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'''
Problem 18
Quantum Bouncing Ball
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a) Find the average energy  $\langle E \rangle$  in units of  $mgl_g$ 
(Plot  $\langle E \rangle / mgl_g$  vs.  $kbT/mgl_g$ )

b) Find the average position of the particle above the floor,  $\langle z \rangle$ ,
(Plot  $\langle z \rangle / l_g$  vs.  $kbT/mgl_g$ )

c) Find the probability distribution  $P(z')$  for finding the particle
between  $z'$  and  $z'+dz'$  vs.  $z'=z/l_g$ . For  $kbT/mgl_g = 10$ , compare
result to the expected classical distribution.
'''

import numpy as np
from scipy.special import airy, ai_zeros
from scipy.integrate import quad_vec
from scipy.integrate import quad
import matplotlib.pyplot as plt

class problem_18:
    def __init__(self, x):
        # let  $x = kbT/mgl_g$ 
        self.x = x

        # Finds first 1000  $z_n$  terms
        self.zn = ai_zeros(100)[0]

        #  $E_n$  in units of  $mgl_g$  is simply  $-z_n$ 
        self.En = -1.0 * self.zn

        # Normalization Constant
        self.Nn = self._normalization()

    def _Pn(self, x):
        '''
        This function find the probability  $\exp(zn/x)/Q$ 
        given  $x=kbT/mgl_g$ 

        Returns an array of  $P_n$ 
        '''

        # Find the partition function
        Q = np.sum(np.exp(-self.En / x))

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    # An array of probability Pn
    Pn = np.exp(-self.En / x) / Q

    # Make sure probabilities add up to 1
    assert np.isclose(np.sum(Pn), 1.0)

    return Pn

def _average_energy(self, x):
    '''
    This function finds the average energy in the unit mgl_g
    <E>/mgl_g = Sum Pn * En/mgl_g
                  = Sum Pn * -zn
    '''

    average_energy = np.sum(self._Pn(x) * self.En)
    return average_energy

def _normalization(self):
    # Find Normalization constant using sympy

    Ai2_func = lambda z : airy(z - np.abs(self.zn))[0]**2
    invNn2 = quad_vec(Ai2_func, 0, np.inf)[0]
    Nn = 1.0/np.sqrt(invNn2)

    return Nn

def _average_z(self, x):
    '''
    Finds the <z>, given x.
    Returns a single value <z>
    '''

    z_n_func = lambda z : z * (airy(z - np.abs(self.zn))[0])**2
    average_z = np.sum(self._Pn(x) * self.Nn**2 * quad_vec(z_n_func, 0, np.inf)[0])

    return average_z

def _quantum_prob_dist(self, z, x):
    '''
    This function finds the probability distribution
    Returns a single point of probability distribution given z and x.
    '''

    P_dist = np.sum(self._Pn(x) * (self.Nn*airy(z - np.abs(self.zn))[0])**2)

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        return P_dist

def _classical_prob_dist(self, z, x):
    # Classical distribution

    func = lambda z, x: np.exp(-z/x)
    Q = quad(func, 0, np.inf, args=(x))[0]
    P_dist = np.exp(-z / x) / Q

    return P_dist

def _plot(self, x, y, xlabel, ylabel, label, dotsize=25, style="scatter"):
    '''
    This function does scatter plot given:
    y = data points for y-axis
    ylabel: label for the y-axis
    '''
    fig, ax1 = plt.subplots()

    for i in range(len(y)):
        if style == "scatter":
            ax1.scatter(x, y[i], label=label[i], s=dotsize)

            elif style == "linear":
                ax1.plot(x, y[i], label=label[i])

    ax1.legend(loc="upper left")
    ax1.set_xlabel(xlabel)
    ax1.set_ylabel(ylabel)
    plt.show()

    return fig

def do_part_a(self):
    # Do part a of the problem, plot <E>

    average_energies = []

    # Loop over different x

    for i in range(len(self.x)):
        average_energies.append(self._average_energy(self.x[i]))

    y = [average_energies]

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xlabel = r"$\frac{k_{BT}}{mgl_g}$"
ylabel = r"$\frac{\langle E \rangle}{mgl_g}$"
label = ["Part A"]

self._plot(self.x, y, xlabel, ylabel, label)

def do_part_b(self):
    # Do part a of the problem, plot <z> = Pn * integral(z * (N*Ai(z-|zn|))**2)

    average_zs = []

    # Loop over different x
    for i in range(len(self.x)):
        average_zs.append(self._average_z(self.x[i]))

    y = [average_zs]

    xlabel = r"$\frac{k_{BT}}{mgl_g}$"
    ylabel = r"$\frac{\langle z \rangle}{l_g}$"
    label = ["Part B"]

    self._plot(self.x, y, xlabel, ylabel, label)

def do_part_c(self, x):
    # Do part c of the problem, plot probability distribution with given x

    quantum_ps = []
    zs = np.linspace(1e-5, 50, 1000)

    # Loop over different x
    for i in range(len(zs)):
        quantum_ps.append(self._quantum_prob_dist(zs[i], x))

    classical_ps = self._classical_prob_dist(zs, x)
    y = [quantum_ps, classical_ps]

    xlabel = "z'"
    ylabel = r"$P(z')$"
    label = [f"Quantum", f"Classical"]
    self._plot(zs, y, xlabel, ylabel, label, dotsize=5, style="linear")

prob_18 = problem_18(np.array([0.5, 1.0, 3.0, 5.0, 10.0]))
prob_18.do_part_a()
prob_18.do_part_b()
prob_18.do_part_c(10.0)

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