

→ Core: Existence proofs

- construct a network with human-chosen weights that is able to solve a problem

exhaustive,  
so no consideration  
of generalization

- does not shine light  
on how we should find  
out these weights

1.1 1

- a) P: have reading assignment  
Q: have HW problems  
R: have a test

$$(P \vee Q) \wedge (\neg(Q \wedge R))$$

- b) P: go skiing  
Q: there is snow

$$(\neg P) \vee (P \wedge \neg Q)$$

c)  $P: \sqrt{7} \leq 2$

$$\neg P$$

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1.1

4)

- a)  $P$ : Ralph is tall  
 $Q$ : Ed is tall  
 $R$ : Ralph is handsome  
 $S$ : Ed is handsome

$$(P \wedge Q) \vee (R \wedge S)$$

b)  $(P \vee R) \wedge (Q \vee S)$

c)  $(\neg(P \vee R)) \wedge (\neg(Q \vee S))$

d)  $\neg((P \wedge R) \vee (Q \wedge S))$

1.1 6)

a)  $\neg(P \wedge \neg S)$  (by De Morgan's law)  
 $= \neg P \vee S$

I will not buy pants or I will buy shirts

b)  $\neg P \wedge \neg S = \neg(P \vee S)$  (by De Morgan's law)  
I will not buy pants or shirt.

c)  $\neg P \vee \neg S = \neg(P \wedge S)$  (by De Morgan's law)  
I will not buy pants and shirt.

→ every class's decision boundary is a hyperplane

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1.2

1. a)  $\neg P \vee Q$

P	Q	$\neg P$	$\neg P \vee Q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

b)  $(S \vee G) \wedge (\neg S \vee \neg G)$

S	G	$S \vee G$	$\neg S$	$\neg G$	$\neg S \vee \neg G$	$(S \vee G) \wedge (\neg S \vee \neg G)$
T	T	T	F	F	F	F
T	F	T	F	T	T	F
F	T	T	T	F	T	F
F	F	F	T	T	T	F

1.2 4

PVG

According to De Morgan's law

$$PVG = \neg(P) \vee \neg(G)$$

$$= \neg(P \wedge G)$$

Truth tables

• PVG

P	G	PVG
T	T	F
T	F	T
F	T	T
F	F	T

•  $\neg(P \wedge G)$

$\neg P$	$\neg G$	$\neg(P \wedge G)$
F	F	F
F	T	T
T	F	T
T	T	F



→ convex (see page 81)

→ every class's decision boundary must touch (see page 80) and is a hyperplane

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1.2 8)

a)  $(P \wedge Q) \vee (\neg P \wedge \neg Q)$

b)  $\neg P \vee Q$

c)  $(P \vee \neg Q) \wedge (Q \vee \neg P)$

d)  $\neg(P \vee Q)$

e)  $(Q \wedge P) \vee \neg P$

P	Q	$P \wedge Q$	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$	$(P \wedge Q) \vee (\neg P \wedge \neg Q)$
T	T	T	F	F	F	T
T	F	F	F	T	F	F
F	T	F	T	F	F	F
F	F	F	T	T	T	T

Using truth tables, we see that b) & e) are equivalent.

P	Q	$\neg P$	$\neg P \vee Q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

$(P \vee \neg Q) \wedge (Q \vee \neg P)$

P	Q	$\neg Q$	$P \vee \neg Q$	$\neg P$	$Q \vee \neg P$	$(P \vee \neg Q) \wedge (Q \vee \neg P)$
T	T	F	T	F	T	T
T	F	T	T	F	F	F
F	T	F	F	T	T	F
F	F	T	T	T	T	T

P	Q	$P \vee Q$	$\neg(P \vee Q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

P	Q	$Q \wedge P$	$\neg P$	$(Q \wedge P) \vee \neg P$
T	T	T	F	T
T	F	F	F	F
F	T	F	T	T
F	F	F	T	T

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## 1.2 #9

a)  $(P \vee Q) \wedge (\neg P \vee \neg Q)$

P	Q	$P \vee Q$	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$	$(P \vee Q) \wedge (\neg P \vee \neg Q)$
T	T	T	F	F	F	F
T	F	T	F	T	T	T
F	T	T	T	F	T	T
F	F	F	T	T	T	F

neither

b)  $(P \vee Q) \wedge (\neg P \wedge \neg Q)$

(using  
truth table  
in a))

$\neg P \wedge \neg Q$	$(P \vee Q) \wedge (\neg P \wedge \neg Q)$
F	F
F	F
F	F
T	F

contradiction

c) using truth table from a)

$(P \vee Q) \vee (\neg P \vee \neg Q)$

T
T
T
T

tautology

d) tautology or  $[P \wedge (Q \vee R)] \vee [\neg P \wedge \neg R]$

P	Q	R	$\neg R$	$Q \vee R$	$P \wedge (Q \vee R)$	$\neg P$	$\neg R$	$[\neg P \wedge \neg R]$
T	T	T	F	T	T	F	F	F
T	T	F	T	T	T	F	T	F
T	F	T	F	F	F	F	F	F
T	F	F	T	F	F	F	T	F
F	T	T	F	T	F	T	F	F
F	T	F	T	F	F	T	T	T
F	F	T	F	T	F	T	F	F
F	F	F	T	F	F	T	T	T

1.2 #12

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$$\begin{aligned} a) & \neg(\neg P \vee Q) \vee (P \wedge \neg R) \\ & \equiv (P \wedge \neg Q) \vee (P \wedge \neg R) \quad (\text{De Morgan's law 2}) \\ & \equiv P \wedge (\neg Q \vee \neg R) \quad (\text{distributive law}) \end{aligned}$$

$$\begin{aligned} b) & \neg(\neg P \wedge Q) \vee (P \wedge \neg R) \\ & \equiv (P \vee \neg Q) \vee (P \wedge \neg R) \quad (\text{De Morgan's law 1}) \end{aligned}$$

$$\begin{aligned} c) & (P \wedge R) \vee [\neg R \wedge (P \vee Q)] \\ & \equiv [(P \wedge R) \vee (\neg R \wedge P)] \vee (\neg R \wedge Q) \quad (\text{distribute}) \\ & \equiv P \wedge (R \vee \neg R) \vee (\neg R \wedge Q) \\ & \quad \text{tautology} \end{aligned}$$

$$\rightarrow \equiv P \vee (\neg R \wedge Q) \quad (\text{absorp.})$$



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1.5 #1

- a) P: gas has unpleasant smell  
Q: gas is explosive  
R: gas is hydrogen

$$P \vee \neg Q \rightarrow \neg R$$

- b) P: George has fever  
Q: George has headache  
R: George should go to doctor

$$P \wedge Q \rightarrow R$$

- c) using the same propositions defined  
in part b)

$$P \vee Q \rightarrow R$$

- d) P:  $x = 2$   
Q:  $x$  is odd  
R:  $x$  is prime

$$\neg P \rightarrow (Q \rightarrow R)$$

For question 4, see  
the end.

1.5 #7

a)  $(P \rightarrow R) \wedge (Q \rightarrow R)$

$$\cong (\neg P \vee R) \wedge (\neg Q \vee R) \quad (\text{conditional law 1})$$

$$\cong (\neg P \wedge \neg Q) \vee R \quad (\text{distributive law})$$

$$\cong \neg(P \vee Q) \vee R \quad (\text{De Morgan's law 2})$$

$$\cong (P \vee Q) \rightarrow R \quad (\text{conditional law 1})$$

b) Show that  $(P \rightarrow R) \vee (Q \rightarrow R) \cong (P \wedge Q) \rightarrow R$ .

$$(P \rightarrow R) \vee (Q \rightarrow R)$$

$$\cong (\neg P \vee R) \vee (\neg Q \vee R) \quad (\text{conditional law 1})$$

$$\cong \neg P \vee \neg Q \vee R \vee R \quad (\text{commutative law})$$

$$\cong (\neg P \vee \neg Q) \vee R \quad (\text{idempotent law})$$

$$\cong \neg(P \wedge Q) \vee R \quad (\text{De Morgan's law 2})$$

$$\cong (P \wedge Q) \rightarrow R \quad (\text{conditional law 1})$$



for question 8, see  
the end

1.5 #9

$$P \wedge Q$$

$$\equiv \neg(\neg P) \wedge \neg(\neg Q)$$

$$\equiv \neg(\neg P \vee (\neg Q)) \quad (\text{De Morgan's law 2})$$

$$\equiv \neg(P \rightarrow \neg Q) \quad (\text{conditional law 1})$$