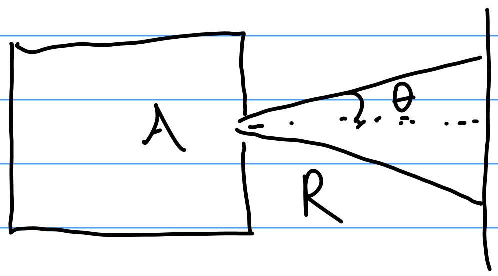
Zhi Chen HW# 6 PHY540

23) Molecular Beams:

a) Consider Chamber with const temperature T_0, assume monoatomic ideal gas with # density ρ_0 atomic mass m, $K_n = \frac{1}{\rho_0 \sigma a} \gg 1$, take hole area $A = \pi a^2$

Derive formula for flux distribution, $f(v,\theta,\phi) = \frac{dN}{dt\, d\Omega\, dv}$, measured a distance $R \gg \sqrt{A}$ away from hole.



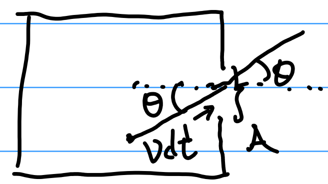
For monoatomic ideal gas, from class, we have:

$$dN = N \underbrace{4\pi v^2 \left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{\frac{-mv^2}{2k_BT}}}_{=f(v)} dv$$

$$= N f(v)\, dv \frac{d\Omega}{4\pi} \leftarrow \text{\# of molecules traveling within } d\Omega, \Omega + d\Omega$$

here $d\Omega = \sin\theta\, d\theta\, d\phi$
$\Downarrow 4\pi$

with $N = \rho_0\, dV$
$\overset{!}{=} \rho_0\, A\, v\cos\theta\, dt$



$$dN = \rho_0 A v\cos\theta\, dt \frac{d\Omega}{4\pi} f(v)\, dv$$

$$\overset{!}{=} \rho_0 A v \cos\theta \frac{1}{4\pi} 4\pi v^2 \left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{\frac{-mv^2}{2k_BT}}\, dt\, d\Omega\, dv$$

$$\boxed{f(v,\theta,\phi) = \frac{dN}{dv\,dt\,d\Omega} = \rho_0 A v^3 \cos\theta \left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{\frac{-mv^2}{2k_BT}}}$$

b) $$\frac{dN}{dt} = f(v,\theta,\phi)\, d\Omega\, dv \qquad \text{where } d\Omega = \sin\theta\, d\theta\, d\phi$$

$$= \rho_0 v^3 \left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{\frac{-mv^2}{2k_BT}} A \sin\theta\cos\theta\, d\theta\, dv\, d\phi$$

$$= \rho_0 A \left(\frac{m}{2\pi k_B T}\right)^{3/2} \underbrace{\int_0^\infty v^3 e^{\frac{-mv^2}{2k_BT}}\, dv}_{\frac{1}{2}\left(\frac{2k_BT}{m}\right)^2} \underbrace{\int_0^{\frac{\pi}{2}} \cos\theta\sin\theta}_{\frac{1}{2}} \underbrace{\int_0^{2\pi} d\phi}_{2\pi}$$

$$= \rho_0 A \left(\frac{m}{2\pi k_B T}\right)^{3/2} \left(\frac{2k_BT}{m}\right)^2 \frac{1}{4}\, 2\pi$$

$$= \rho_0 A \underbrace{\sqrt{\frac{8k_BT}{m\pi}}}_{\langle v\rangle} \frac{1}{4}$$

$$\boxed{\frac{dN}{dt} = \frac{\rho_0 \langle v\rangle}{4} A}$$

c) Calculate T of atoms that escape the source chamber to the vaccum chamber with T'

\# of atoms escaped : $N \overset{!}{=} \frac{\rho_0 \langle v \rangle}{4} A \tau$

$$E = \frac{1}{2} m \langle v^2 \rangle$$

$$\overset{!}{=} \frac{1}{2} m \int v^2 f(v,\theta,\phi) \, d\Omega \, dv \, dt$$

$$\overset{!}{=} \frac{1}{2} m \int v^2 \rho_0 A v^3 \cos\theta \left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{\frac{-mv^2}{2k_B T}} \sin\theta \, d\theta \, d\phi \, dv \, dt$$

$$\overset{!}{=} \frac{1}{2} m \rho_0 A \left(\frac{m}{2\pi k_B T}\right)^{3/2} \underbrace{\int_0^\infty v^5 e^{\frac{-mv^2}{2k_B T}}}_{\left(\frac{2k_B T}{m}\right)^3} \underbrace{\int_0^{\pi/2} \cos\theta \sin\theta \, d\theta}_{\frac{1}{2}} \underbrace{\int_0^{2\pi} d\phi}_{2\pi} \underbrace{\int_0^{\tau} dt}_{\tau}$$

$$\overset{!}{=} \frac{1}{2} m \rho_0 A \left(\frac{m}{2\pi k_B T}\right)^{3/2} \left(\frac{2k_B T}{m}\right)^3 \frac{1}{2} \, 2\pi \, \tau$$

$$E \overset{!}{=} \frac{1}{2} \rho_0 A \tau \frac{1}{\sqrt{\pi}} \left(\frac{2k_B T_0}{m}\right)^{3/2} m$$

know $E = \frac{3}{2} N k_B T'$ at equilibrium

use $N = \frac{\rho_0 \langle v \rangle}{4} A \tau = \frac{\rho_0}{4} A \tau \sqrt{\frac{8 k_B T_0}{m\pi}}$

$$\overset{!}{=} \frac{3}{2} \frac{\rho_0}{4} A \tau \sqrt{\frac{8 k_B T_0}{m\pi}} \, k_B T'$$

Set them equal:

$$\frac{1}{2}\cancel{m}\,\cancel{\rho_0}\cancel{A}\cancel{t}\,\frac{1}{\sqrt{\pi}}\left(\frac{\cancel{2}\cancel{k_B}T_0}{\cancel{m}}\right)^{3/2} = \frac{3}{2}\,\frac{\cancel{\rho_0}}{4}\cancel{A}\cancel{t}\sqrt{\frac{8\cancel{k_B}T_0}{\cancel{m}\pi}}\;\cancel{k_B}T'$$

$$\hookrightarrow \quad \frac{1}{2}T_0^{3/2} = \frac{3}{8}\sqrt{T_0}\,T'$$

$$\boxed{\frac{4}{3}T_0 = T'}$$

d) Assume the gas in source chamber stays in thermal equilibrium as the atoms escape, find the temperature in the source chamber after half of the gas escaped.

Initial: $N(t=0) = \rho_0 V$ with $T = T_0$, $E_0 = \frac{3}{2}\rho_0 V k_B T_0$

Final: $N_f = \frac{\rho_0 V}{2}$ find T_f, $E_f = \frac{3}{2}\frac{\rho_0 V}{2}k_B T_f$

For equilibrium: $E = \frac{3}{2}Nk_B T$

$$\hookrightarrow \quad \frac{dE}{dt} = \frac{3}{2}k_B\left(\frac{dN}{dt}T + N\frac{dT}{dt}\right)$$

From part c:

$$\frac{dE}{dt} = \frac{1}{2}\rho_0 A\frac{1}{\sqrt{\pi}}\left(\frac{2k_BT}{m}\right)^{3/2} m$$

from part b: $\frac{dN}{dt} = \frac{\rho_0 A}{4} \langle v \rangle$

$$\stackrel{!}{=} \frac{\rho_0 A}{4} \sqrt{\frac{8 k_B T}{m \pi}}$$

we see that $\frac{dE}{dt} = 2 \underbrace{\frac{1}{4} \rho_0 A \sqrt{\frac{8 k_B T}{m \pi}}}_{\frac{dN}{dt}} k_B T$

$$\frac{dE}{dt} \stackrel{!}{=} 2 k_B T \frac{dN}{dt}$$

Setting them equal:

$$2 k_B T \frac{dN}{dt} = \frac{3}{2} k_B \left(N \frac{dT}{dt} + T \frac{dN}{dt} \right)$$

$$\hookrightarrow \frac{1}{\cancel{2}} \cancel{k_B} T \frac{dN}{\cancel{dt}} = \frac{3}{\cancel{2}} \cancel{k_B} N \frac{dT}{\cancel{dt}}$$

$$\hookrightarrow T\, dN = 3 N\, dT$$

$$\hookrightarrow \int_{N_0}^{N_f} \frac{dN}{N} = \int_{T_0}^{T_f} 3 \frac{dT}{T}$$

$$\hookrightarrow \ln \frac{N_f}{N_0} = 3 \ln \frac{T_f}{T_0}$$

$$\hookrightarrow \frac{N_f}{N_0} = \left(\frac{T_f}{T_0} \right)^3$$

$$\hookrightarrow T_f = T_0 \left(\frac{N_f}{N_0} \right)^{1/3}$$

$N_f = \frac{1}{2} N_0$

$$\boxed{T_f = T_0 \left(\frac{1}{2} \right)^{1/3}}$$

24) Entropy Mixing

a) 2 ideal distinguishable gases, each with N molecules and V held at constant T.

Find max work when allowing gases to mix within $2V$, $N \to 2N$

$$dE = TdS - pdV$$

Know $dE = 0$

Use reversible condition since it produces max work.

$$\hookrightarrow \quad -TdS = -pdV = W$$

Find ΔS:

use entropy eq for distinguishable ideal gas:

$$S = Nk_B \ln\left(\frac{V}{\lambda_{th}^3}\right) + \frac{3}{2}Nk_B$$

$$S_1 + S_2 = 2\left(Nk_B \ln \frac{V}{\lambda_{th}^3} + \frac{3}{2}Nk_B\right)$$

$$S_{1+2} = 2Nk_B \ln\left(\frac{2V}{\lambda_{th}^3}\right) + \frac{3}{2}2Nk_B$$

$$\Delta S = S_{1+2} - (S_1 + S_2)$$

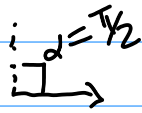
$$\stackrel{!}{=} 2Nk_B \ln\left(\frac{2\bar{V}}{\lambda_{th}^3}\right) - 2Nk_B \ln \frac{\bar{V}}{\lambda_{th}^3}$$

$$\Delta S \stackrel{!}{=} 2Nk_B \ln 2$$

$$W = -TdS = -T\Delta S$$

$$\boxed{W = -2Nk_B \ln(2)\, T}$$

b) Suppose we have $|\alpha = \frac{\pi}{2}\rangle$:



then it is super position of spin up and spin down:

$$|\alpha = \tfrac{\pi}{2}\rangle = \frac{1}{\sqrt{2}}|\uparrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle$$

Then generalize to $|\alpha\rangle$, let:

$$|\alpha\rangle = \cos\left(\frac{\alpha}{2}\right)|\uparrow\rangle + \sin\left(\frac{\alpha}{2}\right)|\downarrow\rangle$$

so that $\cos^2\left(\frac{\alpha}{2}\right) + \sin^2\left(\frac{\alpha}{2}\right) = 1$

with $|\alpha = 0\rangle = |\uparrow\rangle$ and $|\alpha = \pi\rangle = |\downarrow\rangle$

$\lvert\uparrow\rangle$	$\cos\left(\frac{\alpha}{2}\right)\lvert\uparrow\rangle + \sin\left(\frac{\alpha}{2}\right)\lvert\downarrow\rangle$

$$W = \int -p\,dV \qquad , p = \frac{N}{V} k_B T$$

right box : $W_{spin\text{-}down} = \int_{V}^{2V} -k_B T \; N_{spin\text{-}down} \frac{dV}{V}$

$N_{spin\text{-}down} = N\sin^2(\frac{\alpha}{2})$

$$\stackrel{!}{=} -k_B T N \sin^2(\tfrac{\alpha}{2}) \ln V \Big|_{V}^{2V}$$

$$\stackrel{!}{=} -k_B T N \sin^2(\tfrac{\alpha}{2}) \ln\left(\frac{2\cancel{V}}{\cancel{V}}\right)$$

$$W_{spin\text{-}down} \stackrel{!}{=} -k_B T N \sin^2(\tfrac{\alpha}{2}) \ln 2$$

For spin-up, spin-up gas on the left do work on the membrane until there is equal density between left and right boxes.

i.e. $\rho_{left} = \rho_{right}$

$$\hookrightarrow \frac{\cancel{N}}{V_{new}} = \frac{\cancel{N}\cos^2(\frac{\alpha}{2})}{2V - V_{new}}$$

$$\frac{2V - V_{new}}{V_{new}} = \cos^2\left(\frac{\alpha}{2}\right)$$

$$\frac{2V}{V_{new}} = \cos^2\left(\frac{\alpha}{2}\right) + 1$$

$$\Rightarrow \quad V_{new} = \frac{2V}{\cos^2(\frac{\alpha}{2}) + 1}$$

Then work done by left, spin-up gas is:

$$W_{spin\text{-}up} = -\int_{\bar{V}}^{\bar{V}_{new}} p\,d\bar{V}$$

$$\overset{!}{=} -\int \frac{N}{\bar{V}} k_B T \, d\bar{V}$$

$$\overset{!}{=} -N k_B T \ln\left(\frac{\bar{V}_{new}}{\bar{V}}\right)$$

$$\overset{!}{=} -N k_B T \ln\left(\frac{\frac{2\cancel{\bar{V}}}{\cos^2\left(\frac{\alpha}{2}\right)+1}}{\cancel{\bar{V}}}\right)$$

$$\overset{!}{=} -N k_B T \left[\ln(2) - \ln\left(\cos^2\left(\frac{\alpha}{2}\right)+1\right)\right]$$

Then $W_{tot} = W_{spin\text{-}up} + W_{spin\text{-}down}$

$$\boxed{W_{tot} \overset{!}{=} -N k_B T \left[\ln 2 - \ln\left(\cos^2\left(\frac{\alpha}{2}\right)+1\right) + \ln(2)\sin^2\frac{\alpha}{2}\right]}$$

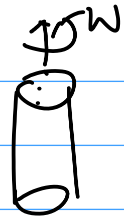
If $\alpha = \pi$, then we return to part a solution:

$$W_{tot} = -N k_B T \, 2\ln 2$$

If $\alpha = 0$, then all gas are the same, and no work is done.

$$W_{tot} = 0$$

25) Uranium Enrichment:



a) $\mathcal{L} = \frac{1}{2} m \left[\vec{V}_r^{\,2} + 2\vec{V}_r \cdot (\vec{\omega} \times \vec{r}) + (\vec{\omega} \times \vec{r})^2 \right]$

$$\vec{\omega} \times \vec{r} = \omega \hat{z} \times (r\hat{r} + z\hat{z}) = \omega r \hat{\theta}$$

$$\vec{V}_r = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + \dot{z}\hat{z}$$

$$\vec{V}_r \cdot (\vec{\omega} \times \vec{r}) = \omega r^2 \dot{\theta}$$

$$\boxed{\mathcal{L} = \frac{1}{2} m \left[\dot{r}^2 + (r\dot{\theta})^2 + \dot{z}^2 + 2\omega r^2 \dot{\theta} + \omega^2 r^2 \right]}$$

b) $\frac{\partial \mathcal{L}}{\partial \dot{r}} = P_r = m\dot{r}$

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = P_\theta = mr^2\dot{\theta} + m\omega r^2 \Rightarrow \left(\frac{P_\theta - m\omega r^2}{mr^2} \right) = \dot{\theta}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{z}} = P_z = m\dot{z}$$

$$\mathcal{H} = \sum p^i \dot{q}_i - \mathcal{L}$$

$$= \frac{P_r^2}{m} + P_\theta \left(\frac{P_\theta - m\omega r^2}{mr^2} \right) + \frac{P_z^2}{m}$$

$$- \left\{ \frac{P_r^2}{2m} + \frac{1}{2} m r^2 \left[\frac{P_\theta - m\omega r^2}{mr^2} \right]^2 + \frac{P_z^2}{2m} + m\omega r^2 \left(\frac{P_\theta - m\omega r^2}{mr^2} \right) \right.$$

$$\left. + \frac{1}{2} m \omega^2 r^2 \right\}$$

$$\mathcal{H} = \frac{p_r^2}{2m} + \frac{p_z^2}{2m} - \frac{1}{2} m w^2 r^2$$

$$+ \frac{p_\theta - m w r^2}{2 m r^2} \left(\underbrace{2p_\theta - \cancel{m} r^2 \frac{p_\theta - m w r^2}{\cancel{m r^2}} - 2 m w r^2}_{(p_\theta - m w r^2)} \right)$$

$$\boxed{\mathcal{H} = \frac{p_r^2}{2m} + \frac{(p_\theta - m r^2 w)^2}{2 m r^2} + \frac{p_z^2}{2m} - \frac{1}{2} m w^2 r^2}$$

c) Explain why rotational and vibrational (Internal DoF) do not matter.

This is because the separation technique depend primarily on the mass difference between Uranium as the substance with higher mass experiences larger centrifugal force and moves toward outer part of the cylinder, i.e. U^{238}. And the internal motion do not depend on the position of the particle so it doesn't contribute to the separation process.

d) If the centrifuge cylinder has radius R and height, H, with const temperature, find the partition function, Q, and A, for N UF_6 molecules. Show ideal gas result as $w \to 0$.

$$q_1 = \sum e^{-\beta \mathcal{H}} \qquad Q = \frac{1}{N!}(q_1)^N$$

$$q_1 = \frac{1}{h^3}\int d^3q\, d^3p\, \exp\left\{-\beta\left[\frac{p_r^2}{2m} + \frac{(p_\theta - mr^2w)^2}{2mr^2} + \frac{p_z^2}{2m} - \frac{mr^2w^2}{2}\right]\right\}$$

$$d^3q\, d^3p = dr\,d\theta\,dz\,dp_r\,dp_\theta\,dp_z$$

$$q_1 = \frac{1}{h^3}\int_0^{2\pi} d\theta \int_0^H dz \int_0^R dr \int_{-\infty}^{\infty} dp_\theta \int_{-\infty}^{\infty} dp_r \int_{-\infty}^{\infty} dp_z\, \exp\left\{-\beta\left[\frac{p_r^2}{2m} + \frac{(p_\theta - mr^2w)^2}{2mr^2} + \frac{p_z^2}{2m} - \frac{mr^2w^2}{2}\right]\right\}$$

$$= \frac{1}{h^3} 2\pi H \int_0^R dr \int_{-\infty}^{\infty} dp_\theta \int_{-\infty}^{\infty} dp_r \sqrt{\frac{2m\pi}{\beta}}\, \exp\left\{-\beta\left[\frac{p_r^2}{2m} + \frac{(p_\theta - mr^2w)^2}{2mr^2} - \frac{mr^2w^2}{2}\right]\right\}$$

$$= \frac{2\pi H}{h^3}\int_0^R dr \int_{-\infty}^{\infty} dp_\theta\, \frac{2m\pi}{\beta} \exp\left\{-\beta\left[\frac{(p_\theta - mr^2w)^2}{2mr^2} - \frac{mr^2w^2}{2}\right]\right\}$$

$$= \frac{2\pi H}{h^3}\int_0^R dr \left(\frac{2m\pi}{\beta}\right)\sqrt{\frac{2m\pi r^2}{\beta}}\, \exp\left\{\frac{\beta m r^2 w^2}{2}\right\}$$

$$q_1 = \frac{2\pi H}{h^3}\left(\frac{2m\pi}{\beta}\right)^{3/2}\left[\frac{\exp\left\{\frac{\beta m R^2 w^2}{2}\right\} - 1}{\beta m w^2}\right]$$

with $\lambda_{th} = \sqrt{\dfrac{h^2}{2\pi m k_B T}}$

$$q_1 = \frac{2\pi H}{\lambda_{th}^3} \frac{1}{\beta m \omega^2} \left[\exp\left\{ \frac{\beta m \omega^2 R^2}{2} \right\} - 1 \right]$$

$$\boxed{Q = \frac{1}{N!} (q_1)^N = \frac{1}{N!} \left[\frac{2\pi H}{\lambda_{th}^3} \frac{1}{\beta m \omega^2} \left[\exp\left\{ \frac{\beta m \omega^2 R^2}{2} \right\} - 1 \right] \right]^N}$$

$$\boxed{\begin{aligned} A &= -k_B T \ln Q \\ &\overset{!}{=} -k_B T \ln \left[\frac{1}{N!} \left(\frac{2\pi H}{\lambda_{th}^3} \frac{1}{\beta m \omega^2} \left[\exp\left\{ \frac{\beta m \omega^2 R^2}{2} \right\} - 1 \right] \right)^N \right] \end{aligned}}$$

with limit $\omega \to 0$:

classical results: $Q = \dfrac{1}{N!} \left(\dfrac{V}{\lambda_{th}^3} \right)^N$

$$\exp\{x\} \cong 1 + x$$

$$\exp\left\{ \frac{\beta m R^2}{2} \omega^2 \right\} = 1 + \frac{\beta m r^2 \omega^2}{2}$$

same classical ✓ result

$$\boxed{\begin{aligned} q_1 &= \frac{2\pi H}{\lambda_{th}^3} \frac{1}{\cancel{\beta m \omega^2}} \left[\cancel{1} + \frac{\cancel{\beta m} R^2 \cancel{\omega^2}}{2} \cancel{- 1} \right] \\ &\overset{!}{=} \frac{\pi H R^2}{\lambda_{th}^3} \\ &\overset{!}{=} \frac{V}{\lambda_{th}^3} \implies Q = \frac{1}{N!} (q_1)^N = \frac{1}{N!} \left(\frac{V}{\lambda_{th}^3} \right)^N \end{aligned}}$$

e) Find $\rho(r)$: $\rho(r) = \frac{dN}{dV} = \frac{dN}{r\,dr\,d\theta\,dz} = \frac{dN}{2\pi H r\,dr}$

$\hookrightarrow \rho(r) = \frac{1}{2\pi H r}\frac{dN}{dr}$

$$\frac{dN}{dr} = N \frac{\frac{1}{h^3}\int_0^H dz \int_0^{2\pi} d\theta \int_{-\infty}^{\infty} dp_r \int_{-\infty}^{\infty} dp_\theta \int_{-\infty}^{\infty} dp_z \; e^{-\beta \mathcal{H}}}{\underbrace{\frac{1}{h^3}\int_0^R dr \int_0^H dz \int_0^{2\pi} d\theta \int_{-\infty}^{\infty} dp_r \int_{-\infty}^{\infty} dp_\theta \int_{-\infty}^{\infty} dp_z \; e^{-\beta \mathcal{H}}}_{= q_1}}$$

$$\overset{!}{=} \frac{2\pi H}{\lambda_{th}^3}\frac{1}{\beta m \omega^2}\left[\exp\left\{\frac{\beta m \omega^2 R^2}{2}\right\} - 1\right]$$

$$\rightarrow \frac{1}{h^3}\int_0^H dz \int_0^{2\pi} d\theta \int_{-\infty}^{\infty} dp_r \int_{-\infty}^{\infty} dp_\theta \int_{-\infty}^{\infty} dp_z \; \exp\left\{-\beta\left[\frac{p_r^2}{2m} + \frac{(p_\theta - m r^2\omega)^2}{2mr^2} + \frac{p_z^2}{2m} - \frac{m\omega^2 r^2}{2}\right]\right\}$$

get result from part d)

$$\hookrightarrow = \frac{2\pi H}{\lambda_{th}^3}\exp\left\{\frac{\beta m \omega^2 r^2}{2}\right\} r$$

then

$$\frac{dN}{dr} = \frac{N \frac{2\pi \cancel{H}}{\cancel{\lambda_{th}^3}}\exp\left\{\frac{\beta m \omega^2 r^2}{2}\right\} r}{\frac{2\pi \cancel{H}}{\cancel{\lambda_{th}^3}}\frac{1}{\beta m \omega^2}\left[\exp\left\{\frac{\beta m \omega^2 R^2}{2}\right\} - 1\right]}$$

$$\boxed{\text{then } \rho(r) = \frac{1}{2\pi H r}\frac{dN}{dr} = \frac{N}{2\pi H}\frac{\beta m \omega^2}{\exp\left\{\frac{\beta m \omega^2 R^2}{2}\right\} - 1}\exp\left\{\frac{\beta m \omega^2 r^2}{2}\right\}}$$

f) If $\omega R = 500$ m/s, find ratio of $^{235}UF_6$ and $^{238}UF_6$

$$\frac{\rho(r=0)_{^{235}UF_6}}{\rho(r=0)_{^{238}UF_6}} = \frac{\frac{N_{235}}{\cancel{2\pi H}} \frac{\cancel{\beta m_{235} \omega^2}}{\exp\left\{\frac{\beta m_{235} \omega^2 R^2}{2}\right\} - 1} \cancel{\exp\{0\}}}{\frac{N_{238}}{\cancel{2\pi H}} \frac{\cancel{\beta m_{238} \omega^2}}{\exp\left\{\frac{\beta m_{238} \omega^2 R^2}{2}\right\} - 1} \cancel{\exp\{0\}}}$$

$$= \frac{N_{235} \; m_{235}}{N_{238} \; m_{238}} \frac{\exp\left\{\frac{\beta m_{238} \omega^2 R^2}{2}\right\} - 1}{\exp\left\{\frac{\beta m_{235} \omega^2 R^2}{2}\right\} - 1}$$

$\Rightarrow$ With $\omega R = 500$ m/s $\rightarrow$ $\omega^2 R^2 = 2{,}500$ m/s

$\Rightarrow$ $T \sim 300$K $\Rightarrow$ $\beta \cong 2.41432 \times 10^{20}$ J^{-1}

$\Rightarrow$ $m_{235UF_6} = (235.043)(1.66 \times 10^{-27} kg) + (6)(18.9984)(1.66 \times 10^{-27} kg)$
$\doteq 5.79395 \times 10^{-25}$ kg

$\Rightarrow$ $mass_{238UF_6} = (238.050)(1.66 \times 10^{-27} kg) + (6)(18.9984)(1.66 \times 10^{-27} kg)$
$\doteq 5.84387 \times 10^{-25}$ kg

Just plug #'s in, I get!

$$\boxed{\frac{\rho(r=0)_{235}}{\rho(r=0)_{238}} \cong 1.1527 \frac{N_{235}}{N_{238}}}$$

g) Find # of centrifugation needed so that $> 90\%$ ^{235}U

Initially, N_{tot} Uranium, with 99.27% ^{238}U and 0.72% ^{235}U

$$r_{init} = r(0) = \frac{N_{235}}{N_{238}} = \frac{N_{235}}{N_{tot} - N_{235}} = \frac{0.72}{99.27}$$

Each centrifugation give

$$r_{next} = 1.1527 \frac{N_{235,current}}{N_{tot} - N_{235,current}}.$$

For N - centrifugation:

$$r(N) = 1.1527^N \, r(N=0)$$

since $\quad r = \dfrac{N_{253}}{N_{tot} - N_{253}}$

$$\hookrightarrow (N_{tot} - N_{253}) = \frac{1}{r} N_{253}$$

$$\text{or} \quad \frac{N_{253}}{N_{tot}} = \left(\frac{1}{r} + 1\right)^{-1} > 0.9$$

with $r = r(N) = 1.153^N \, r(N=0) = 1.1527^N \dfrac{0.72}{99.27}$

$$\hookrightarrow \frac{1}{0.9} - 1 > \frac{1}{r(N)}$$

$$\hookrightarrow 1.1527^N \frac{0.72}{99.27} > \left(\frac{1}{0.9} - 1\right)^{-1}$$

$$\hookrightarrow N > \log_{1.1527}\left(\left[\frac{1}{0.9} - 1\right]^{-1} \frac{99.27}{0.72}\right)$$

$$\hookrightarrow N > 50.128$$

since N is integer, round up and require $N = 51$ at least.