Zhilong Xue

Curriculum Vitae

(Nov. 2024)

Personal Details

Name: Zhilong Xue
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Research Interests

Analysis, partial differential equations and fluid mechanics

Education

Sep. 2020—Current	**Beijing Normal University**, PhD candidate in mathematics	**Supervisor:** Liutang Xue
Sep. 2016—Jun. 2020	**Chang'an University**, BS in mathematics	Ranked: 1st/29

Research Stays

Oct. 2023 — Oct. 2024	**Visiting PhD student**, University of Seville, Spain Supervisor: Francisco Gancedo Funded by China Scholarship Council (No. 202306040124)
Sep. 2024 — Oct. 2024	**Visiting PhD student**, University of Granada, Spain Host: Claudia García

Publications and Preprints

1. Qianyun Miao, Changhui Tan, Liutang Xue, and Zhilong Xue. *Local regularity and finite-time singularity for a class of generalized SQG patches on the half-plane.* ArXiv:2410.19273.
2. Taoufik Hmidi, Liutang Xue, and Zhilong Xue. *Unified theory on V-states structures for active scalar equations.* ArXiv:2312.02874.
3. Taoufik Hmidi, Liutang Xue, and Zhilong Xue. *Emergence of time periodic solutions for the generalized surface quasi-geostrophic equation in the disc.* Journal of Functional Analysis **285** (2023), no. 10, Paper No. 110142.

Invited Talks

16/10/2024	PhD seminar, IMUS, University of Seville, Spain
21/03/2024	Ecuaciones Diferenciales seminar, IMAG, University of Granada, Granada, Spain
13/11/2023	Fluid Conversations seminar, IMUS, University of Seville, Spain

Conferences

Nov. 2024	*Mixing, enhanced dissipation and stability effects in fluid dynamics*, Kunming, China
Jul. 2024	*Alhambra PDE days*, IMAG, University of Granada, Spain
Jul. 2024	*9th European Congress of Mathematics*, University of Seville, Spain

Awards

Dec. 2023	First-class Scholarship, Beijing Normal University, China
Jul. 2023	Scholarship for study abroad, China Scholarship Council, China
Dec. 2022	First-class Scholarship, Beijing Normal University, China
Dec. 2021	First-class Scholarship, Beijing Normal University, China
Dec. 2019	National Scholarship for undergraduate students, Ministry of Education of China, China

Research Proposal

At present, I have the following research plans.

1. I will keep exploring the topic about active scalar equations, especially with patch initial data, which is given by

$$\begin{cases} \partial_t \omega + (u \cdot \nabla)\omega = 0, & (t, \mathbf{x}) \in (0, +\infty) \times \mathbf{D} \\ u = \nabla^{\perp}\psi, & (t, \mathbf{x}) \in (0, +\infty) \times \mathbf{D}, \\ \omega(\mathbf{x}, 0) = \omega_0(\mathbf{x}), & \mathbf{x} \in \mathbf{D}, \end{cases} \tag{1}$$

   where $\mathbf{D}$ is either the whole space $\mathbb{R}^2$ or some radial domain with boundary, $\nabla^{\perp} = (\partial_2, -\partial_1)$, $u = (u_1, u_2)$ refers to the velocity field, ω is a scalar field understood as vorticity or temperature or buoyancy of the fluid and the stream function ψ is prescribed through the following relation

$$\psi(\mathbf{x}, t) = \int_{\mathbf{D}} K(\mathbf{x}, \mathbf{y})\omega(\mathbf{y}, t)\mathrm{d}y. \tag{2}$$

   The most famous hydrodynamic models in the form (1)-(2) are 2D Euler equation in the vorticity form and surface quasi-geostrophic (abbr. SQG) equation. The patch initial data is in the following form,

$$\omega_0(x) = \sum_{j=1}^{N} a_j \mathbf{1}_{D_j}(x), \quad a_j \in \mathbb{R}, \quad \mathbf{1}_{D_j} \triangleq \begin{cases} 1, & x \in D_j, \\ 0, & x \in \mathbf{D} \setminus D_j, \end{cases} \tag{3}$$

   where $D_j (j = 1, \cdots, N)$ are disjoint simply connected bounded domains with regular boundaries ∂D_j on $\mathbf{D}$. The regularity of ω_0 refer to the regularity of ∂D_j.
   For the Euler equation, the global persistence of $C^{k,\gamma}$ patch boundaries in $\mathbb{R}^2$ with $k \in \mathbb{N}^\star$ and $0 < \gamma < 1$ was solved by Chemin [6] in 1993. Recently, Kiselev and his collaborators [18, 20] completely solved this problem for $C^{1,\gamma}$ patch initial data on half plane $\mathbb{R}^2_+$ and bounded domain with C^4 boundary. But it's still an interesting problem to show the propagation of higher boundary regularity on half plane $\mathbb{R}^2_+$ or bounded domain with smooth boundary. Furthermore, last year, Kiselev and Luo[19] proved the illposedness of Euler equation with C^2 patch initial data and believe that their approach also works to show the illposedness for $C^k (k \geq 3)$ patch initial data. Hence, a deep question arise. Can we show the wellposedness or illposedness in the space C^1 or other critical space for Euler equation in the plane $\mathbb{R}^2$? For SQG equation or generalized SQG equation, there are more open problems. Indeed, up to now, the global regularity vs finite time singularity issue for generalized SQG equation with smooth patch initial data or smooth initial data is still open in the plane $\mathbb{R}^2$. Even the local regularity of generalized SQG equation with patch initial data is not clear on bounded domain.
   Except these regularity issue for the Cauchy problem (1)-(3), it's also very important to construct relative equilibrium and analyze the stability or instability around relative equilibrium. In particular, the stationary solution is belong to relative equilibrium. In my previous works [16, 17], I and my collaborators construct nontrivial rotating patches with different topology through applying local bifurcation tools to the equation (1)-(2). Specifically speaking, rotating patches take the form, $\omega(t, \mathbf{x}) = \mathbf{1}_{D_t}$ with $D_t = e^{i\Omega t} D_0$, which is a time periodic solution and rotate in time with angular velocity $\frac{\Omega}{2\pi}$. Though in the past decades, there are a lot of works (e.g. [4, 5, 9, 10, 14]) about rotating patches from different view, this area is still very active. Recently, Berti, Hassainia and Masmoudi[3] construct quasi-periodic vortex patches for Euler equation in plane by using KAM theory and Nash-Moser iteration. At the same time, Alazard and Shao[1, 2] purpose a new approach to prove KAM type theorems. I am very curious if we can simplify the proof in [3] by the approach developed in [1, 2]. Since we have developed a unified theory for the constrution of rotating patches for active scalar equation with completely monotone kernel, an ambitious problem is if we can prove the existence of quasi-periodic patch solutions for active scalar equations with completely monotone kernel. Furthermore, the existence of nontrivial time quasi-periodic smooth solutions for active scalar equation also need to be explored. Recently, there are also many good works about non-rigid time periodic patch solutions [12, 13] and stability problem around relative equilibrium [7, 21]. All in all, there are many important and interesting problems in this field and I will continue to work on them.

2. I also want to do some problems in other area of partial differential equations and analysis, like kinetic equation, fluid stability and KAM theory for PDEs. I have learn some basic knowledge and read a series papers for these topic. And now these areas are very active and have many unsolved problems. In the future, I will spend more time on these important topics.

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