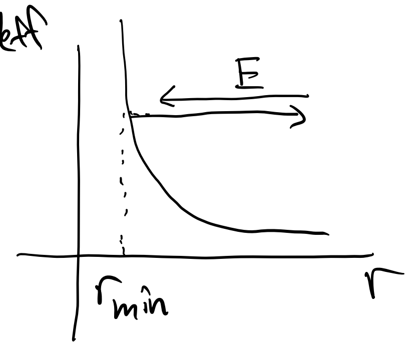
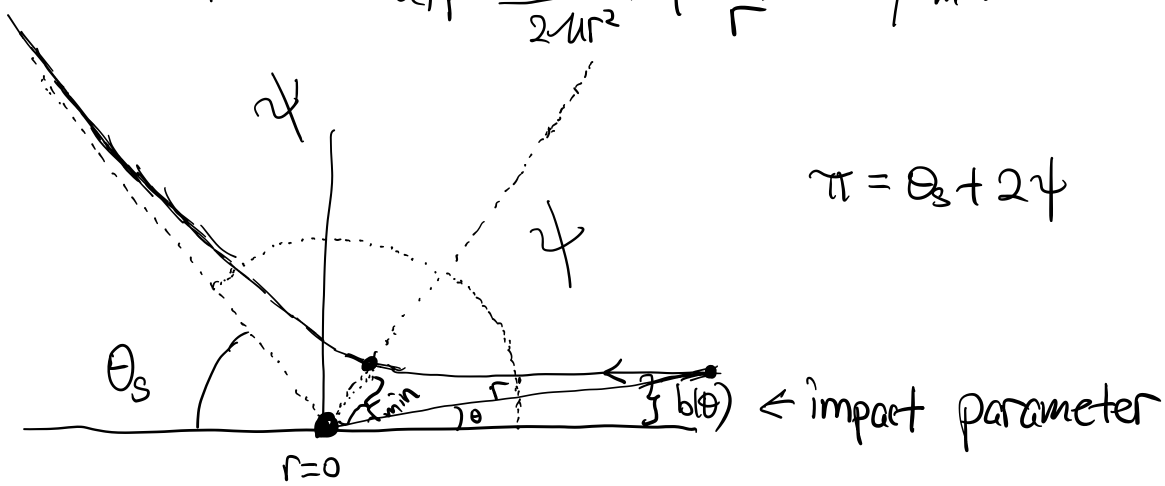
Coulomb Scattering:



$$U(r) = \frac{k}{r}$$

From kepler: $\quad V_{eff} = \frac{P_\phi^2}{2\mu r^2} + \frac{k}{r}$



$$\pi = \theta_s + 2\psi$$

From kepler problem, we know:

$$\psi = \int_{\phi(r=r_{min})}^{\phi(r=\infty)} d\phi = \int_{r_{min}}^{\infty} \frac{dr/r^2}{\sqrt{E - V_{eff}}} \frac{P_\phi}{\sqrt{2\mu}}$$

At $r = \infty$: $\quad V_{eff}(r=\infty) = 0$

$$E = \frac{1}{2}\mu v_0^2$$

$$|L| = P_\phi = \vec{r} \times \vec{p} = \mu v_0 r \sin\theta \qquad b = r\sin\theta$$

$$\stackrel{|}{=} \mu v_0 b$$

$$l \stackrel{|}{=} \sqrt{2\mu E}\, b \qquad v_0 = \sqrt{\frac{2E}{m}}$$

$$\boxed{b = \frac{P_\phi}{}}$$

P_ϕ^2 ... k

$$V_{eff} = \frac{p_\phi^2}{2\mu r^2} + \frac{k}{r} \qquad \boxed{\sqrt{2\mu E}}$$

Make V_{eff} dimensionless.

$\therefore$ Look at when does components equal in magnitude,

$$\frac{\partial V_{eff}}{\partial r} = \frac{-p_\phi^2}{\mu r^3} - \frac{k}{r^2} = 0$$

$$r_0 = \frac{p_\phi^2}{\mu k} \qquad \varepsilon_0 = T(r=r_0, \dot{r}=0)$$

$$\varepsilon_0 \overset{!}{=} \frac{p_\phi^2}{2\mu r_0^2} = \frac{k}{2r_0} = \frac{\mu k^2}{2p_\phi^2}$$

then $\frac{E}{\varepsilon_0} = \varepsilon \qquad \bar{r} = \frac{r}{r_0}$

$$\frac{V_{eff}}{\varepsilon_0} = \frac{1}{\bar{r}^2} + \frac{2}{\bar{r}} = u^2 + 2u = (u+1)^2 - 1$$

for $u = \frac{1}{\bar{r}} \qquad du = \frac{-1}{\bar{r}^2} d\bar{r}$

then

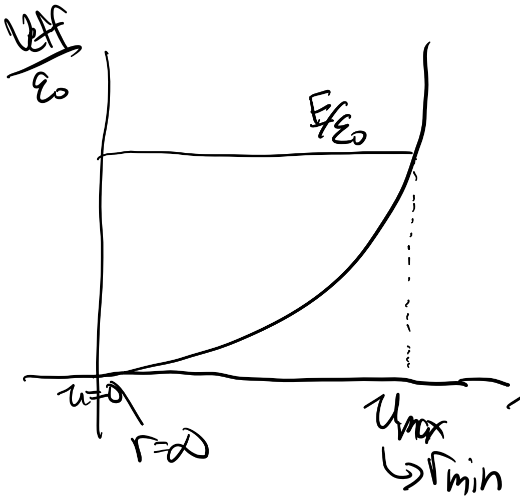
$$\psi = \int_{r_{min}}^{r=\infty} \frac{dr/r^2}{\sqrt{E - V_{eff}}} \frac{p_\phi}{\sqrt{2\mu}}$$

$$\overset{!}{=} \frac{p_\phi}{\sqrt{2\mu}} \int_{r_{min}}^{r=\infty} \frac{\cancel{r_0}\, d\bar{r} / \bar{r}^2 r_0^{\cancel{2}}}{\sqrt{\varepsilon - \left(\frac{1}{\bar{r}^2} + \frac{2}{\bar{r}}\right)}} \frac{1}{\sqrt{\varepsilon_0}}$$

$$\overset{!}{=} \frac{p_\phi}{\sqrt{2\mu}} \frac{\mu k}{p_\phi^2} \sqrt{\frac{2}{k}} \sqrt{\frac{p_\phi^2}{\mu k}} \int_{r_{min}}^{r=\infty} \frac{-du}{\sqrt{(\varepsilon+1)(u+1)^2}}$$

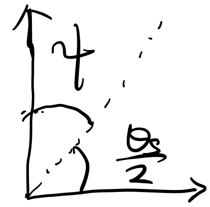
$$\overset{!}{=} \int_{u(r=\infty)=0}^{u(r_{min})=u_{max}} \frac{du}{\sqrt{(\varepsilon+1)(u+1)^2}}$$

$$\psi \stackrel{!}{=} \operatorname{atan}\left(\sqrt{\mathcal{E}}\right) \quad \text{where} \quad \mathcal{E} = \frac{E}{\mathcal{E}_0} = \frac{2P_\psi^2 E}{\mu k^2}$$



$$\theta_s + 2\psi = \pi$$

or $$\frac{\theta_s}{2} + \psi = \frac{\pi}{2}$$



Since $\psi = \operatorname{atan}(\sqrt{\mathcal{E}})$

$$\frac{\pi}{2} - \frac{\theta_s}{2} = \operatorname{atan}(\sqrt{\mathcal{E}})$$

$$\tan\left(\frac{\pi}{2} - \frac{\theta_s}{2}\right) = \sqrt{\mathcal{E}}$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$$

$$\hookrightarrow \cot\left(\frac{\theta_s}{2}\right) = \sqrt{\mathcal{E}} = \sqrt{\frac{2EP_\psi^2}{\mu k^2}}$$

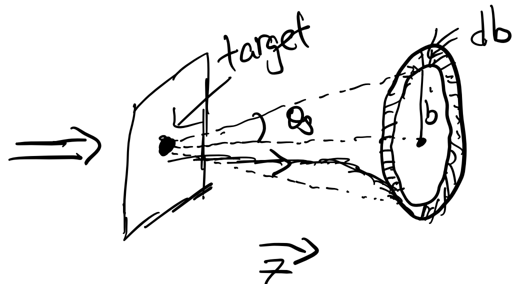
$$\cot\left(\frac{\theta_s}{2}\right) = \sqrt{\frac{2E}{\mu k^2} 2\mu E b^2}$$

$$L \sqrt{\left(\frac{2Eb}{k}\right)^2}$$

$$\boxed{\cot\frac{\theta_s}{2} = \frac{2Eb}{k}} \quad b = r\sin\theta$$

Cross sections:

$$L = \text{Intensity} = \frac{\#\text{ of particles}}{\text{Area Time}}.$$



$d\Gamma$ = rate at which particles arrive between b and $b+db$.

$$d\Gamma = \underbrace{L}_{\text{proportionality constant}} d\sigma$$

$$\overset{!}{=} L \underbrace{\frac{d\sigma}{d\Omega}}_{\text{differential cross section}} d\Omega$$

Solid angle: $d\Omega = \frac{dA}{r^2} = \frac{r^2 \sin\theta \, d\theta \, d\phi}{r^2} = 2\pi \sin\theta \, d\theta$

$$d\Gamma = L \, dA$$

$$\overset{!}{=} L \, 2\pi b \, db$$

$$\overset{!}{=} L \, 2\pi b \left|\frac{db}{d\theta}\right| d\theta$$

$$\overset{!}{=} L \, \frac{2\pi b}{\sin\theta_s} \left|\frac{db}{d\theta}\right| \sin\theta_s \, d\theta$$

$$\overset{!}{=} L \, \underbrace{\frac{b}{\sin\theta} \left|\frac{db}{d\theta}\right|}_{\text{Find } \frac{d\sigma}{d\Omega}} \underbrace{2\pi \sin\theta_s \, d\theta}_{d\Omega}$$

then $\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta_s} \frac{db}{d\theta_s}$

since $b = \frac{k}{2E} \cot\left(\frac{\theta_s}{2}\right)$

$$\frac{d\sigma}{d\Omega} = \left(\frac{k}{4E}\right)^2 \frac{1}{\sin^4\left(\frac{\theta_s}{2}\right)} \propto \frac{1}{\theta_s^4} \quad \text{for } \theta_s \ll 1$$