Homework for Math Structures Monday, Week 1

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1 1.5 Question 4a

Propositions:

- P: Sales go up.
- Q: Expenses go up.
- R: Boss is happy.

Premises:

- Either sales or expenses will go up: $P \vee Q$.
- If sales go up, then the boss will be happy: $P \to R$.
- If expenses go up, then the boss will be unhappy: $Q \to \neg R$.

Argument: Sales and expenses will not both go up: $\neg(P \land Q)$.

p	q	r	(((p	\vee	q)	&	(p	\rightarrow	r))	&	(q	\rightarrow	\sim	r))	\sim	(
\top	T	Т				Т	Т	Т		Т		Т	Т	Т			\perp		T	\perp	\perp	Т			\perp	
T	T	\perp				T	T	\top		\perp		T	\perp	\perp			\perp		Τ	T	T	\perp			丄	
T	\perp	Т				\top	\top	\perp		\top		\top	Т	\top			T		\perp	T	\perp	T			Т	
Τ	\perp	\perp				\top	T	\perp		\perp		\top	\perp	\perp			\perp		\perp	T	T	\perp			Т	
\perp	\top	Т				\perp	\top	Т		\top		\perp	\top	Τ			\perp		Т	\perp	\perp	Т			Т	
\perp	\top	\perp				\perp	T	\top		\top		\perp	T	\perp			T		Τ	T	T	\perp			Т	
\perp	\perp	\top				\perp	\perp	\perp		\perp		\perp	T	T			\perp		\perp	T	\perp	T			Т	
\perp	\perp	\perp				\perp	\perp	\perp		\perp		\perp	Т	\perp			\perp		\perp	Т	Т	\perp			T	

The 4th column shows the truth values for the 3 premises to be right at the same time. As we can see, when all premises are true (row 3 and 6), the argument is always true. Therefore, the argument is valid.

2 1.5 Question 4b

Propositions:

- P: Tax rate goes up.
- Q: Unemployment rate goes up.
- R: There is a recession.
- S: GDP goes up.

Premises:

- If the tax rate and the employment rate both go up, then there will be a recession: $P \wedge Q \rightarrow R$.
- If the GDP goes up, then there will not be a recession: $S \to \neg R$.
- The GDP and taxes are both going up: $S \wedge P$.

Argument: The employment rate is not going up: $\neg Q$.

Р	Q	R	S	((Р	&	Q)	\rightarrow	R)	(\mathbf{S}	\rightarrow	\sim	R)	(S	&	Р)	\sim	Q
T	Т	Т	Т			T	Т	Т		Т	Т			Т	\perp	\perp	Т			Т	T	T		1	Т
Τ	Τ	\top	\perp			T	Т	\top		T	T			\perp	T	\perp	\top			\perp	\perp	T		上	Τ
T	T	\perp	T			Т	T	T		\perp	\perp			T	T	\top	\perp			Т	T	Т		上	Τ
T	T	\perp	\perp			Т	T	T		\perp	\perp			\perp	T	\top	\perp			\perp	\perp	Т		上	Τ
Т	\perp	T	Т			Т	\perp	\perp		T	Т			T	\perp	\perp	Т			Т	T	T		T	\perp
Т	\perp	T	\perp			Т	\perp	\perp		T	Т			\perp	Т	\perp	Т			\perp	\perp	T		T	\perp
Т	\perp	\perp	T			Т	\perp	\perp		T	\perp			T	T	\top	\perp			T	T	Τ		T	\perp
Т	\perp	\perp	\perp			Т	\perp	\perp		T	\perp			\perp	Т	Т	\perp			\perp	\perp	T		T	\perp
\perp	Т	Т	T			\perp	\perp	T		T	Т			T	\perp	\perp	T			T	\perp	\perp		上	Т
\perp	T	T	\perp			\perp	\perp	\top		T	\top			\perp	\top	\perp	T			\perp	\perp	\perp		上	Т
\perp	Т	\perp	T			\perp	\perp	T		T	\perp			T	T	\top	\perp			T	\perp	\perp		上	Т
\perp	T	\perp	\perp			\perp	\perp	\top		T	\perp			\perp	\top	Τ	\perp			\perp	\perp	\perp		上	Τ
\perp	\perp	T	Т			\perp	\perp	\perp		T	Т			T	\perp	\perp	Т			Т	\perp	\perp		T	\perp
\perp	\perp	T	\perp			\perp	\perp	\perp		T	\top			\perp	\top	\perp	T			\perp	\perp	\perp		T	\perp
\perp	\perp	\perp	T			\perp	\perp	\perp		T	\perp			T	T	\top	\perp			T	\perp	\perp		T	\perp
	- 1						- 1			Т	- 1			- 1	Т	Т	- 1							ΙT	

The 7 seventh row shows that the argument $\neg Q$ is always true when all premises are true. Therefore, $\neg Q$ is a valid argument. (The truth values of the premises are shown in red.)

3 1.5 Question 4c

Propositions:

- P: Warning light comes.
- Q: The pressure is too high.
- R: The relief valve is clogged.

Premises:

- The warning light will come if and only if the pressure is too high and the relief valve is clogged: $P \iff Q \land R$.
- The relief valve is not clogged: $\neg R$.

Argument: The warning light will come on if and only if the pressure is too high: $P \iff Q$.

Р	Q	R	(Р	\leftrightarrow	(Q	&	R))	\sim	R	(Р	\leftrightarrow	Q)
T	Т	T		T	T		T	T	T				Т		T	Т	T	
T	T	\perp		T	\perp		T	\perp	\perp			T	\perp		T	T	T	
Т	\perp	T		T	\perp		\perp	\perp	T				T		T	\perp	\perp	
T	\perp	\perp		\top	\perp		\perp	\perp	\perp			T	\perp		T	\perp	\perp	
\perp	\top	\top		\perp	\perp		\top	\top	\top				Т		\perp	\perp	T	
\perp	Т	\perp		\perp	T		Т	\perp	\perp			T	\perp		\perp	\perp	T	
\perp	\perp	T		\perp	T		\perp	\perp	T			上	T		\perp	T	\perp	
\perp	\perp	\perp		\perp	T		\perp	\perp	\perp			T	\perp		\perp	T	\perp	

The 6th row shows that, even when both premises are truth, it is possible for the argument to be false. Therefore, the argument $P \iff Q$ is invalid.

4 1.5 Question 8a

The truth table for $(P \to Q) \land (Q \to R)$ looks like:

P Q R	((Р	\rightarrow	Q)	&	(Q	\rightarrow	R))
T T T		Т	T	T		T		T	T	T		
\top \top \bot		T	T	T		\perp		T	\perp	\perp		
\top \bot \top		\top	\perp	\perp		\perp		\perp	\top	\top		
\top \bot \bot		Т	\perp	\perp		\perp		\perp	Т	\perp		
\perp \top \top		\perp	Т	\top		T		\top	\top	\top		
\bot \top \bot		\perp	Т	T		\perp		Т	\perp	\perp		
\bot \bot \top		\perp	Т	\perp		T		\perp	\top	\top		
\bot \bot \bot		\perp	Т	\perp		Т		\perp	Т	\perp		

The truth table for $(P \to R) \wedge [(P \iff Q) \vee (R \iff Q)]$ looks like:

P Q R	((Р	\rightarrow	R)	&	((Р	\rightarrow	Q)	\vee	(R	\rightarrow	Q)))
T T T		Т	T	Т		Т			Т	T	Т		Т		Т	Т	T			
\top \top \bot		\top	\perp	\perp		\perp			\top	T	T		\top		\perp	T	T			
\top \bot \top		\top	T	T		\perp			T	\perp	\perp		\perp		T	\perp	\perp			
\top \bot \bot		T	\perp	\perp		\perp			Т	\perp	\perp		Т		\perp	Т	\perp			
\perp \top \top		\perp	Т	T		T			\perp	Т	Т		Т		T	Т	T			
\bot \top \bot		\perp	Т	\perp		Τ			\perp	Т	Т		T		\perp	Т	Т			
\perp \perp \top		\perp	Т	T		T			\perp	Т	\perp		Т		T	\perp	\perp			
\perp \perp \perp		\perp	Т	\perp		Τ			\perp	Т	\perp		Τ		\perp	Т	\perp			

Since their truth tables look identical, we have shown that the two logical expressions are equivalent.

5 1.5 Question 8b

The truth table for $(P \to Q) \lor (Q \to R)$ looks like:

Р	Q	\mathbf{R}	((Р	\rightarrow	Q)	\vee	(Q	\rightarrow	\mathbf{R}))
T	T	T			Т	T	T		T		T	Т	Т		
T	T	\perp			T	T	T		T		T	\perp	\perp		
\top	\perp	Т			\top	\perp	\perp		T		\perp	\top	\top		
T	\perp	\perp			Т	\perp	\perp		T		\perp	\top	\perp		
\perp	T	Т			\perp	Т	T		T		Т	\top	Т		
\perp	Т	\perp			\perp	\top	T		\top		Т	\perp	\perp		
\perp	\perp	Т			\perp	Т	\perp		T		\perp	\top	Т		
\perp	\perp	\perp			\perp	\top	\perp		T		\perp	\top	\perp		

Since all truth values are true, we have shown that $(P \to Q) \lor (Q \to R)$ is a tautology.