

***1.** Analyze the logical forms of the following statements:

- (a) We'll have either a reading assignment or homework problems, but we won't have both homework problems and a test.
- (b) You won't go skiing, or you will and there won't be any snow.
- (c) $\sqrt{7} \neq 2$.

4. Analyze the logical forms of the following statements:

- (a) Either both Ralph and Ed are tall, or both of them are handsome.
- (b) Both Ralph and Ed are either tall or handsome.
- (c) Both Ralph and Ed are neither tall nor handsome.
- (d) Neither Ralph nor Ed is both tall and handsome.

***6.** Let P stand for the statement “I will buy the pants” and S for the statement “I will buy the shirt.” What English sentences are represented by the following formulas?

(a) $\neg(P \wedge \neg S)$.

(b) $\neg P \wedge \neg S$.

(c) $\neg P \vee \neg S$.

***1.** Make truth tables for the following formulas:

(a) $\neg P \vee Q$.

(b) $(S \vee G) \wedge (\neg S \vee \neg G)$.

4. Find a formula using only the connectives \wedge and \neg that is equivalent to $P \vee Q$. Justify your answer with a truth table.

8. Use truth tables to determine which of the following formulas are equivalent to each other:

- (a) $(P \wedge Q) \vee (\neg P \wedge \neg Q)$.
- (b) $\neg P \vee Q$.
- (c) $(P \vee \neg Q) \wedge (Q \vee \neg P)$.
- (d) $\neg(P \vee Q)$.
- (e) $(Q \wedge P) \vee \neg P$.

*9. Use truth tables to determine which of these statements are tautologies, which are contradictions, and which are neither:

- (a) $(P \vee Q) \wedge (\neg P \vee \neg Q)$.
- (b) $(P \vee Q) \wedge (\neg P \wedge \neg Q)$.
- (c) $(P \vee Q) \vee (\neg P \vee \neg Q)$.
- (d) $[P \wedge (Q \vee \neg R)] \vee (\neg P \vee R)$.

12. Use the laws stated in the text to find simpler formulas equivalent to these formulas. (See [Examples 1.2.5](#) and [1.2.7](#).)

(a) $\neg(\neg P \vee Q) \vee (P \wedge \neg R)$.

(b) $\neg(\neg P \wedge Q) \vee (P \wedge \neg R)$.

(c) $(P \wedge R) \vee [\neg R \wedge (P \vee Q)]$.

***1.** Analyze the logical forms of the following statements:

- (a) If this gas either has an unpleasant smell or is not explosive, then it isn't hydrogen.
- (b) Having both a fever and a headache is a sufficient condition for George to go to the doctor.
- (c) Both having a fever and having a headache are sufficient conditions for George to go to the doctor.
- (d) If $x \neq 2$, then a necessary condition for x to be prime is that x be odd.

***4.** Use truth tables to determine whether or not the following arguments are valid:

- (a) Either sales or expenses will go up. If sales go up, then the boss will be happy. If expenses go up, then the boss will be unhappy. Therefore, sales and expenses will not both go up.
- (b) If the tax rate and the unemployment rate both go up, then there will be a recession. If the GDP goes up, then there will not be a recession. The GDP and taxes are both going up. Therefore, the unemployment rate is not going up.
- (c) The warning light will come on if and only if the pressure is too high and the relief valve is clogged. The relief valve is not clogged. Therefore, the warning light will come on if and only if the pressure is too high.

- *7.** (a) Show that $(P \rightarrow R) \wedge (Q \rightarrow R)$ is equivalent to $(P \vee Q) \rightarrow R$.
- (b) Formulate and verify a similar equivalence involving $(P \rightarrow R) \vee (Q \rightarrow R)$.

8. (a) Show that $(P \rightarrow Q) \wedge (Q \rightarrow R)$ is equivalent to $(P \rightarrow R) \wedge [(P \leftrightarrow Q) \vee (R \leftrightarrow Q)]$.

(b) Show that $(P \rightarrow Q) \vee (Q \rightarrow R)$ is a tautology.

***9.** Find a formula involving only the connectives \neg and \rightarrow that is equivalent to $P \wedge Q$.