Homework for Math Structures

Monday, Week 1

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1 1.5 Question 4a

Propositions:

- P: Sales go up.
- Q: Expenses go up.
- R: Boss is happy.

Premises:

- Either sales or expenses will go up: $P \vee Q$.
- If sales go up, then the boss will be happy: $P \to R$.
- If expenses go up, then the boss will be unhappy: $Q \to \neg R$.

Argument: Sales and expenses will not both go up: $\neg(P \wedge Q)$.

p	q	r	(	(	(	p	$\vee$	q	)	&	(	p	$\to$	r	)	)	&	(	q	$\to$	$\sim$	r	)	)	$\sim$	(
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$\bot$	$\bot$	$\top$				$\bot$	$\bot$	$\bot$		$\bot$		$\bot$	$\top$	$\top$			$\bot$		$\bot$	$\top$	$\bot$	$\top$			$\top$	
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The 4th column shows the truth values for the 3 premises to be right at the same time. As we can see, when all premises are true (row 3 and 6), the argument is always true. Therefore, the argument is valid.

2 1.5 Question 4b

Propositions:

- P: Tax rate goes up.
- Q: Unemployment rate goes up.
- R: There is a recession.
- S: GDP goes up.

Premises:

- If the tax rate and the employment rate both go up, then there will be a recession: $P \wedge Q \rightarrow R$.
- If the GDP goes up, then there will not be a recession: $S \rightarrow \neg R$.
- The GDP and taxes are both going up: $S \wedge P$.

Argument: The employment rate is not going up: $\neg Q$.

P	Q	R	S	(	(	P	&	Q	)	→	R	)	(	S	→	∼	R	)	(	S	&	P	)	∼	Q
⊤	⊤	⊤	⊤			⊤	⊤	⊤		⊤	⊤			⊤	⊥	⊥	⊤			⊤	⊤	⊤		⊥	⊤
⊤	⊤	⊤	⊥			⊤	⊤	⊤		⊤	⊤			⊥	⊤	⊥	⊤			⊥	⊥	⊤		⊥	⊤
⊤	⊤	⊥	⊤			⊤	⊤	⊤		⊥	⊥			⊤	⊤	⊤	⊥			⊤	⊤	⊤		⊥	⊤
⊤	⊤	⊥	⊥			⊤	⊤	⊤		⊥	⊥			⊥	⊤	⊤	⊥			⊥	⊥	⊤		⊥	⊤
⊤	⊥	⊤	⊤			⊤	⊥	⊥		⊤	⊤			⊤	⊥	⊥	⊤			⊤	⊤	⊤		⊤	⊥
⊤	⊥	⊤	⊥			⊤	⊥	⊥		⊤	⊤			⊥	⊤	⊥	⊤			⊥	⊥	⊤		⊤	⊥
⊤	⊥	⊥	⊤			⊤	⊥	⊥		⊤	⊥			⊤	⊤	⊤	⊥			⊤	⊤	⊤		⊤	⊥
⊤	⊥	⊥	⊥			⊤	⊥	⊥		⊤	⊥			⊥	⊤	⊤	⊥			⊥	⊥	⊤		⊤	⊥
⊥	⊤	⊤	⊤			⊥	⊥	⊤		⊤	⊤			⊤	⊥	⊥	⊤			⊤	⊥	⊥		⊥	⊤
⊥	⊤	⊤	⊥			⊥	⊥	⊤		⊤	⊤			⊥	⊤	⊥	⊤			⊥	⊥	⊥		⊥	⊤
⊥	⊤	⊥	⊤			⊥	⊥	⊤		⊤	⊥			⊤	⊤	⊤	⊥			⊤	⊥	⊥		⊥	⊤
⊥	⊤	⊥	⊥			⊥	⊥	⊤		⊤	⊥			⊥	⊤	⊤	⊥			⊥	⊥	⊥		⊥	⊤
⊥	⊥	⊤	⊤			⊥	⊥	⊥		⊤	⊤			⊤	⊥	⊥	⊤			⊤	⊥	⊥		⊤	⊥
⊥	⊥	⊤	⊥			⊥	⊥	⊥		⊤	⊤			⊥	⊤	⊥	⊤			⊥	⊥	⊥		⊤	⊥
⊥	⊥	⊥	⊤			⊥	⊥	⊥		⊤	⊥			⊤	⊤	⊤	⊥			⊤	⊥	⊥		⊤	⊥
⊥	⊥	⊥	⊥			⊥	⊥	⊥		⊤	⊥			⊥	⊤	⊤	⊥			⊥	⊥	⊥		⊤	⊥

The 7 seventh row shows that the argument $\neg Q$ is always true when all premises are true. Therefore, $\neg Q$ is a valid argument. (The truth values of the premises are shown in red.)

3 1.5 Question 4c

Propositions:

- P: Warning light comes.
- Q: The pressure is too high.
- R: The relief valve is clogged.

Premises:

- The warning light will come if and only if the pressure is too high and the relief valve is clogged: $P \iff Q \wedge R$.
- The relief valve is not clogged: $\neg R$.

Argument: The warning light will come on if and only if the pressure is too high: $P \iff Q$.

P	Q	R	(	P	↔	(	Q	&	R	)	)	∼	R	(	P	↔	Q	)
⊤	⊤	⊤		⊤	⊤		⊤	⊤	⊤			⊥	⊤		⊤	⊤	⊤	
⊤	⊤	⊥		⊤	⊥		⊤	⊥	⊥			⊤	⊥		⊤	⊤	⊤	
⊤	⊥	⊤		⊤	⊥		⊥	⊥	⊤			⊥	⊤		⊤	⊥	⊥	
⊤	⊥	⊥		⊤	⊥		⊥	⊥	⊥			⊤	⊥		⊤	⊥	⊥	
⊥	⊤	⊤		⊥	⊥		⊤	⊤	⊤			⊥	⊤		⊥	⊥	⊤	
⊥	⊤	⊥		⊥	⊤		⊤	⊥	⊥			⊤	⊥		⊥	⊥	⊤	
⊥	⊥	⊤		⊥	⊤		⊥	⊥	⊤			⊥	⊤		⊥	⊤	⊥	
⊥	⊥	⊥		⊥	⊤		⊥	⊥	⊥			⊤	⊥		⊥	⊤	⊥	

The 6th row shows that, even when both premises are truth, it is possible for the argument to be false. Therefore, the argument $P \iff Q$ is invalid.

4 1.5 Question 8a

The truth table for $(P \to Q) \wedge (Q \to R)$ looks like:

P	Q	R	(	(	P	→	Q	)	&	(	Q	→	R	)	)
⊤	⊤	⊤			⊤	⊤	⊤		⊤		⊤	⊤	⊤		
⊤	⊤	⊥			⊤	⊤	⊤		⊥		⊤	⊥	⊥		
⊤	⊥	⊤			⊤	⊥	⊥		⊥		⊥	⊤	⊤		
⊤	⊥	⊥			⊤	⊥	⊥		⊥		⊥	⊤	⊥		
⊥	⊤	⊤			⊥	⊤	⊤		⊤		⊤	⊤	⊤		
⊥	⊤	⊥			⊥	⊤	⊤		⊥		⊤	⊥	⊥		
⊥	⊥	⊤			⊥	⊤	⊥		⊤		⊥	⊤	⊤		
⊥	⊥	⊥			⊥	⊤	⊥		⊤		⊥	⊤	⊥		

The truth table for $(P \to R) \wedge [(P \iff Q) \vee (R \iff Q)]$ looks like:

P	Q	R	(	(	P	→	R	)	&	(	(	P	→	Q	)	∨	(	R	→	Q	)	)	)
⊤	⊤	⊤			⊤	⊤	⊤		⊤			⊤	⊤	⊤		⊤		⊤	⊤	⊤			
⊤	⊤	⊥			⊤	⊥	⊥		⊥			⊤	⊤	⊤		⊤		⊥	⊤	⊤			
⊤	⊥	⊤			⊤	⊤	⊤		⊥			⊤	⊥	⊥		⊥		⊤	⊥	⊥			
⊤	⊥	⊥			⊤	⊥	⊥		⊥			⊤	⊥	⊥		⊤		⊥	⊤	⊥			
⊥	⊤	⊤			⊥	⊤	⊤		⊤			⊥	⊤	⊤		⊤		⊤	⊤	⊤			
⊥	⊤	⊥			⊥	⊤	⊤		⊥			⊥	⊤	⊤		⊤		⊥	⊤	⊤			
⊥	⊥	⊤			⊥	⊤	⊥		⊤			⊥	⊤	⊥		⊤		⊤	⊥	⊥			
⊥	⊥	⊥			⊥	⊤	⊥		⊤			⊥	⊤	⊥		⊤		⊥	⊤	⊥			

Since their truth tables look identical, we have shown that the two logical expressions are equivalent.

5 1.5 Question 8b

The truth table for $(P \to Q) \vee (Q \to R)$ looks like:

P	Q	R	(	(	P	$\to$	Q	)	$\vee$	(	Q	$\to$	R	)	)
$\top$	$\top$	$\top$			$\top$	$\top$	$\top$		$\top$		$\top$	$\top$	$\top$		
$\top$	$\top$	$\bot$			$\top$	$\top$	$\top$		$\top$		$\top$	$\bot$	$\bot$		
$\top$	$\bot$	$\top$			$\top$	$\bot$	$\bot$		$\top$		$\bot$	$\top$	$\top$		
$\top$	$\bot$	$\bot$			$\top$	$\bot$	$\bot$		$\top$		$\bot$	$\top$	$\bot$		
$\bot$	$\top$	$\top$			$\bot$	$\top$	$\top$		$\top$		$\top$	$\top$	$\top$		
$\bot$	$\top$	$\bot$			$\bot$	$\top$	$\top$		$\top$		$\top$	$\bot$	$\bot$		
$\bot$	$\bot$	$\top$			$\bot$	$\top$	$\bot$		$\top$		$\bot$	$\top$	$\top$		
$\bot$	$\bot$	$\bot$			$\bot$	$\top$	$\bot$		$\top$		$\bot$	$\top$	$\bot$		

Since all truth values are true, we have shown that $(P \to Q) \vee (Q \to R)$ is a tautology.