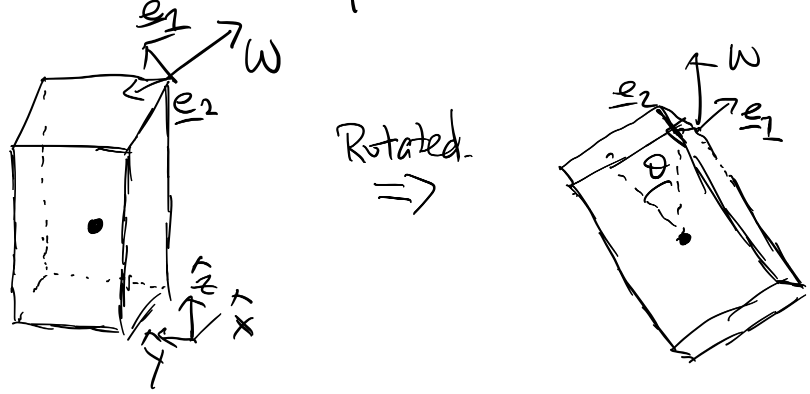
Problem 1: Torque on a box.



a) Compute moment of inertia tensor around center of mass.

$$I_{cm} = I^0 - \left(M\, d^2 \delta_{ab} - d_a d_b\right) \quad , \rho = \frac{M}{2L^3}$$

$$\vec{R}_{cm} = \frac{\int d^3r\, \rho\; x\hat{x} + y\hat{y} + z\hat{z}}{M}$$

$$= \frac{1}{2L^3}\left(\frac{L^2}{2}\right)(L)(2L)\hat{x}$$
$$+ \frac{1}{2L^3}\left(\frac{L^2}{2}\right)(L)(2L)\hat{y}$$
$$+ \frac{1}{2L^3}\left(\frac{(2L)^2}{2}\right)(L)(L)\hat{z}$$

$$\vec{R}_{cm} = \frac{L}{2}\hat{x} + \frac{L}{2}\hat{y} + L\hat{z}$$

$$I^{cm} = \int d^3r\, \rho\left(r^2\delta_{ab} - r_a r_b\right)$$

$$= \int d^3r\, \frac{M}{\quad}\begin{pmatrix} y^2+z^2 & -xy & -xz \\ \end{pmatrix}$$

$$= \int dm \, \frac{M}{2L^3} \begin{pmatrix} -yx & x^2+z^2 & -yz \\ -zx & -zy & x^2+y^2 \end{pmatrix}$$

$$= \int_{-L}^{L}\int_{-L/2}^{L/2}\int_{-L/2}^{L/2} dx\,dy\,dz \, \frac{M}{2L^3} \begin{pmatrix} y^2+z^2 & -xy & -xz \\ -yx & x^2+z^2 & -yz \\ -zx & -zy & x^2+y^2 \end{pmatrix}$$

$$\overset{!}{=} \frac{M}{2L^3} \begin{pmatrix} L\left(2\frac{(L/2)^3}{3}(2L) + 2\frac{(L)^3}{3}L\right) & 0 & 0 \\ 0 & L\left(\frac{2(L/2)^3}{3}2L + 2\frac{(L)^3}{3}L\right) & 0 \\ 0 & 0 & 2L\left(\frac{2(L/2)^3}{3}L + \frac{2(L/2)^3}{3}L\right) \end{pmatrix}$$

$$\boxed{I^{cm} \overset{!}{=} \frac{ML^2}{12} \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{pmatrix}}$$

b) Compute components of angular momentum as function of time in body basis and in lab basis.

Body basis: $\vec{e}_1, \vec{e}_2, \vec{e}_3$ which are aligned with $\hat{x}, \hat{y}, \hat{z}$ at $t=0$

$$L = I^{o}_{ab} w_b = I^{cm}_{ab} w_b + \vec{R}_{cm} \times M \vec{V}_{cm}$$

$$\vec{R}_{cm} = \frac{L}{2}\hat{x} + \frac{L}{2}\hat{y} + L\hat{z}$$

$$\vec{V}_{cm} = \dot{\vec{R}}_{cm} = \dot{x}_{cm}\hat{x} + \dot{y}_{cm}\hat{y} + \dot{z}_{cm}\hat{z}$$

Diagonal vector: $(L\hat{x} + 2L\hat{z}) - (L\hat{y}) = L\hat{x} - L\hat{y} + 2L\hat{z}$

$$\hookrightarrow \underline{\vec{e}_3} = \frac{1}{\sqrt{6}}(\hat{x} - \hat{y} + 2\hat{z})$$

$$\hookrightarrow \quad \vec{\omega} = \frac{\omega}{\sqrt{6}}(\hat{x} - \hat{y} + 2\hat{z})$$

$$I^0 = I_{cm} + M(d^2\delta_{ab} - d_a d_b)$$

$$\vec{d} = \frac{L}{2}\hat{x} + \frac{L}{2}\hat{y} + L\hat{z}$$

$$\hookrightarrow I^0 = \frac{ML^2}{6}\begin{pmatrix} 5/2 & 0 & 0 \\ 0 & 5/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} + M\begin{pmatrix} y^2+z^2 & -xy & -xz \\ -yx & x^2+z^2 & -yz \\ -zx & -zy & x^2+y^2 \end{pmatrix}$$

$$\stackrel{!}{=} \frac{ML^2}{6}\begin{pmatrix} 5/2 & 0 & 0 \\ 0 & 5/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} + ML^2\begin{pmatrix} 5/4 & -\frac{1}{4} & -\frac{1}{2} \\ -1/4 & 5/4 & -1/2 \\ -1/2 & -1/2 & \frac{1}{2} \end{pmatrix}$$

$$\stackrel{!}{=} ML^2\begin{pmatrix} \frac{20}{12} & -\frac{1}{4} & -\frac{1}{2} \\ -1/4 & 20/12 & -1/2 \\ -1/2 & -1/2 & 2/3 \end{pmatrix}$$

$$L_a = I^{o}_{ab}\, w_b$$

$$\doteq \frac{ML^2 w}{\sqrt{6}} \left\{ \left(\frac{20}{12} + \frac{1}{4} - 1\right)\hat{x} + \left(\frac{-1}{4} - \frac{20}{12} - 1\right)\hat{y} + \left(\frac{1}{2} + \frac{1}{2} + \frac{4}{3}\right)\hat{z} \right\}$$

$$L_a \doteq \frac{wML^2}{\sqrt{6}} \left\{ \frac{11}{12}\hat{x} + \frac{-35}{12}\hat{y} + \frac{4}{3}\hat{z} \right\}$$

$$L^{cm}_a = I^{cm}_{ab}\, w_b \quad \leftarrow \text{in rotating frame.}$$

$$= \frac{ML^2}{6} \begin{pmatrix} 5/2 & 0 & 0 \\ 0 & 5/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \frac{W_0}{\sqrt{6}} \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

$$L^{cm.}_a = \frac{w_0 ML^2}{\sqrt{6}} \left(\frac{5}{12}\hat{e}_1 - \frac{5}{12}\hat{e}_2 + \frac{1}{3}\hat{e}_3 \right)$$

Rotating Frame.

In Lab frame : convert to

$$\vec{e}_1 = \frac{1}{\sqrt{2}}(\hat{x}+\hat{y})$$

$$\vec{e}_2 = \underline{e}_3 \times \underline{e}_1$$

$$\doteq \frac{1}{\sqrt{3}}(-\hat{x}+\hat{y}+\hat{z})$$

$$\vec{e}_3 = \frac{1}{\sqrt{6}}(\hat{x}-\hat{y}+2\hat{z})$$

$$L_a^{cm.} = \frac{W_0 ML^2}{\sqrt{6}}\left(\frac{5}{12}\hat{x} - \frac{5}{12}\hat{y} + \frac{4}{12}\hat{z}\right)$$

$$\doteq \frac{W_0 ML^2}{\sqrt{6}}\Big\{ \quad 0 \quad \sin wt\, \vec{e}_1$$

$$+ \frac{1}{\sqrt{3}}\left(\frac{-6}{12}\right)\cos wt\, \vec{e}_2$$

$$+ \frac{1}{\sqrt{6}}\left(\frac{18}{12}\vec{e}_3\right)\Big\}$$

$$L_a^{cm}\Big|_{t=0} \doteq \frac{W_0 ML^2}{12\sqrt{6}}\left(\frac{-6}{\sqrt{3}}\vec{e}_2 + \frac{18}{\sqrt{6}}\vec{e}_3\right)$$

$$\doteq \frac{W_0 ML^2}{12}\left(-\sqrt{2}\,\vec{e}_2 + 3\vec{e}_3\right)$$

at later time: $\vec{e}_2 \rightarrow -\sin wt\, \vec{e}_1 + \cos wt\, \vec{e}_2$

$$L_{\underline{a}}^{cm}(t) = \frac{w_0 ML}{12}\left(\sqrt{2}\left(\sin wt\,\underline{\vec{e}}_1 - \cos \Omega t\,\underline{\vec{e}}_2\right) + 3\,\underline{\vec{e}}_3\right)$$

$$\vec{L} = L_a \vec{e}_a(t) = L_{\underline{a}}(t)\,\underline{\vec{e}}_a$$

c) Compute Torque in rotating basis and fixed basis.

$$\vec{T} = \underset{\text{Fixed Frame.}}{\frac{d\vec{L}}{dt}} = \underbrace{\left(\frac{d\vec{L}}{dt}\right)_r + \vec{w}\times\vec{L}}_{\text{Rotating Frame.}}$$

Rotating Frame:

$$T = \vec{w}\times\vec{L} = \epsilon_{abc}\, w_a L_b\, \hat{e}_c$$

$$= \frac{w_0}{\sqrt{6}}\left(\hat{e}_1 - \hat{e}_2 + 2\hat{e}_3\right) \times \left\{\frac{w_0 ML^2}{12\sqrt{6}}\left(5\,\hat{e}_1 - 5\,\hat{e}_2 + 4\,\hat{e}_3\right)\right\}$$

$$T_1 = \frac{w_0^2 ML^2}{72}\left((-1)(4) - 2(-5)\right)$$

$$\stackrel{!}{=} \frac{6}{72} w_0^2 ML^2$$

$$T_2 = \frac{w_0^2 ML^2}{72}\left((2)(5) - (1)(4)\right)$$

$$\stackrel{!}{=} \frac{6\, w_0^2 ML^2}{72}$$

$$\tau_3 = \frac{w_0^2 ML}{12}\left((1)(-5) - (-1)(5)\right) = 0$$

$$\tau = \frac{w_0^2 ML^2}{12}\left(\hat{e}_1 + \hat{e}_2\right) \leftarrow \text{From Rotating Frame.}$$

From lab frame:

$$\frac{d}{dt} L = \frac{d}{dt}\left(\frac{w_0 ML^2}{12}\left(\sqrt{2}\left(\sin w_0 t\, \vec{e}_1 - \cos w_0 t\, \vec{e}_2\right) + 3\vec{e}_3\right)\right)$$

$$\stackrel{!}{=} \frac{w_0^2 ML^2}{12}\left(\sqrt{2}\cos w_0 t\, \vec{e}_1 + \sqrt{2}\sin w_0 t\, \vec{e}_2\right)$$

$$\stackrel{!}{=} \frac{w_0^2 ML^2}{12}\left(\sqrt{2}\cos w_0 t\, \vec{e}_1 + \sqrt{2}\sin w_0 t\, \vec{e}_2\right)$$

at $t=0$:

$$\tau = \frac{w_0^2 ML^2}{12}\sqrt{2}\,\vec{e}_1 = \frac{w_0^2 ML^2}{12}\sqrt{2}\,\frac{1}{\sqrt{2}}(\hat{x} + \hat{y})$$

$$\stackrel{!}{=} \frac{w_0^2 ML^2}{12}\left(\hat{e}_1 + \hat{e}_2\right)$$

d) Use lagrangian to find torque in body frame.

$$\vec{r} = r^a \vec{e}_a$$

$$\vec{v} = \frac{d}{dt}\vec{r} = \left(\frac{dr}{dt}\right)_r + \vec{w} \times \vec{r}$$

$T = \frac{1}{2} w_a I_{ab} w_b$

$$\boxed{I^{cm} \stackrel{!}{=} \frac{ML^2}{12} \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{pmatrix}}$$

$$L = \frac{1}{2} w_x^2 I_{xx} + \frac{1}{2} w_y^2 I_{yy} + \frac{1}{2} w_z^2 I_{zz} - Mgz$$

$I_{xx} = I_{yy} \qquad I_{zz} = \frac{2}{5} I_{xx}$

$$\stackrel{!}{=} \frac{1}{2}\left(\underset{1}{w_x^2} + \underset{1}{w_y^2} + \frac{2}{5}\underset{4}{w_z^2}\right) I_{xx} - mgz$$

$w^x = w_{yz} = \dot\phi \sin\theta \sin\phi + \dot\theta \cos\psi$

$w^y = w_{zx} = \dot\phi \sin\theta \cos\phi - \dot\theta \sin\psi$

$w^z = w_{xy} = \dot\phi \cos\theta + \dot\psi$

$$\stackrel{!}{=} \frac{1}{2}\left(\sin^2\theta\,\dot\phi^2 + \dot\theta^2 + \frac{2}{5}\left(\dot\phi\cos\theta + \dot\psi\right)^2\right) I_{xx}$$

$\dot\theta = 0$

$-mgz$

2) Done in green function.

3) Motion in a magnetic field.

Consider a homogeneous magnetic field $B_0 \hat{z}$ and a particle of charge q. moving in 3-D in a harmonic potential well, $U = \frac{1}{2} m \omega_0^2 \rho^2$ where $\rho^2 = x^2 + y^2$.

a) Show that for a homogeneous magnetic field, the vector potential $\vec{A}$:

$$\vec{A} = \frac{1}{2} B_0 (-y, x, 0)$$

$$\vec{B} = \epsilon_{abc} \frac{\partial}{\partial x_a} A^b \hat{e}_c$$

$$= \frac{\partial}{\partial y} A^z - \frac{\partial}{\partial z} A^y \hat{x}$$

$$+ \frac{\partial}{\partial z} A^x - \frac{\partial}{\partial x} A^z \hat{y}$$

$$+ \frac{\partial}{\partial x} A^y - \frac{\partial}{\partial y} A^x \hat{z}$$

$$\overset{!}{=} 0\hat{x} + 0\hat{y} + \frac{B_0}{2} - \left(-\frac{B_0}{2}\right)\hat{z}$$

$$\vec{B} \overset{!}{=} B_0 \hat{z}.$$

c) write down the lagrangian for the particle in cylindrical coordinates.

$$L = \frac{1}{2} m \dot{\vec{r}}^2 - q\phi + \frac{q\dot{\vec{r}}}{c} \cdot \vec{A} - U(r)$$

$$\overset{!}{=} \frac{1}{2} m \left(\dot{\rho}^2 + (\rho\dot{\theta})^2 + \dot{z}^2\right) - \cancel{q\phi} + \frac{q}{c}\dot{\vec{r}} \cdot \vec{A} - \frac{1}{2} m \omega_0^2 \rho^2$$

$$\overset{!}{=} \frac{1}{2} m \left(\dot{\rho}^2 + \rho^2\dot{\theta}^2 + \dot{z}^2\right) + \frac{q}{c}\frac{B_0}{2}(\dot{y}x - \dot{x}y) - \frac{1}{2} m \omega_0^2 \rho^2$$

$$\dot{y}x = \frac{d}{dt}(\rho\sin\theta)\rho\cos\theta = (\dot{\rho}\sin\theta + \rho\cos\theta\dot{\theta})\rho\cos\theta$$

$$\dot{x}y = \frac{d}{dt}(\rho\cos\theta)\rho\sin\theta = (\dot{\rho}\cos\theta - \rho\sin\theta\dot{\theta})\rho\sin\theta$$

$$\hookrightarrow \dot{\rho}\rho\sin\theta\cos\theta + \rho^2\cos^2\theta\,\dot{\theta} - \dot{\rho}\rho\cos\theta\sin\theta + \rho^2\sin^2\theta\,\dot{\theta}$$

$$\hookrightarrow \rho^2\dot{\theta}$$

$$L \overset{!}{=} \frac{1}{2} m \left(\dot{\rho}^2 + \rho^2\dot{\theta}^2 + \dot{z}^2\right) + \frac{q}{c}\frac{B_0}{2}\rho^2\dot{\theta} - \frac{1}{2} m \omega_0^2 \rho^2$$

d)

4) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\rho}}\right) = \frac{d}{dt}\left(m\dot{\rho}\right) = \frac{d}{dt}\left(P_\rho\right)$

$$\frac{\partial L}{\partial \rho} = m\rho\dot{\theta}^2 + \frac{qB_0}{c}\rho\dot{\theta} - m\omega_0^2\rho$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \frac{d}{dt}\left(m\rho^2\dot{\theta} + \frac{qB_0}{2c}\rho^2\right) = \text{const.}$$

$$\frac{\partial L}{\partial \theta} = 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}}\right) = \frac{d}{dt}\left(m\dot{z}\right) = \text{const}$$

$$h = P_\rho\dot{\rho} + P_\theta\dot{\theta} + P_z\dot{z} - L$$

$$= m\dot{\rho}^2 + m\rho^2\dot{\theta}^2 + \frac{qB_0}{2c}\rho^2\dot{\theta} + m\dot{z}^2$$

$$-\left\{\frac{1}{2}m\left(\dot{\rho}^2 + \rho^2\dot{\theta}^2 + \dot{z}^2\right) + \frac{q}{c}\frac{B_0}{2}\rho^2\dot{\theta} - \frac{1}{2}m\omega_0^2\rho^2\right\}$$

$$E = \frac{1}{2}m\left(\dot{\rho}^2 + \rho^2\dot{\theta}^2 + \dot{z}^2\right) + \frac{1}{2}m\omega_0^2\rho^2$$

$$\frac{dE}{dt} = 0.$$

Conserved quantity:

$$P_\theta = m\rho^2\dot{\theta} + \frac{qB_0}{2c}\rho^2 \Rightarrow \dot{\theta} = \frac{P_\theta - \frac{qB_0}{2c}\rho^2}{m\rho^2} = \frac{P_\theta}{\rho^2 m} - \frac{\omega_B}{c}$$

$$P_z = m\dot{z}$$

$p_z = m\dot{z}$

$$E = h = \frac{1}{2}m\left(\dot{\rho}^2 + \rho^2\dot{\theta}^2 + \dot{z}^2\right) + \frac{1}{2}m\omega_0^2\rho^2$$

e) $R = p_\theta\dot{\theta} + p_z\dot{z} - L$

$$\stackrel{!}{=} \cancel{\frac{p_z^2}{m}} + p_\theta\left(\frac{p_\theta}{\rho^2 m} - \frac{\omega_B}{c}\right)$$

$$-\left\{\frac{1}{2}m\left(\dot{\rho}^2 + \rho^2\left[\frac{p_\theta}{m\rho^2} - \frac{\omega_B}{c}\right]^2 + \cancel{\left(\frac{p_z}{m}\right)^2}\right)\right.$$

$$\left.\frac{m\omega_B}{c}\rho^2\left[\frac{p_\theta}{m\rho^2} - \frac{\omega_B}{c}\right] - \frac{1}{2}m\omega_0^2\rho^2\right\}$$

$$\stackrel{!}{=} \cancel{\frac{p_z^2}{2m}} - \frac{1}{2}m\dot{\rho}^2 + \frac{1}{2}m\omega_0^2\rho^2$$

$$\frac{p_\theta^2}{\rho^2 m} - \cancel{\frac{p_\theta\omega_B}{c}} - \frac{p_\theta^2}{2m\rho^2} + \frac{1}{2}m\rho^2\, 2\,\cancel{\frac{p_\theta\omega_B}{m\rho^2 c}}$$

$$-\frac{1}{2}m\rho^2\left(\frac{\omega_B}{c}\right)^2$$

$$\stackrel{!}{=} -\frac{1}{2}m\dot{\rho}^2 + \frac{1}{2}m\omega_0^2\rho^2 + \frac{p_\theta^2}{2m\rho^2} - \frac{1}{2}m\rho^2\left(\frac{\omega_B}{c}\right)^2$$

$$\stackrel{!}{=} -\frac{1}{2}m\dot{\rho}^2 + \frac{1}{2}m\rho^2\left(\omega_0^2 - \left(\frac{\omega_B}{c}\right)^2\right) + \frac{p_\theta^2}{2m\rho^2}$$

$-R = L_{\text{effective}}$

$$\frac{d}{dt}\frac{\partial -R}{\partial\dot{\rho}} = m\ddot{\rho}$$

$$\frac{\partial(-R)}{\partial\rho} = -\frac{\partial}{\partial\rho}\left(\frac{1}{2}m\rho^2\left(\omega_0^2 - \left(\frac{\omega_B}{c}\right)^2\right) + \frac{p_\theta^2}{2m\rho^2}\right)$$

$$\underbrace{\qquad\qquad\qquad}_{V_{eff}(\rho)}$$

$$V_{eff}(\rho) = \frac{1}{2}m\left(\omega_0^2 - \left(\frac{\omega_B}{c}\right)^2\right)\rho^2 + \frac{P_\theta^2}{2m\rho^2}$$

$$E = h = \frac{1}{2}m\left(\dot{\rho}^2 + \rho^2\dot{\theta}^2 + \dot{z}^2\right) + \frac{1}{2}m\omega_0^2\rho^2$$

$$\doteq \frac{1}{2}m\dot{\rho}^2 + \frac{1}{2}m\rho^2\left(\frac{P_\theta}{m\rho^2} - \frac{\omega_B}{c}\right)^2 + \frac{1}{2}m\omega_0^2\rho^2$$

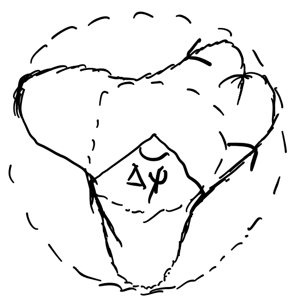
$$E \doteq \frac{1}{2}m\dot{\rho}^2 + \frac{1}{2}\frac{P_\theta^2}{m\rho^2} - \frac{P_\theta\omega_B}{c} + \frac{1}{2}m\rho^2\left(\frac{\omega_B}{c}\right)^2 = \frac{1}{2}m\dot{\rho}^2 - \frac{P_\theta\omega_B}{c} + V_{eff}(\rho)$$

$$\hookrightarrow \quad \mathcal{E} = \frac{1}{2}m\dot{\rho}^2 + V_{eff}(\rho) = E_\perp + \frac{P_\theta\omega_B}{c}$$

f) For different values of initial conditions (or conserved quantities), motion will be different.

Describe the range of parameters correspond to different motion:

a)



$$\underbrace{\left(\frac{1}{2}m\rho^2\left(\omega_0^2 - \left(\frac{\omega_B}{c}\right)^2\right) + \frac{P_\theta^2}{2m\rho^2}\right)}_{V_{eff}(\rho)}$$

$$\dot{\theta} = \frac{P_\theta}{m\rho^2} - \frac{\omega_B}{c}$$

What we know:

① There is a region in middle, so

ρ ?

$$\frac{p_\theta^2}{2m\rho^2} \neq 0 \quad \text{or} \quad p_\theta \neq 0$$

② $\dot{\theta}$ is always positive since its moving in counterclockwise.

$$\dot{\theta} = \frac{p_\theta}{m\rho^2} - \frac{\omega_B}{c} > 0$$

or

$$\rho < \sqrt{\frac{p_\theta c}{m\omega_B}} \quad \text{always.}$$

The turning point is when $\dot{\rho} = 0$ or $E = V_{eff}(\rho)$

then

$$\varepsilon_{crit} = V_{eff}\left(\rho = \sqrt{\frac{p_\theta c}{m\omega_B}}\right)$$

$$\stackrel{!}{=} \frac{1}{2}m\frac{p_\theta c}{m\omega_B}\left(\omega_0^2 - \left(\frac{\omega_B}{c}\right)^2\right) + \frac{p_\theta^2 \, m\omega_B}{2m \, p_\theta \, c}$$

$$\stackrel{!}{=} \frac{1}{2}\frac{p_\theta c}{\omega_B}\omega_0^2 - \frac{1}{2}\frac{p_\theta \omega_B}{c} + \frac{p_\theta \omega_B}{2c}$$

$$\varepsilon_{crit} \stackrel{!}{=} \frac{p_\theta c \, \omega_0^2}{2\omega_B}$$

Now get minimum energy where $\frac{\partial V_{eff}}{\partial \rho} = 0$

$$\frac{\partial V_{eff}}{\partial \rho} = m\rho\left(\omega_0^2 - \left(\frac{\omega_B}{c}\right)^2\right) - \frac{p_\theta^2}{m\rho^3} = 0$$

$$\rho_{min} = \sqrt[4]{\frac{p_\theta^2}{m^2}\frac{1}{\omega_0^2 - \left(\frac{\omega_B}{c}\right)^2}} = \sqrt{\frac{p_\theta}{m}}\sqrt[4]{\frac{1}{\omega_0^2 - \left(\frac{\omega_B}{c}\right)^2}}$$

So

$$\varepsilon_{min} = V_{eff}(\rho_{min}) = \frac{1}{2}m\left(\frac{p_\theta}{m}\right)\sqrt{\frac{1}{\omega_0^2 - \left(\frac{\omega_B}{c}\right)^2}}\left(\omega_0^2 - \left(\frac{\omega_B}{c}\right)^2\right)$$

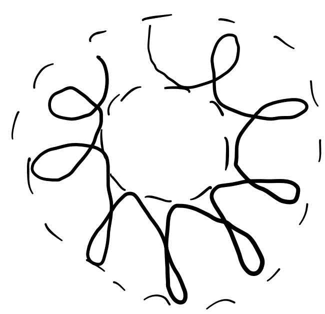
$$+ \frac{p_\theta^2}{2m}\frac{m}{p_\theta}\sqrt{\omega_0^2 - \left(\frac{\omega_B}{c}\right)^2}$$

$$\varepsilon_{min} \stackrel{!}{=} p_\theta\sqrt{\omega_0^2 - \left(\frac{\omega_B}{c}\right)^2}$$

So for a) require

$$\boxed{\mathcal{E}_{min} < \mathcal{E} < \mathcal{E}_{crit} \quad \text{and} \quad P_\theta > 0}$$

c)



Here we see $\dot{\theta}$ changes sign, which can only happen when $P_\theta > 0$

$$\dot{\theta} = \frac{P_\theta}{m\rho^2} - \frac{w_B}{c} < 0 \quad \text{at } \rho_{max}.$$

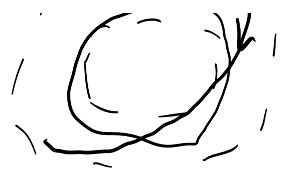
$$\rho_{max} > \sqrt{\frac{P_\theta c}{m w_B}}$$

$$\dot{\theta} = \frac{P_\theta}{m\rho^2} - \frac{w_B}{c} > 0 \quad \text{at } \rho_{min}.$$

$$\rho_{min} < \sqrt{\frac{P_\theta c}{m w_B}}$$

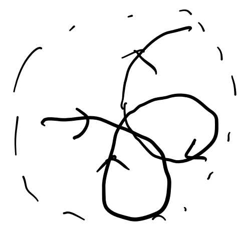
So $\underline{\mathcal{E} > \mathcal{E}_{crit}}$ and $P_\theta > 0$

d)  Here $\dot{\theta} < 0$ always.



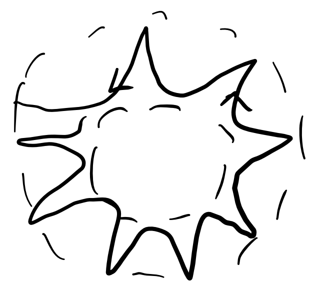
which can happen when $P_\theta < 0$.

e)



Here there is no middle area so $P_\theta = 0$

b)



clearly $\dot{\theta} = 0$ at ρ_{max} or when $\dot{\rho} = 0$

so $V_{eff}(\rho_{max}) \simeq E_{crit}$. for $P_\theta > 0$

$$E = \frac{P_\theta C W_0^2}{2 W_B} = E_{crit} \quad \text{and} \quad P_\theta > 0$$