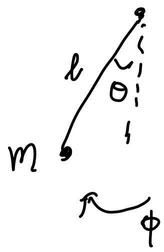


a) $x = l\sin\theta\cos\phi$

$\dot{x} = l(\cos\theta\cos\phi\,\dot{\theta} - \sin\theta\sin\phi\,\dot{\phi})$

$y = l\sin\theta\sin\phi$

$\dot{y} = l(\cos\theta\sin\phi\,\dot{\theta} + \sin\theta\cos\phi\,\dot{\phi})$

$z = -l\cos\theta$

$\dot{z} = l\sin\theta\,\dot{\theta}$

$$\mathcal{L} = \frac{1}{2}m(\dot{x}^2+\dot{y}^2+\dot{z}^2) - mgz$$

$$= \frac{1}{2}ml^2\big(\cos^2\theta\cos^2\phi\,\dot{\theta}^2 \cancel{-2\cos\theta\cos\phi\sin\theta\sin\phi\,\dot{\phi}} + \sin^2\theta\sin^2\phi\,\dot{\phi}^2$$
$$\cos^2\theta\sin^2\phi\,\dot{\theta}^2 \cancel{+2\cos\theta\cos\phi\sin\theta\sin\phi\,\dot{\phi}} + \sin^2\theta\cos^2\phi\,\dot{\phi}^2$$
$$+\sin^2\theta\,\dot{\theta}^2\big)$$
$$+mgl\cos\theta$$

$$\mathcal{L} = \frac{1}{2}ml^2(\dot{\theta}^2 + \sin^2\theta\,\dot{\phi}^2) + mgl\cos\theta$$

b) $\dfrac{\partial\mathcal{L}}{\partial\dot{\phi}} = ml^2\sin^2\theta\,\dot{\phi} = P_\phi \quad$ = constant

$\dfrac{\partial\mathcal{L}}{\partial\dot{\theta}} = P_\theta = ml^2\dot{\theta}$

$h = P_\phi\dot{\phi} + P_\theta\dot{\theta} - \mathcal{L}$

$$= \frac{p_\phi^2}{2ml^2\sin^2\theta} + \frac{p_\theta^2}{2ml^2} - mgl\cos\theta$$ ← energy is constant.

c) Find $\theta(t)$ and $\phi(\theta)$

$$\dot\phi = \frac{p_\phi}{ml^2\sin^2\theta}$$

$$E = \frac{1}{2}ml^2\dot\theta^2 + \frac{p_\phi^2}{2ml^2\sin^2\theta} - mgl\cos\theta$$

$$\sqrt{\left[E + mgl\cos\theta - \frac{p_\phi^2}{2ml^2\sin^2\theta}\right]\frac{2}{ml^2}} = \frac{d\theta}{dt}$$

$$\int_{\theta_0}^{\theta}\sqrt{\frac{ml^2}{2}}\frac{d\theta}{\sqrt{E + mgl\cos\theta - \frac{p_\phi^2}{2ml^2\sin^2\theta}}} = \int_{t_0}^{t} dt = t - t_0$$

$$\int_{\theta_0}^{\theta}\sqrt{\frac{ml^2}{2}}\frac{d\theta}{\sqrt{E + mgl\cos\theta - \frac{p_\phi^2}{2ml^2\sin^2\theta}}} = \int_{t_0}^{t}\frac{dt}{d\phi}\,d\phi = \int_{\phi_0}^{\phi}\frac{1}{\dot\phi}\,d\phi$$

$$\int_{\theta_0}^{\theta}\sqrt{\frac{ml^2}{2}}\frac{d\theta}{\sqrt{E + mgl\cos\theta - \frac{p_\phi^2}{2ml^2\sin^2\theta}}} = \int_{\phi_0}^{\phi}\frac{ml^2\sin^2\theta}{p_\phi}\,d\phi$$

$$\int^{\theta} \quad \frac{p_\phi \quad d\theta}{\quad}$$

$$\int_{\theta_0} \sqrt{\frac{1}{2ml^2}}\ \sin^2\theta \sqrt{E + mgl\cos\theta - \frac{P_\phi^2}{2ml^2\sin^2\theta}} \quad + \varphi_0 - \varphi$$

d) $$R = -\frac{1}{2}ml^2\dot{\theta}^2 + \frac{P_\phi^2}{2ml^2\sin^2\theta} - mgl\cos\theta$$

$$L = -R = \frac{1}{2}ml^2\dot{\theta}^2 - \frac{P_\phi^2}{2ml^2\sin^2\theta} + mgl\cos\theta$$

$$= \frac{1}{2}ml^2\dot{\theta}^2 - V_{eff}(\theta)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = ml^2\ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = -\frac{\partial}{\partial \theta}\left(\frac{P_\phi^2}{2ml^2\sin^2\theta} - mgl\cos\theta\right)$$

$$ml^2\ddot{\theta} = -mgl\sin\theta + \frac{-P_\phi^2}{2ml^2}(-2)(\sin^{-3}\theta)\cos\theta$$

$$ml^2\ddot{\theta} = -mgl\sin\theta + \frac{P_\phi^2\cos\theta}{ml^2\sin^3\theta}$$

$$= -mgl\sin\theta + \frac{P_\phi^2}{ml^2\sin^2\theta}\cot\theta$$

let $\ddot{\theta} = 0$ to have θ circular orbit.

$$-mgl\sin\theta - \frac{P_\phi^2\cos\theta}{ml^2\sin^3\theta} = 0$$

$$mgl\sin^4\theta = \frac{P_\phi^2}{ml^2}\cos\theta$$

$$\frac{m^2gl^3}{P_\phi^2} = \frac{\cos\theta_c}{\sin^4\theta_c} \Rightarrow \frac{P_\phi^2}{m^2gl^3} = \frac{\sin^4\theta_c}{\cos\theta_c}$$

e) Check stability, expand $V_{eff}(\theta = \theta_c + \delta\theta)$

$$V_{eff}(\theta) = V_{eff}(\theta_c) + \frac{\partial V_{eff}(\theta_c)}{\partial\theta}\delta\theta + \frac{1}{2}\frac{\partial^2 V_{eff}(\theta_c)}{\partial\theta^2}\delta\theta^2$$

$$ml^2\ddot{\theta} = -\frac{\partial}{\partial\theta}V_{eff}(\theta)$$

$$\stackrel{!}{=} -\frac{\partial}{\partial\theta}\left(V_{eff}(\theta_c) + \frac{\partial V_{eff}(\theta)}{\partial\theta}\delta\theta\right)$$

$$ml\,\delta\ddot{\theta} \stackrel{!}{=} -\frac{\partial^2 V_{eff}(\theta)}{\partial\theta^2}\delta\theta$$

$$V_{eff}(\theta) = \frac{P_\phi^2}{2ml^2\sin^2\theta} - mgl\cos\theta$$

$$\frac{\partial V_{eff}}{\partial\theta} = \frac{-P_\phi^2}{ml^2\sin^3\theta}\cos\theta + mgl\sin\theta$$

$$\frac{\partial^2}{\partial\theta^2}V_{eff} = \frac{-P_\phi^3}{ml^2}\left(\frac{-\sin^4\theta - 3\sin^2\theta\cos^2\theta}{\sin^6\theta}\right) + mgl\cos\theta$$

$$\stackrel{!}{=} \frac{P_\phi^3}{ml^2}\frac{1}{\sin^2\theta} + \frac{3P_\phi^3}{ml^2}\frac{\cos^2\theta}{\sin^4\theta} + mgl\cos\theta$$

$$\stackrel{!}{=} mgl\left(\frac{P_\phi^3}{m^2gl^3}\frac{1}{\sin^2\theta_c} + \frac{3P_\phi^3}{m^2gl^3}\frac{\cos^2\theta_c}{\sin^4\theta_c}\right) + mgl\cos\theta$$

$$\stackrel{!}{=} mgl\left(\frac{\sin^4\theta_c}{\cos\theta_c}\frac{1}{\sin^2\theta_c} + 3\frac{\sin^4\theta_c}{\cos\theta_c}\frac{\cos^2\theta_c}{\sin^4\theta_c}\right) + mgl\cos\theta$$

$$\stackrel{!}{=} mgl\left(\frac{\sin^2\theta_c}{\cos\theta_c} + 4\cos\theta_c\right)$$

$$= mgl\cos\theta_c \left(4 + \tan^2\theta_c\right)$$

stable when $\frac{\partial^2 V_{eff}}{\partial\theta^2} > 0$.

so when $\cos\theta_c > 0$ or when

$$0 \le \theta_c \le \pi/2$$

$$ml^2\,\delta\ddot{\theta} = -\frac{\partial^2}{\partial\theta} V_{eff}\Big|_{\theta=\theta_c} \delta\theta$$

$$\delta\ddot{\theta} + \underbrace{\frac{1}{ml^2}\frac{\partial^2}{\partial\theta} V_{eff}\Big|_{\theta=\theta_c}}_{\omega^2} \delta\theta = 0.$$

$$\omega^2 = \frac{1}{ml^2} mgl\cos\theta_c\left(4 + \tan^2\theta_c\right)$$

$$\omega^2 = \frac{g}{l}\cos\theta_c\left(4 + \tan^2\theta_c\right)$$

for $\theta_c \ll 1$

$$\omega^2 \cong 4\,g/l$$

$$\omega \cong 2\sqrt{g/l}$$ ← it is 2 times higher than simple pendulum, because we also have $\dot{\phi}$.

by setting $P_\phi = 0$

$$\frac{\partial^2}{\partial\theta^2} V_{eff} = mgl\cos\theta.$$

then we have the $\omega^2 = \frac{1}{m\ell^2} mg\ell\cos\theta_c$
$= g/\ell.$

2) <u>Isotropic oscillator in a magnetic field.</u>

Charge q, mass m, move in x-y plane. with
$V(x,y) = \frac{1}{2} m\omega_0^2 (x^2+y^2)$, $\vec{B} = B_0\hat{z}$

$x = r\cos\phi$ $\qquad\qquad$ $y = r\sin\phi$
$\dot{x} = \dot{r}\cos\phi - r\sin\phi\,\dot{\phi}$ $\qquad\qquad$ $\dot{y} = \dot{r}\sin\phi + r\cos\phi\,\dot{\phi}$

Cartesian:

$$\mathcal{L} = \frac{1}{2}m(\dot{x}^2+\dot{y}^2) - \cancel{q\phi} + \frac{q}{c}\dot{\vec{r}}\cdot\vec{A} - \frac{1}{2}m\omega_0^2(x^2+y^2)$$

Let $\vec{A} = \frac{1}{2}B_0\hat{z} \times (x\hat{x}+y\hat{y})$
$\stackrel{!}{=} \frac{1}{2}B_0(x\hat{y} - y\hat{x})$

$$\boxed{\mathcal{L} = \frac{1}{2}m(\dot{x}^2+\dot{y}^2) + \frac{1}{2}\omega_B m(x\dot{y} - \dot{x}y) - \frac{1}{2}m\omega_0^2(x^2+y^2)}$$

Cylindrical:

$$\mathcal{L} = \frac{1}{2}m\left[(\dot{r}\cos\phi - r\sin\phi\,\dot{\phi})^2 + (\dot{r}\sin\phi + r\cos\phi\,\dot{\phi})^2\right]$$
$$+ \frac{q}{c}\dot{\vec{r}}\cdot\vec{A} - \frac{1}{2}m\omega_0^2 r^2$$

$$\vec{A} = \frac{1}{2}B_0(\hat{z} \times r\hat{r})$$
$$\stackrel{!}{=} \frac{1}{2}B_0(\hat{z} \times (\cos\phi\,\hat{x} + \sin\phi\,\hat{y})\,r)$$

$= \frac{1}{2} B_0 (r\cos\phi\,\hat{y} - r\sin\phi\,\hat{x})$

$$\mathcal{L} = \frac{1}{2} m (\dot{r}^2 + r^2\dot{\phi}^2) + \frac{q}{c}\frac{1}{2}B_0 \left[r\cos\phi(\dot{r}\sin\phi + r\cos\phi\,\dot{\phi}) - r\sin\phi(\dot{r}\cos\phi - r\sin\phi\,\dot{\phi})\right] - \frac{1}{2} m\omega_0^2 r^2$$

$$\boxed{\mathcal{L} = \frac{1}{2} m (\dot{r}^2 + r^2\dot{\phi}^2) + \frac{1}{2}\omega_B m \left[r^2 \dot{\phi} \right] - \frac{1}{2} m\omega_0^2 r^2}$$

b) $\frac{\partial \mathcal{L}}{\partial \dot{x}} = m\dot{x} - \frac{1}{2}\omega_B m y$

$\frac{\partial \mathcal{L}}{\partial x} = \frac{1}{2}\omega_B m \dot{y} - m\omega_0^2 x$

$m\ddot{x} - \frac{1}{2}\omega_B m\dot{y} = \frac{1}{2}\omega_B m\dot{y} - m\omega_0^2 x$

$$\boxed{m\ddot{x} + m\omega_0^2 x - \omega_B m\dot{y} = 0}$$

$\frac{\partial \mathcal{L}}{\partial \dot{y}} = m\dot{y} + \frac{1}{2}\omega_B m x$

$\frac{\partial \mathcal{L}}{\partial y} = -\frac{1}{2}\omega_B m \dot{x} - m\omega_0^2 y$

$$\boxed{m\ddot{y} + m\omega_0^2 y + \omega_B m\dot{x} = 0}$$

$$\boxed{\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = mr^2\dot{\phi} + \frac{1}{2}\omega_B m r^2 = P_\phi = \text{constant.}}$$

$$\frac{\partial L}{\partial \phi} = 0. \qquad \dot{\phi} = \frac{p_\phi}{mr^2} - \frac{1}{2}\omega_B$$

$$\frac{\partial L}{\partial \dot{r}} = m\dot{r}$$

$$\frac{\partial L}{\partial r} = mr\dot{\phi}^2 + \omega_B m r\dot{\phi} - m\omega_0^2 r$$

$$R = p_\phi \dot{\phi} - L$$

$$= \frac{p_\phi^2}{mr^2} - \frac{1}{2}p_\phi\omega_B - \Big(\frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\Big(\frac{p_\phi}{mr^2} - \frac{1}{2}\omega_B\Big)^2$$
$$+ \frac{1}{2}m\omega_B r^2\Big(\frac{p_\phi}{mr^2} - \frac{1}{2}\omega_B\Big)$$
$$- \frac{1}{2}m\omega_0^2 r^2\Big)$$

$$= -\frac{1}{2}m\dot{r}^2 + \cancel{\frac{p_\phi^2}{mr^2}} - \cancel{\frac{1}{2}p_\phi\omega_B} - \cancel{\frac{p_\phi^2}{2mr^2}} - \frac{1}{8}mr^2\omega_B^2 + \cancel{\frac{1}{2}mr^2\frac{p_\phi}{mr^2}}\omega_B$$
$$- \frac{1}{2}\omega_B p_\phi + \frac{1}{4}m\omega_B^2 r^2 + \frac{1}{2}m\omega_0^2 r^2$$

$$= \frac{p_\phi^2}{2mr^2} + \frac{1}{8}m\omega_B^2 r^2 - \frac{1}{2}\omega_B p_\phi + \frac{1}{2}m\omega_0^2 r^2 - \frac{1}{2}m\dot{r}^2$$

$$L_{eff} = -R$$

$$\frac{\partial L}{\partial r} = -\Big[\frac{-p_\phi^2}{mr^3} + \Big(\frac{1}{4}m\omega_B^2 + m\omega_0^2\Big)r\Big]$$

$$\boxed{m\ddot{r} = \frac{p_\phi^2}{mr^3} - \Big(\frac{1}{4}m\omega_B^2 + m\omega_0^2\Big)r}$$

c)

$$\frac{\partial L}{\partial \dot\phi} = mr^2\dot\phi + \frac{1}{2}\omega_B m r^2 = P_\phi = \text{constant}.$$ angular momentum.

Energy is also constant

$h = P_x\dot x + P_y \dot y - L$

$\overset{!}{=} m\dot x^2 - \frac{1}{2}m\omega_B y\dot x + m\dot y^2 + \frac{1}{2}m\omega_B x\dot y$

$- \left[\frac{1}{2}m(\dot x^2+\dot y^2) + \frac{1}{2}\omega_B m\,(x\dot y - \dot x y) - \frac{1}{2}m\omega_0^2(x^2+y^2)\right]$

$\overset{!}{=} \frac{1}{2}m\dot x^2 + \frac{1}{2}m\dot y^2 + \frac{1}{2}m\omega_0^2(x^2+y^2)$

$h = P_r\dot r + P_\phi\dot\phi - L$

$\overset{!}{=} m\dot r^2 + mr^2\dot\phi^2 + \frac{1}{2}m\omega_B r^2\dot\phi -$

$\left(\frac{1}{2}m(\dot r^2 + r^2\dot\phi^2) + \frac{1}{2}\omega_B m\left[r^2\dot\phi\right] - \frac{1}{2}m\omega_0^2 r^2\right)$

$\overset{!}{=} \frac{1}{2}m\dot r^2 + \frac{1}{2}mr^2\dot\phi^2 + \frac{1}{2}m\omega_0^2 r^2$

d) circular orbit when $\dot r = \ddot r = 0$

$$m\ddot{r} = \frac{P_\phi^2}{mr^3} - \left(\frac{1}{4}m\omega_B^2 + m\omega_0^2\right)r = 0$$

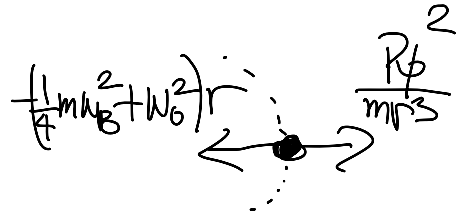
$$\frac{P_\phi^2}{mr^3} = \left(\frac{1}{4}m\omega_B^2 + m\omega_0^2\right)r$$

$$\pm\frac{|P_\phi|}{m}\left(\left[\frac{1}{4}\omega_B^2 + \omega_0^2\right]^{-1}\right)^{1/2} = r_c^2$$

$$\dot{\phi} = \frac{P_\phi}{mr_c^2} - \frac{1}{2}\omega_B$$

$$\dot{\phi} = \frac{P_\phi}{m}\cdot\frac{m}{P_\phi}\left[\frac{1}{4}\omega_B^2 + \omega_0^2\right]^{1/2} - \frac{1}{2}\omega_B$$

$$\dot{\phi} = \pm\frac{P_\phi}{|P_\phi|}\sqrt{\frac{1}{4}\omega_B^2 + \omega_0^2} - \frac{1}{2}\omega_B$$



If P_ϕ is negative, $\dot{\phi}$ moves in clockwise fast.

If P_ϕ is positive, $\dot{\phi}$ moves in counterclockwise, slowly.

e) Determine $x(t)$, $y(t)$

$$m\ddot{x} + m\omega_0^2 x - m\omega_B\dot{y} = 0$$

$$m\ddot{y} + m\omega_0^2 y + \omega_B m\dot{x} = 0$$

$$z = x + iy$$
$$\dot{z} = \dot{x} + i\dot{y}$$
$$\ddot{z} = \ddot{x} + i\ddot{y}$$

$$\ddot{z} = -w_0^2 x + w_B \dot{y} + i\left(-w_B \dot{x} - w_0^2 y\right)$$
$$\stackrel{!}{=} -w_0^2(x + iy) - iw_B(\dot{x} + i\dot{y})$$

$$\ddot{z} = -w_0^2 z - iw_B \dot{z}$$

$$\ddot{z} + iw_B \dot{z} + w_0^2 z = 0$$

$$z = Ae^{iwt}$$
$$\dot{z} = iwAe^{iwt}$$
$$\ddot{z} = -w^2 Ae^{iwt}$$

$$-w^2 + -ww_B + w_0^2 = 0$$

$$w^2 + ww_B - w_0^2$$

$$w = \frac{-w_B \pm \sqrt{w_B^2 + 4w_0^2}}{2} = \frac{-w_B}{2} \pm \sqrt{\left(\frac{w_B}{2}\right)^2 + w_0^2}$$

$$z = Ae^{i\left(\frac{-w_B}{2} + \sqrt{\left(\frac{w_B}{2}\right)^2 + w_0^2}\right)t} + Be^{-i\left(\frac{-w_B}{2} - \sqrt{\left(\frac{w_B}{2}\right)^2 + w_0^2}\right)t}$$
$$\stackrel{!}{=}$$

$$Re[z] = x = A_1 \cos\left(\frac{-w_B}{2} + \sqrt{\left(\frac{w_B}{2}\right)^2 + w_0^2}\; t + \phi_1\right) + A_2 \cos\left(\frac{-w_B}{2} - \sqrt{\left(\frac{w_B}{2}\right)^2 + w_0^2}\; t + \phi_2\right)$$

$$Im[z] = y = A_1 \sin\left(\frac{-w_B}{2} + \sqrt{\left(\frac{w_B}{2}\right)^2 + w_0^2}\; t + \phi_1\right) + A_2 \sin\left(\frac{-w_B}{2} - \sqrt{\left(\frac{w_B}{2}\right)^2 + w_0^2}\; t + \phi_2\right)$$

e) angular momentum P_ϕ:

$$P_\phi = \vec{r} \times \vec{p} + \frac{1}{2} m \omega_B (x^2+y^2)$$

$$= (x\hat{x} + y\hat{y}) \times (m\dot{x}\,\hat{x} + m\dot{y}\,\hat{y}) + \frac{1}{2} m\omega_B (x^2+y^2)$$

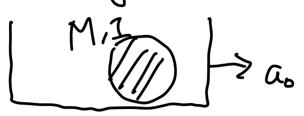
$$\stackrel{!}{=} (xm\dot{y} - ym\dot{x})\hat{z} + \frac{1}{2} m\omega_B (x^2+y^2)$$

$$\stackrel{!}{=} m\big(A_1\cos(\omega_+ t+\phi_1) + A_2\cos(\omega_- t+\phi_2)\big)\big(A_1\omega_+\cos(\omega_+ t+\phi_1) + A_2\omega_-\cos(\omega_- t+\phi_2)\big)$$

$$+ \big(A_1\sin(\omega_+ t+\phi_1) + A_2\sin(\omega_- t+\phi_2)\big)\big(A_1\omega_+\sin(\omega_+ t+\phi_1) + A_2\omega_-\sin(\omega_+ t+\phi_2)\big).$$

$$+ \frac{1}{2} m\omega_B \left(|A_1|^2 + |A_2|^2\right)$$

3) An accelerating frame:



$$L = \frac{1}{2} M\dot{x}^2 + \frac{1}{2} I\dot{\phi}^2 - \cancel{Mgy} \leftarrow \text{constant.}$$

$$\dot{x} = R\dot{\phi} + a_0 t$$

$$L = \frac{1}{2} M (R\dot{\phi} + a_0 t)^2 + \frac{1}{2} I \dot{\phi}^2$$

$$\stackrel{!}{=} \frac{1}{2} M (R^2\dot{\phi}^2 + 2R\dot{\phi}a_0 t + a_0^2 t^2) + \frac{1}{2} I\dot{\phi}^2$$

$$\stackrel{!}{=} \frac{1}{2} MR^2\dot{\phi}^2 + MR\dot{\phi}a_0 t + \frac{1}{2} M a_0^2 t^2 + \frac{1}{2} I\dot{\phi}^2$$

$$\stackrel{!}{=} \left(\frac{1}{2} MR^2 + \frac{1}{2} I\right)\dot{\phi}^2 + MR\dot{\phi}a_0 t + \cancel{\underbrace{\frac{1}{2} M a_0^2 t^2}_{\frac{d}{dt}\left(\frac{1}{2} M \frac{1}{3} a_0^2 t^3\right)}}$$

$$\stackrel{|}{=} \left(\frac{1}{2} MR^2 + \frac{1}{2} I\right)\dot{\phi}^2 + MRa_0\left[\frac{d}{dt}(\phi t) - \phi\right]$$

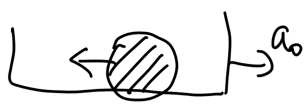
$$= \left(\frac{1}{2}MR^2+\frac{1}{2}I\right)\dot\phi - \underbrace{MRa_0\phi}_{\text{potential.}}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\phi}\right) = \left(\frac{1}{2}MR^2+\frac{1}{2}I\right)\ddot\phi$$

$$\frac{\partial L}{\partial \phi} = -MRa_0$$

$$\left(MR^2+I\right)\ddot\phi = -MRa_0$$

a)



$$F = ma$$

$$-FR = I\ddot\phi \Rightarrow -maR = I\ddot\phi$$

$$-m(a_0+R\ddot\phi)R = I\ddot\phi$$

without slipping condition:

$$x = \frac{1}{2}a_0t^2 + R\phi \Rightarrow a = a_0 + R\ddot\phi$$

$$m(a_0+R\ddot\phi) = -\frac{I}{R}\ddot\phi$$

$$mRa_0 + mR^2\ddot\phi = -I\ddot\phi$$

$$(mR^2+I)\ddot\phi = -mRa_0 \Rightarrow R\ddot\phi\left(mR+\frac{I}{R}\right) = -mRa_0$$

$$R\ddot\phi = \frac{-a_0}{1+\frac{I}{mR^2}}$$

$$\ddot x = a = a_0 + R\ddot\phi$$

$$a = a_0 + \frac{-a_0}{1+\frac{I}{MR^2}}$$

$$= a_0\left(1 - \frac{1}{1+\frac{I}{MR^2}}\right) = a_0\frac{\cancel{1}+\frac{I}{MR^2}-\cancel{1}}{1+\frac{I}{MR^2}}$$

$$a = a_0\frac{1}{MR^2 \dots}$$

$$\frac{MR^2}{I}+1$$

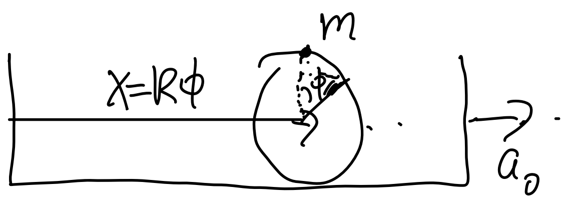
b) In moving frame.

$$F_{eff} = F - ma_0$$

$$\stackrel{!}{=} a_0 + R\ddot{\phi} - a_0$$

$$\stackrel{!}{=} R\ddot{\phi} = \frac{-a_0}{1+\frac{I}{MR^2}} = -a_0\frac{\frac{MR^2}{I}}{1+\frac{MR^2}{I}}$$

d)



$$X_{cylinder} = \frac{1}{2}a_0t^2 + R\phi \qquad Y_{cylinder} = 0$$

$$X_{bead} = X_{cylinder} + R\sin\phi \qquad Y_{bead} = R\cos\phi + R$$

$$L = \frac{1}{2}M\dot{X}_{cy}^2 + \frac{1}{2}m\left(\dot{X}_{bead}^2 + \dot{Y}_{bead}^2\right) - mg(Y_{bead})$$
$$+\frac{1}{2}I\dot{\phi}^2$$

$$\dot{X}_{cy} = a_0t + R\dot{\phi}$$
$$\dot{X}_{bead} = a_0t + R\dot{\phi} + R\cos\phi\,\dot{\phi} \qquad I = \int\frac{M}{2\pi R}R^2\,R\,d\theta$$
$$\dot{Y}_{bead} = -R\sin\phi\,\dot{\phi} \qquad I \stackrel{!}{=} MR^2$$

$$L = \frac{1}{2}M(a_0t + R\dot{\phi})^2 + \frac{1}{2}m\left(\left[a_0t + R\dot{\phi} + R\cos\phi\,\dot{\phi}\right]^2 + R^2\sin^2\phi\,\dot{\phi}^2\right) + \frac{1}{2}MR^2\dot{\phi}^2$$
$$-mgR(1+\cos\phi)$$

$$\stackrel{!}{=} MR^2\dot{\phi}^2 + \cancel{\tfrac{1}{2}M a_0^2 t^2} + MRa_0 t\dot{\phi} + \tfrac{1}{2}m\Big(\cancel{R^2\cos^2\phi\dot{\phi}^2} + \cancel{a_0^2t^2} + \cancel{R^2\dot{\phi}^2}$$

$$+ \cancel{R^2\sin^2\phi\dot{\phi}^2} + 2a_0Rt\dot{\phi} + 2a_0Rt\cos\phi\dot{\phi} + 2R^2\cos\phi\dot{\phi}^2\Big)$$

$$- mgR(1+\cos\theta)$$

$$+ MRa_0t\dot{\phi}$$

$$\stackrel{!}{=} MR^2\dot{\phi}^2 + mR^2\dot{\phi}^2 + ma_0Rt\dot{\phi} + ma_0R\cos\phi t\dot{\phi} + mR^2\cos\phi\dot{\phi}^2$$

$$- mgR\sin\theta$$

$$\stackrel{!}{=} \big(M+m(1+\cos\phi)\big)\dot{\phi}^2R^2 + \underbrace{(m+M)a_0Rt\dot{\phi}}_{(m+M)a_0R\left[\frac{d}{dt}(\phi t) - \phi\right]} + \underbrace{ma_0R\cos\phi t\dot{\phi}}_{ma_0R\left(\frac{d}{dt}[\sin\phi t] - \sin\phi\right)} - mgR\cos\phi$$

$$\stackrel{!}{=} \big(M+m(1+\cos\phi)\big)\dot{\phi}^2R^2 - (m+M)a_0R\phi - ma_0R\sin\phi - mgR\cos\phi$$

$$m_{eff}(\phi) = 2\big(M+m(1+\cos\phi)\big)$$

$$V_{eff}(\phi) = (m+M)a_0R\phi + ma_0R\sin\phi + mgR\cos\phi$$

$$\stackrel{!}{=} ma_0R(\phi+\sin\phi) + Ma_0R\phi + mgR\cos\phi$$

e) After 2 turns $\phi = -4\pi$ ← negative direction.

when $\phi = 0$

$$\frac{\partial L}{\partial\dot{\phi}} = 2\big(M+m(1+\cos\phi)\big)R^2\dot{\phi}$$

$$h = p_\phi \dot{\phi} = 2\left(M+m(1+\cos\phi)\right)R^2\dot{\phi}^2 - \left(M+m(1+\cos\phi)\right)R^2\dot{\phi}^2$$

$$+(m+M)a_0R\phi + ma_0R\sin\phi + mgR\cos\phi$$

$$E_{init}(\phi=0) = mgR$$

$$E_{final}(\phi=4\pi) = (M+2m)R^2\dot{\phi}^2 - (m+M)4\pi a_0R$$

$$+\cancel{mgR} = \cancel{mgR}$$

$$(M+2m)R^2\dot{\phi}^2 = (m+M)4\pi a_0R$$

$$\dot{\phi} = \sqrt{\frac{(m+M)4\pi a_0R}{(M+2m)R^2}}$$

$$\dot{x} = a_0t + R\dot{\phi}$$