Network-based exit strategy in COVID-19 epidemics

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1 Introduction

Lockdown is currently in place in many countries, at a large economic and social cost, including unemployment on a massive scale. If a lockdown is successful in reducing the number of critical cases [1], a decision must be reached on when and how to exit it. In the absence of herd immunity or vaccine, the main concern is the risk of resurgence of the epidemic. Currently, many countries has decided to launch exit procedure but with different criteria. China provided the access of lockdown release of a province until this province has no infected increment for 14 days. America has decided to recovery working of a state if the infected increment decreases for several days in this state. Although some previous research has focused on the prediction of epidemic spreading [2][3] and the impacts of lockdown policy [4][5], no clear exit strategy has been proposed [6]. Both a framework to measure the performance of different strategy and a sustainable exit strategy are urgent at this moment under the high pressures of epidemic control and economic recession.

Some researchers [7] suggest that governments could identify territories which have been free of cases after 14 days of lockdown, which is the maximum incubation period of COVID-19. Many previous approaches have attempted to control epidemic spreading through timed intervention [8], i.e. restarting lockdown timely. One strategy proposes reinstating lockdown when a critical number of cases is exceeded in a resurge, and stopping lockdown again once cases drop below a low threshold [9]. The research [10] defines such kind of intervention as the *timed intervention policy*, and proposes a multi-shot epidemic intervention based on the SIQR model (the state Q implies quarantine). The methods switches the lockdown state and normal state frequently by considering a cost combining epidemic growth and societal cost. Different from the strategy that restarting lockdown when epidemic resurgences, the study [11] proposes a cyclic work-lockdown strategy (e.g., working 4 days per two weeks), which can control the epidemic and offer predictable part-time employment. Simulation shows that the cyclic strategy presents better performance in the SEIR-Erlang deterministic model. Similarly but less aggressively, the work [12] investigates to apply "a phased lift of control" to achieve herd immunity. The strategy regards the limited IC beds and the regional restriction as curing resources, and dynamically (or with a time interval) allocates them to different regions, in order to achieve regional herd immunity one by one, with a low death rate.

Several researchers have addressed the problem of epidemic control by optimization model and deep learning. A robust economic model predictive controller for the containment of stochastic susceptible-exposed-infected-vigilant (SEIV) epidemic processes is developed in [13], which drives the process to extinction quickly, while minimizing the rate at which control resources are used. The nonlinear model predictive control (NMPC) has also been applied to a pairwise SIS model on nontrivial contact networks [14], which aims to eradicate the disease while keeping the network well connected. The research [15] extends the SIR model to a model with vulnerable subpopulation, where the vulnerable implies the elder or the people with pre-existing chronic diseases. A targeted isolation approach is proposed to provide a more efficient and robust strategy at a lower economic and social cost. The control decision in recovery processes can also be formulated as a Markov decision process (MDP) and be addressed by the approach of deep reinforcement learning [16].

2 Network-based framework for exit process

We first revisit the time table of the lockdown process and the exit process in China, as shown in Figure 1. We cannot deny the efficacy of Chinese policies on controlling the spread of COVID-19 though the same measures could be infeasible in other countries.

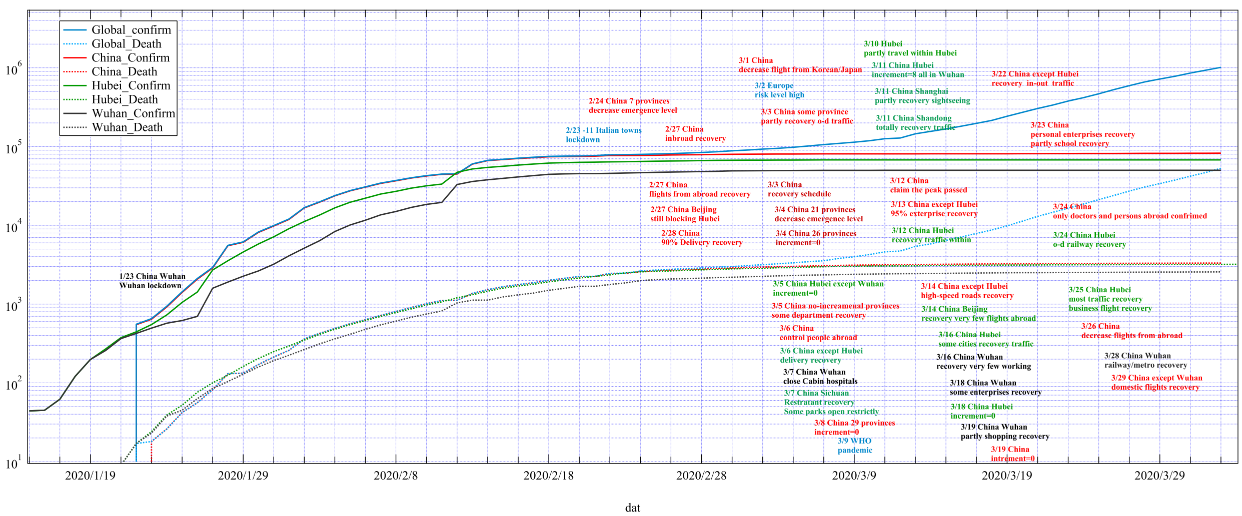

Figure 1: Time table of the lockdown and exit process for COVID-19 in China

We may highlight three inspirations from the time table:

1. **Fast responses:** The measures are taken frequently (as almost every day). We can observe that China follows a swift lockdown measures as well as a very strict exiting condition (Releasing the traffic until no infection increment for several days). Also, we observe a feedback response to the latest situation. i.e., restarting travel restrictions once the number of the infected increases.

2. **Regional granularity:** The exiting steps are divided by different regional granularity, e.g., city, province and country. This fact reminds us that the traffic and the contact on a network model can tune the granularity of exit measures. The respective measures on entering and leaving a region (e.g., limitation in population flow from abroad and people going abroad) implies a directed network features the contact behaviors among regions better.

3. **Object granularity:** The object, namely people group, is also a crucial factor in the exiting policy. For example, the re-opening of stores for daily necessities, the transportation for delivery among different regions, the public government departments usually have the priority to be

recovered to maintain the city functions. Some measures are turned out for specific groups, e.g., school starting for students.

Focusing on exit processes, each exit measure can be defined by an ontology with three factors: $\{object, regions, level\}$, which is determined by an effective exit strategy.

3 Network-based framework for exit process

3.1 SIR epidemic model on networks

We consider a heterogeneous SIR epidemic model on networks. Every individual is in either one of the compartments among susceptible (healthy), infectious or removed. Due to hospitalisation, quarantine measures, death or recovery, infectious individuals become removed individuals, which cannot infect susceptible individuals as well as re-infected any longer. We denote the state $\mathcal{S}_i(t), \mathcal{I}_i(t), \mathcal{R}_i(t)$ by the fraction of susceptible, infectious, and removed individuals at region i at time t, respectively. We abstract the contact relation by a underlying network where each node represents a region and the links represent the population flow between two regions.

In the N-Intertwined Mean-Field Approximation (NIMFA), the SIR model on a network G_N with N nodes following the governing equations in a vector form:

$$\begin{aligned}
\frac{d\boldsymbol{\mathcal{I}}(t)}{dt} &= diag(\boldsymbol{\mathcal{S}}) \cdot B(t) \cdot \boldsymbol{\mathcal{I}} - diag(\boldsymbol{\delta}(t))\boldsymbol{\mathcal{I}}(t) \\
\frac{d\boldsymbol{\mathcal{S}}(t)}{dt} &= -diag(\boldsymbol{\mathcal{S}}) \cdot B(t) \cdot \boldsymbol{\mathcal{I}} \\
\frac{d\boldsymbol{\mathcal{R}}(t)}{dt} &= diag(\boldsymbol{\delta})\boldsymbol{\mathcal{I}}
\end{aligned} \tag{1}$$

where $\boldsymbol{\mathcal{S}}(t), \boldsymbol{\mathcal{I}}(t), \boldsymbol{\mathcal{R}}(t)$ are state vectors, the matrix $B(t)$ with the entry $\beta_{ij}(t)$ denotes the infection rate from region j to region i at time t, and $\delta_i(t)$ denotes the curing probability of region i.

In this report, we assume that the infection rate $B(t)$ purely depends on the contact probability. More realistically, the infection rate $B(t)$ and the curing rate $\boldsymbol{\delta}(t)$ could change due to external perturbations, e.g., hospital capacity, virus mutation, or vaccine development.

3.2 An exemplified process

We then show an exemplified process.We assume a SIR model for COVID-19 epidemics on the NL network with $N = 12$ provinces. All numerical results are calculated by the solution of NIMFA equations (1) via Matlab. The entry B_{ij} in the matrix B denotes the infection rates of province i from province j, and the diagonal element B_{ii} denotes the infection rate within each province. (The matrix B is provided by NIPA.) For simplicity, we set the curing rate in each province is the same, i.e., $\delta_i = 0.1$ for $i = 1, 2, \dots, N$.

We now present the prevalence $y(t) = \frac{1}{N}\sum_{i=1}^{N} \mathcal{I}_i(t)$ to illustrate the recoverability framework following Algorithm 1. The x-axis is the time line. The whole process can be regarded as three phases:

1. **Phase I:** In the first phase (by blue part), we take no intervention for a networked SIR process, started with a province with $\mathcal{I}_1(0) = 0.01$. The prevalence $y(t)$ increases until the lockdown threshold $y_L = 0.1$ is reached, i,e., we start to take lockdowm if $y(t) \geq y_L$ .The lockdown threshold y_L acts as an alert that suggests the government should take lockdown measures.

2. **Phase II:** The second phase is the lockdown phase (red part). We take lockdown measures one by one with a time interval $\Delta t = 1$, acting as attack challenges [17]. For each challenge, we first find the link between two provinces with the maximum product of infected fraction, i.e., $\ell = (i, j) = \arg\max_{i \in N, j \in N} \mathcal{I}_i \mathcal{I}_j$. Then we decrease the link weights to $B_{ij} = 0.001$ and $B_{ji} = 0.001$ (we apply 0.001 instead of 0 to imply a minimal-level contact). Meanwhile, we set the self-infection rate $B_{ii} = 0.001$ and $B_{jj} = 0.001$ at these two regions. The lockdown phase will continue until the exit/recovery condition is satisfied.

3. **Phase III:** Once the exit condition is satisfied, we stop further lockdown and start the recovery process (as green part). We also take recovery measures one by one with a time interval $\Delta t = 1$, acting as recovery challenges. The matrix $\hat{B}$ denotes the original infection rate matrix before lockdown. In each challenge, we recovery the link with the maximal decrement in the attack process to the original weight, i.e., we recovery the link weight $B_{ij} = \hat{B}_{ij}$ and $B_{ji} = \hat{B}_{ji}$ for the link $\ell = (i, j) = \arg\max_{i \in N, j \in N} (\hat{B}_{ij} - B_{ij})$. Meanwhile, we recovery the self-infection rate B_{ii} and B_{jj} in these two regions to the original infection rates $\hat{B}_{ii}$ and $\hat{B}_{jj}$, respectively.

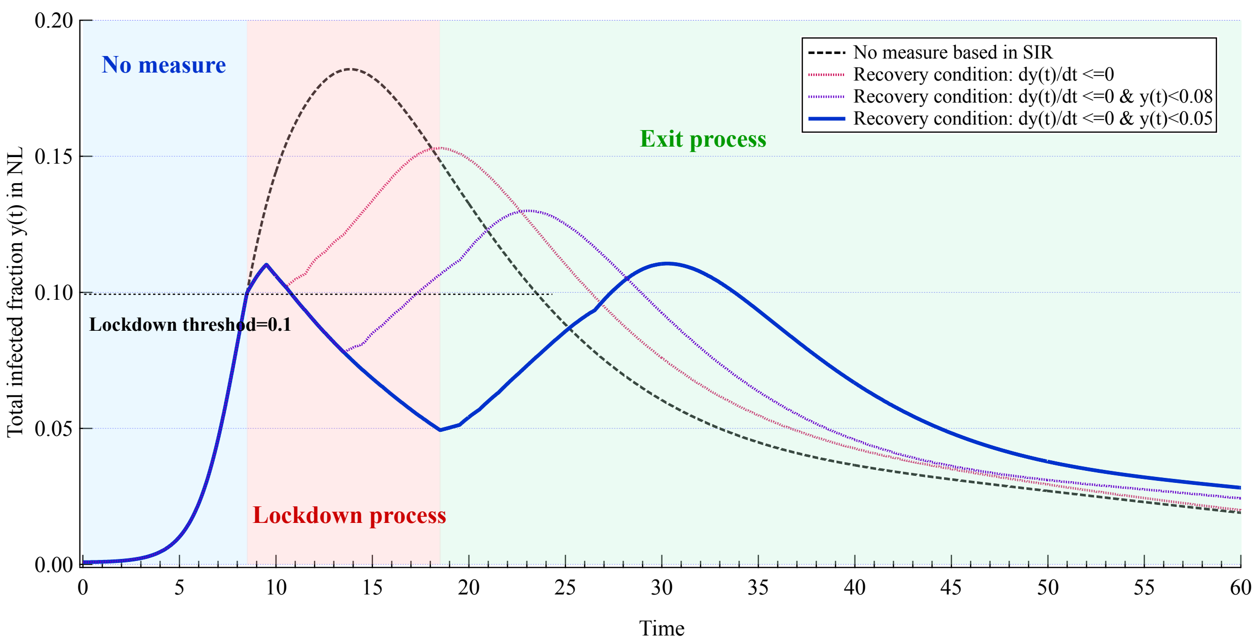

Figure 2: Average infected fraction versus time in NL with 12 provinces. The colorful areas are illustrated for the recovery condition (of solid blue line): $\frac{dy(t)}{dt} < 0$ and $y(t) < 0.05$.

We are curious about the effect of exit condition and exit strategy on the recovery performance. In Figure 2, we compare three different exit conditions (i) once $\frac{dy(t)}{dt} < 0$, (ii) once $\frac{dy(t)}{dt} < 0$ and $y(t) < 0.08$, (iii) once $\frac{dy(t)}{dt} < 0$ and $y(t) < 0.05$. An intuitive inspiration is that both a too-early exit time and an aggressive exit strategy could stage a comeback of epidemics.

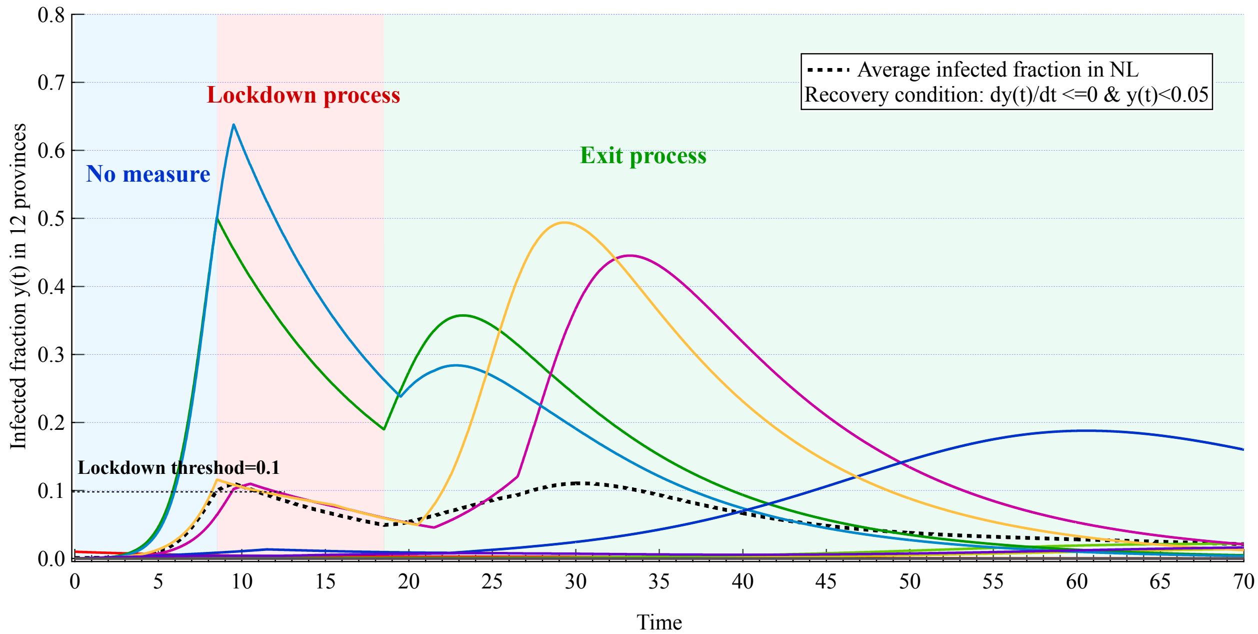

Figure 3: Infected fraction versus time in each province. The colorful areas are illustrated for the recovery condition: $\frac{dy(t)}{dt} < 0$ and $y(t) < 0.05$.

Further, Figure 3 illustrates the infected fraction versus time in each province, which exhibits different behaviour during the exit process. The province with a faster increasing rate in the initial stage usually have a higher probability to stage a resurgence in the exit process. This result suggests that a heterogeneous exit strategy (including the exit condition and the link recovery method) is necessary, instead of regarding the total infection fraction as a pure indicator.

3.3 Performance of exit strategy

We can measure performances of an exit strategy by different aspects:

1. **Expected death toll:** The number of deaths is one of the most crucial metrics for an exit strategy. Moreover, some study shows that COVID-19 can result lung in long-term sequelae for critical patients. We should mention that the death rate is couple to the medical capacity, e.g, the death probability increases sharply if the number of infectious exceeds the ICU capacity.

2. **Expected elimination time:** A long-term, even not very strict, lockdown environment could led to social panic and influence to mental healthy. People prefer a fully-recovery of normal life, instead of half-normal life with cyclic lockdown.

3. **Economic recovery:** It is highly likely that many businesses will not re-open immediately following lockdown termination, would rehire in smaller numbers, or fail to reopen at all. Hence, many workers would face long-term unemployment until global economy recovers. even if the actual lockdown and the exiting are shorter than expected. Unfortunately, there seems no exit strategy can benefit both epidemic controlling and economy development.

4. **Reliability:** Since theoretical epidemic models may fail to feature practical spreading behaviors with stochastic perturbations. The uncontrollable external factors, (such as asymptomatic

patients, infected travelers from abroad and few re-infected cases) could mislead the government to underestimate the risk of epidemic resurgence during the exit process. A reliable exit strategy is necessary for guaranteeing the decay of epidemic spreading under a determined range of perturbations.

4 Further issues

4.1 Problem formulation

Considering the performances of exit strategy mentioned in Section 3.3, we can formulate the problem as:

1. **Topology-based heterogeneous approach:** We can recovery the population flow (traffic) according to the topological metric of links, which is of operational simplicity. For example, increasing network efficiency [17] by link restoring under a greedy strategy helps recovering the social communication effectively.

2. **Adaptive dynamic programming:** Adaptive dynamic programming (ADP) is a novel approximate optimal control scheme. Employing the principle of optimality, we can adjust the link weight (implying contact probability) in real-time based on the latest state. Following ADP, we may reach a near-optimal measure scheme to exit lockdown under limited medical resources and preventing resurgence.

3. **Reinforcement learning:** An further approach over ADP is reinforcement learning, which has a more powerful learning capability to handle a more complicated scenario or optimization goals.

4. **Model predictive control (MPC) problem:** The main advantage of MPC is the fact that it allows the current timeslot to be optimized, while keeping future timeslots in account. Also MPC has the ability to anticipate future events and can take control actions accordingly. We may combine MPC and some predictive approaches (e.g. NIPA) to achieve better performance.

5. **Multi-object optimization or Non-cooperative game problem:** We mention that the performance metrics such as economy under a conservative exit measures may conflict with a fast epidemic elimination. Thus, we may formulate the problem as a multi-object optimization to get a trade-off between different performance metrics.

6. **Dynamic evolution:** We can assume that the contact rate follows a state-based adaptive principle in recovery processes. For example, we can assume that the contact rate $\beta_{ij}(t)$ depends on the infection fraction product $\mathcal{S}_i\mathcal{S}_j$ of two regions as

$$\beta_{ij}(t) = \hat{\beta}_{ij}(t) \cdot e^{-\alpha \int_{t-\Delta t}^{t} \mathcal{S}_i(t')\mathcal{S}_j(t')dt'} \tag{2}$$

   where α is a recovery parameter, $\hat{\beta}_{ij}(t)$ is the original contact rate before lockdown, and Δt is a time delay for an exit step. Then, we can formulate (2) and original epidemic model (1) as a new dynamic process. The time delay Δt may be couple to a phase transition for resurgences.

4.2 More realistic scenarios

Also, we assume that the curing probability $\delta_i(t)$ is a pairwise function as the ICU capacity based.

5 Summary

References

[1] N. Ferguson, D. Laydon, G. Nedjati Gilani, N. Imai, K. Ainslie, M. Baguelin, S. Bhatia, A. Boonyasiri, Z. Cucunuba Perez, G. Cuomo-Dannenburg *et al.*, "Report 9: Impact of non-pharmaceutical interventions (npis) to reduce covid19 mortality and healthcare demand," 2020.

[2] P. Magal and G. Webb, "Predicting the number of reported and unreported cases for the covid-19 epidemic in south korea, italy, france and germany," *Italy, France and Germany (March 19, 2020)*, 2020.

[3] F. Casella, "Can the covid-19 epidemic be managed on the basis of daily data?" *arXiv preprint arXiv:2003.06967*, 2020.

[4] P. Shao, "Impact of city and residential unit lockdowns on prevention and control of covid-19," *medRxiv*, 2020.

[5] Y. Zhang, B. Jiang, J. Yuan, and Y. Tao, "The impact of social distancing and epicenter lockdown on the covid-19 epidemic in mainland china: A data-driven seiqr model study," *medRxiv*, 2020.

[6] C. Barbieri and J.-P. Darnis, "Technology: An exit strategy for covid-19?" *Commentaries Istituto Affari Internazionali, Commentaries, March*, pp. 1–4, 2020.

[7] R. Sarwal and T. Sarwal, "Mitigating covid-19 with lockdowns: A possible exit strategy," *Available at SSRN 3563538*, 2020.

[8] F. Di Lauro, I. Z. Kiss, and J. Miller, "The timing of one-shot interventions for epidemic control," *medRxiv*, 2020.

[9] S. M. Kissler, C. Tedijanto, M. Lipsitch, and Y. Grad, "Social distancing strategies for curbing the covid-19 epidemic," *medRxiv*, 2020.

[10] M. Bin, P. Cheung, E. Crisostomi, P. Ferraro, C. Myant, T. Parisini, and R. Shorten, "On fast multi-shot epidemic interventions for post lock-down mitigation: Implications for simple covid-19 models," *arXiv preprint arXiv:2003.09930*, 2020.

[11] O. Karin, Y. M. Bar-On, T. Milo, I. Katzir, A. Mayo, Y. Korem, B. Dudovich, E. Yashiv, A. J. Zehavi, N. Davidovich *et al.*, "Adaptive cyclic exit strategies from lockdown to suppress covid-19 and allow economic activity," *medRxiv*, 2020.

[12] S. J. de Vlas and L. E. Coffeng, "A phased lift of control: a practical strategy to achieve herd immunity against covid-19 at the country level," *medRxiv*, 2020.

[13] N. J. Watkins, C. Nowzari, and G. J. Pappas, "Robust economic model predictive control of continuous-time epidemic processes," *IEEE Transactions on Automatic Control*, 2019.

[14] F. Sélley, Á. Besenyei, I. Z. Kiss, and P. L. Simon, "Dynamic control of modern, network-based epidemic models," *SIAM Journal on applied dynamical systems*, vol. 14, no. 1, pp. 168–187, 2015.

[15] Z. Neufeld and H. Khataee, "Targeted adaptive isolation strategy for covid-19 pandemic," *medRxiv*, 2020.

[16] P. Libin, A. Moonens, T. Verstraeten, F. Perez-Sanjines, N. Hens, P. Lemey, and A. Nowé, "Deep reinforcement learning for large-scale epidemic control," *arXiv preprint arXiv:2003.13676*, 2020.

[17] Z. He, P. Sun, and P. Van Mieghem, "Topological approach to measure network recoverability," in *2019 11th International Workshop on Resilient Networks Design and Modeling (RNDM)*. IEEE, 2019, pp. 1–7.

A Numerical method for the exemplified process

Algorithm 1 Numerical simulation for prevalence with intervention

Inputs:
 Original infection rate matrix $\hat{B}$, curing rate vector $\boldsymbol{\delta}$, Lockdown threshold y_L,
 Recovery threshold y_R, Measure interval Δt
Initialization:
 Set $\mathcal{I}_1 \leftarrow 0.01$ and $\mathcal{I}_i \leftarrow 0$ for $i = 2 - N$
 Set $\mathcal{S}_1 \leftarrow 0.99$ and $\mathcal{S}_i \leftarrow 1$ for $i = 2 - N$
 Set $\mathcal{R}_i \leftarrow 0$ for $i = 1 - N$
 Calculate SIR prevalence $\hat{y}(t)$ without intervention by (1) with $\hat{B}$
 Time handle $t \leftarrow 0$, Infection rate matrix $B \leftarrow \hat{B}$
 % *Phase I:*
while $\hat{y}(t) < y_L$ **do**
 $t \leftarrow t + 1$
end while
Obtain the time for lockdown measures $t_L \leftarrow t$
 % *Phase II:*
while not (recovery condition: $\frac{dy(t)}{dt} < 0$ and $y(t) < y_R$) **do**
 Find the link $\ell = (i, j) = \arg\max_{i \in N, j \in N} \mathcal{I}_i \mathcal{I}_j$
 Set $B_{ij} \leftarrow 0.001$,$B_{ji} \leftarrow 0.001$, $B_{ii} \leftarrow 0.001$ and $B_{jj} \leftarrow 0.001$
 Calculate SIR prevalence from time t to $t + \Delta t$ by (1) with B
 Set $t \leftarrow t + \Delta t$
end while
 % *Phase III:*
while $B \neq \hat{B}$ **do**
 Find the link $\ell = (i, j) = \arg\max_{i \in N, j \in N} (\hat{B}_{ij} - B_{ij})$
 Set $B_{ij} \leftarrow \hat{B}_{ij}$, $B_{ji} \leftarrow \hat{B}_{ji}$, $B_{ii} \leftarrow \hat{B}_{ii}$ and $B_{jj} \leftarrow \hat{B}_{jj}$
 Calculate SIR prevalence from time t to $t + \Delta t$ by (1) with B
 Set $t \leftarrow t + \Delta t$
end while
Calculate SIR prevalence after intervention by (1) with $\hat{B}$

References

[1] N. Ferguson, D. Laydon, G. Nedjati Gilani, N. Imai, K. Ainslie, M. Baguelin, S. Bhatia, A. Boonyasiri, Z. Cucunuba Perez, G. Cuomo-Dannenburg *et al.*, "Report 9: Impact of non-pharmaceutical interventions (npis) to reduce covid19 mortality and healthcare demand," 2020.

[2] P. Magal and G. Webb, "Predicting the number of reported and unreported cases for the covid-19 epidemic in south korea, italy, france and germany," *Italy, France and Germany (March 19, 2020)*, 2020.

[3] F. Casella, "Can the covid-19 epidemic be managed on the basis of daily data?" *arXiv preprint arXiv:2003.06967*, 2020.

[4] P. Shao, "Impact of city and residential unit lockdowns on prevention and control of covid-19," *medRxiv*, 2020.

[5] Y. Zhang, B. Jiang, J. Yuan, and Y. Tao, "The impact of social distancing and epicenter lockdown on the covid-19 epidemic in mainland china: A data-driven seiqr model study," *medRxiv*, 2020.

[6] C. Barbieri and J.-P. Darnis, "Technology: An exit strategy for covid-19?" *Commentaries Istituto Affari Internazionali, Commentaries, March*, pp. 1–4, 2020.

[7] R. Sarwal and T. Sarwal, "Mitigating covid-19 with lockdowns: A possible exit strategy," *Available at SSRN 3563538*, 2020.

[8] F. Di Lauro, I. Z. Kiss, and J. Miller, "The timing of one-shot interventions for epidemic control," *medRxiv*, 2020.

[9] S. M. Kissler, C. Tedijanto, M. Lipsitch, and Y. Grad, "Social distancing strategies for curbing the covid-19 epidemic," *medRxiv*, 2020.

[10] M. Bin, P. Cheung, E. Crisostomi, P. Ferraro, C. Myant, T. Parisini, and R. Shorten, "On fast multi-shot epidemic interventions for post lock-down mitigation: Implications for simple covid-19 models," *arXiv preprint arXiv:2003.09930*, 2020.

[11] O. Karin, Y. M. Bar-On, T. Milo, I. Katzir, A. Mayo, Y. Korem, B. Dudovich, E. Yashiv, A. J. Zehavi, N. Davidovich *et al.*, "Adaptive cyclic exit strategies from lockdown to suppress covid-19 and allow economic activity," *medRxiv*, 2020.

[12] S. J. de Vlas and L. E. Coffeng, "A phased lift of control: a practical strategy to achieve herd immunity against covid-19 at the country level," *medRxiv*, 2020.

[13] N. J. Watkins, C. Nowzari, and G. J. Pappas, "Robust economic model predictive control of continuous-time epidemic processes," *IEEE Transactions on Automatic Control*, 2019.

[14] F. Sélley, Á. Besenyei, I. Z. Kiss, and P. L. Simon, "Dynamic control of modern, network-based epidemic models," *SIAM Journal on applied dynamical systems*, vol. 14, no. 1, pp. 168–187, 2015.

[15] Z. Neufeld and H. Khataee, "Targeted adaptive isolation strategy for covid-19 pandemic," *medRxiv*, 2020.

[16] P. Libin, A. Moonens, T. Verstraeten, F. Perez-Sanjines, N. Hens, P. Lemey, and A. Nowé, "Deep reinforcement learning for large-scale epidemic control," *arXiv preprint arXiv:2003.13676*, 2020.

[17] Z. He, P. Sun, and P. Van Mieghem, "Topological approach to measure network recoverability," in *2019 11th International Workshop on Resilient Networks Design and Modeling (RNDM)*. IEEE, 2019, pp. 1–7.