Postulates of QM:

1) State = ray in Hilbert Space.

2) Observable is $\hat{A}$ for $\hat{A}$ is Hermitian

3) $i\hbar \frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$ H: Hamiltonian operator.

4) Measurement and Collapse: get eigenvalues of $\hat{A}$ as measurement with probability.

Probability of measuring a in the state $|\alpha\rangle$

$$\langle\alpha|P_a|\alpha\rangle_{norm} = \frac{\langle\alpha|P_a|\alpha\rangle}{\langle\alpha|\alpha\rangle}$$ P_a: projector to eigenvalues of $|\alpha\rangle$

with $|\alpha\rangle$ already normalized.

Ex:

$$P_a = \sum_j |a^{(j)}\rangle\langle a^{(j)}|$$

$$\sum_i \sum_j |a_i^{(j)}\rangle\langle a_i^{(j)}| = 1$$

ith eigen

$$A|\alpha\rangle = \hat{A} c_i |a_i\rangle = c_i A|a_i\rangle = c_i\, a|a_i\rangle$$

$$\doteq a|a_i\rangle\underbrace{\langle a_i|\alpha\rangle}_{c_i}$$

$\langle\alpha| = c_i^* \langle a_i|$

$$\langle\alpha|A|\alpha\rangle = \sum a_i \underbrace{\langle\alpha|a_i\rangle}_{c_i^*}\underbrace{\langle a_i|\alpha\rangle}_{c_i}$$

$$\overset{!}{=} |c_i|^2 a_i$$

5) Composite system:

outer product.

$$\mathcal{H} = \underset{|\alpha\rangle}{\mathcal{H}_1} \otimes \underset{|\beta\rangle}{\mathcal{H}_2} \quad = |\alpha, \beta\rangle$$

6) Physical symmetries act on $\mathcal{H}$ are unitary or antiunitary.

Ex: $|\alpha\rangle = \frac{1}{\sqrt{2}}|a_1\rangle + \frac{i}{\sqrt{2}}|a_2\rangle$

$$A|\alpha\rangle = a_{1,2}\,|a_{1,2}\rangle$$

$|\alpha\rangle$ <
- a_1 with $p = 1/2$ get $|a_1\rangle$
- a_2 with $p = 1/2$ get $|a_2\rangle$

SG Experiment:

$\mathcal{H}$: $\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha |S_z +\rangle + \beta |S_z -\rangle$

$\qquad\qquad\qquad \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad\qquad \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

let $S_z = \frac{\hbar}{2} \sigma_z \rightarrow \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$S_x = \frac{\hbar}{2} \sigma_x \rightarrow \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$S_y = \frac{\hbar}{2} \sigma_y \rightarrow \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow \frac{1}{\sqrt{\alpha^2+\beta^2}} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \alpha' \\ \beta' \end{pmatrix} = \begin{pmatrix} \sin\frac{\theta}{2} e^{i\phi} \\ \cos\frac{\theta}{2} \end{pmatrix}$$

Properties of Pauli - Matrices:

1) $[\sigma_i, \sigma_j] = 2i \epsilon^{ijk} \sigma_k$

2) $\{\sigma_i, \sigma_j\} = 2\delta_{ij}$