Linear Algebra 1

fundamental problem

1 The Geometry of linear equations

linear algebra ⇒ solve m equations and n unknowns

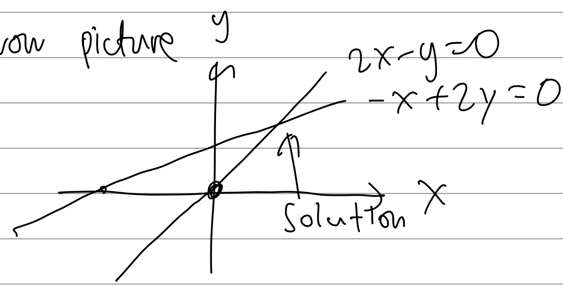
for now, n eqs and n unknowns

Example 1

$$\begin{cases} 2x - y = 0 \\ -x + 2y = 3 \end{cases}$$

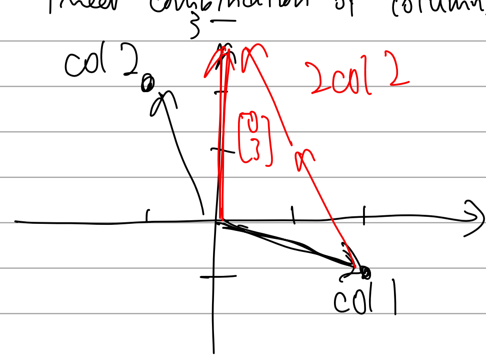
$$\underbrace{\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} 0 \\ 3 \end{bmatrix}}_{\mathbf{b}}$$

row picture



column picture

$$\underbrace{x\begin{bmatrix} 2 \\ -1 \end{bmatrix} + y\begin{bmatrix} -1 \\ 2 \end{bmatrix}}_{\text{linear combination of columns}} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$



☆ What are all the combinations?
What possible b's can we get?

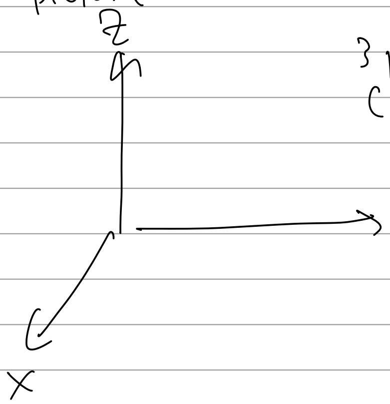
Example 2
3 eqs.
3 unknowns

$$2x - y = 0$$
$$-x + 2y - z = -1$$
$$-3y + 4z = 4$$

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -3 & 4 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

get a little unclear graphically for higher dimensions

Row picture

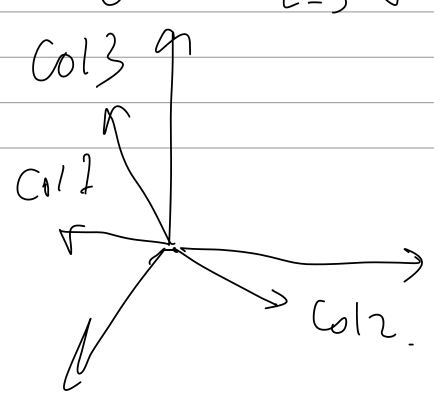


3 planes
(I didn't bother drawing).
turns out 3 planes intersect at a single point

Column picture

$$x\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix} + z\begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

same



so, $x = 0$,
$y = 0$
$z = 1$

Again, can I solve $Ax=b$
for every b?

i.e., do the linear combinations
of columns of A fill $\mathbb{R}^3$?

For this A, the answer is yes.

☆ How could it go wrong?

case is singular
mat not invertible
no solution for every b

e.g.: 3 columns lie in the same plane

Ax is a linear combination of
the columns of A.

Next time: ~~sys~~ a systematic way,
using elimination, to find the solution(s),
if any, to a system of any size
and find out, if elim fails,
when there isn't a solution.