# MODULATION DESIGN IN AMPLIFY-AND-FORWARD TWO-WAY RELAY HARQ CHANNEL

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## ABSTRACT

As a practical transmission enhancement technique for relay and HARQ system, Modulation Diversity (MoDiv) uses distinct mappings to the same constellation for different (re)transmissions. In this work, we study the MoDiv optimization in a Amplify-and-Forward (AF) Two-Way Relay Channel (TWRC). The design of MoDivto minimize the bit-error rate (BER) is formulated into a successive Koopmans-Beckmann Quadratic Assignment Problem (QAP), which is solved sequatially with a robust taboo search method. The performance gain of our MoDiv scheme over retransmission without remapping and a heuristic MoDiv scheme is demonstrated with numerical ressults.

***Index Terms*—** Modulation diversity, two-way relay, amplify-and-forward, HARQ, QAP

## 1. INTRODUCTION

As an advanced technique to improve the robustness of high-rate wireless transmissions against poor channel conditions, Hybrid Automatic Repeat reQuest (HARQ) has found its application in various communication systems [1]. HARQ works on both PHY layer and MAC sublayer to mitigate packet loss due to channel fading and link-adaptation accuracy. Recently, substantial research interest has been drawn to HARQ in Two-Way Relay Channel (TWRC) [2-4]. In [2], the average throughput of naive Type-I HARQ policy for both Amplify and Forward (AF) and Decode and Forward (RF) TWRC schemes have been analyzed. The energy-delay tradeoff, and the diversity-multiplexing tradeoff of type-II HARQ policy, also known as full Incremental Redundancy (IR), for AF TWRC scheme have been studied in [3] and [4], respectively. Related works about TWRC with ARQ for different relay schemes and retransmission policies can also be found in [5, 6, 7] and the references therein.

Apart from the naive Type-I HARQ and HARQ-IR, Type-I HARQ with maximal ratio combining (MRC), also known as HARQ-Chase Combining (HARQ-CC) [8], is another simple and practical HARQ scheme supported by such standands as HSPA [9], LTE [10] and so forth. As practical transmissions often admit linear modulations of finite-alphabet constellation (e.g. Q-ary QAM), the performance of HARQ-CC can be improved with Modulation Diversity (MoDiv) [11], in which a same group of $\log_2 Q$ bits are mapped to different symbols in a same constellation in different round of (re)transmissions. MoDiv has been studied for HARQ [12], relay networks [13, 14] and relay-HARQ systems [15, 16].

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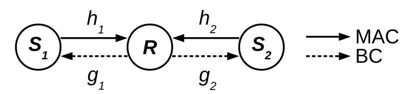



**Fig. 1**. Two-way relay channel with analog network coding.

In this paper, we study the MoDiv design for the TWRC under a simple AF scheme and HARQ-CC protocol. We first derive an approximation for the uncoded bit-error rate (BER) of TWRC-AF channel under the Rayleigh fading condition, given $M$ different mapping schemes corresponding to each (re)transmission. Based on this approximation, we formulate a successive BER minimization MoDiv design into a series of Quadratic Assignment Problem (QAP) in Koopmans-Beckmann (KB) form [17]. Although QAP is NP-hard, efficient numerical algorithms have been extensively researched [18], some of which have shown extremely high performance over QAPLIB [19]. We adopt a taboo search algorithm [20] to solve each QAP in our formulation. Moreover, the coefficients of QAP problem can be also be computed efficiently in a successive manner based on the solution to the preceding QAP problem. Our numerical results demonstrate significant BER reduction over both non-MoDiv and a simple heuristic MoDiv retransmission scheme for 16-QAM, 32-QAM and 64-QAM constellation, even under mismatching design parameters.

The paper is organized as follows. Section 2 introduces the TWRC-AF model and the HARQ protocol we are using. Section 3 presents the successive BER minimization MoDiv design problem. In Section 4, we present the numerical results to show the performance gain of our MoDiv scheme. Finally, Section 5 concludes the paper.

## 2. SYSTEM MODEL

Consider a TWRC with analog network coding (ANC) protocol [3], a generalization the AF protocol, as shown in Fig. 5.

the relay node $R$ is totally unaware of the HARQ procedure and simply performs ANC. Each round of ANC transmission is composed of two phases. In the multiple access (MAC) phase, the two source nodes $S_1$ and $S_2$ transmit to $R$ simultaneously. In the broadcast (BC) phase, $R$ amplify and broadcast the signal received during the MAC phase to both $S_1$ and $S_2$. Denote the uplink channel from $S_s$ to $R$ and downlink channel from $R$ to $S_s$ as $h_s$ and $g_s$, respectively, where $s = 1, 2$. We assume that all the channels follow Rayleigh distribution, i.e. $h_s \sim \mathcal{CN}(0, \beta_{h_s})$ and $g_s \sim \mathcal{CN}(0, \beta_{g_s})$, $s = 1, 2$. Denote the transmitted symbol from $S_s$ as $x_s$ whose average power $\mathbb{E}[|x_s|^2] = P_s$. Then the signal received by $R$ during the MAC phase is

$$y_R = h_1 x_1 + h_2 x_2 + n_R, \tag{1}$$

where $n_R \sim \mathcal{CN}(0, \sigma_R^2)$ is the received noise at $R$. Assuming that the relay $R$ has an expected power constraint of $P_R$, and that $S_1$ and $S_2$ perform perfect self-interference cancellation (SIC), then the received signal at $S_s$ after SIC is

$$y_s = \alpha g_s y_R + n_s, \; s = 1, 2, \tag{2}$$

where $n_s \sim \mathcal{CN}(0, \sigma_s^2)$ is the received noise at $S_s$, and

$$\alpha = \sqrt{\frac{P_R}{|h_1|^2 P_1 + |h_2|^2 P_2 + \sigma_R^2}} \tag{3}$$

is the power normalization factor at $R$.

On top of this settings, $S_1$ and $S_2$ performs the HARQ-CC protocol in an unsynchronized manner. Consequently, the MoDiv design at $S_1$ and $S_2$ can be handled independently. Without loss of generality, we study the HARQ transmission from $S_1$ to $S_2$. Denote $\mathcal{C}$ as the constellation used by $S_1$ whose cardinality equals $Q = |\mathcal{C}|$. As a convention, during the initial transmission of a packet, $S_1$ converts a bit sequence of length $\log_2 Q$ into symbols with Gray mapping $\psi_0 : \{0, \ldots, Q-1\} \to \mathcal{C}$. The bit sequence is is labeled by its decimal equivalence $p \in \{0, \ldots, Q-1\}$. What distinct HARQ-CC with MoDiv from conventional HARQ-CC is that, during the $m$-th retransmission, $S_1$ is allowed to use a mapping function $\psi_m \neq \psi_0$ to remap the same label $p$. We assume $m \leq M$ where $M$ is the maximum number of retransmissions. According to Eq. (1)(2), the signal received by $S_2$ after SIC during the $m$-th (re)transmission of $p$ is

$$y_2^{(m)} = \alpha^{(m)} g_2^{(m)} h_1^{(m)} \psi_m[p] + \alpha^{(m)} g_2^{(m)} n_R^{(m)} + n_2^{(m)}, \tag{4}$$

where $X^{(m)}$ is the $m$-th realization of random variable $X$.

Assume that $S_2$ acquires perfect channel state information (CSI). After the $m$-th retransmission, it attempts to demodulate the received symbols by identifying label $p$ with $y_2^{(0)}, \ldots, y_2^{(m)}$ via the maximum likelihood (ML) detection:

$$p^* = \arg\min_p \sum_{k=0}^{m} \frac{|y_2^{(k)} - \alpha^{(k)} g_2^{(k)} h_1^{(k)} \psi_k[p]|^2}{\sigma_2^2 + (\alpha^{(k)})^2 \sigma_R^2 |g_2^{(k)}|^2}. \tag{5}$$

## 3. SUCCESSIVE CONSTELLATION MAPPING DESIGN FOR MODULATION DIVERSITY

In this section, we first derive an closed-form approximation of the reception bit-error rate in our TWRC channel with HARQ-CC. Based on this result, we formulate the BER-minimization MoDiv design into a successive QAP (S-QAP).

### 3.1. A BER approximation

Assume that the label $p$ follows a uniform distribution. The BER of the ML demodulator after the $m$-th retransmission can be upper-bounded and approximated with the pair-wise error probability (PEP) [12]:

$$P_{BER}^{(m)} = \sum_{p=0}^{Q-1} \sum_{q=0}^{Q-1} \frac{B[p,q]}{Q} P_{PEP}^{(m)}(q|p), \tag{6}$$

where $B[p, q]$ represents the Hamming distance between the binary representation of $p$ and $q$ normalized by $\log_2 Q$, and $P_{PEP}^{(m)}(q|p)$ is the probability that the ML demodulator prefer $q$ over $p$ conditioned on the transmission of $p$. From Eq. (5), we have

$$P_{PEP}^{(m)}(q|p) = \mathbb{E}\left[ Q\left( \sqrt{\sum_{k=0}^{m} \frac{(\alpha^{(k)})^2 \epsilon_k[p,q] \gamma_2^{(k)} \delta_1^{(k)}}{2(\tilde{\sigma}_2^{(k)})^2}} \right) \right], \tag{7}$$

where $\gamma_2^{(k)} = \|g_2^{(k)}\|^2$, $\delta_1^{(k)} = \|h_1^{(k)}\|^2$ , $\epsilon_k[p,q] = \|\psi_k[p] - \psi_k[q]\|^2$, and $(\tilde{\sigma}_2^{(k)})^2 = \sigma_2^2 + (\alpha^{(k)})^2 \sigma_R^2 \gamma_2^{(k)}$ is the instantaneous variance of the noise received by $S_2$. By addopting the Chernoff upper bound $Q(x) \leq e^{-x^2/2}/2$ [21], an approximation to $P_{PEP}^{(m)}(q|p)$ is

$$\tilde{P}_{PEP}^{(m)}(q|p) = \frac{1}{2} \prod_{k=0}^{m} \mathbb{E}\left[ \exp\left( -\frac{(\alpha^{(k)})^2 \epsilon_k[p,q] \gamma_2^{(k)} \delta_1^{(k)}}{4(\tilde{\sigma}_2^{(k)})^2} \right) \right]. \tag{8}$$

Although the Chernoff bound is a rather rough appoximation, it enables efficient iterative computation of $P_{PEP}^{(m)}(q|p)$ as $m$ varies. Moreover, as shown in Section 3.2, this approximation results in a simple KB-form QAP. Nevertheless, the Chernoff bound can be replaced with a more accurate approximation as in Eq.(14) of [22], As will be explained in Section 3.2, however, this will lead to a more complex general-form QAP.

Denote $E_k[p, q]$ as the expectation in Eq.(8), which can be evaluated as follows:

**Proposition 1.** *An approximation to $E_k[p, q]$ is*

$$\tilde{E}_k[p,q] = \frac{4\sigma_R^2 + \beta_{h_1} \epsilon_k[p,q] v \exp(v) Ei(v)}{u} \tag{9}$$

*where*

$$u = 4\sigma_R^2 + \beta_{h_1}\epsilon_k[p,q], \tag{10a}$$

$$v = \frac{4\sigma_2^2}{\tilde{\alpha}^2\beta_{g_2}u}, \tag{10b}$$

$$\tilde{\alpha} = \sqrt{\frac{P_R}{\beta_{h_1}P_1 + \beta_{h_2}P_2 + \sigma_R^2}}, \tag{10c}$$

*and* $Ei(x) = \int_x^\infty e^{-t}/tdt$ *is the exponential integral function [23].*

*Proof.* See Appendix. □

### 3.2. The Successive Quadratic Assignment Problem

Our MoDiv design is based on the approximated BER minimization criterion. As it is impossible to know the number of actual retransmission $m$ in advance, we formulate a sequence of $M$ optimization problems as in [12], in which $\psi_m$ is optimized to minimize the approximated BER given $\psi_1, \dots, \psi_{m-1}$ without expecting future retransmissions. The

$$\min_{\psi^{(m)}|\psi^{(k)},k=0,\dots,m-1} \tilde{P}_{BER}^{(m)},\ m = 1,\dots,M \tag{11}$$

where $\tilde{P}_{BER}^{(m)}$ denotes the approximated version of Eq.(6) evaluated with Eq.(8)(9).

In order to rewrite Eq.(11) into a S-QAP formulation, we denote $\mathbf{x}^{(m)} = \{x_{pi}^{(m)}|p,i = 0,\dots,Q-1\}$ as the permutation matrix representing $\psi_m$:

$$x_{pi}^{(m)} = \begin{cases} 1, & \text{if } \psi_m[p] = \psi_0[i] \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

Denote the constraint sets

$$\mathcal{P} = \left\{\mathbf{x} : \sum_{p=0}^{Q-1} x_{pi} = 1, x_{pi} \in \{0,1\}\right\}, \tag{13a}$$

$$\mathcal{I} = \left\{\mathbf{x} : \sum_{i=0}^{Q-1} x_{pi} = 1, x_{pi} \in \{0,1\}\right\}. \tag{13b}$$

Then the MoDiv design problems in Eq.(11) can be formulated into a S-QAP as follows:

$$\min_{\mathbf{x}^{(m)}} \sum_{p=0}^{Q-1}\sum_{i=0}^{Q-1}\sum_{q=0}^{Q-1}\sum_{j=0}^{Q-1} f_{pq}^{(m)} d_{ij} x_{pi}^{(m)} x_{qj}^{(m)}, \tag{14}$$
$$\text{s.t. } \mathbf{x}^{(m)} \in \mathcal{P} \cap \mathcal{I}.$$

in which the "flow" matrix $f_{pq}^{(m)}$ and the "distance" matrix $d_{ij}$ are defined as

$$f_{pq}^{(m)} = \frac{B[p,q]}{Q}\tilde{P}_{PEP}^{(m-1)}(q|p) \tag{15a}$$

$$d_{ij} = \tilde{E}_0[i,j] \tag{15b}$$

Note that here we assume all channel and noises to be stationary across all retransmissions, so $d_{ij}$ only needs to be evaluated once. On the other hand, $f_{pq}^{(m)}$ can be computed recursively along solving the S-QAP, since

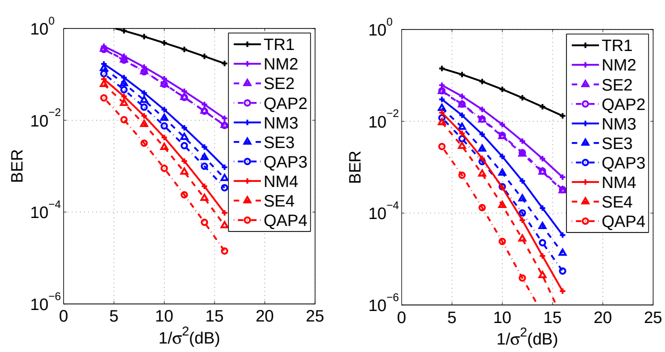
$$\tilde{P}_{PEP}^{(m)}(q|p) = \sum_{i=0}^{Q-1}\sum_{j=0}^{Q-1} \tilde{P}_{PEP}^{(m-1)}(q|p) d_{ij} \hat{x}_{pi}^{(m)} \hat{x}_{qj}^{(m)} \tag{16a}$$

$$\tilde{P}_{PEP}^{(-1)}(q|p) = \frac{1}{2} \tag{16b}$$

where $\hat{\mathbf{x}}^{(m)}$ is the solution to Eq.(14).

In our S-QAP fromulation, each KB-form QAP is defined with two $Q$-by-$Q$ matrices, and only one of them needs to be updated. Should we adopt the more accurate approximation [22] in Eq.(7), each QAP would be in general-form which is defined with one $Q^4$ matrix. Although this 4-dimensional matrix can still be updated iteratively using a few $Q$-by-$Q$ matrices in a sequential manner, the solution to the general-form QAP is ususally more complicated. With the S-QAP in KB form, we are able to handle much larger constellation with an efficient robust taboo search algorithm [20]. Other numerical approaches to solve KB-QAP include simulated anneling (SA) [24], ant-colony [25] and so forth. Finally, we note that the MoDiv design can be precomputed offline and stored in $S_1$ and $S_2$ as it depends only on statistical CSI.

## 4. NUMERICAL RESULTS



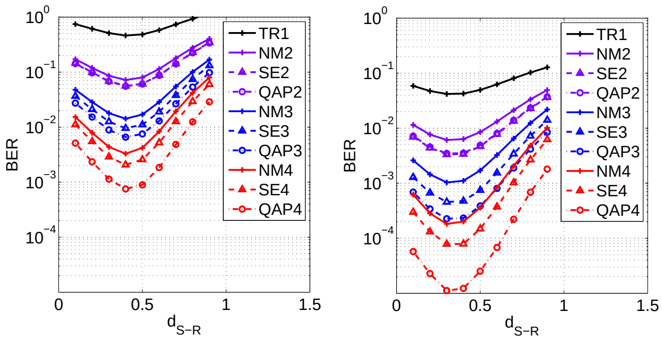

(a) Approximated (b) Monte-Carlo

**Fig. 2**. Uncoded BER vs the noise power for 16-QAM.

## 5. CONCLUSION

## 6. APPENDIX: PROOF OF PROPOSITION 1

The proof of Proposition 1 is generally based on Eq.(43) of [26]. Firstly, by adopting the heuristic approximation in [27], the random variable $\alpha^{(k)}$ is replaced with constant $\tilde{\alpha}$




(a) Approximated (b) Monte-Carlo

**Fig. 3**. Uncoded BER vs the position of the relay node for 16-QAM.

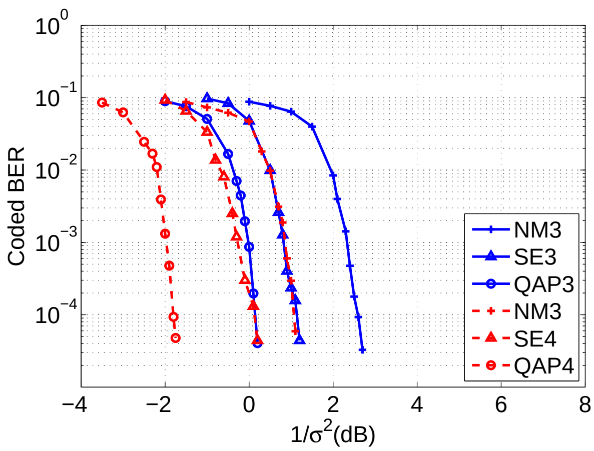



**Fig. 4**. Coded BER for 16-QAM.

in $E_k[p,q]$, then we have

$$E_k[p,q] \approx \mathbb{E}_{\gamma_2}\left[\mathbb{E}_{\delta_1|\gamma_2}\left[\exp\left(-\frac{\tilde{\alpha}^2\epsilon_k[p,q]\gamma_2\delta_1}{4(\sigma_2^2+\tilde{\alpha}^2\sigma_R^2\gamma_2)}\right)\right]\right]$$
$$= \mathbb{E}_{\gamma_2}\left[\left(1+\frac{\tilde{\alpha}^2\epsilon_k[p,q]\beta_{h_1}\gamma_2}{4(\sigma_2^2+\tilde{\alpha}^2\sigma_R^2\gamma_2)}\right)^{-1}\right]. \qquad (17)$$

As $\delta_1, \gamma_2$ both follow exponential distribution, Eq.(9) is derived by evaluating the above expectation with Eq.(3.352.4) of [23].

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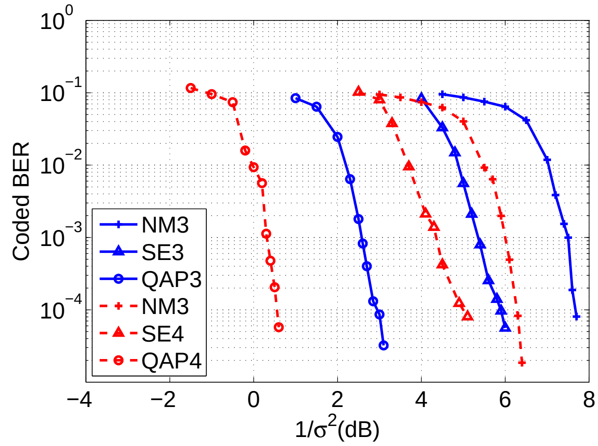


**Fig. 5**. Coded BER for 64-QAM.


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