

$$y_{j+1} = y_j + h \left(1 + \frac{1}{2} \nabla + \frac{5}{12} \nabla^2 + \frac{3}{8} \nabla^3 + \frac{251}{720} \nabla^4 + \dots \right) f_j$$

$$\left\{ \begin{aligned} y_1 &= y_0 + h f_0 = -1 + 0.1 \times 1 = -0.9 \\ f_1 &= -y_1 - 2x_1 = -(-0.9) - 2 \times 0.1 = 0.7 \\ \nabla f_1 &= f_1 - f_0 = 0.7 - 1 = -0.3 \end{aligned} \right.$$

$$\left\{ \begin{aligned} y_2 &= y_1 + h \left(f_1 + \frac{1}{2} \nabla f_1 \right) \\ &= -0.9 + 0.1 \times (0.7 + \frac{1}{2} \times (-0.3)) \\ &= -0.845 \end{aligned} \right.$$

$$\left\{ \begin{aligned} f_2 &= -y_2 - 2x_2 = -(-0.845) - 2 \times 0.2 = 0.445 \\ \nabla f_2 &= f_2 - f_1 = 0.445 - 0.7 = -0.255 \\ \nabla^2 f_2 &= \nabla f_2 - \nabla f_1 = -0.255 - (-0.3) = 0.045 \end{aligned} \right.$$

$$\left\{ \begin{aligned} y_3 &= y_2 + h \left(f_2 + \frac{1}{2} \nabla f_2 + \frac{5}{12} \nabla^2 f_2 \right) \\ &= -0.845 + 0.1 \times (0.445 + \frac{1}{2} \times (-0.255) + \frac{5}{12} \times 0.045) \\ &= -0.811375 \end{aligned} \right.$$

$$\left\{ \begin{aligned} f_3 &= -y_3 - 2x_3 = -(-0.811375) - 2 \times 0.3 = 0.211375 \\ \nabla f_3 &= f_3 - f_2 = 0.211375 - 0.445 = -0.233625 \\ \nabla^2 f_3 &= \nabla f_3 - \nabla f_2 = -0.233625 - (-0.255) = 0.021375 \\ \nabla^3 f_3 &= \nabla^2 f_3 - \nabla^2 f_2 = 0.021375 - 0.045 = -0.023625 \end{aligned} \right.$$

$$\begin{aligned}
 y_4 &= y_3 + h(f_3 + \frac{1}{2}\nabla f_3 + \frac{5}{12}\nabla^2 f_3 + \frac{3}{8}\nabla^3 f_3) \\
 &= -0.811375 + 0.1 \times [0.211375 + \frac{1}{2} \times (-0.233625) \\
 &\quad + \frac{5}{12} \times 0.021375 + \frac{3}{8} \times (-0.023625)] \\
 &= -0.8019140625
 \end{aligned}$$

$$\begin{aligned}
 f_4 &= -y_4 - 2x_4 = -(-0.8019140625) - 2 \times 0.4 \\
 &= 1.9140625 \times 10^{-3}
 \end{aligned}$$

$$\begin{aligned}
 \nabla f_4 &= f_4 - f_3 = 1.9140625 \times 10^{-3} - 0.211375 \\
 &= -0.2094609375
 \end{aligned}$$

$$\begin{aligned}
 \nabla^2 f_4 &= \nabla f_4 - \nabla f_3 = -0.2094609375 - (-0.233625) \\
 &= 0.0241640625
 \end{aligned}$$

$$\begin{aligned}
 \nabla^3 f_4 &= \nabla^2 f_4 - \nabla^2 f_3 = 0.0241640625 - 0.021375 \\
 &= 2.7890625 \times 10^{-3}
 \end{aligned}$$

$$\begin{aligned}
 \nabla^4 f_4 &= \nabla^3 f_4 - \nabla^3 f_3 = 2.7890625 \times 10^{-3} - (-0.023625) \\
 &= 0.0264140625
 \end{aligned}$$

$$\begin{aligned}
 y_5 &= y_4 + h(f_4 + \frac{1}{2}\nabla f_4 + \frac{5}{12}\nabla^2 f_4 + \frac{3}{8}\nabla^3 f_4 + \frac{25}{720}\nabla^4 f_4) \\
 &= -0.8019140625 + 0.1 \times [1.9140625 \times 10^{-3} \\
 &\quad + \frac{1}{2} \times (-0.2094609375) + \frac{5}{12} \times 0.0241640625 \\
 &\quad + \frac{3}{8} \times 2.7890625 \times 10^{-3} + \frac{25}{720} \times 0.0264140625] \\
 &= -0.8101634538
 \end{aligned}$$

