

linear comb of
cols of A space of RHS for which $Ax=b$ has solution(s)

6 Column Space and Nullspace

- vector spaces and subspaces
- column space of A : solving $Ax=b$
- nullspace of A

Union of subspaces is NOT a subspace
(necessarity)

Intersections of subspaces is a subspace

subspace
2

column space & its relationship to solving $Ax=b$

e.g.

$$Ax = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

$c(A)$ is a 2D subspace of \mathbb{R}^4

★ Which b does $Ax=b$ have a solution?
The ones in $c(A)$!

subspace
2

Nullspace $\in \mathbb{R}^3$ for this example

All solutions to $Ax=0$

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

vectors in nullspace

$$c \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

How do we know that the nullspace is a subspace?

If \vec{v} $A\vec{v} = 0$, $A\vec{w} = 0$

then $A(\vec{v} + \vec{w}) = 0 \Rightarrow$
 $\vec{v} + \vec{w}$ is in
nullspace