Durham University

Digital Electronics
Number Systems

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Overview of today's lecture

- Number Systems
  - Binary; decimal; hex
- Translation between bases
- Intro to our next topic

Numeral (a.k.a. number) systems

Writing **system** for expressing **numbers**

- using digits or other symbols in a consistent manner

Same sequence of symbols may represent different numbers in different numeral systems!

Examples?

Numeral (a.k.a. number) systems

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Examples?

Decimal 11 vs Binary 11

“There are only 10 types of people in the world: those who understand binary, and those who don't.”

Binary

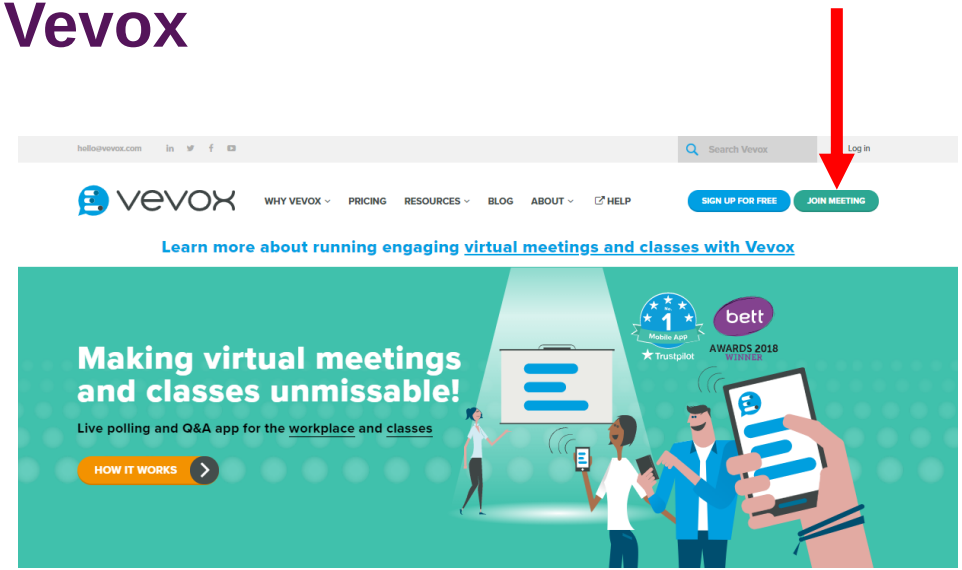
In a real CPU, everything is in **binary**…

All the characters are stored by **computers** as **binary** data numbers.

So we need to know how to:

- represent numbers in binary
- represent negatives
- add, subtract, multiply and divide binary numbers

Vevox



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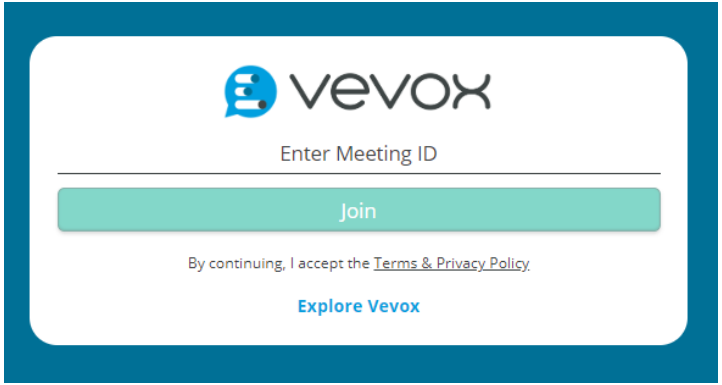

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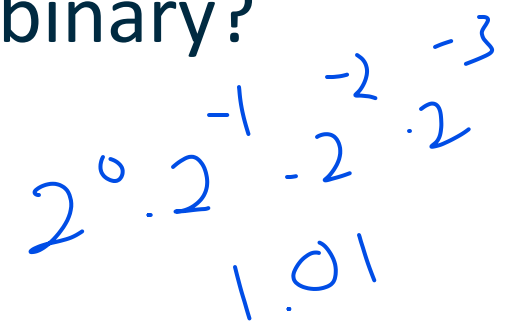
What is decimal 10 in binary?



1. 2
   14.06%
2. 10
   0%
3. 1010
   85.16%
4. 1
   0%
5. 11
   0.78%

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What is binary 111 in decimal?

1101111

seven

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What is hex 1B in decimal?

Number systems

This lecture's objective: develop an understanding of the different number systems used when discussing computers including:

- **Decimal** – used by humans
- **Binary** – used when discussing issues close to the machine
- **Hexadecimal** – used when humans try to interpret what is happening in the machine

Decimal: 232

Binary: 11101000

Hex: E8

Roman: CCXXXII All represent the **same number of objects.**

Decimal

Ten unique symbols 0 1 2 3 4 5 6 7 8 9

What does the decimal notation 432.75 **mean**?

Decimal point or **radix point**: symbol used in numerical representations to separate the integer part of a number.

The position of the ‘units’ is immediately to the left of the radix point.

So we have 2 ‘units’

What is the 3? 10s

What is the 4? 100s

The 7? 1/10ths

The 5? 1/100ths

Positional number systems

We start with a particular ordered set of symbols. E.g. a,b,c or 0,1,2

The **base** (or **radix**) of the number system is the number of symbols (including 0).

E.g. 10 for decimal (0,1,2,3,4,5,6,7,8,9), 2 for binary (0,1).

We use **positional number systems** to represent values

$$cab.bc_3 \quad \text{or} \quad 201.12_3$$

Note: subscript after a number gives the base

The contribution of a symbol *x,* which is the i^{th} symbol in the order, is $(i-1)*base^{position}$, where position is number of places to the **left** of the units.

E.g $cab.bc_3$ is $1*3^0+0*3^1+2*3^2 + 1*3^{-1} + 2*3^{-2}$

Same as 201.12_3

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E.g $cab.bc_3$ is $1*3^0+0*3^1+2*3^2 + 1*3^{-1} + 2*3^{-2}$

$= 1+0+18+1/3+2/9 = 19\ 5/9$

Same as 201.12_3

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Binary

2-symbol positional number system: symbols 0,1

Position	2	1	0	.	-1	-2
Base^{position}	2^2	2^1	2^0	.	2^{-1}	2^{-2}
Decimal value	4	2	1	.	.5	.25
Example	1	1	0	.	1	1

110.11_2 =

$1 * 2^2$ = 1 * 4 = 4.

$1 * 2^1$ = 1 * 2 = 2.

$0 * 2^0$ = 0 * 1 = 0.

$1 * 2^{-1}$ = 1 * .5 = 0.5

$1 * 2^{-2}$ = 1 * .25 + = 0.25

6.75_{10}

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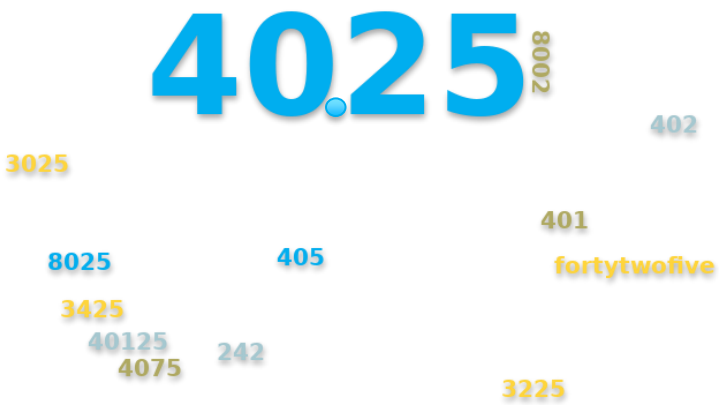
What is binary 101000.01 in decimal?



Binary

Exercise:

What is 11101000.101_2 in decimal?

It is $0+0*2+0*2^2+1*2^3+0*2^4+1*2^5+1*2^6+1*2^7 \; + \; 1*2^{-1}+0*2^{-2}+1*2^{-3}$

i.e. 8 + 32 + 64 + 128 + 1/2 + 1/8 = 232 5/8 in decimal + fraction

Binary

Each digit in a binary number system is known as a bit

- **B**inary dig**it**

A bit can have only one of two possible values

- 0 or 1 (sometimes referred to as false / true or off / on).

Groups of bits are known as:

- **Nibble** – 4 bits, 2^4 = 16 possible values.
- **Bytes** – 8 bits, 2^8 = 256 possible values.
- **Half word** – 16 bits, 2^{16} = 65,536 possible values.
- **Word** – 32 bits, 2^{32} = 4,294,967,296 possible values.
- **Double word** – 64 bits, 2^{64} = 18,446,744,073,709,551,616 values.

Note: “Word” is CPU dependent – e.g. sometimes refers to 64 bits

Hexadecimal

16 distinct symbols: **0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F**

Why do we need Hexadecimal?

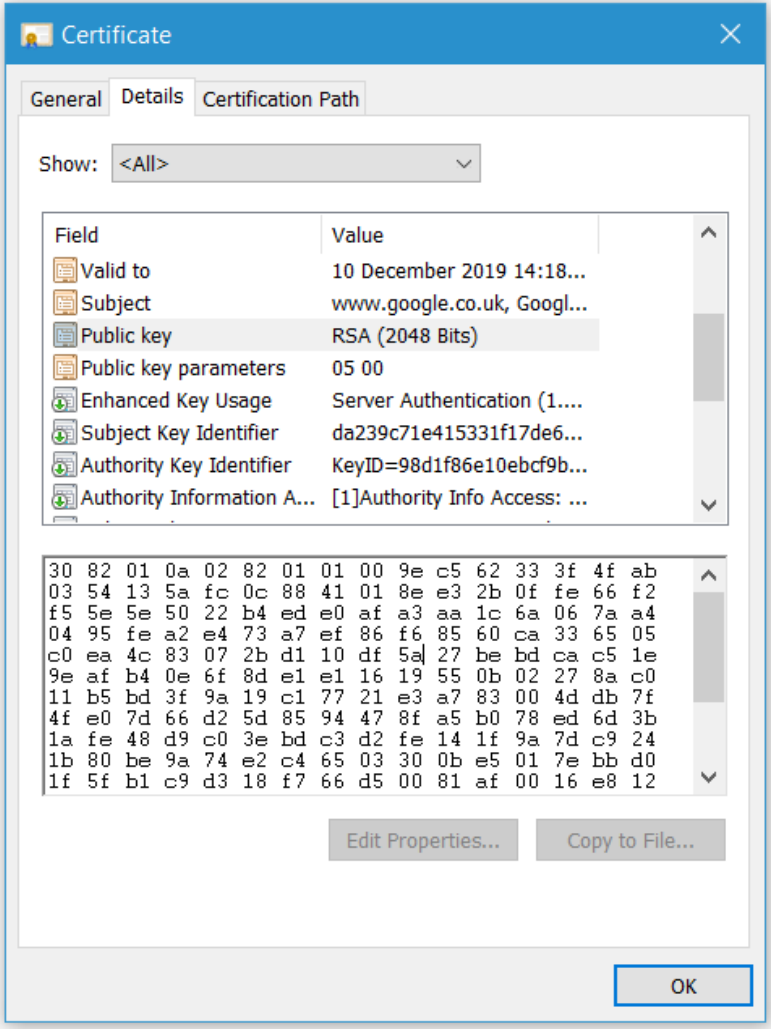
- Reading and writing binary values is difficult for humans

Advantages to using Hexadecimal

- **More compact** that other number systems
- **Easy to convert** between binary and hexadecimal

Programmers must be aware of what they are writing

- BEEF and $BEEF_{16}$ have very different meanings
- In Java use a prefix to denote a hexadecimal value: **0x**BEEF = $BEEF_{16}$


Certificate
General
Details
Certification Path
Show:
<All>
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Valid to
10 December 2019 14:18...
Subject
www.google.co.uk, Googl...
Public key
RSA (2048 Bits)
Public key parameters
05 00
Enhanced Key Usage
Server Authentication (1....
Subject Key Identifier
da239c71e415331f17de6...
Authority Key Identifier
KeyID=98d1f86e10ebcf9b...
Authority Information A...
[1]Authority Info Access: ...
30 82 01 0a 02 82 01 01 00 9e c5 62 33 3f 4f ab
03 54 13 5a fc 0c 88 41 01 8e e3 2b 0f fe 66 f2
f5 5e 5e 50 22 b4 ed e0 af a3 aa 1c 6a 06 7a a4
04 95 fe a2 e4 73 a7 ef 86 f6 85 60 ca 33 65 05
c0 ea 4c 83 07 2b d1 10 df 5a 27 be bd ca c5 1e
9e af b4 0e 6f 8d e1 e1 16 19 55 0b 02 27 8a c0
11 b5 bd 3f 9a 19 c1 77 21 e3 a7 83 00 4d db 7f
4f e0 7d 66 d2 5d 85 94 47 8f a5 b0 78 ed 6d 3b
1a fe 48 d9 c0 3e bd c3 d2 fe 14 1f 9a 7d c9 24
1b 80 be 9a 74 e2 c4 65 03 30 0b e5 01 7e bb d0
1f 5f b1 c9 d3 18 f7 66 d5 00 81 af 00 16 e8 12
Edit Properties...
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OK

Hexadecimal

Position	2	1	0	.	-1	-2
Base^{position}	16^2	16^1	16^0	.	16^{-1}	16^{-2}
Decimal Value	256	16	1	.	.0625	.00390625
Example	C	2	D	.	1	0

$C * 16^2 = 12 * 256 = 3072.$

$2 * 16^1 = 2 * 16 = 32.$

$D * 16^0 = 13 * 1 = 13.$

$1 * 16^{-1} = 1 * .0625 = .0625$

3117.0625_{10}

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What is hex 10A.8 in decimal?

1. 266.5

89.01%

2. 266.25

3.3%

3. 268.5

4.4%

4. 300.75

0%

5. 1010.8

3.3%

Hexadecimal

Exercise:

What is $1F8C.C_{16}$ in decimal?

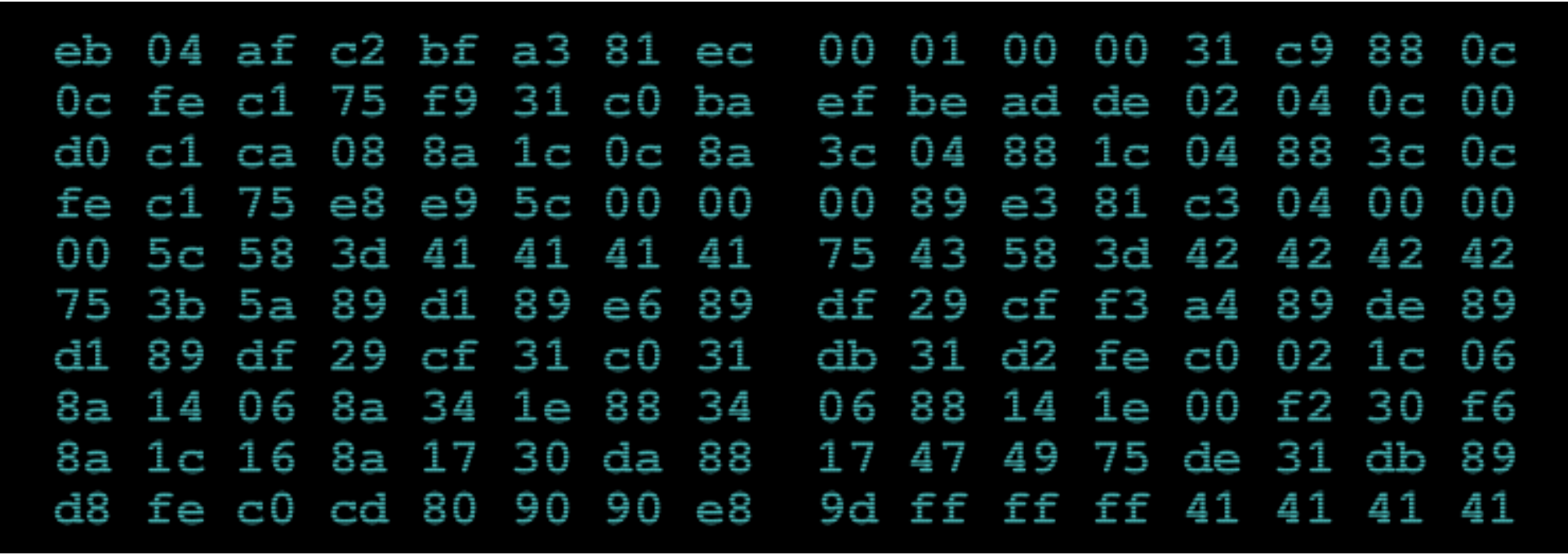
It is $12+8*16+15*16^2+1*16^3 + 12*16^{-1}$

i.e. 12 + 8*16 + 15*256 + 1*4096 + 12/16

= 12 + 128 + 3840 + 4096 + 3/4 = 8076 3/4 in decimal + fraction

GCHQ challenge

Can you crack it?



This is machine code written in hexadecimal.

Translation from Binary to Hex

1. Starting from the **radix point**, separate the binary number into groups of **four** binary digits (nibbles)
2. Then **translate each group** (nibble) into it hexadecimal equivalent, group by group, maintaining right to left order

Example:

$11\ 0101\ 1101\ 1000.001_2 = 35D8.2_{16}$

0011 0101 1101 1000. 0010

3 5 D 8 . 2

Translation from Hex to Binary

1. Starting from the **radix point**, separate the hexadecimal number into digits.
2. Then translate each digit into a **4-digit binary nibble**, maintaining right to left order

Example:

$$EF02A.B4_{16} = 1110\ 1111\ 0000\ 0010\ 1010\ .\ 1011\ 0100_2$$

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What is binary 111100.01 in hex?

1. B0.1

1.25%

2. 2B.01

1.25%

3. 3C.4

91.25%

4. 3C.1

6.25%

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What is hex BB.8 in binary?

1. 11110000.1

2.38%

2. 10111011.1

96.43%

3. 10011001.01

1.19%

Converting Decimal to Binary

- Repeatedly divide the number by 2, until you reach 0 / 2
- Put the remainders down **right to left** from radix point

Example: Convert 13_{10} to its binary representation

		remainder	
13/2	= 6	1	digit closest to radix point
6/2	= 3	0	
3/2	= 1	1	
1/2	= 0	1	left most digit

Result is 1101_2

Decimal Fractions to Binary

- Repeatedly multiply the number by 2, until fractional part is 0.
- If the *i*th result is greater than or equal to 1, place 1 in the *i*th position to the right of the radix, retain only fractional part.
- Else, place a 0 in the *i*th position to the right of the radix.

Example: 0.40625_{10}

0.40625*2	= 0.8125	0
0.8125*2	= 1.625	1
0.625*2	= 1.25	1
0.25*2	= 0.5	0
0.5*2	= 1.0	1

Answer 0.01101_2

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Other bases:

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What is 404_7 in decimal?

1. 20
   0%
2. 112
   0%
3. 200
   
   100%
4. 56
   0%

Other bases:

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What is 119_{10} in base 6?

1. 513

11.69%

2. 1930

9.09%

3. 315

77.92%

4. 325

1.3%

Other bases:

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What is 232_4 in base 5?

1. 46

2. 241

3. 141

4. 32

Animated examples

http://courses.cs.vt.edu/~csonline/NumberSystems/Lessons/index.html

Intro to Binary Arithmetic

Adding in binary

```
  111
   11100
+  01110
  101010
```

Based on 8 simple rules:

0 + 0 = 0	Carry + 0 + 0 = 1
0 + 1 = 1	Carry + 0 + 1 = 0 with Carry
1 + 0 = 1	Carry + 1 + 0 = 0 with Carry
1 + 1 = 0 with Carry	Carry + 1 + 1 = 1 with Carry

Multiplication

The same as decimal long multiplication – but easier!

```
   11100
  *01110
  -----
   00000
  111000
 1110000
11100000
--------
00000000
--------
110001000
```

Can be efficiently accomplished with **left-shift** and **add** operations