# Tracing and Forecasting of Congested Patterns for Highway Traffic Management

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Abstract— Based on recent empirical findings of spatial-temporal features of patterns in congested traffic, a new approach for a tracing and prediction of congested patterns on highways and the related possibilities for highway traffic management are discussed.

Keywords—congested traffic, synchronized flow, wide moving jams, models for tracing and prediction of congested patterns, highway management strategy.

# I. Introduction. About Forecasting Models for Highway Traffic

Traffic on highways can be either "free" or "congested". It is well known that in congested traffic diverse spatial-temporal patterns, in particular "stop-and-go" patterns are observed (e.g., the classical works of Treiterer [1] and Koshi et al. [2]). To avoid the congestion, different management methods have been proposed and applied. For a successful highway management it is important to make a traffic forecasting which should allow to predict the behavior of patterns in congested traffic.

One of the approaches and trials in traffic technology to estimate and predict traffic patterns is an application of either microscopic, mesoscopic or macroscopic traffic flow models which calculate the movement of individual vehicles and/or the average vehicle speed and the density spatialtemporal distributions in highway networks (e.g. [3] and a review by Helbing [4]). Unfortunately, practical onlineapplications of this approach have some principle problems. One of the problems is the necessity of a validation of model parameters. These parameters are strongly dependent on the infrastructure, the weather and other environmental conditions. Therefore, it is hard to find and adapt a set of the model parameters which are valid for real traffic flow in all possible, totally different, conditions. This may be the reason why the mentioned models, are up to now, far from being used for practical online applications on highways.

Recently Kerner, in collaboration with Rehborn, has found out that two qualitatively different traffic phases should be distinguished in the congested traffic: "synchronized flow" and "wide moving jams" [5]. In measurements of congested traffic, the traffic phases "synchronized flow" and "wide moving jam" can be distinguished only through a spatial-temporal investigation of traffic. This means that simultaneous measurements of traffic at several different locations on a highway are necessary. Based on these empirical findings, Kerner developed the three-phase-traffictheory [6], [7], which is the basis for new models ASDA

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and FOTO for tracing and forecasting of patterns in congested traffic on highways [8], [9], [10], [11]. The model "ASDA" (Automatische Staudynamikanalyse: Automatic Tracing of Moving Traffic Jams) [8] performs the automatic tracing and prediction of the propagation of moving traffic jams. The model "FOTO" (Forecasting of Traffic Objects) [10] identifies the traffic phases and performs the tracing and prediction of the patterns of the traffic phase "synchronized flow". It must be noted that the models ASDA and FOTO [8], [10] perform without any validation of model parameters in different environmental and traffic conditions. In this paper, some results of the three-phase-traffic-theory by the author, some results of models ASDA and FOTO and possible ways of their application for the highway management are discussed.

#### II. THREE-PHASE-TRAFFIC-THEORY

## A. Objective Criteria for Identification of Traffic Phases

In the three-phase-traffic-theory [6], [7], [12], [13], [14], [15], [16], the complexity of highway traffic is explained based on the spatial-temporal phase transitions between three traffic phases:

- 1. Free flow.
- 2. Synchronized flow.
- 3. Wide moving jam.

Recall that a moving jam is a local congested pattern which is spatially restricted by two upstream moving fronts where the average vehicle speed and the density sharply change; both fronts of the moving jam propagate upstream continuously; inside the jam the vehicle speed is low (sometimes zero) and the vehicle density is high. A wide moving jam is a moving jam whose width (in the longitudinal direction), i.e., the distance between the jam fronts, is considerably higher than the widths of the fronts.

To distinguish between wide moving jams and synchronized flow in the congested regime the following objective criteria can be applied [15]. After a wide moving jam has emerged, it propagates through either free flow or any states of synchronized flow or any kinds of bottlenecks (e.g. on- and off ramps) keeping the velocity of the jam's downstream front. In contrast to a wide moving jam, after synchronized flow has occurred at a bottleneck the downstream front of the pattern of synchronized flow is fixed at the effective location of the bottleneck (at the effective bottleneck). In general, all states and spatial-temporal patterns in congested traffic which do not possess this characteristic feature of a wide moving jam - the keeping of the velocity of the jam's downstream front - belong to the traffic phase "synchronized flow".

This principle difference between synchronized flow and a wide moving jam can be seen in Fig. 1 and Fig. 2. In Fig. 1 (c), the sequence of two wide moving jams propagates through at least three bottlenecks (in the intersections I1, I2 and I3, Fig. 1 (a)) and through different very complex states of synchronized flow (Fig. 1 (e), bottom). In contrast to the wide moving jams, after synchronized flow has occurred at an on-ramp (D7), the downstream front of the synchronized flow is fixed at the on-ramp. This effect is shown in Fig. 1 (c), where the downstream front of the synchronized flow is shown by the dotted line. The location of this fixed downstream front determines the effective location of the bottleneck at the on-ramp (D7 in Fig. 1 (a)).

A different example is shown in Fig. 2: a moving jam propagates through states of free flow (e.g., D9-D12) and through several bottlenecks inside the three intersections I1, I2 and I3. Propagating through the effective location of the bottleneck at the detectors D16 (the on-ramp), the jam causes the phase transition from the traffic phase "free flow" to the traffic phase "synchronized flow" (the  $F \to S$ transition in Fig. 2 (b)). The jam plays a role of a nucleus for the  $F \to S$ -transition at the on-ramp near D16. In contrast to the wide moving jam, after the synchronized flow has occurred, the downstream front of the pattern of synchronized flow is fixed at the on-ramp at detectors D16 (the location of the effective bottleneck in Fig. 2 (b)). Therefore, corresponding to the objective criteria mentioned above the moving jam belongs to the traffic phase "wide moving jam" and the pattern which is localized upstream the bottleneck at D16 belongs to the traffic phase "synchronized flow".

To distinguish traffic phases in congested traffic, a spatial-temporal behavior of traffic simultaneously measured at different locations has to be studied (Fig. 1 (b. c) and Fig. 2 (b)) rather than the analysis of measurement points in the flow-density plane. However, after the traffic phases have been distinguished, some of their features can be analyzed in the flow-density plane further more (Fig. 2 (c)). In particular, empirical points related to synchronized flow cover a two-dimensional region in the flow-density plane (points S). The property of a wide moving jam to keep the mean velocity of the downstream jam front can be presented by the characteristic line for the downstream jam front (the line J) in the flow-density plane (Fig. 2 (c), the line J): The slope of the line J equals this characteristic, i.e., unique, predictable and reproducible velocity. If free flow is formed in the outflow of the wide jam, then the flow rate out of the jam,  $q_{out}$ , is also the characteristic parameter which together with the related density  $\rho_{min}$  gives the co-ordinate on the line J which is needed for its determination.

Corresponding to the three-phase-traffic-theory, there is no fundamental diagram even for hypothetical homogeneous and stationary (steady) states of synchronized flow: These states cover a two-dimension region in the flow-density plane (dashed area in Fig. 3 (a)). The maximal capacity of the highway (i.e., at a bottleneck) depends on which phase the traffic is in: For free flow it is  $q_{max}^{(free)}$ , for synchronized flow it is  $q_{max}^{(syn)}$  and in the outflow out of a

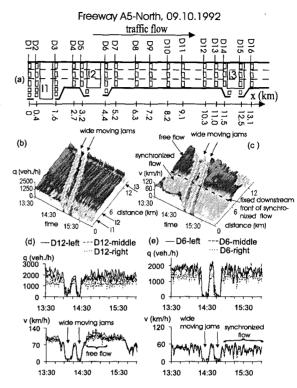


Fig. 1. Explanation of traffic phases: Free flow, wide moving jams and synchronized flow (b-e) on a section of the highway A5-North (a) in Germany [17].

wide moving jam it is  $q_{out}$  (Fig. 3 (b)). For the explanation of these and other hypothesis see [6], [12].

### B. Predictable Features of Congested Patterns

It is well-known and somewhat common sense that congested patterns occur mostly at highway bottlenecks. In the three-phase-traffic-theory it has been found out that the spatial-temporal structure of congested patterns at bottlenecks possesses some common features which are predictable and reproducible [11]. This allows a classification of spatial-temporal patterns at bottlenecks.

It has already been stressed that wide moving jams possess some characteristic i.e., unique, predictable and reproducible parameters which do not depend on initial conditions and on time and are also the same for different wide moving jams [17]. The spatial-temporal structure of the patterns of synchronized flow also possesses some predictable features. First note that a spatial-temporal pattern consisting only of the traffic phase "synchronized flow" is usually localized spatially: There are downstream and upstream fronts (boundaries) of the spatial-temporal pattern of synchronized flow where the vehicle speed and the density spatially sharply change. The downstream front of the pattern of synchronized flow is fixed at the effective location of the bottleneck: This front separates synchronized

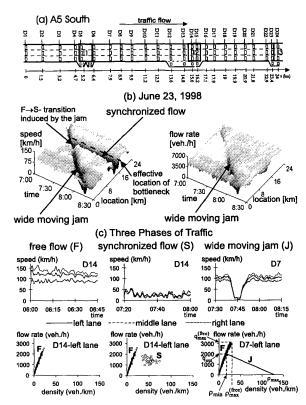


Fig. 2. An overview of congested patterns on 23.06.1998 observed on the section of the highway  $\Lambda5$ -South (a): (b) dependencies of the average (across all lanes) vehicle speed (left) and the total flow rate across the highway (right) on time and space [14].

flow upstream and free flow downstream (Fig. 4 (a)). The upstream front of the pattern synchronized flow separates the synchronized flow downstream either from free flow or from a wide moving jam upstream.

Indeed, in contrast to the wide moving jam whose upstream front propagates upstream continuously (see the wide moving jam in Fig. 2 (b)), the upstream propagation of the upstream front of the pattern of synchronized flow has always a spatial limit. Even if the flow rate upstream of the congested pattern is much higher than the maximal capacity of a bottleneck, the length of the congested pattern is limited by the bottleneck-specific length,  $L_{syn}$ , if this highway bottleneck is an isolated bottleneck (Fig. 4 (a)). The isolated bottleneck is a bottleneck on a highway where both the appearance of spatial-temporal patterns and their properties are not influenced by other bottlenecks and other spatial-temporal congested patterns which can simultaneously occur at other locations on the highway.

At the isolated bottleneck, the general pattern is usually formed (Fig. 4 (a)) [11], [15]. The general pattern consists of (i) the pattern of synchronized flow and (ii) a region of wide moving jams. Inside the pattern of synchronized

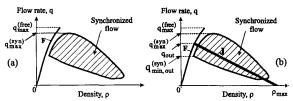


Fig. 3. Concatenation of three traffic phases in the flow-density plane: (a) Free flow (curve F) and hypothetical steady states of synchronized flow (dashed area) on a multilane freeway, (b) the same states of free and synchronized flows and the line J for the downstream front of a wide moving jam [6], [7], [12], [15].

flow the pinch region is formed where growing narrow upstream moving jams emerge. The location of the upstream boundary (front) of the pattern of the synchronized flow and therefore the length  $L_{syn}$  of the pattern is determined by the location, where a narrow moving jam has just transformed into a wide moving jam. An example of the general pattern at an isolated bottleneck is shown in Fig. 5 (the bottleneck exists due to the on-ramp in the vicinity of D6, see the schema in Fig. 2 (a)). The data at the detectors D5 and D4 are related to the pinch region and at D1 to the region of wide moving jams.

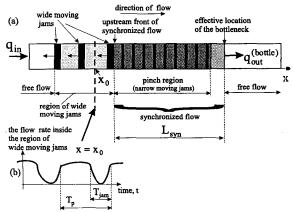


Fig. 4. Symbolical representation of the general pattern at an isolated bottleneck: (a) the spatial structure of the pattern at a fixed moment of time, (b) the flow rate as the function of time measured at the location  $x = x_0$  inside the region of wide moving jams [11], [15].

Usually, the length  $L_{syn}$  of the pattern (Fig. 4 (a)) is approximately determined by the length of the pinch region. In this case, it can be assumed that

$$L_{syn} \approx |v_{narrow,mean}| T_{narrow}, \tag{1}$$

where  $v_{narrow,mean}$  is the mean velocity of narrow jams and  $T_{narrow}$  is the mean time interval needed for the transformation of the narrow jams into the wide jams.

Further we restrict to the consideration of an isolated bottleneck, where no on- and off-ramps upstream of the bottleneck exist. Let us consider the flow rate as the function of time which is measured inside the region of wide

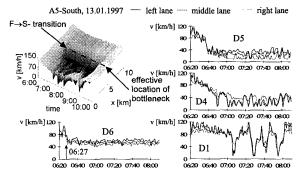


Fig. 5. Empirical general pattern at the isolated bottleneck [6].

moving jams at a fixed location  $x=x_0$  (Fig. 4 (a)). This function is obviously the alternation of the flow rate measured inside wide moving jams  $q_{jam}$  during the duration of wide moving jams  $T_{jam}$  with the flow rate measured in the outflow of the jams  $q_{out}^{(jam)-1}$  during a time  $T_p-T_{jam}$ , where  $T_p$  is the time interval between the downstream fronts of wide moving jams (Fig. 4 (b)). Stress that the flow rate inside the region of wide moving jams (Fig. 4 (b)) determines the flow rate through the pinch region and therefore, it has a great influence on the discharge flow rate at the downstream front of the pattern,  $q_{out}^{(bottle)}$  (Fig. 4 (a)). In particular, let us consider the average discharge flow

In particular, let us consider the average discharge flow rate which is averaged over the time interval T which is considerably higher than  $T_p$ :

$$q_{out,mean}^{(bottle)} = T^{-1} \int_{t-T/2}^{t+T/2} q_{out}^{(bottle)} dt.$$
 (2)

If the number of highway lanes does not change in the vicinity of an isolated bottleneck, then

$$q_{out,mean}^{(bottle)} = q_{out,mean}^{(jam)} \left(1 - \frac{T_{jam,mean}}{T_{p,mean}}\right) +$$

$$+ q_{jam,mean} \frac{T_{jam,mean}}{T_{p,mean}} < q_{out}.$$
(3)

In this case,  $q_{out,mcan}^{(bottle)}$  (3) depends only on the parameters of the region of wide moving jams (Fig. 4 (a)). All values in the right hand of (3) are the related arithmetical mean values.

If an isolated bottleneck is linked to an on-ramp and the number of highway lanes does not change upstream of the on-ramp, then

$$q_{out,mean}^{(bottle)} = q_{ramp,mean} + q_{out,mean}^{(jam)} (1 - \frac{T_{jam,mean}}{T_{p,mean}}) + q_{jam,mean} \frac{T_{jam,mean}}{T_{p,mean}}.$$
(4)

In this case, the discharge flow rate can change in the range  $[q_{max}^{(syn)}, q_{min,out}^{(syn)}]$  ((Fig. 3 (b)) [15].

There is feedback from the downstream boundary of the congested pattern to the upstream boundary: First the F→S-transition occurs, then the pattern of synchronized flow is formed and later the wide moving jams emerge. There is another feedback from the upstream boundary to the downstream boundary: In particular, the discharge flow rate (3) depends on the parameters of wide moving jams,  $T_p$ ,  $T_{jam}$ ,  $q_{jam}$  and  $q_{out}^{(jam)}$ . Therefore, a spatialtemporal competition between both mentioned feedbacks determines features of congested patterns at bottlenecks. Thus, the discharge flow rate  $q_{out}^{bottle}$  must not be either given or predefined even as a function of time. These empirical features of congested traffic are in contrast to the basic assumptions which are made (see, e.g., [18]) by the application of the classical queuing and kinematic wave theories for the congested traffic analysis. For these reasons, the terms "kinematic waves" or "queued traffic" are not used in the three-phase-traffic-theory [15].

The wide moving jam at the upstream boundary of the pattern of synchronized flow can be considered as a region where 'superfluous' vehicles which can not immediately pass through the pinch region are virtually stored. As long as traffic demand, exactly the flow rate upstream of the region of the wide moving jams  $q_{in}$ , is higher than the flow rate out from the wide moving jam,  $q_{out}$ , traffic demand upstream does not influence the length of the pattern of synchronized flow  $L_{syn}$  (Fig. 4 (a)). In particular, an increase in traffic demand  $q_{in}$  leads to the related increase in the width of the most upstream wide moving jam. Thus, even if traffic demand  $q_{in}$  is for a long time considerably higher than the maximal capacity of the congested bottleneck, the length of the congested pattern  $L_{syn}$  is not influenced. When in contrast the traffic demand  $q_{in}$  becomes during a long time noticeably lower than  $q_{out}$ , first wide moving jams and then the pattern of synchronized flow successively disappear.

In some cases, instead of the general pattern, the *short-ened* congested patterns can be formed at the isolated bot-tleneck. The shortened pattern of type 1) consists of a region of synchronized flow with the pinch region inside. The shortened pattern of type 2) consists only of a region of synchronized flow [11].

If several bottlenecks exist on a highway, then expanded spatial-temporal patterns can be formed. The pinch region in an expanded pattern can cover several bottlenecks. Besides, foreign wide moving jams, i.e., the jams which have earlier occurred within other patterns downstream and due to their upstream propagation have reached the expanded pattern, can propagate through the whole expanded pattern keeping the velocity of the jam's downstream fronts [11].

For each effective bottleneck the characteristic, i.e., reproducible and predictable spatial-temporal structure of congested patterns at different days can be found. This pattern is either the general pattern or one of the shortened patterns or else an expanded pattern [11]. These

<sup>&</sup>lt;sup>1</sup>Note that  $q_{out}^{(jam)}$  reaches its maximum value  $q_{out}$  (Fig. 3 (b)) only if free flow is formed between the wide moving jams, i.e.,  $q_{out}^{(jam)} \leq q_{out}^{(jam)}$ 

results have been used for a development of methods for traffic forecasting of spatial-temporal features of congested patterns at highway bottlenecks [11]. In these methods, historical spatial-temporal features of patterns at different bottlenecks are investigated first. Then, the typical features for each of the bottlenecks are extracted. These features can be matched later with actual features of patterns using the models ASDA and FOTO for a reliable dynamic traffic forecast.

#### III. APPLICATION OF MODELS ASDA AND FOTO

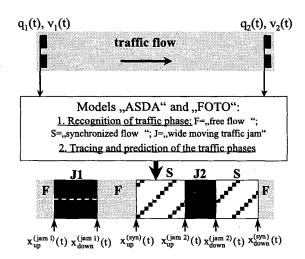


Fig. 6. A scheme of the recognition of different traffic phases and the spatial boundaries (fronts) between these phases through ASDA/FOTO [10].

Based on the results of the three-phase-traffic-theory, Kerner has proposed an approach to a traffic prediction, where macroscopic parameters of spatial-temporal traffic patterns are determined based on traffic measurements. An example of these macroscopic parameters is the velocity of the fronts which separate different traffic phases within traffic patterns, e.g., the velocity of the fronts of moving jams [8], [10]. Fig. 6 illustrates this approach: Based on local measurements of traffic the recognition of the traffic phases is performed first with the FOTO model. Second the initial fronts of wide moving jams  $x_{up}^{(jam)}(t_0), x_{down}^{(jam)}(t_0)$  and of synchronized flow  $x_{up}^{(syn)}(t_0), x_{down}^{(syn)}(t_0)$  are determined. These fronts define the spatial size and location of the related "synchronized flow" object and "wide moving jam" object in congested regime. Finally, with the models ASDA and FOTO the tracing and forecasting of these objects' fronts in time and space is done, i.e. the positions of all fronts  $x_{up}^{(jam)}(t), x_{down}^{(jam)}(t), x_{up}^{(syn)}(t), x_{down}^{(syn)}(t)$  are found as functions of time. Note that for traffic forecasting historical time series at least for the flow rates are necessary. The knowledge of the parameters of traffic patterns allow us to calculate trip travel times, vehicle trajectories, etc.

In the field-tested application FOTOwin [8], which has

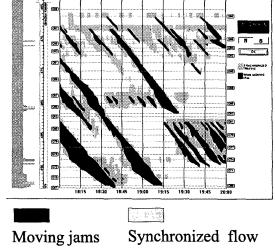


Fig. 7. Results of the on-line application of the models ASDA and FOTO [8]: Histogram A5-North on 12.04.2000 from 18:00-20:00.

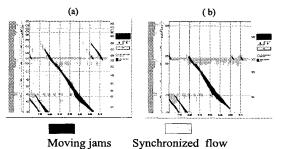


Fig. 8. Results of the on-line application of the models ASDA and FOTO [10]: (a) Histogram A5-South on 17.07.2000 from 07:30-09:30 using all available data (31 sets of detectors). (b) the same day and time interval as in (a but using reduced input data (9 of 31 sets of detectors).

been realized in a research project for the Federal State of Hessen in Germany, both the models ASDA and FOTO have been integrated. Examples of this application are shown in Figs. 7 and 8.

The output information of the models ASDA and FOTO can be used for traffic control systems. It is also possible to predict the dissolution of moving jams and of patterns of synchronized flow. For the verification of the model results, empirical data of an infrastructure with many detection sites has been chosen. First, the data of all detectors has been used (Fig. 8 (a)). Then, the data of some detectors have been omitted and the process of the traffic pattern recognition has been repeated (Fig. 8 (b)). The data which has not been used are taken for comparison to the model results. Fig. 8 (b) shows the results of ASDA and FOTO with a strongly reduced configuration of the input values: instead of 31 detection sites for the models only 9 detection sites are used, i.e. only 30% of the detec-

tion site infrastructure. The histogram A5-South at 17th July, 2000 from 07:30-09:30a.m. (Fig. 8 (b)) shows a very similar result at the same situation as in Fig. 8 (a) with all detectors as input: in spite of the strongly reduced input information in Fig. 8 (b), the generation of traffic objects with ASDA and FOTO is very similar.

#### IV. ABOUT POSSIBLE MANAGEMENT STRATEGIES

Based on the results of the-three-phase-traffic-theory and the models ASDA and FOTO, it is possible to predict the pattern formation. First, it is possible to predict the propagation of the wide moving jam as a whole local structure on the highway. Indeed, the jam possesses the characteristic velocity of its downstream front. Therefore, if the time series for the flow rates are known then either a possible dissolution or the time of the arrival of the jam at each location of the highway can be calculated. Second, it is possible to predict whether the jam causes the F-S-transition at a bottleneck when the jam propagates through the bottleneck. For example, let us assume that the flow rate in the vicinity of an on-ramp is higher than some minimal flow rate at which the F→S-transition can occur. Then, it can be predicted (with a very high probability) that when the jam propagates through the bottleneck, the jam causes the F→S-transition. As it can be seen in Fig. 2, these assumptions are confirmed by empirical investigations.

The three-phase-traffic-theory allows to distinguish the features of congested patterns which can be influenced in an effective way [6], [12], [15]. Therefore, the possible management strategies for the prevention of congested pattern formation or for the dissolution of the existing congested patterns may include the steps in Fig. 9.

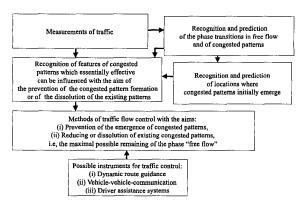


Fig. 9. Possible steps of management strategies for the prevention of congested pattern formation or for the dissolution of the existing congested patterns.

One of the important steps is the recognition of empirical features of the  $F\rightarrow S$ -transitions and of congested patterns which effectively can be influenced and the development of methods for the prevention of the emergence of the congested pattern or for the dissolution of the existing patterns. For example, the method of a time-limited rampmetering proposed in [9], which is based on the empirical

features of the F $\rightarrow$ S-transitions, should ensure the maximal duration of free flow conditions. For the dissolution of the existing patterns a spatial-temporal combination of a reducing of the inflows to congested patterns and of an increasing of the outflows from the patterns would be applied. Possible instruments for such methods of traffic control can be dynamic route guidance, vehicle-vehicle communication and driver assistance systems.

#### V. Conclusions

The three-phase-traffic-theory [6], [7], [12], [13], [14] allows to determine the main spatial-temporal features of patterns in congested traffic. Together with the models ASDA and FOTO for a tracing and prediction of congested patterns it can be a useful instrument for management strategies for the prevention of congested pattern formation or for the dissolution of the existing congested patterns on highway networks in the future.

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