

```
'''
```

Part a)

$g(x) = 1/L$ for $0 < x < L$

The mean position of all particles is:

$\langle x \rangle = 1/N \sum(x_i)$

mean = $L/2$

variance = $(L^2/12)/100$

Part b)

$g(x) = 10/L \exp(-10x/L)$

Find mean and standard deviation of this exponential distribution.

Part c)

Repeat part a) with exponential function

```
'''
```

```
import numpy as np
import matplotlib.pyplot as plt
from sympy import *
from IPython.display import display
```

```
init_printing(use_unicode=True)
```

```
class problem_9:
```

```
    def __init__(self, L):
```

```
        # Length of the Tube
```

```
        self.L = L
```

```
        self.generator = np.random.default_rng()
```

```
    def _draw_uniform(self, size):
```

```
        # Draws size=(M, N) samples from uniform distribution and output as array
```

```
        # N is the number of samples to find the mean
```

```
        # M is the M times that we take N samples
```

```
        return self.generator.uniform(low=0.0, high=self.L, size=size)
```

```
    def _draw_exponential(self, size):
```

```
        # Draws size=(M, N) samples from exponential distribution and output as array
```

```
        # N is the number of samples, we take to find the mean
```

```
        # M is the M times that we take N samples
```

```
        # scale for the exponential distribution
```

```
        beta = self.L / 10.0
```

```
        return self.generator.exponential(scale = beta, size=size)
```

```
    def _compute_exponential_mean(self):
```

```
        # Compute the mean of exponential distribution
```

```
        x, L = symbols("x L")
```

```
        exp_dist = 10/L*exp(-10*x/L)
```

```
        mean = integrate(exp_dist*x, (x, 0, L))
```

```
        return mean
```

```
    def _compute_exponential_sigma(self):
```

```
        # Compute the variance of exponential distribution
```

```
        x, L, N = symbols("x L N")
```

```
        exp_dist = 10/L*exp(-10*x/L)
```

```
        mean = self._compute_exponential_mean()
```

```

variance = integrate(exp_dist*(x - mean)**2, (x, 0, L)) / N
sigma = sqrt(variance)

return sigma

def _compute_mean_from_sample(self, size, dist_type):
    # Compute mean value from the distribution for a given size=(M, N)
    # Output gives an array of size M,

    if dist_type == "uniform":
        samples = self._draw_uniform(size)

    elif dist_type == "exponential":
        samples = self._draw_exponential(size)

    else:
        ValueError("Invalid Distribution Type")

    return np.sum(samples, axis=1) / size[1]

def _compute_gaussian(self, x, mean, sigma):
    # compute Normalized Gaussian distribution given mean and sigma

    return np.exp(-0.5*((x - mean)/sigma)**2) / (sigma*np.sqrt(2.0*np.pi))

def _plot(self, Ns, dist_type, plot_gaussian=False):
    # Plot Probability Distribution and Gaussian distribution:

    # size=(M, N), where N is the number of samples we use to find the mean
    #                                     M is the number of means we have.
    M = 1000
    size = (M, Ns)

    if dist_type == "uniform":

        mean = 0.5 * self.L
        sigma = np.sqrt((self.L * self.L / 12) / Ns)
        xbars = self._compute_mean_from_sample(size, "uniform")

    elif dist_type == "exponential":

        L, N = symbols("L N")

        # Convert sympy expression to actual function
        mean_func = lambdify([L], self._compute_exponential_mean())
        mean = mean_func(self.L)

        sigma_func = lambdify([L, N], self._compute_exponential_sigma())
        sigma = sigma_func(self.L, Ns)

        xbars = self._compute_mean_from_sample(size, "exponential")
    else:
        ValueError("Invalid Distribution Type")

    # Number of bins
    bins = 20

    fig, ax1 = plt.subplots()

    # Plot probability density using matplotlib with density=True
    ax1.hist(xbars, bins=bins, density=True, label=r"<x>(N) ")
    ax1.legend(loc="upper right")
    ax1.set_xlabel("<x>(N) ")
    ax1.set_ylabel("Probability Density")

    if plot_gaussian:

```

```

x = np.linspace(0.0, self.L, M)
y = self._compute_gaussian(x, mean, sigma)

# ax2 = ax1.twinx()
# ax2.plot(x, y, color="red", label="Gaussian")
# ax2.legend(loc="upper left")
# ax2.set_ylim([0.0, np.amax(y)])
# ax2.set_ylabel("Probability Density")
ax1.plot(x, y, color="red", label="Gaussian")

```

```

plt.title(f"N = {size[1]}")
plt.show()

```

```

return fig

```

```

def part_a(self):
    # Do Part A of the problem.
    # Plot Distribution of  $\langle X \rangle(N)$  using uniform distribution.

```

```

    # N = 1
    N_1 = self._plot(1, "uniform", plot_gaussian=True)

    # N = 3
    N_3 = self._plot(3, "uniform", plot_gaussian=True)

    # N = 10
    N_10 = self._plot(10, "uniform", plot_gaussian=True)

    # N = 100
    N_100 = self._plot(100, "uniform", plot_gaussian=True)

```

```

def part_b(self):
    # Do Part B of the problem.
    # Solve for mean and standard deviation of the exponential distribution

```

```

    mean = self._compute_exponential_mean()
    sigma = self._compute_exponential_sigma()

    print("Mean of the exponential distribution is: \n")
    display(simplify(mean))
    print("Standard Deviation of the exponential distribution is: \n")
    display(simplify(sigma))

```

```

def part_c(self):
    # Do Part C of the problem.
    # Plot Distribution of  $\langle X \rangle(N)$  using exponential distribution.

```

```

    # N = 1
    N_1 = self._plot(1, "exponential", plot_gaussian=True)

    # N = 3
    N_3 = self._plot(3, "exponential", plot_gaussian=True)

    # N = 10
    N_10 = self._plot(10, "exponential", plot_gaussian=True)

    # N = 100
    N_100 = self._plot(100, "exponential", plot_gaussian=True)

```

```

# Solve problem 9 for L = 1.0

```

```

prob9 = problem_9(L=1.0)
prob9.part_a()
prob9.part_b()
prob9.part_c()

```

