Richard Yang Hw 1.

Question 1

(a) $Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y)$

$Cov(X,Y) = Corr(X,Y) \cdot \sqrt{Var(X) \cdot Var(Y)}$

$= 0.25 \times \sqrt{36}$

$= 1.5$

$Var(X+Y) = 9 + 4 + 2 \cdot 1.5$

$Var(X+Y) = 16$

(b) $Cov(X, X+Y) = Cov(X,X) + Cov(X,Y)$

$= Var(X) + Cov(X,Y)$

$= 9 + 1.5$

$= 10.5$

(c) $Corr(X+Y, X-Y) = \dfrac{Cov(X+Y, X-Y)}{\sqrt{Var(X+Y) \cdot Var(X-Y)}}$

$Cov(X+Y, X-Y) = Cov(X,X) + Cov(X,-Y) + Cov(Y,X) + Cov(Y,-Y)$

$= Var(X) - Cov(X,Y) + Cov(Y,X) - Var(Y)$

$= 9 - 4 = 5$

$Var(X+Y) = 16$

$Var(X-Y) = Var(X) + Var(Y) - 2Cov(X,Y)$

$= 9 + 4 - 2 \times 1.5$

$= 10$

$\Rightarrow Corr(X+Y, X-Y) = \dfrac{5}{\sqrt{16 \cdot 10}}$

Question 2

$cov(X+Y, X-Y) = cov(X, X-Y) + cov(Y, X-Y)$

$= cov(X,X) - cov(X,Y) + cov(Y,X) + cov(Y,-Y)$

$= var(X) - cov(X,Y) + cov(Y,X) - var(Y)$

$= Var(X) - var(Y)$

Since $Var(X) = var(Y)$

$cov(X+Y), X-Y) = Var(X) - var(Y)$

$= var(X) - var(X)$

$= 0$

Question 3.

(a) $E[Y_t] = E[5+2t+X_t]$

since $E(X_t)=0 \Rightarrow \; = 5+2t+0$

$= 5+2t.$

(b) $cov(Y_t, Y_{t-k}) =$

$= cov(5+2t+X_t, 5+2(t+k)+X_{t-k})$

$= cov(X_t, X_{t-k})$

$= Corr(X_t, X_{t-k})\sqrt{Var(X_t)\,Var(X_{t-k})}$

$= \rho_k \sqrt{var(X_t)\,var(X_{t-k})}$

$\Rightarrow \gamma_k = \rho_k \sqrt{var(X_t)\,var(X_{t-k})}$

(c) Since the mean function $5+2t$ depend on t and autocovariance γ_k depend on $X_t, \cdots X_{t-k}$

Therefore, Y_t is not stationary

Question 4.

(1) - (A) , since (1) has cycle of 12 months

(2) - (C) , since (2) is non-stationary

(3) - (B) , sice (3) has cycle of 10 years

AS1 Question 5

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Question 5

```r
# Set the seed for reproducibility
set.seed(123)

# Simulate a normal white noise sample of size 10 with mean 2.3 and standard deviation 1.2
sample_size_10 <- rnorm(10, mean = 2.3, sd = 1.2)

# Calculate the sample mean and sample standard deviation for sample_size_10
mean_10 <- mean(sample_size_10)
sd_10 <- sd(sample_size_10)

# Print the results
cat("Sample Mean (n = 10):", mean_10, "\n")
```

```
## Sample Mean (n = 10): 2.389551
```

```r
cat("Sample Standard Deviation (n = 10):", sd_10, "\n")
```

```
## Sample Standard Deviation (n = 10): 1.144541
```

```r
# Simulate a normal white noise sample of size 10,000 with mean 2.3 and standard deviation 1.2
sample_size_10000 <- rnorm(10000, mean = 2.3, sd = 1.2)

# Calculate the sample mean and sample standard deviation for sample_size_10000
mean_10000 <- mean(sample_size_10000)
sd_10000 <- sd(sample_size_10000)

# Print the results
cat("Sample Mean (n = 10,000):", mean_10000, "\n")
```

```
## Sample Mean (n = 10,000): 2.297445
```

```r
cat("Sample Standard Deviation (n = 10,000):", sd_10000, "\n")
```

```
## Sample Standard Deviation (n = 10,000): 1.198637
```

Yes, I think the result is overall satisfied.

For the sample size of 10, the estimates might be less accurate due to the smaller sample size. As a result, the sample mean and sample standard deviation might deviate more from the true population parameters.

On the other hand, for the larger sample size of 10,000, the estimates tend to be more accurate as the sample size approaches the population size. Thus, the sample mean and sample standard deviation are expected to be closer to the true population parameters.

Ultimately, the satisfaction with the computed results depends on the specific context and requirements of your analysis. However, in general, larger sample sizes tend to provide more reliable estimates of the population parameters.