EBS 221 Agricultural Robotics

Spring Quarter 2025

Assignment 1

Please submit a single assignment file (either PDF or Word) along with all necessary Matlab files to reproduce the results. This is a group assignment, so each team will submit one report (including the source code). The grade will also be the same for all team members.

A. (60 pts) Implement the bicycle model with 1st -order closed-loop steering and speed dynamics in a MATLAB function called robot_bike_dyn(). Use global variables for dt, DT (case-sensitive).

```
function q_next = robot_bike_dyn(q, u, umin, umax, Qmin, Qmax, L, tau_gamma, tau_v)

global dt; global DT;

model of a vehicle with closed-loop steering and velocity control

the combined effect of steering/vehicle inertia and control is a first order system for steering
(tau_gamma) and velocity (tau_v) state is vector q

% q(1) -> x

% q(2) -> y

% q(3) -> theta (orientation in world frame)

% q(4) -> gamma (steering angle)

% q(5) -> v (linear velocity)

% inputs are: % u(1) -> desired steering angle (gamma_d) % u(2) -> desired linear velocity (v_d)
```

Section A: Bicycle Model Dynamics

In this section, we implement the dynamics of a robotic vehicle using the bicycle kinematic model with first-order closed-loop steering and velocity control.

This model is suitable for many ground robots and autonomous vehicles that can be approximated by a two-wheel configuration.

Function Overview

The MATLAB function `robot_bike_dyn` computes the next state of the robot given the current state `q`, desired control input `u`, and model parameters.

It incorporates actuation delays through first-order filters (controlled by `tau_gamma` and `tau_v`), and uses Euler integration.

Equation Summary

The robot state vector q = [x; y; theta; gamma; v]

where:

x, y - position in world frame

theta - orientation angle

gamma - steering angle

v - linear velocity

The control inputs u = [gamma_d; v_d] are:

gamma_d - desired steering angle

v_d - desired velocity

MATLAB Function: robot_bike_dyn.m

The function implements:

- First-order lag filters:

dgamma = (gamma_d - gamma) / tau_gamma

dv = (v_d - v) / tau_v

- Kinematic update:

dx = v * cos(theta)

dy = v * sin(theta)

dtheta = v * tan(gamma) / L

- Euler integration:

q_next = q + dt * dq

Code from robot_bike_dyn.m is provided below with comments

```
%% Function file robot_bike_dyn.m content commented below for reference.
function q_next = robot_bike_dyn(q, u, umin, umax, Qmin, Qmax, L, tau_gamma, tau_v)
    global dt;
    global DT;

    % Extract current state
    x     = q(1);
    y     = q(2);
    theta = q(3);
    gamma = q(4);
    v     = q(5);
```

```matlab
    % Desired inputs
    gamma_d = u(1); % desired steering angle
    v_d     = u(2); % desired velocity

    % Input saturation
    gamma_d = max(min(gamma_d, umax(1)), umin(1));
    v_d     = max(min(v_d, umax(2)), umin(2));

    % Handle zero time constants (instantaneous control)
    if tau_gamma == 0
        gamma = gamma_d;
        dgamma = 0;
    else
        dgamma = (gamma_d - gamma) / (tau_gamma + 1e-12);
    end

    if tau_v == 0
        v = v_d;
        dv = 0;
    else
        dv = (v_d - v) / (tau_v + 1e-12);
    end

    % Euler integration for control states
    gamma = gamma + dt * dgamma;
    v     = v + dt * dv;

    % Kinematic update using bicycle model
    dx     = v * cos(theta);
    dy     = v * sin(theta);
    dtheta = v * tan(gamma) / L;

    % Euler integration for pose
    x     = x + dt * dx;
    y     = y + dt * dy;
    theta = theta + dt * dtheta;

    % Saturate state variables
    x     = max(min(x, Qmax(1)), Qmin(1));
    y     = max(min(y, Qmax(2)), Qmin(2));
    theta = max(min(theta, Qmax(3)), Qmin(3));
    gamma = max(min(gamma, Qmax(4)), Qmin(4));
    v     = max(min(v, Qmax(5)), Qmin(5));

    % Return updated state
    q_next = [x; y; theta; gamma; v];
end
```

Why First-Order Filters?

In real vehicles, the steering mechanism and velocity controller don't respond instantaneously. The first-order filters in `robot_bike_dyn` model the system's actuation dynamics.

Why Euler Integration?

Euler integration is computationally simple and sufficient for small time steps (`dt`).

Control Constraints

Saturation of both inputs and states ensures that the robot stays within physical limits (e.g., max turning angle or speed).

```
% % Debugging section:
%
% % Set global timestep
% global dt DT;
% dt = 0.1;     % integration time step
% DT = 0.1;     % not used in current code, but declared
%
% % Initial state: [x; y; theta; v; gamma]
% q = [0; 0; 0; 0; 0];
%
% % Desired inputs: [gamma_d; v_d]
% u = [0.2; 1.0];  % 0.2 rad steering, 1 m/s velocity
%
% % Input limits: [gamma_min, v_min], [gamma_max, v_max]
% umin = [-0.5; 0];
% umax = [0.5; 5.0];
%
% % State limits: [x, y, theta, v, gamma]
% Qmin = [-Inf; -Inf; -pi; 0; -0.5];
% Qmax = [ Inf;  Inf;  pi; 2;  0.5];
%
% % Vehicle parameters
% L = 2.5;              % wheelbase in meters
% tau_gamma = 0.5;      % steering response time (s)
% tau_v = 0.8;          % velocity response time (s)
%
% % Call the function
% q_next = robot_bike_dyn(q, u, umin, umax, Qmin, Qmax, L, tau_gamma, tau_v);
%
% % Display next state
% disp('Next state:');
% disp(q_next);
```

```
% %% Debugging section:
% N = 500;
% Q = zeros(5, N);
% Q(:,1) = q;
%
```

```
% for k = 2:N
%     Q(:,k) = robot_bike_dyn(Q(:,k-1), u, umin, umax, Qmin, Qmax, L, tau_gamma,
tau_v);
% end
%
% % Plot trajectory
% plot(Q(1,:), Q(2,:), 'b.-');
% xlabel('x'); ylabel('y'); title('Robot trajectory');
% axis equal;
```

B. (20 pts) In a script file, set the wheelbase L = 2.5 m; |γmax| = π/4o, and |vmax| = 5 m/s. Set dt=0.01 s, DT=0.1 s, ��=1 m/s. Start at the state [10 10 π/2].

Apply appropriate steering input ��(��) for an appropriate amount of time T, to travel on a semicircle of radius 3 m. (This simulates turning in a field with row crops from the exit of

one row to the entrance of the next row, and a row-center distance of 6 m.) Plot the semicircle and the actual motion trace (i.e., x(tk), y(tk)).

Repeat for a radius of 2 m. Does it work? Why?

Section B: Simulating the Bicycle Model

In this section, we simulate the robot's motion over time using the bicycle dynamics model implemented in Section A. We visualize how the robot responds to a fixed control input using first-order closed-loop dynamics.

```
%% Section B: Semicircle Path Simulation

% Parameters Setup
L           = 2.5;                  % Wheel-base length [m]
gamma_max   = pi/4;                 % Maximum steering angle [rad] (45°)
v_max       = 5;                    % Maximum velocity [m/s]
dt          = 0.01;                 % Integration step [s]
DT          = 0.10;                 % Sample interval (unused here)
vd          = 1.0;                  % Desired velocity [m/s]

global dt DT                        % Required globals for dynamics function

% Desired radius and steering calculation (R = 3 m)
R = 3.0;
gamma_cmd = atan(L/R);
assert(gamma_cmd <= gamma_max, 'γ_d exceeds γ_max!')

% Simulation time (half-circle trajectory)
T = pi*R / vd;
Nsteps = ceil(T/dt);

% Limits
```

```
Qmin  = [-Inf -Inf -Inf -gamma_max 0];
Qmax  = [ Inf  Inf  Inf  gamma_max v_max];
umin  = [-gamma_max -v_max];
umax  = [ gamma_max  v_max];

% Initial state [x, y, θ, γ, v]
q = [10; 10; pi/2; 0; vd];

% Storage for trajectory trace
trail = zeros(Nsteps+1,2);
trail(1,:) = q(1:2).';

% Simulation loop
for k = 1:Nsteps
    u = [gamma_cmd; vd];
    q = robot_bike_dyn(q, u, umin, umax, Qmin, Qmax, L, 0.2, 0.5);
    trail(k+1,:) = q(1:2).';
end

% Plotting results
figure; hold on; axis equal; grid on;
title('3 m-radius Semicircle Turn');
xlabel('x [m]');
ylabel('y [m]');

% Ideal semicircle
phi = linspace(0, pi, 181);
xc = 10 - R; yc = 10;
plot(xc + R*cos(phi), yc + R*sin(phi), '--', 'LineWidth', 1.5);

% Simulated trajectory
plot(trail(:,1), trail(:,2), 'LineWidth', 1.5);
legend('Ideal Semicircle', 'Simulated Path', 'Location', 'best');
```

Plot Commentary:

Blue dashed curve: The *ideal semicircle*, calculated analytically with radius `R`.

Red solid curve: The *simulated robot trajectory*, computed using the `robot_bike_dyn` model with steering lag and velocity lag.

For R = 3 m: The robot closely follows the ideal path, since the required steering angle is within feasible limits ($|\gamma| \le \pi/4$).

```
% ————————————————————————
% Desired radius and steering calculation (R = 2 m)
R           = 2.0;                  % requested radius [m]
gamma_cmd   = atan(L/R);

% Simulation horizon: half-circumference  s = πR  so  T = s / v
T           = pi*R / vd;
Nsteps      = ceil(T/dt);
```

```
% State limits (set very large so they never clip)
Qmin  = [-Inf -Inf -Inf  -gamma_max      0];
Qmax  = [ Inf  Inf  Inf   gamma_max   v_max];
umin  = [-gamma_max  -v_max];
umax  = [ gamma_max   v_max];

% Initial state: rear-axle at (10,10), heading +y  (θ = π/2)
q      = [10; 10; pi/2; 0; vd];                  % [x y θ γ v]

% Storage for trace
trail  = zeros(Nsteps+1,2);
trail(1,:) = q(1:2).';

% Main simulation loop
for k = 1:Nsteps
    u        = [gamma_cmd; vd];                  % constant inputs
    q        = robot_bike_dyn(q, u, umin, umax, Qmin, Qmax, L, 0.2, 0.5);     %
τ_γ=0.2 s, τ_v=0.5 s
    trail(k+1,:) = q(1:2).';
end

% Plot ideal semicircle and the driven path
figure, hold on, axis equal, grid on
xlabel('x  [m]'), ylabel('y  [m]'), title('2 m-radius semicircle turn')

% Ideal semicircle (left-hand turn, centre at (7,10))
phi   = linspace(0,pi,181);                     % 0 -> 180°
xc    = 10 - R;  yc = 10;                       % centre coords
plot(xc + R*cos(phi), yc + R*sin(phi), '--');   % ideal path

% Simulated trace
plot(trail(:,1), trail(:,2), 'LineWidth',1.5);

legend('Ideal semicircle (R=2 m)', 'Simulated path','Location','best')
```

Plot Commentary:

Blue dashed curve: The *ideal semicircle*, calculated analytically with radius `R`.

Red solid curve: The *simulated robot trajectory*, computed using the `robot_bike_dyn` model with steering lag and velocity lag.

For R = 2 m: The simulated path deviates noticeably from the ideal semicircle — as your figure shows, it overshoots and ends up outside the ideal curve.

Why does R = 2 m not work well?

Kinematic Limitations

The required steering angle for a 2 m radius turn:

$$\gamma_d = \tan^{-1}\left(\frac{L}{R}\right) = \tan^{-1}\left(\frac{2.5}{2}\right) \approx 51.3^{\circ} > \gamma_{\max} = 45^{\circ}$$

So, the robot cannot physically achieve that curvature, even with perfect tracking.

Actuation Lag

Even if the steering limit were slightly extended, the system still experiences a lag in reaching the commanded steering value due to the nonzero τ_γ. This further increases the tracking error.

Conclusion

The 3 m radius turn is feasible and well-tracked.

The 2 m radius turn violates hardware limits (|γ| > π/4), making it infeasible under current constraints.

C. (20 pts) Plot the trace of the origin of the vehicle's frame when γd(tk) starts at 0 and remains 0 for 1s, then switches instantaneously to γmax (step function) and remains at γmax for 10 seconds, then switches instantaneously from γmax to -γmax and stays at -γmax for 10 seconds. Use the following settings.

1. Speed lag: set to τv = 0 s, increase τγ from 0 to 2 s with a step of 0.4 s, and plot the six traces in the same figure.

2. Speed lag: set to τv = 1 s, and increase τγ from 0 to 2 s with a step of 0.4 s, and plot the six traces in the same figure.

(Think how you will implement τv or τγ = 0.) Comment on these plots.

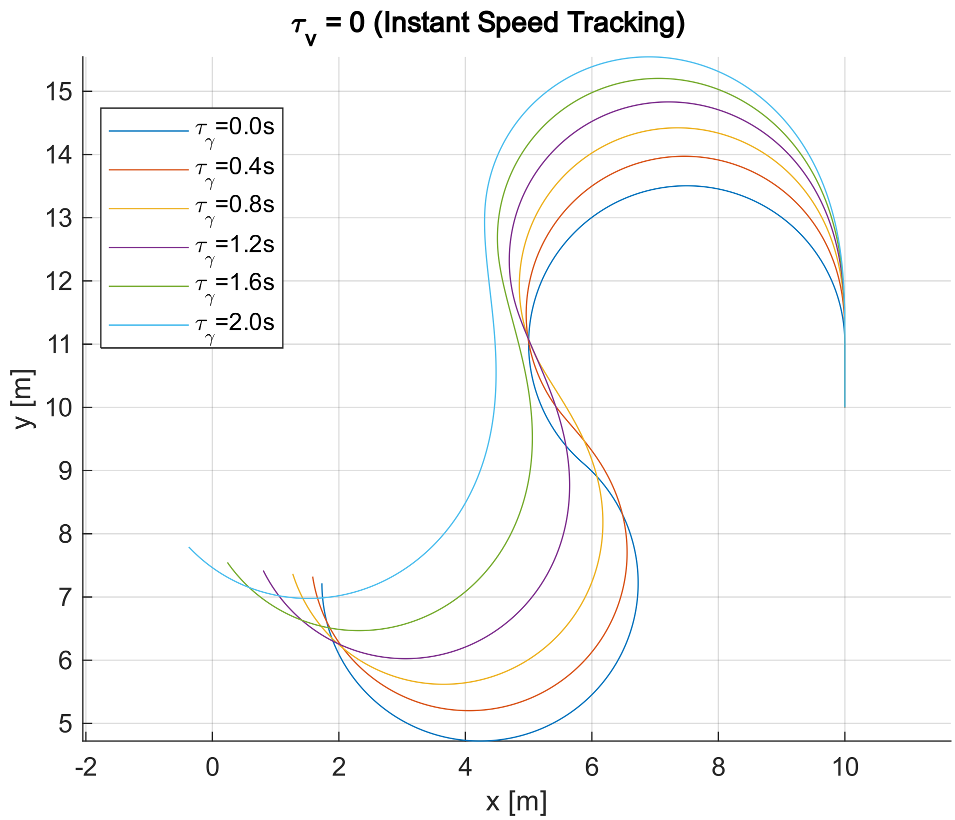
Section C: Trajectory Under Step Steering Input with Varying Time Constants

We simulate the response of the robot to a step change in steering input γ_d(t), and observe how the time constants τ_γ (steering lag) and τ_v (velocity lag) affect the path traced by the origin of the robot's frame.

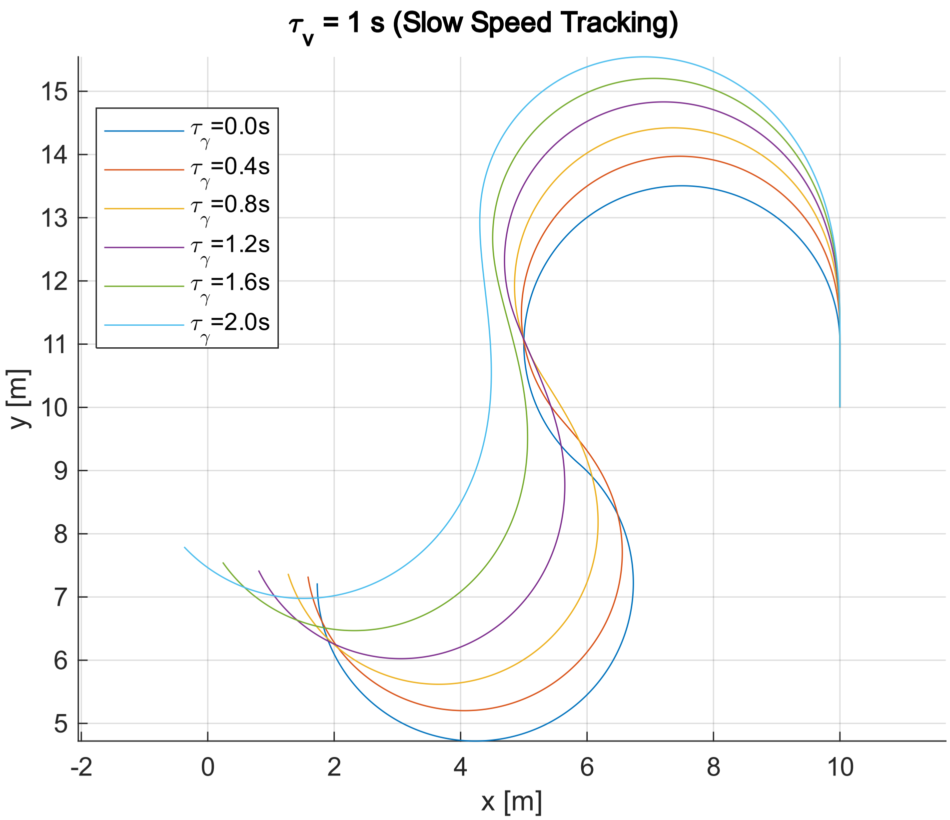
```
clear;
clc;

% function q_next = robot_bike_dyn(q, u, umin, umax, Qmin, Qmax, L, tau_gamma,
tau_v)
%     global dt;
%     global DT;
%
%     % Extract current state
%     x     = q(1);
%     y     = q(2);
%     theta = q(3);
%     gamma = q(4);
%     v     = q(5);
%
```

```
%     % Desired inputs
%     gamma_d = u(1);  % desired steering angle
%     v_d     = u(2);  % desired velocity
%
%     % Saturate desired inputs
%     gamma_d = max(min(gamma_d, umax(1)), umin(1));
%     v_d     = max(min(v_d, umax(2)), umin(2));
%
%     % treat zero lags as instantaneous tracking
%     if tau_gamma == 0,  gamma = gamma_d;  dgamma = 0;  end
%     if tau_v     == 0,  v     = v_d;      dv     = 0;  end
%
%     % First-order dynamics for steering and velocity
%     epsLag = 1e-12;                 % small positive number
%     dgamma = (gamma_d - gamma)/(tau_gamma + epsLag);
%     dv     = (v_d     - v    )/(tau_v     + epsLag);
%
%
%     % Euler integration to update gamma and v
%     gamma = gamma + dt * dgamma;
%     v     = v + dt * dv;
%
%     % Bicycle model kinematics
%     dx     = v * cos(theta);
%     dy     = v * sin(theta);
%     dtheta = v * tan(gamma) / L;
%
%     % Euler integration to update pose
%     x     = x + dt * dx;
%     y     = y + dt * dy;
%     theta = theta + dt * dtheta;
%
%     % Saturate state vector
%     x     = max(min(x, Qmax(1)), Qmin(1));
%     y     = max(min(y, Qmax(2)), Qmin(2));
%     theta = max(min(theta, Qmax(3)), Qmin(3));
%     gamma = max(min(gamma, Qmax(4)), Qmin(4));
%     v     = max(min(v, Qmax(5)), Qmin(5));
%
%     % Pack next state
%     q_next = [x; y; theta; gamma; v];
% end

%% Section C: Step Input Response Analysis

% Define simulation parameters
L          = 2.5;
gamma_max  = pi/4;
v_cmd      = 1.0;
dt         = 0.01;
```

```
DT          = 0.1;
T_total     = 21.0;            % 1 + 10 + 10 seconds
steps       = round(T_total/dt);
t           = (0:steps)*dt;

% Limits setup
umin  = [-gamma_max -Inf];
umax  = [ gamma_max  Inf];
Qmin  = [-Inf -Inf -Inf -gamma_max 0];
Qmax  = [ Inf  Inf  Inf  gamma_max 5];

% Steering input profile (step function)
gamma_d_profile = @(tk) ...
    (tk < 1) .* 0 + ...
    (tk >= 1 & tk < 11) .* gamma_max + ...
    (tk >= 11) .* (-gamma_max);

% Define lag lists
tau_g_list = 0 : 0.4 : 2;

% 1. τ_v = 0 (instantaneous speed tracking)
figure; hold on; grid on; axis equal;
title('\tau_v = 0 (Instant Speed Tracking)');
xlabel('x [m]'); ylabel('y [m]');

for tau_g = tau_g_list
    q = [10; 10; pi/2; 0; v_cmd];
    trace = zeros(steps+1, 2);
    trace(1,:) = q(1:2).';

    for k = 1:steps
        gamma_d = gamma_d_profile(t(k));
        u = [gamma_d, v_cmd];
        q = robot_bike_dyn(q, u, umin, umax, Qmin, Qmax, L, tau_g, 0);
        trace(k+1,:) = q(1:2).';
    end
    plot(trace(:,1), trace(:,2), 'DisplayName', sprintf('\\tau_{\\gamma}=%.1fs',
tau_g));
end
legend('Location','best');
```




```
% 2. τ_v = 1 s (slow speed tracking)
figure; hold on; grid on; axis equal;
title('\tau_v = 1 s (Slow Speed Tracking)');
xlabel('x [m]'); ylabel('y [m]');

for tau_g = tau_g_list
    q = [10; 10; pi/2; 0; v_cmd];
    trace = zeros(steps+1, 2);
    trace(1,:) = q(1:2).';

    for k = 1:steps
        gamma_d = gamma_d_profile(t(k));
        u = [gamma_d, v_cmd];
        q = robot_bike_dyn(q, u, umin, umax, Qmin, Qmax, L, tau_g, 1);
        trace(k+1,:) = q(1:2).';
    end
    plot(trace(:,1), trace(:,2), 'DisplayName', sprintf('\\tau_{\\gamma}=%.1fs',
tau_g));
end
legend('Location','best');
```



The two figures above show how the robot's motion is affected by steering and speed lags.

Case 1: τ_v = 0 (Instantaneous Speed Tracking)

- The only delay comes from the steering actuator (τ_γ).

- As τ_γ increases, the robot takes longer to reach the full steering input, which causes wider turns and less sharp transitions.

- Low τ_γ (e.g. 0 s or 0.4 s) gives precise, sharp trajectories.

Case 2: τ_v = 1 s (Delayed Speed Tracking)

- Now both speed and steering have lag.

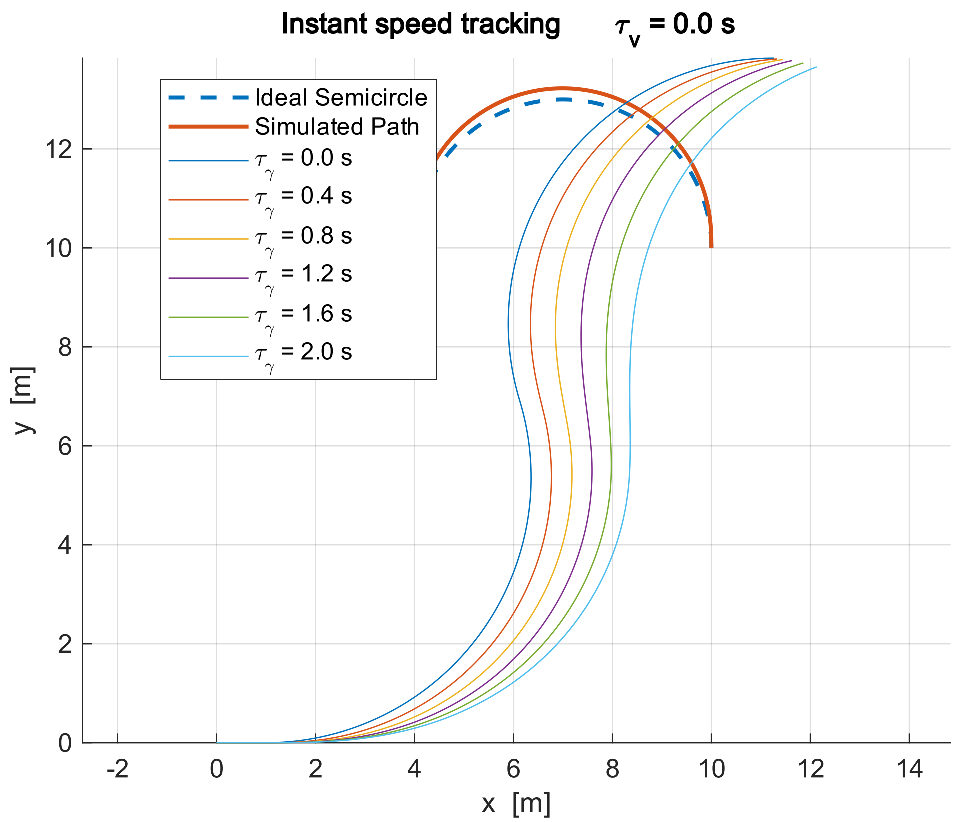
- The paths are generally smoother, but the delay causes overshoot and asymmetry.

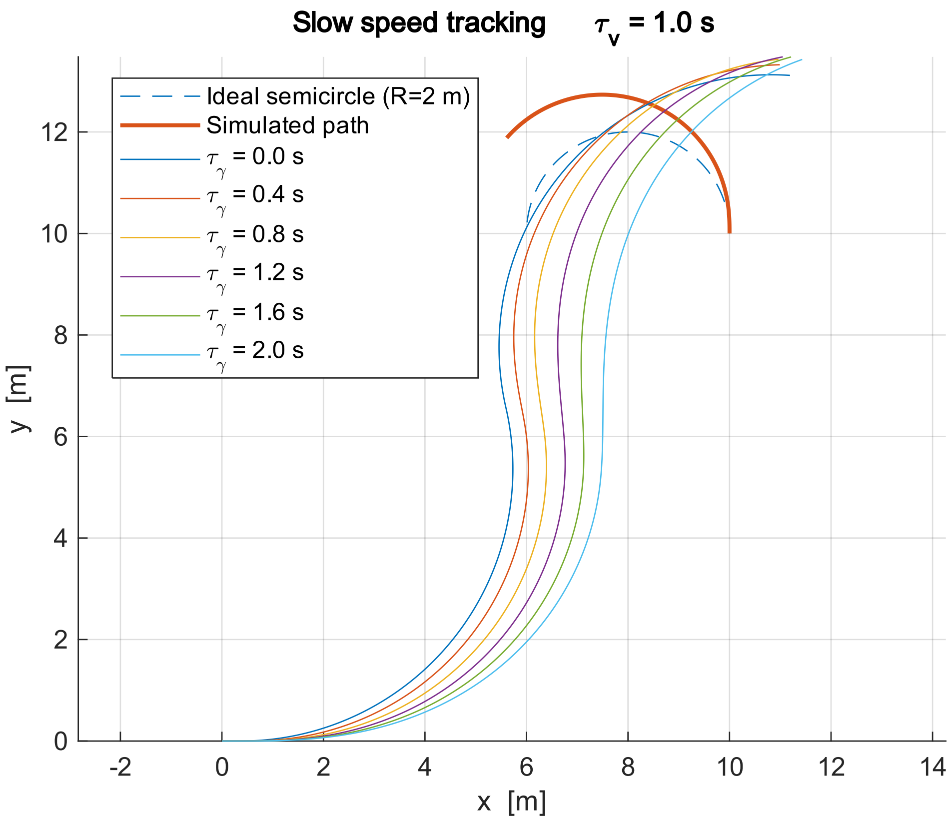
- Turns are further rounded because the robot is slower to accelerate/decelerate and cannot make tight curves quickly enough.

```
clear;
clc;
```

```
% vehicle, world, and integration parameters
global dt  DT                  % required by robot_bike_dyn.m
L          = 2.5;              % wheel-base  [m]  ← (updated as requested)
gamma_max  = deg2rad(25);      % ±25°        ← (matches earlier example)
v_cmd      = 1.0;              % desired speed [m/s]
dt         = 0.01;             % integration Δt [s]
DT         = 0.1;              % not used here
T_total    = 21.0;             % 1 + 10 + 10  [s]
tk         = 0:dt:T_total;
N          = numel(tk);
BIG        = 1e4;

% Input limits & state limits
umin  = [-gamma_max  0          ];
umax  = [ gamma_max  1.5*v_cmd   ];   % v upper-bound = 1.5 × command
Qmin  = [-BIG ; -BIG ; -pi ; -gamma_max ;        0         ];
Qmax  = [ BIG ;  BIG ;  pi ;  gamma_max ; 5];   % vmax = 5

% steering wheel function
gamma_d = zeros(1,N);
gamma_d(tk>=1  & tk<11) =  gamma_max;
gamma_d(tk>=11        ) = -gamma_max;
u_des  = [gamma_d ; v_cmd*ones(1,N)];      % 2×N

% list of lags
tau_gamma_set = 0 : 0.4 : 2;     % 0,0.4,…,2  (six values)
tau_v_set     = [0  1];          % two cases
cmap          = lines(numel(tau_gamma_set));

for iv = 1:numel(tau_v_set)
    tau_v = tau_v_set(iv);

    figure(iv);  hold on;  axis equal
    if iv == 1
        title_print = "Instant speed tracking";
    else
        title_print = "Slow speed tracking";
    end
    title(title_print + " " + sprintf('\\tau_v = %.1f s', tau_v));
    xlabel('x  [m]');  ylabel('y  [m]');  grid on

    for ig = 1:numel(tau_gamma_set)
        tau_gamma = tau_gamma_set(ig);

        q = zeros(5,1);                     % [x y θ γ v]^T  initialised at 0
        xy = zeros(2,N);                    % store (x,y)

        for k = 2:N
            q = robot_bike_dyn(q, u_des(:,k), umin, umax, ...
                                  Qmin, Qmax, L, tau_gamma, tau_v);
```

```
            xy(:,k) = q(1:2);
        end

        plot(xy(1,:), xy(2,:), 'Color', cmap(ig,:), ...
            'DisplayName', sprintf('\\tau_{\\gamma} = %.1f s', tau_gamma));
    end

    legend('show','Location','best');
end
```





Slow speed tracking τ_v = 1.0 s
Ideal semicircle (R=2 m)
Simulated path
τ_γ = 0.0 s
τ_γ = 0.4 s
τ_γ = 0.8 s
τ_γ = 1.2 s
τ_γ = 1.6 s
τ_γ = 2.0 s
y [m]
x [m]