Question 1

```python
import numpy as np
import math
```

```python
def analytical(b,h,w):
    z = (math.exp(-b*h*w*0.5)/(1-math.exp(-b*h*w)))
    return z

def en(h,w,n):
    e = h*w*(n+0.5)
    return e

def sum_equation(b,h,w,n):
    results_sum = 0
    for i in range(n + 1):
        results = math.exp(-b*en(h,w,i))
        results_sum = results_sum + results
    return results_sum

energy_levels = [1,10,100]
```

a)

```python
print("Analytical Result: " + str(analytical(1,1,1)))
print(" ")
print("Numerical Results:")
for i in energy_levels:
    print("for " + str(i) + " energy levels: " + str(sum_equation(1,1,1,i)))
```

```
Analytical Result: 0.9595173756674719

Numerical Results:
for 1 energy levels: 0.8296608198610632
for 10 energy levels: 0.9595013500953602
for 100 energy levels: 0.9595173756674712
```

The analytical result is greater than the numerical results. The numerical results increases as the energy level increases. The numerical results seem to converge around 0.9595 at energy level of n = 10

b)

```python
print("Analytical Result: " + str(analytical(0.1,1,1)))
print(" ")
print("Numerical Results:")
for i in energy_levels:
    print("for " + str(i) + " energy levels: " + str(sum_equation(0.1,1,1,i)))
```

```
Analytical Result: 9.995834548290834

Numerical Results:
for 1 energy levels: 1.8119374009257718
for 10 energy levels: 6.668510269734563
for 100 energy levels: 9.995423923853489
```

The analytical result is greater than the numerical results. The numerical results increases as the energy level increases. The numerical results seem to converge around 9.995 at energy level of n = 100

For $\beta = 1$, The numerical results converge at lower energy levels (n = 10) compared to $\beta = 0.1$ where the numerical results converge at a higher energy level (n = 100). Since the equation for β is $\beta = 1/KT$ where T is temperature, as the temperature increases, the numerical results will converge at higher energy levels.