Question 1:

a) The expected number of triangles= $\lim_{n\to\infty}\binom{n}{3}p^3 = \lim_{n\to\infty}\frac{n(n-1)(n-2)}{6}p3 \approx \lim_{n\to\infty}\frac{n^3p^3}{6} = \lim_{n\to\infty}\frac{c^3}{6} = \frac{c^3}{6}$. As $c = (n-1)p \approx np$

b) The expected number of connected triples= $\lim_{n\to\infty}\binom{n}{3}\binom{3}{1}(1-p)p^2 = \big(n(n-1)(n-2)\big)/2p^2 \approx \frac{1}{2}nc^2$: as $1-p \to 0\ as\ n \to \infty$ and $c = (n-1)p \approx np$

c) C=$\frac{c^3}{6} * \frac{3}{\frac{1}{2}nc^2} = \frac{c}{n}\quad as\ n \to \infty\ \frac{c}{n} = \frac{c}{n-1}$

Question 2:

a) For each connected vertice of the a vertex of degree k, it should not belong to giant component, so the probability is (1-S), and this probability should happen k times, the probability is $(1-S)^k$.

b) $P(vertice\ with\ degree\ K|vertice \in small\ component) = \frac{P(vertice\in small\ component|vertice\ with\ degree\ K)P(vertice\ with\ degree\ K)}{P(ertice\in small\ component)} = \frac{(1-S)^k}{1-S}\binom{n-1}{k}p^k(1-p)^{n-1-k} = (1-S)^{k-1}e^{-c}\frac{c^k}{k!}$ According to $\boldsymbol{G}(\boldsymbol{n},\boldsymbol{p})$ model is a Poisson degree distributions.

Question 3:

a) For random selected edge, the probability of they link node I with ki degree is k_i/m, their value should be given $1/2k_i/mx_i$ as the edge link two nodes, so $< x > edge = \sum_i \frac{1}{n}k_i x_i / \frac{2m}{n} = \sum_i \frac{\frac{1}{n}k_i x_i}{<k>}$

b) $< x > edge - < x > = \sum_i \frac{\frac{1}{n}k_i x_i}{<k>} - < x > = \frac{<kx>}{<k>} - < x > = \frac{cov(k,\ x)}{<k>}$

Question 4:

a) $a = 30,\ < q > = \int_0^\infty (q+30)^{-3}$q=0.17. The average number of citations received by a paper is 0.17

b) $P(0) = \frac{1+a/c}{a+1+a/c} = \frac{1+30/30}{30+1+30/30} = 6.25\%$. On average 6.25% of papers receive no citations at all.

c) P(>=100)= $\int_{100}^\infty (q+30)^{-3} = 0.06\%$. On average 0.06% of papers receive 100 or more citations.

d) For the n+1th paper, $P(100\ th\ paper\ not\ cited\ by\ n+1\ th\ paper) = \frac{0+30}{n(30+30)} = \frac{1}{2n}$. So the $P(100th\ paper\ not\ cited\ by\ other\ paper) = \frac{1}{2*100*2*101*...2*9999} = 2^{-9900} * \frac{1}{\prod_{x=100}^{x=9999} x}$.

$P(100th\ to\ 10000th\ paper\ not\ cited\ by\ other\ paper) = 2^{-9900} * \frac{1}{\prod_{x=100}^{x=9999} x} * 2^{-9899} * \frac{1}{\prod_{x=101}^{x=9999} x} * \ldots * 2^{-1} * \frac{1}{9999}$.