→ con: existence proofs

· construct a network with human-chosen weights that is able to solve a problem

exhaustive,
so no consideration of generalization

· does not shine light on how we should find out those weights

1.1 1

a) P : have reading assigment

Q : have HW problems

R : have a test

$$(P \vee Q) \wedge (\neg(Q \wedge R))$$

b) P : go skiing

Q : there is snow

$$(\neg P) \vee (P \wedge \neg Q)$$

c) P : $\sqrt{7} \leq 2$

$\neg P$

core : existence proofs
· construct a network with human-chosen weights that is able to solve a problem

exhaustive, so no consideration of generalization
• does not shine light on how we should find out those weights

1.1
4)

a) P : Ralph is tall
Q : Ed is tall
R : Ralph is handsome
S : Ed is handsome

$(P \wedge Q) \vee (R \wedge S)$

b) $(P \vee R) \wedge (Q \vee S)$

c) $(\neg(P \vee R)) \wedge (\neg(Q \vee S))$

d) $\neg((P \wedge R) \vee (Q \wedge S))$

1.1 6)

a) $\neg(P \wedge \neg S)$ (by De Morgan's law)
$= \neg P \vee S$
I will not buy pants or I will by shirts

b) $\neg P \wedge \neg S = \neg(P \vee S)$ (by De Morgan's law)
I will not buy pants or shirt.

c) $\neg P \vee \neg S = \neg(P \wedge S)$ (by De Morgan's law)
I will not buy pants and shirt.

→ every class's

and is a hyperplane

abilities

ork

sigmoidal act.

→ core : existence proofs

· construct a network with human-chosen weights that is able to solve a problem

for training

al and estimate

e

Limitations

- usually exhaustive, so no consideration of generalization
- does not shine light on how we should find out those weigh

1.2

1. a) $\neg P \vee Q$

P	Q	$\neg P$	$\neg P \vee Q$ (or)
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

b) $(S \vee G) \wedge (\neg S \vee \neg G)$ (or, and, or)

S	G	$S \vee G$	$\neg S$	$\neg G$	$\neg S \vee \neg G$ (or)	$(S \vee G) \wedge (\neg S \vee \neg G)$ (and)
T	T	T	F	F	F	F
T	F	T	F	T	T	F
F	T	T	T	F	T	T
F	F	F	T	T	T	F

1.2 4

$P \vee G$ (or)

According to De Morgan's law

$$P \vee G = \neg(\neg P) \vee \neg(\neg G)$$
$$= \neg(\neg P \wedge \neg G)$$

Truth tables

- $P \vee G$

P	G	$P \vee G$
T	T	T
T	F	T
F	T	T
F	F	F

- $\neg(\neg P \wedge \neg G)$

$\neg P$	$\neg G$	$\neg P \wedge \neg G$ (and)	$\neg(\neg P \wedge \neg G)$
F	F	F	T
F	T	F	T
T	F	F	T
T	T	T	F

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abilities

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sigmoidal act.

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→ convex (see page 81)

→ every class's decision boundary must touch (see page 80) and is a hyperplane

→ core : existence proofs

- construct a network with human-chosen weights that is able to solve a problem

Limitations

- usually exhaustive, so no consideration of generalization
- does not shine light on how we should find out those weights

1.2 87)

a) $(P \wedge Q) \vee (\neg P \wedge \neg Q)$

b) $\neg P \vee Q$

c) $(P \vee \neg Q) \wedge (Q \vee \neg P)$

d) $\neg(P \vee Q)$

e) $(Q \wedge P) \vee \neg P$

a)

P	Q	$P \wedge Q$ (&)	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$ (&)	$(P\wedge Q) \vee (\neg P \wedge \neg Q)$ (or)
T	T	T	F	F	F	T
T	F	T	F	T	F	T
F	T	T	T	F	F	T
F	F	F	T	T	T	T

Using truth tables, we see that b) & e) are equivalent

b)

P	Q	$\neg P$	$\neg P \vee Q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

&

$(P \vee \neg Q) \wedge (Q \vee \neg P)$

c)

P	Q	$\neg Q$	$P \vee \neg Q$ (or)	$\neg P$	$Q \vee \neg P$	$\wedge$
T	T	F	T	F	T	T
T	F	T	T	F	F	F
F	T	F	F	T	T	F
F	F	T	T	T	T	T

d)

P	Q	$P \vee Q$ (or)	$\neg(P \vee Q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

e)

P	Q	$Q \wedge P$ (&)	$\neg P$	$(Q \wedge P) \vee \neg P$ (or)
T	T	T	F	T
T	F	F	F	F
F	T	F	T	T
F	F	F	T	T

→ core : existence proofs

act.

- construct a network with human-chosen weights that is able to solve a problem

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stimate

exhaustive,
so no consideration of generalization

- does not shine light on how we should find out those weig

1.2 #9

a) $(P \vee Q) \wedge (\neg P \vee \neg Q)$

P	Q	$P \vee Q$	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$	& $(P \vee Q) \wedge (\neg P \vee \neg Q)$
T	T	T	F	F	F	F
T	F	T	F	T	T	T
F	T	T	T	F	T	T
F	F	F	T	T	T	F
		T			T	

neither

b) $(P \vee Q) \wedge (\neg P \wedge \neg Q)$

(using truth table in a))

$\neg P \wedge \neg Q$	$(P \vee Q) \wedge (\neg P \wedge \neg Q)$
F	F
F	F
F	F
T	F

contradiction

c) using truths table from a)

$(P \vee Q) \vee (\neg P \vee \neg Q)$
T
T
T
T

tautology

d) tautology

$[P \wedge (Q \vee \neg R)] \vee [\neg P \vee R]$

P	Q	R	$\neg R$	or $Q \vee \neg R$	& $P \wedge (Q \vee \neg R)$	$\neg P$	$\neg P \vee R$	√
T	T	T	F	T	T	F	T	T
T	T	F	T	T	T	F	F	T
T	F	T	F	F	F	F	T	T
T	F	F	T	T	T	F	F	T
F	T	T	F	T	F	T	T	T
F	T	F	T	T	F	T	T	T
F	F	T	F	F	F	T	T	T
F	F	F	T	T	F	T	T	T

- does not shine light on how we should find out those weig

1.2 #12

a) $\neg(\neg P \vee Q) \vee (P \wedge \neg R)$

$\cong (P \wedge \neg Q) \vee (P \wedge \neg R)$ (De Morgan's law 2)

$\cong P \wedge (\neg Q \vee \neg R)$ (distributive law)

b) $\neg(\neg P \wedge Q) \vee (P \wedge \neg R)$

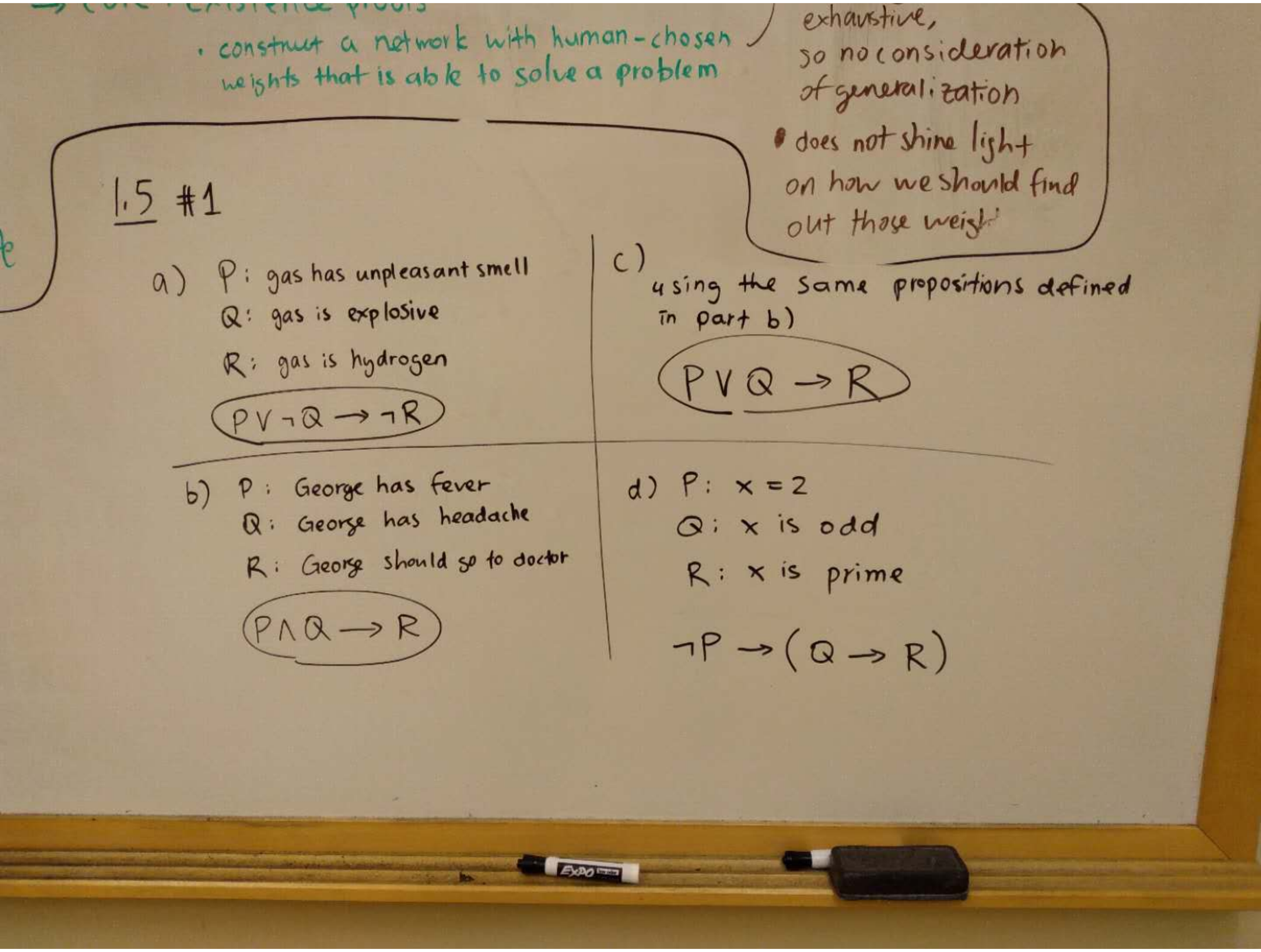
$\cong (P \vee \neg Q) \vee (P \wedge \neg R)$ (De Morgan's law 1)

c) $(P \wedge R) \vee [\neg R \wedge (P \vee Q)]$

$\cong [(P \wedge R) \vee (\neg R \wedge P)] \vee (\neg R \wedge Q)$ (distribute)

$\cong P \wedge \underbrace{(R \vee \neg R)}_{\text{tautology}} \vee (\neg R \wedge Q)$

$\cong P \vee (\neg R \wedge Q)$ (absorp.)



For question 4, see the end.

1.5 #7

a) $(P \to R) \wedge (Q \to R)$

$\cong (\neg P \vee R) \wedge (\neg Q \vee R)$ (conditional law 1)

$\cong (\neg P \wedge \neg Q) \vee R$ (distributive law)

$\cong \neg(P \vee Q) \vee R$ (De Morgan's law 2)

$\boxed{\cong (P \vee Q) \to R}$ (conditional law 1)

b) Show that $(P \to R) \vee (Q \to R) \cong (P \wedge Q) \to R$.

$(P \to R) \vee (Q \to R)$

$\cong (\neg P \vee R) \vee (\neg Q \vee R)$ (conditional law 1)

$\cong \neg P \vee \neg Q \vee R \vee R$ (commutative law)

$\cong (\neg P \vee \neg Q) \vee R$ (idempotent law)

$\cong [\neg (P \wedge Q)] \vee R$ (De Morgan's law 2)

$\boxed{\cong (P \wedge Q) \to R}$ (conditional law 1)

for question 8, see the end

1.5 #9

$$P \wedge Q$$

$$\cong \neg(\neg P) \wedge \neg(\neg Q)$$

$$\cong \neg(\neg P \vee (\neg Q)) \quad \text{(De Morgan's law 2)}$$

$$\cong \neg(P \to \neg Q) \quad \text{(conditional law 1)}$$

Homework for Math Structures
Monday, Week 1

Zhihan Yang

April 7, 2020

1 1.5 Question 4a

Propositions:

- P: Sales go up.
- Q: Expenses go up.
- R: Boss is happy.

Premises:

- Either sales or expenses will go up: $P \vee Q$.
- If sales go up, then the boss will be happy: $P \to R$.
- If expenses go up, then the boss will be unhappy: $Q \to \neg R$.

Argument: Sales and expenses will not both go up: $\neg(P \wedge Q)$.

p	q	r	(	(	(	p	$\vee$	q	)	&	(	p	$\to$	r	)	)	&	(	q	$\to$	$\sim$	r	)	)	$\sim$	q(
$\top$	$\top$	$\top$				$\top$	$\top$	$\top$		$\top$		$\top$	$\top$	$\top$			$\bot$		$\top$	$\bot$	$\bot$	$\top$			$\bot$	
$\top$	$\top$	$\bot$				$\top$	$\top$	$\top$		$\bot$		$\top$	$\bot$	$\bot$			$\bot$		$\top$	$\top$	$\top$	$\bot$			$\bot$	
$\top$	$\bot$	$\top$				$\top$	$\top$	$\bot$		$\top$		$\top$	$\top$	$\top$			$\top$		$\bot$	$\top$	$\bot$	$\top$			$\top$	
$\top$	$\bot$	$\bot$				$\top$	$\top$	$\bot$		$\bot$		$\top$	$\bot$	$\bot$			$\bot$		$\bot$	$\top$	$\top$	$\bot$			$\top$	
$\bot$	$\top$	$\top$				$\bot$	$\top$	$\top$		$\top$		$\bot$	$\top$	$\top$			$\bot$		$\top$	$\bot$	$\bot$	$\top$			$\top$	
$\bot$	$\top$	$\bot$				$\bot$	$\top$	$\top$		$\top$		$\bot$	$\top$	$\bot$			$\top$		$\top$	$\top$	$\top$	$\bot$			$\top$	
$\bot$	$\bot$	$\top$				$\bot$	$\bot$	$\bot$		$\bot$		$\bot$	$\top$	$\top$			$\bot$		$\bot$	$\top$	$\bot$	$\top$			$\top$	
$\bot$	$\bot$	$\bot$				$\bot$	$\bot$	$\bot$		$\bot$		$\bot$	$\top$	$\bot$			$\bot$		$\bot$	$\top$	$\top$	$\bot$			$\top$	

The 4th column shows the truth values for the 3 premises to be right at the same time. As we can see, when all premises are true (row 3 and 6), the argument is always true. Therefore, the argument is valid.

2 1.5 Question 4b

Propositions:

- P: Tax rate goes up.
- Q: Unemployment rate goes up.
- R: There is a recession.
- S: GDP goes up.

Premises:

- If the tax rate and the employment rate both go up, then there will be a recession: $P \land Q \to R$.
- If the GDP goes up, then there will not be a recession: $S \to \neg R$.
- The GDP and taxes are both going up: $S \land P$.

Argument: The employment rate is not going up: $\neg Q$.

P	Q	R	S	(	(	P	&	Q	)	$\to$	R	)	(	S	$\to$	$\sim$	R	)	(	S	&	P	)	$\sim$	Q
$\top$	$\top$	$\top$	$\top$			$\top$	$\top$	$\top$		$\top$	$\top$			$\top$	$\bot$	$\bot$	$\top$			$\top$	$\top$	$\top$		$\bot$	$\top$
$\top$	$\top$	$\top$	$\bot$			$\top$	$\top$	$\top$		$\top$	$\top$			$\bot$	$\top$	$\bot$	$\top$			$\bot$	$\bot$	$\top$		$\bot$	$\top$
$\top$	$\top$	$\bot$	$\top$			$\top$	$\top$	$\top$		$\bot$	$\bot$			$\top$	$\top$	$\top$	$\bot$			$\top$	$\top$	$\top$		$\bot$	$\top$
$\top$	$\top$	$\bot$	$\bot$			$\top$	$\top$	$\top$		$\bot$	$\bot$			$\bot$	$\top$	$\top$	$\bot$			$\bot$	$\bot$	$\top$		$\bot$	$\top$
$\top$	$\bot$	$\top$	$\top$			$\top$	$\bot$	$\bot$		$\top$	$\top$			$\top$	$\bot$	$\bot$	$\top$			$\top$	$\top$	$\top$		$\top$	$\bot$
$\top$	$\bot$	$\top$	$\bot$			$\top$	$\bot$	$\bot$		$\top$	$\top$			$\bot$	$\top$	$\bot$	$\top$			$\bot$	$\bot$	$\top$		$\top$	$\bot$
$\top$	$\bot$	$\bot$	$\top$			$\top$	$\bot$	$\bot$		$\top$	$\bot$			$\top$	$\top$	$\top$	$\bot$			$\top$	$\top$	$\top$		$\top$	$\bot$
$\top$	$\bot$	$\bot$	$\bot$			$\top$	$\bot$	$\bot$		$\top$	$\bot$			$\bot$	$\top$	$\top$	$\bot$			$\bot$	$\bot$	$\top$		$\top$	$\bot$
$\bot$	$\top$	$\top$	$\top$			$\bot$	$\bot$	$\top$		$\top$	$\top$			$\top$	$\bot$	$\bot$	$\top$			$\top$	$\bot$	$\bot$		$\bot$	$\top$
$\bot$	$\top$	$\top$	$\bot$			$\bot$	$\bot$	$\top$		$\top$	$\top$			$\bot$	$\top$	$\bot$	$\top$			$\bot$	$\bot$	$\bot$		$\bot$	$\top$
$\bot$	$\top$	$\bot$	$\top$			$\bot$	$\bot$	$\top$		$\top$	$\bot$			$\top$	$\top$	$\top$	$\bot$			$\top$	$\bot$	$\bot$		$\bot$	$\top$
$\bot$	$\top$	$\bot$	$\bot$			$\bot$	$\bot$	$\top$		$\top$	$\bot$			$\bot$	$\top$	$\top$	$\bot$			$\bot$	$\bot$	$\bot$		$\bot$	$\top$
$\bot$	$\bot$	$\top$	$\top$			$\bot$	$\bot$	$\bot$		$\top$	$\top$			$\top$	$\bot$	$\bot$	$\top$			$\top$	$\bot$	$\bot$		$\top$	$\bot$
$\bot$	$\bot$	$\top$	$\bot$			$\bot$	$\bot$	$\bot$		$\top$	$\top$			$\bot$	$\top$	$\bot$	$\top$			$\bot$	$\bot$	$\bot$		$\top$	$\bot$
$\bot$	$\bot$	$\bot$	$\top$			$\bot$	$\bot$	$\bot$		$\top$	$\bot$			$\top$	$\top$	$\top$	$\bot$			$\top$	$\bot$	$\bot$		$\top$	$\bot$
$\bot$	$\bot$	$\bot$	$\bot$			$\bot$	$\bot$	$\bot$		$\top$	$\bot$			$\bot$	$\top$	$\top$	$\bot$			$\bot$	$\bot$	$\bot$		$\top$	$\bot$

The 7 seventh row shows that the argument $\neg Q$ is always true when all premises are true. Therefore, $\neg Q$ is a valid argument. (The truth values of the premises are shown in red.)

3 1.5 Question 4c

Propositions:

- P: Warning light comes.
- Q: The pressure is too high.
- R: The relief valve is clogged.

Premises:

- The warning light will come if and only if the pressure is too high and the relief valve is clogged: $P \iff Q \wedge R$.
- The relief valve is not clogged: $\neg R$.

Argument: The warning light will come on if and only if the pressure is too high: $P \iff Q$.

P	Q	R	(	P	$\leftrightarrow$	(	Q	&	R	)	)	$\sim$	R	(	P	$\leftrightarrow$	Q	)
$\top$	$\top$	$\top$		$\top$	$\top$		$\top$	$\top$	$\top$			$\bot$	$\top$		$\top$	$\top$	$\top$	
$\top$	$\top$	$\bot$		$\top$	$\bot$		$\top$	$\bot$	$\bot$			$\top$	$\bot$		$\top$	$\top$	$\top$	
$\top$	$\bot$	$\top$		$\top$	$\bot$		$\bot$	$\bot$	$\top$			$\bot$	$\top$		$\top$	$\bot$	$\bot$	
$\top$	$\bot$	$\bot$		$\top$	$\bot$		$\bot$	$\bot$	$\bot$			$\top$	$\bot$		$\top$	$\bot$	$\bot$	
$\bot$	$\top$	$\top$		$\bot$	$\bot$		$\top$	$\top$	$\top$			$\bot$	$\top$		$\bot$	$\bot$	$\top$	
$\bot$	$\top$	$\bot$		$\bot$	$\top$		$\top$	$\bot$	$\bot$			$\top$	$\bot$		$\bot$	$\bot$	$\top$	
$\bot$	$\bot$	$\top$		$\bot$	$\top$		$\bot$	$\bot$	$\top$			$\bot$	$\top$		$\bot$	$\top$	$\bot$	
$\bot$	$\bot$	$\bot$		$\bot$	$\top$		$\bot$	$\bot$	$\bot$			$\top$	$\bot$		$\bot$	$\top$	$\bot$	

The 6th row shows that, even when both premises are truth, it is possible for the argument to be false. Therefore, the argument $P \iff Q$ is invalid.

4 1.5 Question 8a

The truth table for $(P \to Q) \wedge (Q \to R)$ looks like:

P	Q	R	(	(	P	$\to$	Q	)	&	(	Q	$\to$	R	)	)
$\top$	$\top$	$\top$			$\top$	$\top$	$\top$		$\top$		$\top$	$\top$	$\top$		
$\top$	$\top$	$\bot$			$\top$	$\top$	$\top$		$\bot$		$\top$	$\bot$	$\bot$		
$\top$	$\bot$	$\top$			$\top$	$\bot$	$\bot$		$\bot$		$\bot$	$\top$	$\top$		
$\top$	$\bot$	$\bot$			$\top$	$\bot$	$\bot$		$\bot$		$\bot$	$\top$	$\bot$		
$\bot$	$\top$	$\top$			$\bot$	$\top$	$\top$		$\top$		$\top$	$\top$	$\top$		
$\bot$	$\top$	$\bot$			$\bot$	$\top$	$\top$		$\bot$		$\top$	$\bot$	$\bot$		
$\bot$	$\bot$	$\top$			$\bot$	$\top$	$\bot$		$\top$		$\bot$	$\top$	$\top$		
$\bot$	$\bot$	$\bot$			$\bot$	$\top$	$\bot$		$\top$		$\bot$	$\top$	$\bot$		

The truth table for $(P \to R) \wedge [(P \iff Q) \vee (R \iff Q)]$ looks like:

P	Q	R	(	(	P	$\to$	R	)	&	(	(	P	$\to$	Q	)	$\vee$	(	R	$\to$	Q	)	)	)
$\top$	$\top$	$\top$			$\top$	$\top$	$\top$		$\top$			$\top$	$\top$	$\top$		$\top$		$\top$	$\top$	$\top$			
$\top$	$\top$	$\bot$			$\top$	$\bot$	$\bot$		$\bot$			$\top$	$\top$	$\top$		$\top$		$\bot$	$\top$	$\top$			
$\top$	$\bot$	$\top$			$\top$	$\top$	$\top$		$\bot$			$\top$	$\bot$	$\bot$		$\bot$		$\top$	$\bot$	$\bot$			
$\top$	$\bot$	$\bot$			$\top$	$\bot$	$\bot$		$\bot$			$\top$	$\bot$	$\bot$		$\top$		$\bot$	$\top$	$\bot$			
$\bot$	$\top$	$\top$			$\bot$	$\top$	$\top$		$\top$			$\bot$	$\top$	$\top$		$\top$		$\top$	$\top$	$\top$			
$\bot$	$\top$	$\bot$			$\bot$	$\top$	$\bot$		$\top$			$\bot$	$\top$	$\top$		$\top$		$\bot$	$\top$	$\top$			
$\bot$	$\bot$	$\top$			$\bot$	$\top$	$\top$		$\top$			$\bot$	$\top$	$\bot$		$\top$		$\top$	$\bot$	$\bot$			
$\bot$	$\bot$	$\bot$			$\bot$	$\top$	$\bot$		$\top$			$\bot$	$\top$	$\bot$		$\top$		$\bot$	$\top$	$\bot$			

Since their truth tables look identical, we have shown that the two logical expressions are equivalent.

5 1.5 Question 8b

The truth table for $(P \to Q) \vee (Q \to R)$ looks like:

P	Q	R	(	(	P	$\to$	Q	)	$\vee$	(	Q	$\to$	R	)	)
$\top$	$\top$	$\top$			$\top$	$\top$	$\top$		$\top$		$\top$	$\top$	$\top$		
$\top$	$\top$	$\bot$			$\top$	$\top$	$\top$		$\top$		$\top$	$\bot$	$\bot$		
$\top$	$\bot$	$\top$			$\top$	$\bot$	$\bot$		$\top$		$\bot$	$\top$	$\top$		
$\top$	$\bot$	$\bot$			$\top$	$\bot$	$\bot$		$\top$		$\bot$	$\top$	$\bot$		
$\bot$	$\top$	$\top$			$\bot$	$\top$	$\top$		$\top$		$\top$	$\top$	$\top$		
$\bot$	$\top$	$\bot$			$\bot$	$\top$	$\top$		$\top$		$\top$	$\bot$	$\bot$		
$\bot$	$\bot$	$\top$			$\bot$	$\top$	$\bot$		$\top$		$\bot$	$\top$	$\top$		
$\bot$	$\bot$	$\bot$			$\bot$	$\top$	$\bot$		$\top$		$\bot$	$\top$	$\bot$		

Since all truth values are true, we have shown that $(P \to Q) \vee (Q \to R)$ is a tautology.