Modulation Design in Amplify-and-Forward Two-Way Relay HARQ Channel

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***Abstract*—As a practical transmission enhancement technique for relay and HARQ system, Modulation Diversity (MoDiv) uses distinct mappings from information bits to the same constellation for different (re)transmissions. In this work, we study the MoDiv optimization in a amplify-and-forward (AF) two-way relay channel (TWRC). The design of MoDiv design to minimize the bit-error rate (BER) is formulated into a successive Koopmans-Beckmann Quadratic Assignment Problem (QAP), which is solved sequatially with a robust taboo search method. The performance gain of our MoDiv scheme over retransmission without remapping and a heuristic MoDiv scheme is demonstrated with numerical ressults.**

***Index Terms*—Modulation diversity, two-way relay, amplify-and-forward, HARQ, QAP.**

I. Introduction

As an advanced technique to improve the robustness of high-rate wireless transmissions against poor channel conditions, Hybrid Automatic Repeat reQuest (HARQ) has been widely adopted in various communication systems [1]. HARQ works on both PHY layer and MAC sublayer to mitigate packet loss due to channel fading and link-adaptation accuracy. Recently, there has been some research interests in applying HARQ over two-way relay channel (TWRC) [2-4]. In [2], the average throughput of a simple Type-I HARQ policy for both Amplify-and-Forward (AF) and Decode-and-Forward (DF) TWRC schemes have been analyzed. The energy-delay tradeoff, and the diversity-multiplexing tradeoff of type-II HARQ policy, also known as full Incremental Redundancy (IR), for AF TWRC scheme have been studied in [3] and [4], respectively. Related works on TWRC with ARQ for different relay schemes and retransmission policies can also be found in [5], [6], [7] and the references therein.

Apart from Type-I HARQ and HARQ-IR, Type-I HARQ with maximal ratio combining (MRC), also known as HARQ-Chase Combining (HARQ-CC) [8], is another simple and effective HARQ scheme supported by such standards as HSPA [9], LTE [10], among others. As practical transmissions often admit linear modulations of finite-alphabet constellation (e.g. Q-ary QAM), the performance of HARQ-CC can be improved with Modulation Diversity (MoDiv) [11], in which a same group of $\log_2 Q$ information bits are mapped to different symbols in a same constellation in different round of (re)transmissions. MoDiv has been studied for HARQ [12], relay networks [13], [14] and relay-HARQ systems [15], [16].

In this paper, we study the MoDiv design for the TWRC under a simple AF scheme and HARQ-CC protocol. We first derive an approximation for the uncoded bit-error rate (BER) of TWRC-AF channel under Rayleigh fading channel condition, given M different mapping schemes corresponding to each (re)transmission. Based on this approximation, we formulate a successive BER minimization MoDiv design into a series of Quadratic Assignment Problem (QAP) in Koopmans-Beckmann (KB) form [17]. Although QAP is NP-hard, efficient numerical algorithms have been extensively researched [18], some of which have shown extremely high performance over QAPLIB [19]. We adopt a taboo search algorithm [20] to solve each QAP in our formulation. Moreover, the coefficients of QAP problem can be also be computed efficiently in a successive manner based on the solution to the preceding QAP problem. Our numerical results demonstrate significant BER reduction over both non-MoDiv and a simple heuristic MoDiv retransmission scheme for 16-QAM and 64-QAM constellations, even under mismatched design parameters.

The paper is organized as follows. Section II introduces the TWRC-AF model and the HARQ protocol we are using. Section III presents the successive BER minimization MoDiv design problem. In Section IV, we present the numerical results to show the performance gain of our MoDiv scheme. Finally, Section V concludes the paper.

II. System Model

Consider a TWRC with analog network coding (ANC) protocol [3], a generalization the AF protocol, as shown in Fig. 1. The relay node R is totally unaware of the HARQ procedure and simply performs ANC. Each round of ANC transmission consists of two phases. In the multiple access (MAC) phase, the two source nodes S_1 and S_2 transmit to R simultaneously. In the broadcast (BC) phase, node R amplifies and broadcasts the signal received during the MAC phase to both S_1 and S_2. Denote the uplink channel from S_s to R and downlink channel from R to S_s as h_s and g_s, respectively, where $s = 1, 2$. We assume that all the channels follow Rayleigh distribution, i.e. $h_s \sim \mathcal{CN}(0, \beta_{h_s})$ and $g_s \sim \mathcal{CN}(0, \beta_{g_s})$, $s = 1, 2$. Denote the transmitted symbol from S_s as x_s whose average power $\mathbb{E}[|x_s|^2] = P_s$. Then the signal received by R during the MAC phase is

$$y_R = h_1 x_1 + h_2 x_2 + n_R, \tag{1}$$

where $n_R \sim \mathcal{CN}(0, \sigma_R^2)$ is the received noise at R. Assuming that the relay R has an expected power constraint of P_R, and that S_1 and S_2 perform perfect self-interference cancellation (SIC), then the received signal at S_s after SIC is

$$y_s = \alpha g_s y_R + n_s, \ s = 1, 2, \tag{2}$$

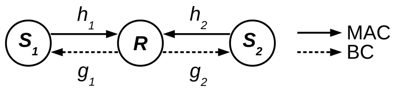


Fig. 1. Two-way relay channel with analog network coding.

where $n_s \sim \mathcal{CN}(0, \sigma_s^2)$ is the received noise at S_s, and

$$\alpha = \sqrt{\frac{P_R}{|h_1|^2 P_1 + |h_2|^2 P_2 + \sigma_R^2}} \tag{3}$$

is the power normalization factor at R.

On top of this settings, S_1 and S_2 performs the HARQ-CC protocol in an unsynchronized manner. Consequently, the MoDiv design at S_1 and S_2 can be handled independently. Without loss of generality, we study the HARQ transmission from S_1 to S_2. Denote $\mathcal{C}$ as the constellation used by S_1 whose cardinality equals $Q = |\mathcal{C}|$. As a convention, during the initial transmission of a packet, S_1 converts a bit sequence of length $\log_2 Q$ into symbols with Gray mapping $\psi_0 : \{0, \ldots, Q-1\} \to \mathcal{C}$. The bit sequence is is labeled by its decimal equivalence $p \in \{0, \ldots, Q-1\}$. What distinct HARQ-CC with MoDiv from conventional HARQ-CC is that, during the m-th retransmission, S_1 is allowed to use a mapping function $\psi_m \neq \psi_0$ to remap the same label p. We assume $m \leq M$ where M is the maximum number of retransmissions. According to Eqs. (1)(2), the signal received by S_2 after SIC during the m-th (re)transmission of p is

$$y_2^{(m)} = \alpha^{(m)} g_2^{(m)} h_1^{(m)} \psi_m[p] + \alpha^{(m)} g_2^{(m)} n_R^{(m)} + n_2^{(m)}, \tag{4}$$

where $X^{(m)}$ is the m-th realization of random variable X.

Assume that S_2 acquires perfect channel state information (CSI). After the m-th retransmission, it attempts to demodulate the received symbols by identifying label p with $y_2^{(0)}, \ldots, y_2^{(m)}$ via the maximum likelihood (ML) detection:

$$p^* = \arg\min_p \sum_{k=0}^{m} \frac{|y_2^{(k)} - \alpha^{(k)} g_2^{(k)} h_1^{(k)} \psi_k[p]|^2}{\sigma_2^2 + (\alpha^{(k)})^2 \sigma_R^2 |g_2^{(k)}|^2}. \tag{5}$$

III. Successive Constellation Mapping Design For Modulation Diversity

In this section, we first derive an closed-form approximation of the reception bit-error rate in our TWRC channel with HARQ-CC. Based on this result, we formulate the BER-minimization MoDiv design into a successive QAP (S-QAP).

A. A BER approximation

Assume that the label p follows a uniform distribution. The BER of the ML demodulator after the m-th retransmission can be upper-bounded and approximated with the pair-wise error probability (PEP) [12]:

$$P_{BER}^{(m)} = \sum_{p=0}^{Q-1} \sum_{q=0}^{Q-1} \frac{B[p,q]}{Q} P_{PEP}^{(m)}(q|p), \tag{6}$$

where $B[p,q]$ represents the Hamming distance between the binary representation of p and q normalized by $\log_2 Q$, and $P_{PEP}^{(m)}(q|p)$ is the probability that the ML demodulator prefer q over p conditioned on the transmission of p. From Eq. (5), we have

$$P_{PEP}^{(m)}(q|p) = \mathbb{E}\left[Q\left(\sqrt{\sum_{k=0}^{m} \frac{(\alpha^{(k)})^2 \epsilon_k[p,q] \gamma_2^{(k)} \delta_1^{(k)}}{2(\tilde{\sigma}_2^{(k)})^2}} \right) \right], \tag{7}$$

where $\gamma_2^{(k)} = \|g_2^{(k)}\|^2$, $\delta_1^{(k)} = \|h_1^{(k)}\|^2$, $\epsilon_k[p,q] = \|\psi_k[p] - \psi_k[q]\|^2$, and $(\tilde{\sigma}_2^{(k)})^2 = \sigma_2^2 + (\alpha^{(k)})^2 \sigma_R^2 \gamma_2^{(k)}$ is the instantaneous variance of the noise received by S_2. By adopting the Chernoff upper bound $Q(x) \leq e^{-x^2/2}/2$ [21], an approximation to $P_{PEP}^{(m)}(q|p)$ is

$$\tilde{P}_{PEP}^{(m)}(q|p) = \frac{1}{2} \prod_{k=0}^{m} \mathbb{E}\left[\exp\left(-\frac{(\alpha^{(k)})^2 \epsilon_k[p,q] \gamma_2^{(k)} \delta_1^{(k)}}{4(\tilde{\sigma}_2^{(k)})^2} \right) \right]. \tag{8}$$

Although the Chernoff bound is a rather coarse appoximation, it enables efficient iterative computation of $P_{PEP}^{(m)}(q|p)$ as m varies. Moreover, as shown in Section III-B, this approximation results in a simple KB-form QAP. Nevertheless, the Chernoff bound can be replaced with a more accurate approximation as in Eq.(14) of [22], As will be explained in Section III-B, however, this will lead to a more complex general-form QAP.

Denote $E_k[p,q]$ as the expectation in Eq.(8), which can be evaluated as follows:

Proposition 1. *An approximation to $E_k[p,q]$ is*

$$\tilde{E}_k[p,q] = \frac{4\sigma_R^2 + \beta_{h_1} \epsilon_k[p,q] v \exp(v) Ei(v)}{u} \tag{9}$$

where

$$u = 4\sigma_R^2 + \beta_{h_1} \epsilon_k[p,q], \; v = \frac{4\sigma_2^2}{\tilde{\alpha}^2 \beta_{g_2} u},$$

$$\tilde{\alpha} = \sqrt{\frac{P_R}{\beta_{h_1} P_1 + \beta_{h_2} P_2 + \sigma_R^2}},$$

and $Ei(x) = \int_x^\infty e^{-t}/t dt$ is the exponential integral function [23].

Proof. See Appendix. □

While $E_k[p,q]$ plays a key role in our MoDiv design based on BER minimization, we comment that it is also closely related to another performance metric, the ergodic mutual information (EMI), via the following proposition.

Proposition 2. *The EMI after the m-th retransmission, denoted as $I^{(m)}$, is lower bounded by*

$$\tilde{I}^{(m)} = \log_2 Q - \log_2 \left[\frac{1}{Q} \sum_{p=0}^{Q-1} \sum_{q=0}^{Q-1} \prod_{k=0}^{m} E_k[p,q] \right]. \tag{11}$$

Proof. See Appendix. □

Proposition 2 bridges MoDiv design based on rate criterion [] and BER criterion []. Moreover, it leads to an efficient KB-form QAP almost identical to that derived from the BER minimization, as will be explained in Section III-B.

B. The Successive Quadratic Assignment Problem

Our MoDiv design is based on the approximated BER minimization criterion. As it is impossible to know the number of actual retransmission m in advance, we formulate a sequence of M optimization problems as in [12], in which ψ_m is optimized to minimize the approximated BER given $\psi_1, \ldots, \psi_{m-1}$ without expecting future retransmissions:

$$\min_{\psi^{(m)}|\psi^{(k)},k=0,\ldots,m-1} \tilde{P}_{BER}^{(m)},\ m = 1, \ldots, M \tag{12}$$

where $\tilde{P}_{BER}^{(m)}$ denotes the approximated version of Eq.(6) evaluated with Eq.(8)(9).

In order to rewrite Eq.(12) into a S-QAP formulation, we denote $\mathbf{x}^{(m)} = \{x_{pi}^{(m)}|p, i = 0, \ldots, Q-1\}$ as the permutation matrix representing ψ_m, in which $x_{pi}^{(m)} = 1(\psi_m[p] = \psi_0[i])$ and $1(\cdot)$ is the indicator function. Denote the constraint sets

$$\mathcal{P} = \left\{ \mathbf{x} : \sum_{p=0}^{Q-1} x_{pi} = 1, x_{pi} \in \{0, 1\} \right\}, \tag{13a}$$

$$\mathcal{I} = \left\{ \mathbf{x} : \sum_{i=0}^{Q-1} x_{pi} = 1, x_{pi} \in \{0, 1\} \right\}. \tag{13b}$$

Then the MoDiv design problems in Eq.(12) can be formulated into a S-QAP as follows:

$$\min_{\mathbf{x}^{(m)} \in \mathcal{P} \cap \mathcal{I}} \sum_{p=0}^{Q-1} \sum_{i=0}^{Q-1} \sum_{q=0}^{Q-1} \sum_{j=0}^{Q-1} f_{pq}^{(m)} d_{ij} x_{pi}^{(m)} x_{qj}^{(m)}, \tag{14}$$

in which the "flow" matrix $f_{pq}^{(m)}$ and the "distance" matrix d_{ij} are defined as

$$f_{pq}^{(m)} = \frac{B[p, q]}{Q} \tilde{P}_{PEP}^{(m-1)}(q|p),\ d_{ij} = \tilde{E}_0[i, j] \tag{15}$$

Note that here we assume all channel and noises to be stationary across all retransmissions, such that d_{ij} only needs to be evaluated once. On the other hand, $f_{pq}^{(m)}$ can be computed recursively while solving the S-QAP, since

$$\tilde{P}_{PEP}^{(m)}(q|p) = \sum_{i=0}^{Q-1} \sum_{j=0}^{Q-1} \tilde{P}_{PEP}^{(m-1)}(q|p) d_{ij} \hat{x}_{pi}^{(m)} \hat{x}_{qj}^{(m)} \tag{16a}$$

$$\tilde{P}_{PEP}^{(-1)}(q|p) = \frac{1}{2} \tag{16b}$$

where $\hat{\mathbf{x}}^{(m)}$ is the solution to Eq.(14).

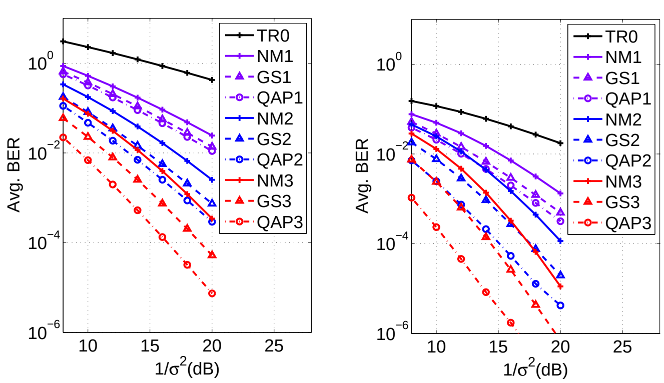
In our S-QAP fromulation, each KB-form QAP is defined with two Q-by-Q matrices, only one of which needs to be updated. Should we adopt the more accurate approximation [22] in Eq.(7), each QAP would be in general-form which is defined with one Q^4 matrix. Although this 4-dimensional matrix can still be updated iteratively using a few Q-by-Q matrices in a sequential manner, the solution to the general-form QAP is usually more complicated. On the other hand, if we adopted a EMI-lowerbound maximization design philosophy according to Proposition 2, the only change to the KB-form S-QAP would be a new "flow" matrix $f_{pq}^{*(m)} = \tilde{P}_{PEP}^{(m-1)}(q|p)$ in Eq.(15). Nevertheless, in terms of practical performance measurement such as coded BER, the approximated EMI-lowerbound is generally an inferior criterion than the approximated BER-upperbound.

With the S-QAP in KB form, we are able to handle larger constellations than examined in previous works with an efficient robust tabu search algorithm [20]. The tabu search heuristic yields slight overestimates or upper bounds of the optimal objective values. To shed some light on this we computed the lower bounds based an semidefinite programming relaxations as in [24]. Typically the gap between lower and upper bounds is on the order of 10-20% with the exact objective value being much closer to the upper bound.Finally, we note that the MoDiv design can be precomputed offline and stored in S_1 and S_2 as it depends only on statistical CSI.

IV. Numerical Results

In our simulation, we consider a TWRC where the distance from S_s to the relay is d_s, $s = 1, 2$, thus $\beta_{h_s} = \beta_{g_s} = d_s^{\nu}$ where $\nu = 3$ is the path-loss coefficient. We also fix $P_1 = P_2 = 0.5P_R = 1$ and assume that $\sigma_1^2 = \sigma_2^2 = \sigma_R^2 = \sigma^2$. In the HARQ protocol, we select $M = 3$. For the m-th retransmission, we compare the performance of three MoDiv strategies: the simple repeated retransmission without MoDiv, our QAP-optimized solution, and a heuristic MoDiv scheme where Gray-mapping and Seddik's heuristic re-mapping [13] are used alternatingly for each (re)transmission (GSm). In our simulation results, the performance of the above three schemes after the m-th retransmission are labeled as NMm, QAPm and GSm, respectively and that of the original transmission using Gray mapping is labeled as TR0.

Firstly, we demonstrate the approximated and the Monte-Carlo simulated uncoded BER results of S_1 and S_2 for 64-QAM constellation. In Fig. 2, we fix $d_1 = d_2 = 0.5$ and plot the uncoded BER against varying σ^2. For the first retransmission, both the the QAP and the heuristic GS MoDiv offers about the same performance gap over NM scheme. However, as m increases, both the gap between GS and NM as well as that between QAP and GS widens.



(a) Approximated (b) Monte-Carlo

Fig. 2. Uncoded BER vs σ^2 for 64-QAM.

To further verify the performance gain and robustness of the QAP-optimized MoDiv scheme, we compare the coded-BER of the three MoDiv schemes in a LDPC-coded communication system based on [25]. We use a LDPC code of length $L = 2400$, coding rate of $3/4$ and a Monte-Carlo run of up to 2000 LDPC frames. Since an important motivation of HARQ is to adopt for link adaptation inaccuracies [26], we deliberately optimize the remappings at $\sigma^2 = 0dB$ for 16-QAM and $\sigma^2 = 4.5dB$ for 64-QAM and test their performances on mismatching σ^2. Again we fix $d_1 = d_2 = 0.5$. The results are shown in Fig. 3 and Fig. 4. Apparently, the QAP scheme offers larger performance gain for higher-order constellation. Specifically, even under design mismatch, QAP2 achieves a equal or better performance than NM3 and GS3 for 64-QAM constellation.

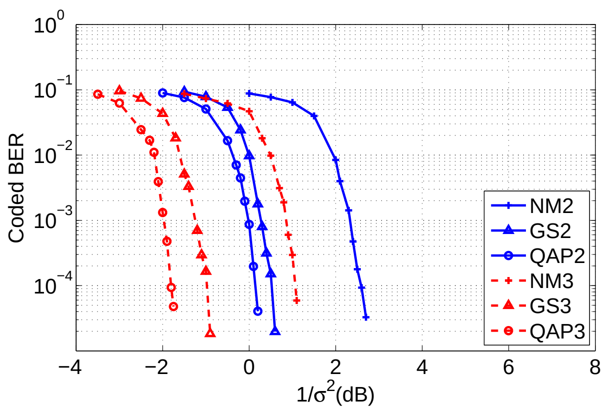


Fig. 3. Coded BER for 16-QAM.

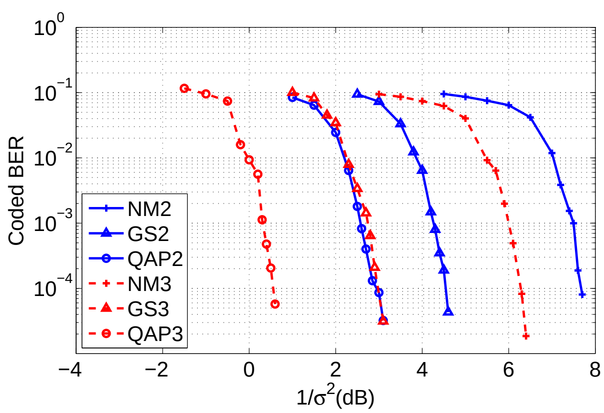


Fig. 4. Coded BER for 64-QAM.

V. Conclusion

In this work, we investigated the modulation diversity (MoDiv) design for Chase Combining (CC) HARQ in amplify and forward (AF) two-way relay channel (TWRC). With the objective of minimizing an approximated bit error rate (BER), the MoDiv design was formulated into a successive Koopmans-Beckmann Quadratic Assignment Problem (QAP) and solved with a robust taboo search algorithm. Our numerical tests demonstrated that the QAP-optimized MoDiv outperformed simple repeated use of Gray mapping and a heuristic MoDiv scheme under various settings and was robust against mismatched design parameters.

Appendix A
Proof of Proposition 1

The proof of Proposition 1 is generally based on Eq.(43) of [27]. Firstly, by adopting the heuristic approximation in [28], the random variable $\alpha^{(k)}$ is replaced with constant $\tilde{\alpha}$ in $E_k[p,q]$, then we have

$$\begin{aligned} E_k[p,q] &\approx \mathbb{E}_{\gamma_2}\left[\mathbb{E}_{\delta_1|\gamma_2}\left[\exp\left(-\frac{\tilde{\alpha}^2\epsilon_k[p,q]\gamma_2\delta_1}{4(\sigma_2^2+\tilde{\alpha}^2\sigma_R^2\gamma_2)}\right)\right]\right] \\ &= \mathbb{E}_{\gamma_2}\left[\left(1+\frac{\tilde{\alpha}^2\epsilon_k[p,q]\beta_{h_1}\gamma_2}{4(\sigma_2^2+\tilde{\alpha}^2\sigma_R^2\gamma_2)}\right)^{-1}\right]. \end{aligned} \tag{17}$$

As δ_1, γ_2 both follow exponential distribution, Eq.(9) is derived by evaluating Eq.(17) with Eq.(3.352.4) of [23].

Appendix B
Proof of Proposition 2

Denote the mutual information conditioned on the channel state informations as $I^{(m)}(\mathbf{h}_1^{(m)}, \mathbf{g}_2^{(m)}, \boldsymbol{\alpha}^{(m)})$, where $\mathbf{h}_1^{(m)} = [h_1^{(0)},\ldots,h_1^{(m)}]^T$, $\mathbf{g}_2^{(m)} = [g_2^{(0)},\ldots,g_2^{(m)}]^T$ and $\boldsymbol{\alpha}^{(m)} = [\alpha^{(0)},\ldots,\alpha^{(m)}]^T$. By assuming a uniform distribution of all constellation symbols, $I^{(m)}(\mathbf{h}_1^{(m)}, \mathbf{g}_2^{(m)}, \boldsymbol{\alpha}^{(m)})$ is lower bounded with [, Eq.(4.3.37)]:

$$\begin{aligned} &\tilde{I}^{(m)}(\mathbf{h}_1^{(m)}, \mathbf{g}_2^{(m)}, \boldsymbol{\alpha}^{(m)}) = \log_2 Q - \\ &\log_2\left[\frac{1}{Q}\sum_{p=0}^{Q-1}\sum_{q=0}^{Q-1}\prod_{k=0}^{m}\exp\left(-\frac{(\alpha^{(k)})^2\epsilon_k[p,q]\gamma_2^{(k)}\delta_1^{(k)}}{4(\tilde{\sigma}_2^{(k)})^2}\right)\right]. \end{aligned} \tag{18}$$

Noting that $\log(x)$ is a concave function and the channels are assumed independent across each round of (re)transmissions, we have $\mathbb{E}\left[\tilde{I}^{(m)}(\mathbf{h}_1^{(m)}, \mathbf{g}_2^{(m)}, \boldsymbol{\alpha}^{(m)})\right] \geq \tilde{I}^{(m)}$, thus Proposition 2 is proved.

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