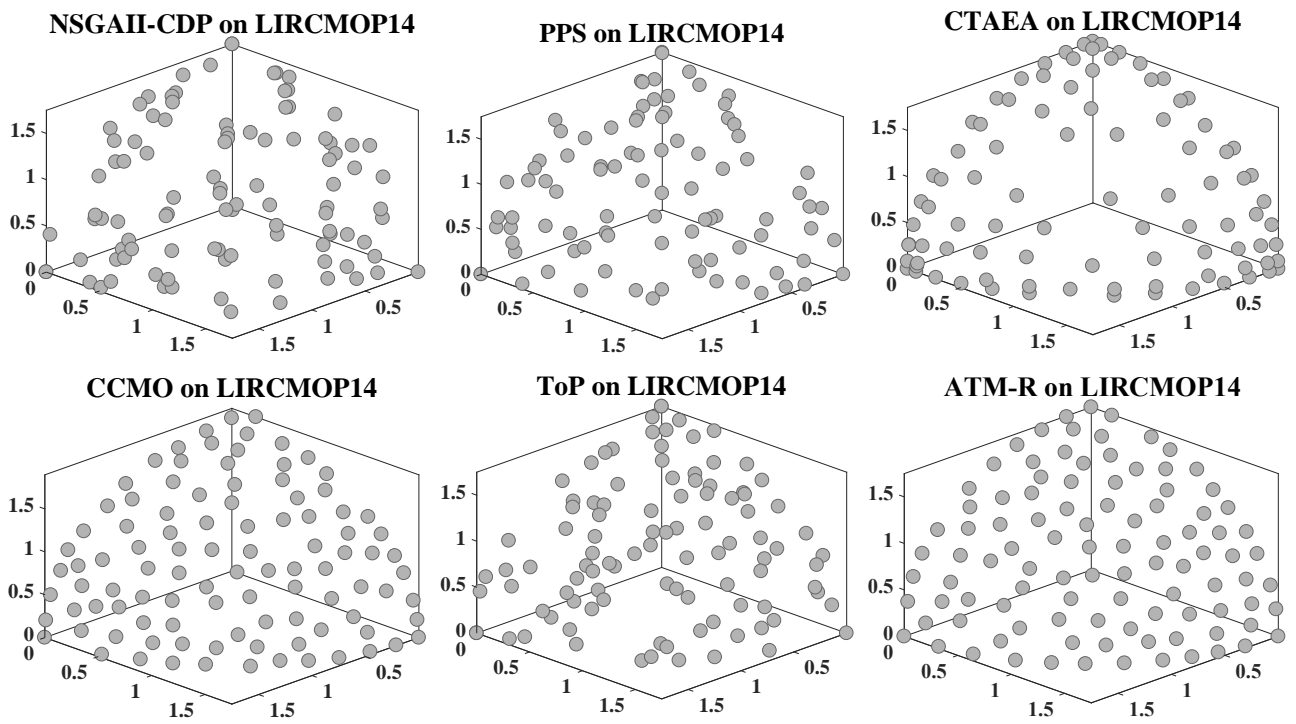


Fig. S-3. The constrained Pareto front with median value among 30 runs obtained by NSGAII-CDP, PPS, CTAEA, CCMO, ToP, and ATM-R on LIRCMOP14.